Dynamic Effects of Labor Income Taxation in an Unequal Schumpeterian Economy

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Abstract

How does taxation affect growth and inequality? To study this question, we develop a Schumpeterian model in which wealth heterogeneity influences the effects of tax policy. The key mechanism is that a change in consumption dispersion across heterogeneous households due to a change in labor income taxation can cause a novel positive effect on the employment of poor households in addition to the usual negative effect on the employment of rich households. Together, these effects yield an overall ambiguous response of employment to labor income taxation. A negative (positive) change of employment causes a negative (positive) change of innovation-driven growth in the short run and also a negative (positive) change of the real interest rate. Consequently, labor income taxation has an ambiguous effect on income inequality (e.g., asset income falls while labor income may rise) but unambiguously increases consumption inequality by reducing disposable wage income even for households that work more. Therefore, the effects on income inequality and consumption inequality are drastically different. We calibrate the model to examine its quantitative implications.

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1 Introduction

Macroeconomists often evaluate the effects of government policies in macroeconomic models that feature a representative household. However, household heterogeneity potentially influences the effects of government policies. In this study, we explore the following question: how does household heterogeneity influence the effects of tax policy on economic growth and income inequality? This question has a long history in economics. Answering it, however, has proven difficult because of the technical challenges that it poses.

To make progress, we introduce heterogeneous households in a Schumpeterian growth model with endogenous market structure. The resulting model has the following advantages. First, the presence of endogenous market structure removes the (strong) scale effect. This property removes the excess sensitivity of the growth rate to employment that weakens the ability of the first-generation endogenous growth model to explain the data.[1] Second, we consider a recent vintage of this variant of the model developed by Peretto (2007, 2011, 2015) that delivers a closed-form solution for the entire transition dynamics of the economy.

The key heterogeneity in the Schumpeterian model augmented for inequality is the distribution of wealth among households. A key novelty of our analysis is that the wealth distribution can influence how tax policy affects the aggregate economy. Our model features iso-elastic utility with respect to leisure which delivers elastic labor supply. This property in turn causes wealth inequality to generate an endogenous distribution of employment and thereby of wage income among households. We find that the elasticity of intertemporal substitution for leisure determines whether the wealth distribution influences the effects of labor income taxation on aggregate employment, and thus on the aggregate economy, because it regulates the different responses of relatively poor and relatively rich households. Therefore, we focus our policy analysis on the labor income tax, which in the model affects employment, innovation, economic growth and income inequality. The model provides a tractable framework for analytically deriving the complete transition dynamic effects of the labor income tax on the distributions of income and consumption, in addition to macroeconomic variables such as employment, innovation and economic growth.

We obtain the following results. If the elasticity of intertemporal substitution for leisure is equal to one, the wealth distribution does not influence the effects of labor income tax, which are the same as in a representative-household model. If the elasticity is not equal to one, instead, the wealth distribution influences the effects of the labor income tax by changing the dispersion of consumption across heterogeneous households. Specifically, relatively poor households experience a reduction in their consumption share and therefore increase their labor supply for a given tax rate. Relatively rich households, in contrast, experience an increase in their consumption share and therefore reduce their labor supply for a given tax rate.

This difference in behavior has important consequences. When the elasticity of intertemporal substitution for leisure is greater than one, the change in the dispersion of consumption amplifies the usual negative effect of labor income taxation on aggregate employment. When the elasticity of intertemporal substitution for leisure is less than one, instead, the change in the dispersion of consumption gives rise to a novel positive effect of labor income taxation on the employment of the relatively poor households in addition to the usual negative effect on the

[1] See Laincz and Peretto (2006) for a discussion of the scale effect in the Schumpeterian model and Ang and Madsen (2011) and Madsen (2008, 2010) for empirical evidence that supports the Schumpeterian model with endogenous market structure.

employment of the relatively rich households. As a result, when the degree of wealth inequality is sufficiently high, the overall effect of the labor income tax on aggregate employment can surprisingly become positive due to the relatively poor households increasing their labor supply by more than the rich households decreasing their labor supply.

The effect of the labor income tax on aggregate employment discussed above gives rise to an effect of the same sign on innovation-driven economic growth in the short run. These ambiguous effects of labor income taxation on growth are consistent with the empirical results in Gale *et al.* (2015), who identify both positive and negative effects in the US.[2] In the long run, the endogenous market structure removes the scale effect so that labor income tax does not affect the steady-state growth rate. The short-run effect on consumption growth, in turn, causes a short-run effect of the same sign on the real interest rate via the households' consumption Euler equation. Because the long-run growth rate is independent of aggregate employment in our scale-invariant Schumpeterian model, the effect of labor income taxation on the real interest rate is neutral in the long run.

When we look at the individual households, labor income taxation affects the distribution of income. In the short run, an increase in the labor income tax that finances government consumption affects gross income inequality partly via the real interest rate, which determines asset income relative to wage income. Therefore, if the labor income tax has an ambiguous effect on the real interest rate in the short run, it also has an ambiguous effect on gross income inequality in the short run. However, the distribution of consumption is based on net income. In the case of consumption inequality, a higher labor income tax rate unambiguously increases consumption inequality in both the short run and the long run because higher labor income tax reduces disposable wage income relative to asset income, which is more unequally distributed. Therefore, the effects of labor income tax on income inequality and consumption inequality are drastically different. To make further progress on this aspect of the model, we calibrate it to data to examine its quantitative implications.

This study relates to the growth-theoretic literature on innovation and fiscal policy. This literature builds on the seminal contributions of Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Subsequent studies by Howitt (1999), Peretto (1998, 1999) and Smulders and van de Klundert (1995) combine quality improvement and variety expansion in the Schumpeterian growth model to remove the scale effect. Different variants of these innovation-driven growth models explore the effects of various fiscal policy instruments on growth and innovation; see for example, Arawatari *et al.* (2023), Chen *et al.* (2017, 2023), Haruyama and Itaya (2006), Jaimovich and Rebelo (2017), Lin and Russo (1999), Peretto (2003, 2007, 2011), Suzuki (2022) and Zeng and Zhang (2002). Our analysis relates most closely to Peretto (2007) who considers a representative household in the Schumpeterian growth model with endogenous market structure. If we remove household heterogeneity from our model, we obtain the same short-run negative effect and the same long-run neutral effect of labor income taxation on growth and innovation. Our novel contribution, thus, is to introduce heterogeneous households to explore how household heterogeneity in the form of wealth inequality changes the effects of tax policy on economic growth and innovation.

Therefore, this study also relates to the literature on income inequality and economic growth. Early studies in this literature explore the relationship between income inequality and economic growth that is driven by the accumulation of capital; see for example, Aghion and Bolton

[2] See also their discussion on the contrasting effects found in the empirical literature.

(1997), Galor and Moav (2004) and Galor and Zeira (1993). More recent studies explore the relationship between income inequality and economic growth that is driven by innovation; see for example, Aghion *et al.* (2019), Chou and Talmain (1996), Chu and Peretto (2023), Foellmi and Zweimuller (2006), Garcia-Penalosa and Wen (2008), Jones and Kim (2018), Schetter *et al.* (2024) and Zweimuller (2000). We follow this branch of the literature by introducing a non-degenerate wealth distribution to the Schumpeterian growth model.[3] Then, we use the resulting heterogeneous-agent Schumpeterian growth model to derive the complete transition dynamic effects of labor income tax on innovation and income inequality and explore how these effects are influenced by the underlying wealth distribution.[4] Arawatari *et al.* (2023) and Jaimovich and Rebelo (2017) show that the effects of productive government spending and capital income tax on innovation become nonlinear in an innovation-driven growth model in which agents have heterogeneous R&D abilities. We complement their analysis by showing that the effects of labor income tax on innovation and income inequality can even become positive under heterogeneous agents with wealth inequality.

Finally, there is a small empirical literature on fiscal policy and income inequality. Barro (2000) is an early study that examines the empirical determinants of income inequality across countries. He considers data on both gross income inequality and net income inequality and finds that income inequality net of taxes tends to be lower than gross income inequality. Roine *et al.* (2009) consider a panel of 16 countries and find that tax progressivity tends to reduce income inequality. A recent study by Eydam and Qualo (2024) considers a cross-section of 61 countries and also finds that personal income tax is negatively associated with income inequality. Although these empirical studies find a negative effect of income tax on income inequality, Troiano (2017) provides quasi-experimental evidence on a positive effect of income tax on income inequality in the US. Our growth-theoretic analysis also suggests that although labor income tax usually reduces income inequality, it is possible for a positive effect to arise.

The rest of this study is organized as follows. Section 2 presents the heterogeneous-agent Schumpeterian growth model. Section 3 explores the effects of tax policy. Section 4 calibrates the model and explores its quantitative properties. The final section concludes.

2 A heterogeneous-agent Schumpeterian growth model

In the Schumpeterian growth model with endogenous market structure, the growing number of products causes a dilution effect that removes the scale effect. We consider the variant in Peretto (2015) and introduce heterogeneous households as in Chu (2010) and Chu and Cozzi (2018). This variant first appeared in Chu and Peretto (2023), which here we extend further to consider a government sector and fiscal policy instruments in order to explore the effects of labor income taxation on economic growth and income inequality.

[3]See also Garcia-Penolosa and Turnovsky (2006, 2011) who develop an AK growth model with a non-degenerate wealth distribution to explore the relationship between growth and inequality.

[4]See also Chu (2010), Chu and Cozzi (2018) and Chu *et al.* (2019, 2021) who explore the effects of patent policy and monetary policy on innovation and income inequality in the heterogeneous-agent Schumpeterian growth model.

2.1 Heterogeneous households

We consider a unit continuum of households indexed by $h \in [0,1]$. They exhibit identical preferences over consumption and leisure but have different levels of wealth. The utility function of household h is given by

$$U(h) = \int_0^\infty e^{-(\rho-\lambda)t} \left\{ \ln c_t(h) + \frac{\eta}{1-1/\omega} [1 - l_t(h)]^{1-1/\omega} \right\} dt, \tag{1}$$

where $\rho > 0$ is the subjective discount rate, $\eta > 0$ measures the importance of leisure and $\omega > 0$ determines the elasticity of intertemporal substitution for leisure $1 - l_t(h)$. Each member of household h devotes $l_t(h)$ units of time to employment and consumes $c_t(h)$ units of final good. Finally, $\lambda \in (0, \rho)$ is the population growth rate, and we set the initial population size equal to one since it plays no important role in our analysis (i.e., $L_t = e^{\lambda t}$).

Household h maximizes (1) subject to

$$\dot{a}_t(h) = (r_t - \lambda)a_t(h) + (1 - \tau_w)w_t l_t(h) - c_t(h) + \iota_t, \tag{2}$$

where r_t is the real interest rate on per capita asset $a_t(h)$ in household h. Each member of household h supplies $l_t(h)$ units of labor to earn a real wage rate w_t and pays labor income tax $\tau_w w_t l_t(h)$ to the government, in which $\tau_w \in (0,1)$ is the labor income tax rate. Each member of household h also faces a lump-sum transfer $\iota_t > 0$ or tax $\iota_t < 0$ set by the government. From dynamic optimization, we derive the growth rate of consumption per capita in household h as

$$\frac{\dot{c}_t(h)}{c_t(h)} = r_t - \rho, \tag{3}$$

which shows that the growth rate of consumption is the same across households such that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t = r_t - \rho$, where $c_t \equiv \int_0^1 c_t(h)dh$ denotes average consumption per capita. Therefore, the growth rate of average consumption is also given by

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{4}$$

Labor supply, which differs across households $h \in [0,1]$,[5] is

$$l_t(h) = 1 - \left[\frac{\eta c_t(h)}{(1-\tau_w)w_t} \right]^\omega, \tag{5}$$

which is increasing in the wage rate w_t but decreasing in the level of consumption $c_t(h)$ and the labor income tax rate τ_w.

[5] Bick *et al.* (2024) provide evidence that differences in labor supply are an important determinant for differences in wage income.

2.2 Final good

Competitive firms produce final good Y_t using the following production function:

$$Y_t = \int_0^{N_t} X_t^\theta(i) \left[Z_t^\alpha(i) Z_t^{1-\alpha} \frac{L_{y,t}}{N_t^{1-\sigma}} \right]^{1-\theta} di, \tag{6}$$

where $\{\theta, \alpha, \sigma\} \in (0,1)$. The quantity of differentiated intermediate good i is denoted as $X_t(i)$, and there are N_t differentiated intermediate goods in the economy at time t. The quality of intermediate good i is denoted as $Z_t(i)$, and $Z_t \equiv \frac{1}{N_t} \int_0^{N_t} Z_t(i) di$ is the average quality of all N_t intermediate goods. The degree of technology spillovers is captured by $1 - \alpha \in (0,1)$. $L_{y,t}$ denotes production labor, and the specification $L_{y,t}/N_t^{1-\sigma}$ captures a congestion effect of variety N_t, which removes the (strong) scale effect for $1 - \sigma > 0$.

We perform profit maximization to derive the conditional demand functions for $L_{y,t}$ and $X_t(i)$ as

$$L_{y,t} = (1-\theta) \frac{Y_t}{w_t}, \tag{7}$$

$$X_t(i) = \left[\frac{\theta}{P_t(i)} \right]^{1/(1-\theta)} Z_t^\alpha(i) Z_t^{1-\alpha} \frac{L_{y,t}}{N_t^{1-\sigma}}, \tag{8}$$

where $P_t(i)$ denotes the price of $X_t(i)$. Due to perfect competition, final-good firms pay $(1 - \theta) Y_t = w_t L_{y,t}$ for production labor and $\theta Y_t = \int_0^{N_t} P_t(i) X_t(i) di$ for intermediate goods.

2.3 Intermediate goods and in-house R&D

The economy features a continuum of differentiated intermediate good $i \in [0, N_t]$. Each differentiated intermediate good i is produced by a monopolistic firm using a linear production function. Specifically, it requires $X_t(i)$ units of final good to produce $X_t(i)$ units of intermediate good i. The monopolistic firm also needs to incur $\phi Z_t^\alpha(i) Z_t^{1-\alpha}$ units of final good as a fixed operating cost, where $\phi > 0$ is an operating cost parameter. To improve the quality $Z_t(i)$ of its product, the monopolistic firm performs in-house R&D by investing $R_t(i)$ units of final good. The process for quality improvement is given by

$$\dot{Z}_t(i) = R_t(i). \tag{9}$$

The before-R&D profit flow of the monopolistic firm at time t is

$$\Pi_t(i) = [P_t(i) - 1] X_t(i) - \phi Z_t^\alpha(i) Z_t^{1-\alpha}. \tag{10}$$

The monopolistic firm maximizes its stock market value of the firm at time t,

$$V_t(i) = \int_t^\infty \exp\left(-\int_t^s r_u du \right) [\Pi_s(i) - R_s(i)] \, ds, \tag{11}$$

subject to the constraints (8)-(10).

We perform this dynamic optimization problem in Appendix A to show that the unconstrained profit-maximizing price $P_t(i)$ is given by $1/\theta$. However, we assume the presence of competitive fringe firms, which can also produce $X_t(i)$ with the same quality $Z_t(i)$ but at a higher marginal cost $\mu \in (1, 1/\theta)$. Bertrand competition then implies that the monopolistic firm sets

$$P_t(i) = \min\{\mu, 1/\theta\} = \mu. \tag{12}$$

The literature has shown that the industry equilibrium is symmetric. Thus, we have $Z_t(i) = Z_t$ and $X_t(i) = X_t$ for $i \in [0, N_t]$. From (8) and (12), quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{l_t L_t}{N_t^{1-\sigma}}, \tag{13}$$

where we have used the labor-market-clearing condition $L_{y,t} = l_t L_t$ in which l_t and $l_t L_t$, respectively, denote average and aggregate employment. For notational convenience, we define the following transformed state variable:

$$x_t \equiv \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}, \tag{14}$$

which determines the dynamics of the economy. Lemma 1 shows that the rate of return on quality-improving R&D is increasing in firm size $x_t l_t$.

Lemma 1 *The rate of return on quality-improving in-house R&D is*

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[(\mu - 1) x_t l_t - \phi\right]. \tag{15}$$

Proof. See Appendix A. ■

2.4 Entrants

Following the standard approach in the literature, we preserve the symmetric equilibrium at any time t by assuming that entrants join the industry with quality equal to the industry average Z_t. To develop a new intermediate good and begin its production, a new firm incurs βX_t units of final good, where β is an entry-cost parameter. We use the asset-pricing equation to determine the rate of return on the value V_t of a monopolistic firm as

$$r_t = \frac{\Pi_t - R_t}{V_t} + \frac{\dot{V}_t}{V_t}, \tag{16}$$

in which monopolistic profit (net of R&D expenses) is $\Pi_t - R_t$ and capital gain is $\dot{V}_t$. The free-entry condition requires that firm value V_t is equal to the entry cost βX_t at any time t:

$$V_t = \beta X_t. \tag{17}$$

We substitute (9), (10), (12), (13), (14) and (17) into (16) to derive the rate of return on entry as

$$r_t^e = \frac{1}{\beta}\left(\mu - 1 - \frac{\phi + z_t}{x_t l_t}\right) + \frac{\dot{l}_t}{l_t} + \frac{\dot{x}_t}{x_t} + z_t, \tag{18}$$

where $z_t \equiv \dot{Z}_t / Z_t$ is the quality growth rate.

2.5 Government

The government sets the labor income tax rate $\tau_w > 0$ and uses the tax revenue to finance its spendings. The government's balanced-budget condition is

$$G_t + T_t = \left[\int_0^1 \tau_w w_t l_t(h) dh \right] L_t = \tau_w w_t l_t L_t, \tag{19}$$

where $G_t > 0$ is government consumption that does not affect productivity and changes endogenously to balance the fiscal budget.[6] Lump-sum transfer $T_t = \iota_t L_t$ is assumed to be proportional to output (i.e., $T_t = \gamma Y_t$), where the policy parameter γ is the ratio of lump-sum transfer $\gamma > 0$ or tax $\gamma < 0$ to output.[7] Finally, $l_t \equiv \int_0^1 l_t(h)dh$ denotes average employment per capita.

2.6 Equilibrium

The equilibrium is a time path of allocations $\{a_t, c_t, Y_t, l_t, L_{y,t}, X_t(i), R_t(i)\}$ and a time path of prices $\{r_t, w_t, P_t(i), V_t(i)\}$ such that at any time t the following conditions hold:

- households maximize (1) taking $\{r_t, w_t\}$ as given;
- competitive firms maximize profit by producing Y_t and taking $\{P_t(i), w_t\}$ as given;
- a monopolistic firm maximizes $V_t(i)$ by producing $X_t(i)$ and choosing $\{P_t(i), R_t(i)\}$ while taking r_t as given;
- the entry condition holds such that $V_t = \beta X_t$;
- the value of existing monopolistic firms is equal to the value of households' assets such that $N_t V_t = \left[\int_0^1 a_t(h)dh \right] L_t \equiv a_t L_t$;
- the government balances its fiscal budget such that $G_t + T_t = \tau_w w_t l_t L_t$;
- the labor-market-clearing condition holds such that $l_t L_t = L_{y,t}$; and
- the final-good-market-clearing condition holds such that $Y_t = C_t + N_t(X_t + \phi Z_t + R_t) + \dot{N}_t \beta X_t + G_t$, where $C_t \equiv c_t L_t$ denotes total consumption.

[6] It is useful to note that we could allow for a mix of productive and unproductive government spendings. Our results would remain unchanged so long as we treat the unproductive government spending as the endogenous balancing item in the fiscal budget.

[7] Alternatively, one can also treat G_t as exogenous and endogenize γ to balance the fiscal budget. For example, if we set $G_t = 0$, then $\gamma = \tau_w w_t l_t L_t / Y_t = (1 - \theta)\tau_w$, which shows that γ rises whenever τ_w rises. In this case, our qualitative results remain the same.

2.7 Aggregation

Substituting (8) and (12) into (6) and imposing symmetry yield aggregate production as

$$Y_t = \left(\frac{\theta}{\mu}\right)^{\theta/(1-\theta)} N_t^{\sigma} Z_t l_t L_t. \tag{20}$$

Therefore, the growth rate of per capita output $y_t \equiv Y_t/L_t$ is

$$\frac{\dot{y}_t}{y_t} = \sigma n_t + z_t + \frac{\dot{l}_t}{l_t}, \tag{21}$$

where n_t is the growth rate of variety N_t and z_t is the growth rate of quality Z_t.

2.8 Dynamics of the aggregate economy

Let $s_{c,t}(h) \equiv c_t(h)/c_t$ denote the consumption share of household h at time t. We integrate $l_t(h)$ in (5) across households to obtain the average employment function as follows:

$$l_t = 1 - \left[\frac{\eta l_t}{(1-\tau_w)(1-\theta)} \frac{c_t}{y_t}\right]^{\omega} (\Delta_{c,t})^{\omega}, \tag{22}$$

which uses $w_t l_t = (1-\theta) y_t$ from (7) whereas $\Delta_{c,t}$ is a consumption dispersion index defined as

$$\Delta_{c,t} \equiv \left\{\int_0^1 [s_{c,t}(h)]^{\omega} dh\right\}^{\frac{1}{\omega}}.$$

Equation (22) shows that average employment l_t depends on the consumption dispersion index $\Delta_{c,t}$ and the consumption-output ratio c_t/y_t. Therefore, we need to first derive the dynamics of the consumption dispersion index and the consumption-output ratio.

The consumption dispersion index $\Delta_{c,t}$ is an aggregate of the consumption share $s_{c,t}(h)$. Taking the log of $s_{c,t}(h) \equiv c_t(h)/c_t$ and differentiating it with respect to time yield

$$\frac{\dot{s}_{c,t}(h)}{s_{c,t}(h)} = \frac{\dot{c}_t(h)}{c_t(h)} - \frac{\dot{c}_t}{c_t}. \tag{23}$$

Given that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$ from (3) and (4), (23) becomes $\dot{s}_{c,t}(h) = 0$ for all time $t > 0$, which implies that $s_{c,t}(h) = s_c^*(h)$ and $\Delta_{c,t} = \Delta_c^*$ remain stationary across time by jumping to their steady-state values. Moreover, in the proof of Lemma 2, we show that the consumption-output ratio c_t/y_t also jumps to its unique steady-state value, which ensures the stationarity of the wealth distribution along the transition path of the aggregate economy, as we will show.

Lemma 2 *The consumption-output ratio c_t/y_t jumps to a unique steady-state value:*

$$\left(\frac{c}{y}\right)^* = (1-\tau_w)(1-\theta) + \gamma + \frac{\theta\beta}{\mu}(\rho - \lambda) > 0. \tag{24}$$

Proof. See Appendix A. ■

Lemma 2 implies the following results: (a) the steady-state value of the consumption-output ratio $(c/y)^*$ is decreasing in the labor income tax rate τ_w; (b) average employment l_t in (22) jumps to its steady-state equilibrium value l^* because the consumption dispersion index $\Delta_{c,t}$ is also stationary and jumps to Δ_c^*; and (c) consumption and output grow at the same rate g_t at any time t such that

$$g_t \equiv \frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho, \tag{25}$$

where the last equality uses (4). Then, we can combine (15) and (25) by setting $r_t = r_t^q$ to derive the equilibrium growth rate of output per capita as

$$g_t = \alpha \left[(\mu - 1) x_t l^* - \phi \right] - \rho, \tag{26}$$

where g_t is increasing in firm size $x_t l^*$. One can also derive the growth rate of variety as[8]

$$n_t = \frac{1}{\beta} \left\{ (1 - \alpha)(\mu - 1) + \beta (\lambda - \rho) - \frac{(1 - \alpha)\phi - \rho}{x_t l^*} \right\}, \tag{27}$$

which is also increasing in firm size $x_t l^*$. Recall that average employment l^* is determined by

$$l^* = 1 - \left\{ \frac{\eta l^*}{(1 - \tau_w)(1 - \theta)} \left[(1 - \tau_w)(1 - \theta) + \gamma + \frac{\theta \beta}{\mu} (\rho - \lambda) \right] \right\}^{\omega} (\Delta_c^*)^{\omega}, \tag{28}$$

which uses (22) and (24). Equation (26) and (27) show that the dynamics of both the growth rate g_t of output per capita and the growth rate n_t of the number of firms is determined by the state variable x_t defined in (14). Its law of motion is given by $\dot{x}_t / x_t = \lambda - (1 - \sigma) n_t$, where the variety growth rate n_t is a function of x_t as shown in (27).

Lemma 3 The dynamics of x_t is given by the following one-dimensional differential equation:

$$\dot{x}_t = \frac{1 - \sigma}{\beta} \left\{ \left[(1 - \alpha)\phi - \left(\rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] \frac{1}{l^*} - \left[(1 - \alpha)(\mu - 1) - \beta \left(\rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] x_t \right\}. \tag{29}$$

Proof. See Appendix A. ■

Lemma 3 shows that the dynamics of x_t is globally stable if the following parameter condition holds:

$$\beta \phi > \frac{1}{\alpha} \left[\mu - 1 - \beta \left(\rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] > \mu - 1.$$

Given this parameter condition, the state variable x_t gradually converges to a unique steady-state value:

$$x^* = \frac{(1 - \alpha)\phi - \left(\rho + \frac{\sigma \lambda}{1 - \sigma} \right)}{(1 - \alpha)(\mu - 1) - \beta \left(\rho + \frac{\sigma \lambda}{1 - \sigma} \right)} \frac{1}{l^*}. \tag{30}$$

[8] See the proof of Lemma 3 in Appendix A.

In other words, given an initial value, x_t gradually converges to its steady-value state x^* in (30). As x_t converges to x^*, the equilibrium growth rate g_t of output per capita in (26) also converges to its steady-value state:

$$g^* = \alpha \left\{ \frac{(\mu - 1)\left[(1-\alpha)\phi - \left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right]}{(1-\alpha)(\mu-1) - \beta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)} - \phi \right\} - \rho > 0, \tag{31}$$

which is independent of the labor income tax rate τ_w. Intuitively, although labor income tax τ_w affects employment l^*, the scale-invariant property of the Schumpeterian growth model with endogenous market structure removes the effects of changes in employment l^* on the steady-state equilibrium growth rate g^*.

2.9 Dynamics of the wealth distribution

In this section, we show the stationarity of the wealth distribution, which in turn is given by its initial distribution that is predetermined at time 0. Intuitively, although the aggregate economy features transition dynamics from the dynamics of the state variable x_t, the wealth distribution always remains stationary because both the consumption-output ratio c_t/y_t and the consumption share $s_{c,t}(h)$ are stationary.

2.9.1 General case $\omega \in (0, \infty)$

Integrating (2) across households yields the following asset-accumulation equation:

$$\dot{a}_t = (r_t - \lambda)a_t + (1-\tau_w)w_t l_t - c_t + \iota_t. \tag{32}$$

Let $s_{a,t}(h) \equiv a_t(h)/a_t$ denote household h's share of wealth in the economy. Taking the log of wealth share $s_{a,t}(h)$ and differentiating the resulting expression with respect to time yield

$$\frac{\dot{s}_{a,t}(h)}{s_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{c_t - (1-\tau_w)\, w_t l_t - \iota_t}{a_t} - \frac{c_t(h) - (1-\tau_w)\, w_t l_t(h) - \iota_t}{a_t(h)}, \tag{33}$$

where we have used (2). Equation (33) can be re-expressed as

$$\dot{s}_{a,t}(h) = \frac{c_t - (1-\tau_w)\, w_t l_t - \gamma y_t}{a_t} s_{a,t}(h) - \frac{s_{c,t}(h) c_t - (1-\tau_w)\, w_t l_t(h) - \gamma y_t}{a_t}, \tag{34}$$

where we have used $s_{c,t}(h) \equiv c_t(h)/c_t$ and $\iota_t = \gamma y_t$.

Recall that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$ and the consumption share $s_{c,t}(h)$ of any household h is stationary such that $s_{c,t}(h) = s_c^*(h)$ and $\Delta_{c,t} = \Delta_c^*$. Given $\{a_t, c_t, y_t, w_t\}$ all grow at the same rate g_t at any point in time due to the stationary consumption-output ratio $c_t/y_t = (c/y)^*$, (34) becomes a one-dimensional differential equation as shown in Proposition 1, which describes the dynamics of $s_{a,t}(h)$ given an initial value $s_{a,0}(h)$. In Appendix A, we show that the coefficient on $s_{a,t}(h)$ is $\rho - \lambda > 0$. Together with the fact $s_{a,t}(h)$ is a pre-determined variable, the only solution of (34) that is consistent with long-run stability is $\dot{s}_{a,t}(h) = 0$ for all time t, which is achieved by the consumption share $s_{c,t}(h)$ jumping to its steady-state value $s_c^*(h)$ (which implies that the consumption dispersion index $\Delta_{c,t}$ also jumps to its steady-state value Δ_c^*) and employment

$l_t(h)$ jumping to its steady-state value $l^*(h)$ (which implies that average employment l_t also jumps to its steady-state value l^*) as shown in the previous section.

Proposition 1 shows that as an equilibrium outcome, the wealth distribution is stationary and remains the same as the initial distribution given at time 0.

Proposition 1 *The dynamics of $s_{a,t}(h)$ is given by an one-dimensional differential equation:*

$$\dot{s}_{a,t}(h) = (\rho - \lambda)\left[s_{a,t}(h) - 1\right] - \frac{\mu}{\theta\beta}\left(\frac{c}{y}\right)^* \left[s_c^*(h) - 1\right] + \frac{\mu(1-\tau_w)(1-\theta)}{\theta\beta}\left[\frac{l^*(h) - l^*}{l^*}\right], \quad (35)$$

where $(c/y)^*$, $s_c^*(h)$ and $l^*(h)$ are stationary and independent of time. Therefore, the wealth share of household $h \in [0,1]$ is given by $s_{a,t}(h) = s_{a,0}(h)$ for all time t.

Proof. See Appendix A. ∎

Imposing $\dot{s}_{a,t}(h) = 0$ on (35) yields the steady-state value of $s_{c,t}(h)$ determined by

$$s_c^*(h) = \frac{(1-\tau_w)(1-\theta)}{l^*\left(\frac{c}{y}\right)^*}\left\{1 - \left[\frac{\eta l^*\left(\frac{c}{y}\right)^*}{(1-\tau_w)(1-\theta)}\right]^{\omega} s_c^*(h)^{\omega}\right\} + \frac{\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)\, s_{a,0}(h)}{\left(\frac{c}{y}\right)^*}, \quad (36)$$

where we have used (5) and (7) and the average level of employment l^* in (28) can be re-expressed as

$$l^* = 1 - \left\{\eta l^*\left[1 + \frac{\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)}{(1-\tau_w)(1-\theta)}\right]\right\}^{\omega}\left(\Delta_c^*\right)^{\omega}. \quad (37)$$

Equation (36) provides an implicit solution for $s_c^*(h)$, which can be integrated across households to obtain the consumption dispersion index Δ_c^*. Given the complexity of (36), we focus on the special case $\omega = 1$ under which $\Delta_c^* = 1$ for some of the analytical results.

2.9.2 Special case $\omega = 1$

Setting $\omega = 1$ in (37) and using $(c/y)^*$ from (24) yield the average level of employment as

$$l^* = \frac{1}{1 + \eta\left\{1 + \frac{1}{(1-\tau_w)(1-\theta)}\left[\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)\right]\right\}}, \quad (38)$$

which is decreasing in the labor income tax rate τ_w given $\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda) > 0$. Then, setting $\omega = 1$ in (36) yields

$$s_c^*(h) = \frac{1}{1+\eta}\,\frac{\frac{(1-\tau_w)(1-\theta)}{l^*} + \gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)\, s_{a,0}(h)}{(1-\tau_w)(1-\theta) + \gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)}, \quad (39)$$

where the average level of employment l^* is given in (38).

2.10 Dynamics of the income distribution

In this section, we derive the dynamics of the income distribution. Although the wealth distribution remains stationary, the transition dynamics of the real interest rate leads to an endogenous evolution of the income distribution. Therefore, upon deriving the transition dynamics of the real interest rate r_t, we can also obtain the transition dynamics of income inequality.

2.10.1 General case $\omega \in (0, \infty)$

Gross income received by each member of household h is

$$I_t(h) \equiv (r_t - \lambda)\, a_t(h) + w_t l_t(h) + \iota_t. \tag{40}$$

Integrating $I_t(h)$ across households yields the average level of gross income per capita as

$$I_t = (r_t - \lambda)\, a_t + w_t l_t + \iota_t. \tag{41}$$

Let $s_{I,t}(h) \equiv I_t(h)/I_t$ denote the share of gross income received by household h. Combining (40) and (41), we have

$$s_{I,t}(h) = \frac{s_{a,0}(h)\,(r_t - \lambda)\, a_t + w_t l_t(h) + \iota_t}{(r_t - \lambda)\, a_t + w_t l_t + \iota_t}, \tag{42}$$

which also uses $a_t(h) = s_{a,t}(h) a_t = s_{a,0}(h) a_t$. Equation (42) determines the dynamics of the share of gross income received by household h and allows us to derive any moment of the income distribution. We measure income inequality by the standard deviation of income share $s_{I,t}(h)$ defined as $\sigma_{I,t} \equiv \sqrt{\int_0^1 \left[s_{I,t}(h) - 1\right]^2 dh}$, which is also the coefficient of variation in income $I_t(h)$.

2.10.2 Special case $\omega = 1$

Proposition 2 derives the equilibrium expression for the degree of income inequality $\sigma_{I,t}$ at any time t for the special case of $\omega = 1$.

Proposition 2 *For $\omega = 1$, the degree of income inequality at any time t is given by*

$$\sigma_{I,t} = \frac{r_t - \lambda - \frac{\eta}{1-\tau_w}\left(\frac{\rho-\lambda}{1+\eta}\right)}{r_t - \lambda + \frac{\mu}{\theta\beta}(1 - \theta + \gamma)}\sigma_{a,0} = \frac{g_t + \rho - \lambda - \frac{\eta}{1-\tau_w}\left(\frac{\rho-\lambda}{1+\eta}\right)}{g_t + \rho - \lambda + \frac{\mu}{\theta\beta}(1 - \theta + \gamma)}\sigma_{a,0}, \tag{43}$$

where the degree of wealth inequality $\sigma_{a,0} \equiv \sqrt{\int_0^1 \left[s_{a,0}(h) - 1\right]^2 dh}$ is determined at time 0.

Proof. See Appendix A. ■

Equation (43) shows that income inequality $\sigma_{I,t}$ depends on the growth rate g_t because of the real interest rate $r_t = \rho + g_t$. This is the *interest-rate* effect on income inequality discussed in Chu and Cozzi (2018). Given this interest-rate effect, the transition dynamics of income inequality $\sigma_{I,t}$ is governed by the transition dynamics of the growth rate g_t in (26) that is driven by the dynamics of the state variable x_t in (29). Moreover, income inequality is increasing in the growth rate g_t for a given degree of wealth inequality $\sigma_{a,0}$ that is determined by the initial wealth distribution at time 0. In addition to the interest-rate effect on income inequality, we also have a *wage-income* effect captured by the term $\frac{\eta}{1+\eta}\left(\frac{\rho-\lambda}{1-\tau_w}\right)$, which disappears under perfectly inelastic labor supply (i.e., $\eta = 0$). Intuitively, unequal wage income also affects income inequality unless all households supply the same amount of labor, which is the case in Chu and Cozzi (2018) in which this wage-income effect on inequality is absent.

2.11 Dynamics of the consumption distribution

In this section, we explore the consumption distribution. To measure consumption inequality, we once again consider the standard deviation of consumption share $s_{c,t}(h)$ defined as $\sigma_{c,t} \equiv \sqrt{\int_0^1 \left[s_{c,t}(h) - 1\right]^2 dh}$, which is also the coefficient of variation in consumption $c_t(h)$. It is useful to recall that the consumption share $s_c^*(h)$ is stationary, so that the degree of consumption inequality σ_c^* is also stationary.

Proposition 3 derives the stationary degree of consumption inequality σ_c^* at any time t for the special case of $\omega = 1$. Equation (44) shows that consumption inequality σ_c^* is stationary because the real interest rate r_t does not affect the consumption share $s_c^*(h)$ in (39). However, labor income tax τ_w affects consumption inequality σ_c^* via the consumption-asset ratio $(c/a)^* = \mu\,(c/y)^*/(\theta\beta)$,[9] in which the consumption-output ratio $(c/y)^*$ is given in (24) and decreasing in labor income tax τ_w.

Proposition 3 *For $\omega = 1$, the degree of consumption inequality at any time t is given by*

$$\sigma_c^* = \frac{1}{1+\eta}\frac{\rho-\lambda}{\left(\frac{c}{a}\right)^*}\sigma_{a,0} = \frac{1}{1+\eta}\frac{\frac{\theta\beta}{\mu}(\rho-\lambda)}{(1-\tau_w)(1-\theta)+\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)}\sigma_{a,0}, \tag{44}$$

which is stationary across time.

Proof. See Appendix A. ■

3 How taxation affects growth and inequality

In this section, we explore the complete dynamic effects of labor income tax on growth and inequality. We first consider the special case of $\omega = 1$. Section 3.1 presents the effects of labor income tax on economic growth. Section 3.2 presents the effects of labor income tax on income inequality. Section 3.3 presents the effects of labor income tax on consumption inequality. Then, Section 3.4 explores the general case of $\omega \neq 1$. Section 3.5 performs a quantitative analysis.

[9] See the proof of Lemma 2 in Appendix A.

3.1 Labor income tax and economic growth

Equation (26) shows that the transitional growth rate g_t of output per capita is increasing in employment l^* for a given x_t. For the special case of $\omega = 1$, the level of average employment l^* is given in (38) and decreasing in the labor income tax rate τ_w. Therefore, an increase in the labor income tax rate reduces the transitional growth rate g_t. However, (31) shows that the steady-state equilibrium growth rate g^* is independent of the labor income tax rate τ_w. Therefore, the reduction in economic growth is temporary, and the equilibrium growth rate g_t eventually returns to the initial steady-state value g^* in (31); see Figure 1 for the time path of economic growth when the labor income tax rate τ_w rises at time t. Proposition 4 summarizes the complete dynamic effects of labor income tax on economic growth.

Proposition 4 *For $\omega = 1$, an increase in the labor income tax rate τ_w leads to a reduction in the transitional growth rate g_t but does not affect the steady-state growth rate g^*.*

Proof. Proven in text. ■

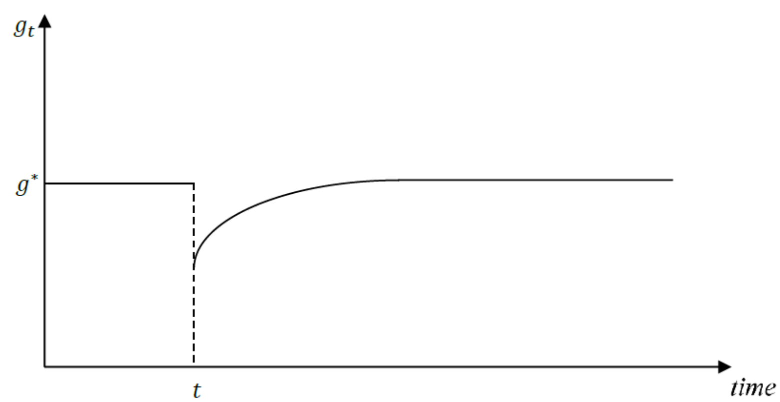

Figure 1: Labor income tax and economic growth

3.2 Labor income tax and income inequality

Equation (43) shows that the degree of income inequality $\sigma_{I,t}$ is decreasing in the labor income tax rate τ_w and increasing in the equilibrium growth rate g_t. Therefore, in addition to a direct negative effect of labor income tax on income inequality via the *wage-income* channel discussed in Section 2.10.2, the negative effect of labor income tax on economic growth also affects income inequality via the *interest-rate* channel. Specifically, an increase in the labor income tax rate τ_w reduces the degree of income inequality $\sigma_{I,t}$ via a reduction in the interest rate r_t and the growth rate g_t; see Proposition 4. However, this indirect negative effect on income inequality is temporary. As the equilibrium growth rate g_t returns to the initial steady-state value g^*, the indirect negative effect on income inequality via the interest-rate channel also disappears.

However, (43) shows that the degree of income inequality $\sigma_{I,t}$ remains below the initial steady-state value σ_I^* due to the direct negative effect of τ_w via the wage-income channel. Intuitively, the negative effect of labor income tax τ_w on labor supply is stronger for wealthier households as shown in (5), and the larger reduction in wealthier households' wage income reduces income inequality. Figure 1 presents the time path of income inequality when the labor income tax rate τ_w rises at time t. Proposition 5 summarizes the complete dynamic effects of labor income tax on income inequality.

Proposition 5 *For $\omega = 1$, an increase in the labor income tax τ_w leads to a reduction in income inequality $\sigma_{I,t}$ but the decrease is larger in the short run than in the long run.*

Proof. Proven in text. ■

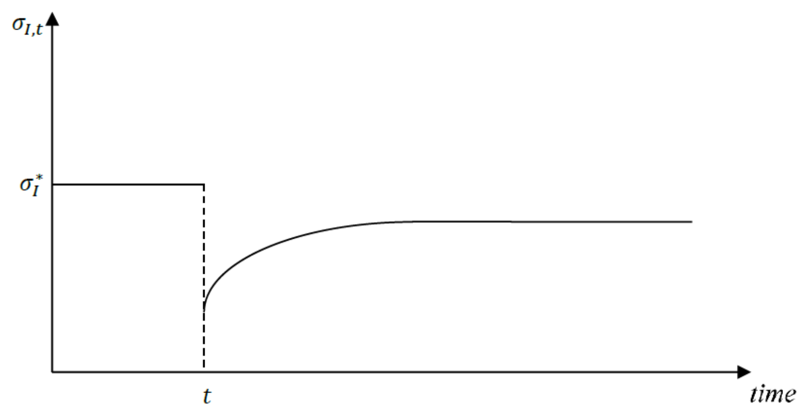

Figure 2: Labor income tax and income inequality

3.3 Labor income tax and consumption inequality

Equation (44) shows that the degree of consumption inequality σ_c^* depends on the labor income tax rate. Interestingly, this effect is positive and permanent. In other words, an increase in the labor income tax rate raises the degree of consumption inequality σ_c^* permanently. Recall that consumption depends on net income. Therefore, an increase in the labor income tax rate reduces disposable wage income relative to asset income, which is more unequally distributed, and gives rise to an increase in consumption inequality. Figure 3 presents the time path of consumption inequality when the labor income tax rate τ_w rises at time t. Proposition 6 summarizes the permanent effect of labor income tax on consumption inequality.

Proposition 6 *For $\omega = 1$, an increase in the labor income tax τ_w leads to a permanent increase in the degree of consumption inequality σ_c^*.*

Proof. Proven in text. ■

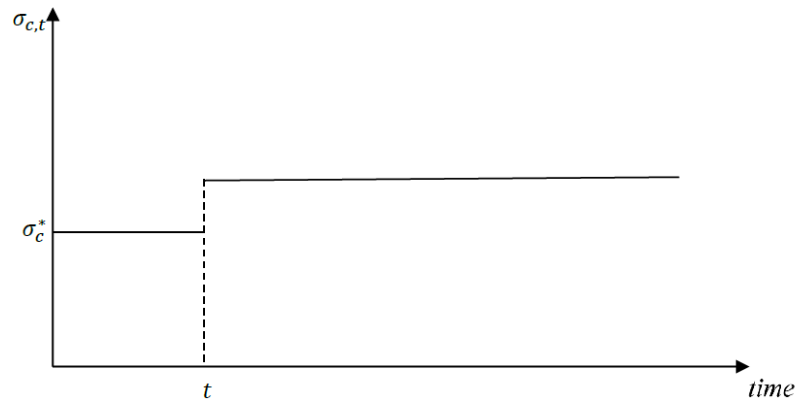

Figure 3: Labor income tax and consumption inequality

3.4 When does the wealth distribution matter?

Equation (37) shows that for a given consumption dispersion index Δ_c^*, an increase in the labor income tax rate τ_w has a direct negative effect on employment l^*. However, labor income tax also affects consumption dispersion unless (a) $s_c^*(h) = 1$ for all $h \in [0, 1]$ under homogeneous households or (b) $\omega = 1$ under which $\Delta_c^* = 1$ even in the case of heterogeneous households. In the general case $\omega \neq 1$ under heterogeneous households, the effects of labor income tax on consumption dispersion is given by

$$\frac{\partial \left(\Delta_c^*\right)^{\omega}}{\partial \tau_w} = \omega \int_0^1 \left[s_c^*(h)\right]^{\omega-1} \frac{\partial s_c^*(h)}{\partial \tau_w} dh, \tag{45}$$

which can be positive or negative.

Given the complexity of (45), we consider the following simple parametric example for the wealth distribution: $s_{a,0}(h) = 1 - \varepsilon$ for $h \in [0, \delta]$ and $s_{a,0}(h) = 1 + \varepsilon\delta/(1-\delta)$ for $h \in (\delta, 1]$, where the parameter $\delta \in (0, 1)$ measures the share of poor households and the parameter $\varepsilon > 0$ measures the degree of wealth inequality. In this case, (45) becomes

$$\begin{aligned} \frac{\partial \left(\Delta_c^*\right)^{\omega}}{\partial \tau_w} &= \omega \left\{ \delta \left[s_c^*(p)\right]^{\omega-1} \frac{\partial s_c^*(p)}{\partial \tau_w} + (1-\delta) \left[s_c^*(r)\right]^{\omega-1} \frac{\partial s_c^*(r)}{\partial \tau_w} \right\} \\ &= \omega\delta \left\{ \left[s_c^*(p)\right]^{\omega-1} - \left[s_c^*(r)\right]^{\omega-1} \right\} \underbrace{\frac{\partial s_c^*(p)}{\partial \tau_w}}_{-}, \end{aligned}$$

in which $s_c^*(p)$ and $s_c^*(r)$ denote, respectively, the consumption share of a poor household with wealth share $s_{a,0}(p) = 1 - \varepsilon$ and the consumption share of a rich household with wealth share $s_{a,0}(r) = 1 + \varepsilon\delta/(1-\delta)$ whereas the second equality uses $\delta s_c^*(p) + (1-\delta)s_c^*(r) = 1$. It is useful to note that a poor household has a lower consumption share than a rich household such that $s_c^*(p) < s_c^*(r)$ and that the consumption share of poor households $s_c^*(p)$ is decreasing in the labor income tax rate τ_w.[10] Intuitively, higher labor income tax τ_w reduces the consumption of poor households relative to rich households because the former has a higher level of wage income and a lower level of asset income than the latter. This reduction in the consumption share of poor households gives rise to a positive effect on their labor supply.

We now examine how labor income tax affects consumption dispersion. If $\omega > 1$, then $[s_c^*(p)]^{\omega-1} < [s_c^*(r)]^{\omega-1}$ and $\partial (\Delta_c^*)^{\omega} / \partial \tau_w > 0$. In other words, labor income tax has a positive effect on consumption dispersion, which in turn amplifies the negative effect of τ_w on employment l^* in (37). On the other hand, if $\omega \in (0,1)$, then $[s_c^*(p)]^{\omega-1} > [s_c^*(r)]^{\omega-1}$ and $\partial (\Delta_c^*)^{\omega} / \partial \tau_w < 0$. In this case, labor income tax has a negative effect on consumption dispersion which in turn gives rise to a novel positive effect on employment l^*, in addition to the direct negative effect of τ_w in (37). In Appendix A, we show that labor supply $l^*(p)$ of poor households is increasing in τ_w if and only if $\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)(1-\varepsilon) < 0$.[11] In other words, when the degree of wealth inequality ε is sufficiently high, the overall effect of labor income tax τ_w on employment l^* can surprisingly become positive due to poor households increasing labor supply by more than rich households decreasing labor supply. Overall, the effects of labor income tax τ_w on employment l^* and the transitional growth rate g_t become ambiguous under $\omega \in (0,1)$.

Proposition 7 summarizes the effects of labor income tax on employment and economic growth under the general case $\omega \neq 1$ with heterogeneous households.

Proposition 7 *For $\omega > 1$, an increase in the labor income tax rate τ_w at time t has a negative effect on employment l^* and the instantaneous growth rate g_t at time t. For $\omega \in (0,1)$, an increase in the labor income tax rate τ_w at time t can have a negative or positive effect on employment l^* and the instantaneous growth rate g_t at time t. In both cases, the increase in the labor income tax rate τ_w does not affect the steady-state growth rate g^*.*

Proof. See Appendix A. ■

3.4.1 What if rich households don't work?

In the previous analysis, we assume that all households supply labor such that $l^*(h) > 0$ for all $h \in [0,1]$. However, when the wealth share $s_{a,0}(r)$ of rich households is sufficiently high, they choose not to supply any labor such that $l^*(r) = 0$. In this case, the supply of labor by poor households is solely determined by the following equation:[12]

$$l^*(p) = 1 - \left\{ \eta\delta l^*(p) \frac{\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)}{(1-\tau_w)(1-\theta)} + \eta l^*(p) \right\}^{\omega}, \tag{46}$$

where we have used $l^* = \delta l^*(p)$ which is now increasing in the labor income tax rate τ_w if and only if $\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)(1-\varepsilon) < 0$. It is useful to note that a high degree ε of wealth inequality makes this parameter condition more likely to hold and that the resulting positive effect of labor

[10] See the proof of Proposition 7 in Appendix A.

[11] Recall that γ can be negative in case of a lump-sum tax. Also, ε can be greater than unity if poor households have negative wealth (i.e., debt). Therefore, even if $\gamma = 0$, $\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)(1-\varepsilon) < 0$ still holds whenever $\varepsilon > 1$.

[12] See the proof of Proposition 7 in Appendix A.

income tax on employment and growth can now be present for any value of $\omega > 0$. Proposition 8 summarizes the effects of labor income tax on employment and economic growth when rich households do not work (i.e., $l^*(r) = 0$).

Proposition 8 *Suppose $l^*(r) = 0$. Then, for any value of $\omega > 0$, an increase in the labor income tax rate τ_w at time t has a positive (negative) effect on employment l^* and the instantaneous growth rate g_t at time t if $\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)(1-\varepsilon) < 0$ ($\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)(1-\varepsilon) > 0$). In both cases, the increase in the labor income tax rate τ_w does not affect the steady-state growth rate g^*.*

Proof. First, use (46) to show that $l^*(p)$ is increasing (decreasing) in τ_w if $\gamma+\frac{\theta\beta}{\mu}(\rho - \lambda)(1-\varepsilon) < 0$ (> 0). Then, note that $l^* = \delta l^*(p)$. Finally, use (26) to show that g_t is increasing in l^* and (31) to show that g^* is independent of l^*. ■

4 Quantitative analysis

In this section, we calibrate the model using US data in order to quantitatively examine the growth and inequality effects of tax policy. The model features the following 14 parameters $\{\omega, \rho, \mu, \alpha, \sigma, \theta, \tau_w, \lambda, \phi, \beta, \eta, \gamma, \delta, \varepsilon\}$. These parameter values are determined as follows. We consider three values of the elasticity of intertemporal substitution for leisure $\omega \in \{0.2, 1, 1.5\}$. Given (5), it can be shown that the elasticity of labor supply is given by $\omega(1-l^*)/l^*$. Under our calibrated parameter values, the values of $\omega \in \{0.2, 1, 1.5\}$ correspond to labor supply elasticity of $\{0.4, 2, 3\}$, which are within the range of empirical estimates in the literature.[13] For the discount rate ρ, we set it to 0.03. For the markup ratio μ, we consider a conventional value of 1.2, which is within the range of estimates summarized in Jones and Williams (2000). We follow Iacopetta and Peretto (2021) to consider a value of 0.67 for the degree of technology spillover $1 - \alpha$. We set the degree of congestion $1 - \sigma$ to 0.5 as in Iacopetta *et al.* (2019). The labor share of output $1 - \theta$ is set to a value of 0.65.[14] The average tax rate τ_w on wage income is 23% in the US.[15] For the population growth rate λ, we set it to 1.58%, which corresponds to the average employment growth rate in the US from 1979 to 2019.[16]

For other parameters, we calibrate them to match empirical moments of the US economy. For the cost parameters $\{\phi, \beta\}$, we calibrate them by using an average growth rate of GDP per capita of 2% and an average R&D share of GDP of 2.6%.[17] We calibrate the leisure parameter η by matching the share of time spent on working to 0.33. For the parameter γ, we calibrate it using an average ratio of government spending to GDP of 16.06%.[18] We define the top 10% households in terms of wealth as rich households (i.e., $1 - \delta = 0.1$) and calibrate ε using their

[13] See for example, Chetty *et al.* (2011) and Keane and Rogerson (2012). It is useful to note that macroeconomic estimates for labor supply elasticity tend to be greater than unity, whereas microeconomic estimates tend to be smaller than unity.

[14] See for example, Karabarbounis and Neiman (2014).

[15] Data source: OECD Database.

[16] Data source: Business Dynamics Statistics.

[17] Data source: Federal Reserve Economic Data and OECD Database.

[18] Data source: Bureau of Economic Analysis.

wealth share. In the US, the top 10% of households owns 65.32% of total wealth. Table 1 summarizes the benchmark parameter values,[19] which satisfy $\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)(1 - \varepsilon) < 0$.

Table 1: Benchmark parameter values

ω	ρ	μ	α	σ	θ	τ_w	λ	ϕ	β	η	γ	δ	ε
0.20	0.03	1.20	0.33	0.50	0.35	0.23	0.016	0.086	2.709	0.395	-0.011	0.90	0.615
1.00	0.03	1.20	0.33	0.50	0.35	0.23	0.016	0.086	2.709	2.000	-0.011	0.90	0.615
1.50	0.03	1.20	0.33	0.50	0.35	0.23	0.016	0.086	2.709	2.289	-0.011	0.90	0.615

Given the parameter values in Table 1, we simulate the effects of labor income tax τ_w on average employment l^*, the growth rate g_t of output per capita, income inequality $\sigma_{I,t}$ and consumption inequality $\sigma_{c,t}$. Figure 4 to 7 simulate the instantaneous effects of tax policy for a given x_t at time t. Figure 4 and 5 show that labor income tax τ_w has negative effects on average employment l^* and the instantaneous growth rate g_t of output per capita for $\omega \in \{1, 1.5\}$. These values of ω correspond to a labor supply elasticity of 2 and 3, which are within the range of macroeconomic estimates. In this case, we obtain the conventional negative effect of labor income tax on employment. However, for $\omega = 0.2$, the effects of labor income tax τ_w on average employment l^* and the instantaneous growth rate g_t of output per capita become positive due to poor households increasing labor supply. This value of $\omega = 0.2$ corresponds to a labor supply elasticity of 0.4, which is within the range of microeconomic estimates. In this case, we obtain the novel positive effect of labor income tax on employment. Figure 6 shows that labor income tax τ_w has a negative instantaneous effect on income inequality $\sigma_{I,t}$ for all $\omega \in \{0.2, 1, 1.5\}$;[20] however, the positive effect of τ_w on the growth rate g_t and the real interest rate r_t under $\omega = 0.2$ implies a smaller negative effect of τ_w on income inequality $\sigma_{I,t}$ in this case. Specifically, increasing the labor income tax rate τ_w from 0.23 to 0.33 reduces income inequality $\sigma_{I,t}$ by 2.58% in the case of $\omega = 0.2$ as compared to 8.39% in the case of $\omega = 1$. Figure 7 shows that labor income tax τ_w has a positive instantaneous effect on consumption inequality for $\omega \in \{0.2, 1, 1.5\}$, and the magnitude is about the same in all cases.

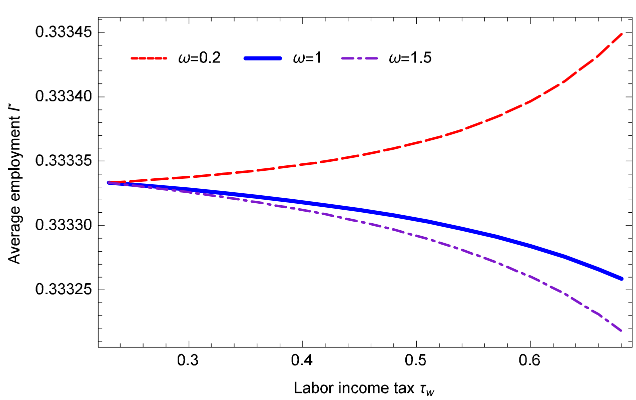

Figure 4: Instantaneous effect of τ_w on l^*

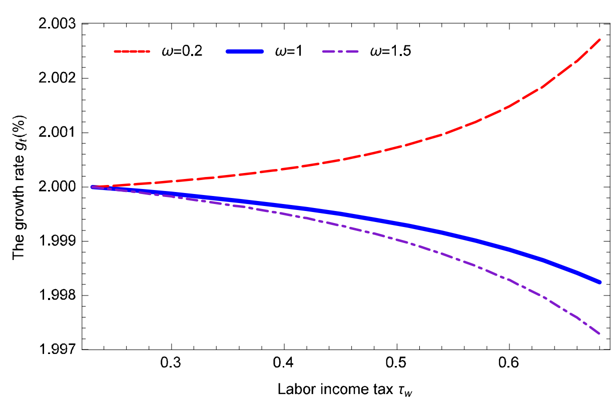

Figure 5: Instantaneous effect of τ_w on g_t

[19]The calibrated value of $\gamma < 0$ is due to government expenditures being greater than labor income tax revenue, which then requires a lump-sum tax in the model, capturing other tax revenues in reality.

[20]Under other parameter values, it is possible for labor income tax τ_w to have a positive instantaneous effect on income inequality $\sigma_{I,t}$ when $\omega < 1$.

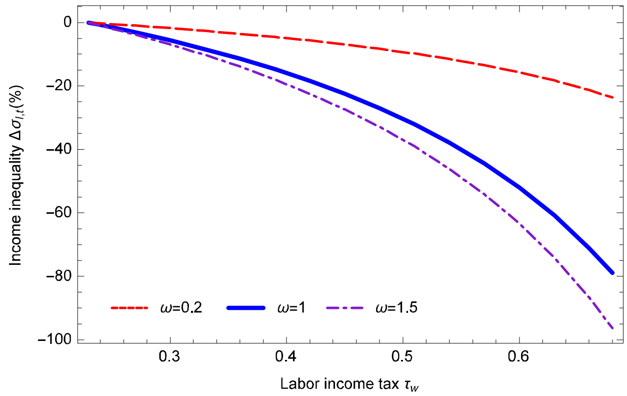

Figure 6: Instantaneous effect of τ_w on $\sigma_{I,t}$

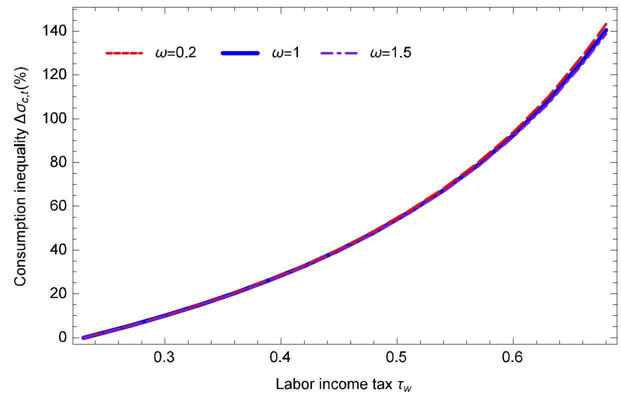

Figure 7: Instantaneous effect of τ_w on $\sigma_{c,t}$

Figure 8 to 10 simulate the transition dynamic effects of labor income tax τ_w on the growth rate g_t of output per capita, income inequality $\sigma_{I,t}$ and consumption inequality $\sigma_{c,t}$. Figure 8 shows that labor income tax τ_w has a temporary negative effect on the growth rate g_t of output per capita in the cases of $\omega \in \{1, 1.5\}$. However, in the case of $\omega = 0.2$, the transitional effect of labor income tax τ_w on the growth rate g_t of output per capita becomes positive due to the increase in average employment l^*. In all cases $\omega \in \{0.2, 1, 1.5\}$, labor income tax τ_w does not affect the steady-state growth rate g^*. Figure 9 shows that labor income tax τ_w reduces income inequality $\sigma_{I,t}$; however, in the cases of $\omega \in \{1, 1.5\}$, this negative effect becomes smaller over time as the growth rate g_t and the real interest rate r_t rise and return to their initial steady-state values. In the case of $\omega = 0.2$, labor income tax τ_w also reduces income inequality $\sigma_{I,t}$; however, this negative effect becomes larger over time as the growth rate g_t and the real interest rate r_t fall and return to their initial steady-state values. Finally, Figure 10 shows that labor income tax τ_w increases consumption inequality permanently by about the same magnitude in all three cases.

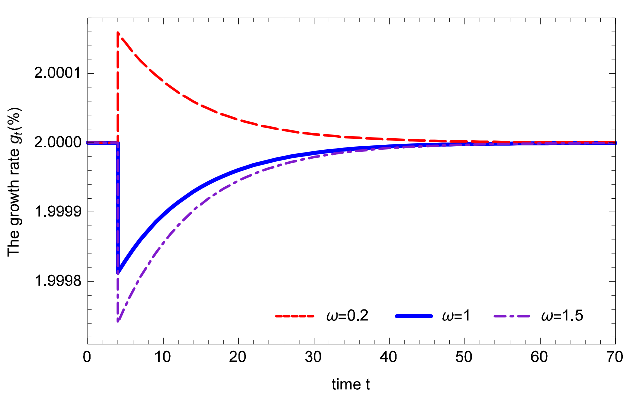

Figure 8: Transition dynamics of g_t

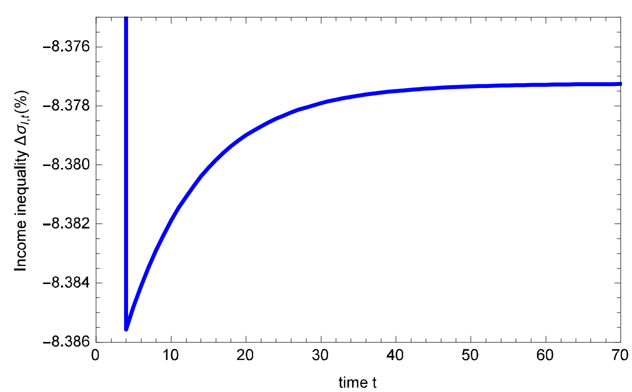

Figure 9a: Transition dynamics of $\sigma_{I,t}$ $(\omega = 1)$

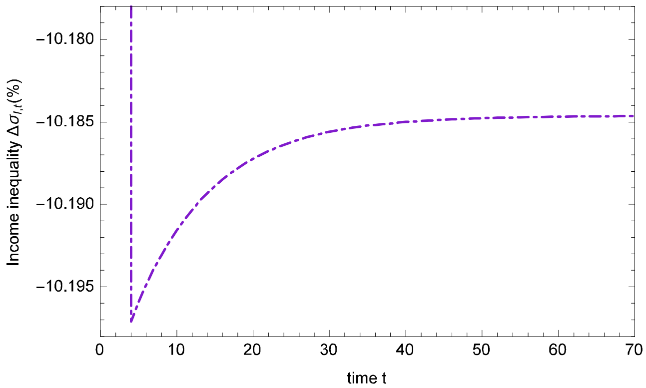

Figure 9b: Transition dynamics of $\sigma_{I,t}$ ($\omega = 1.5$)

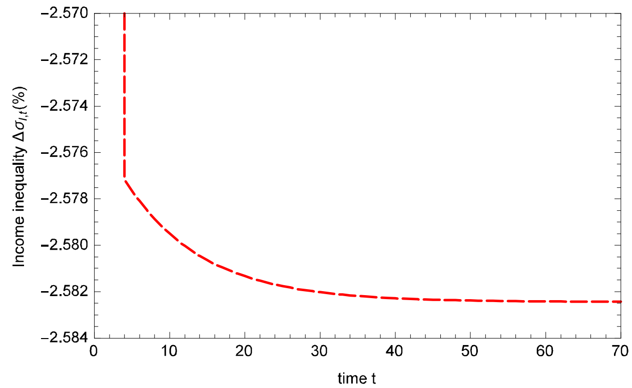

Figure 9c: Transition dynamics of $\sigma_{I,t}$ ($\omega = 0.2$)

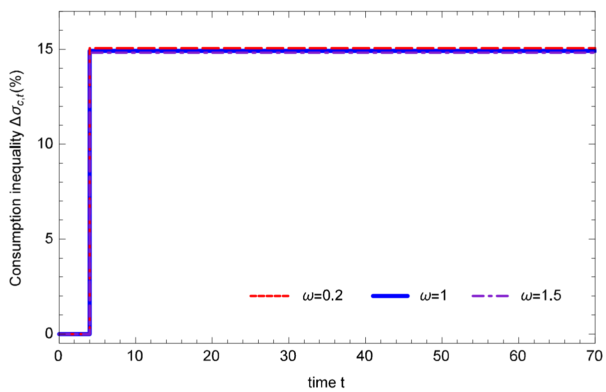

Figure 10: Transition dynamics of $\sigma_{c,t}$

Here we also consider the case of after-tax income inequality. Let $s_{I,t}^{A}(h)$ denote the share of after-tax income received by household h and it is given by

$$s_{I,t}^{A}(h) = \frac{s_{a,0}(h)\,(r_t - \lambda)\,a_t + (1-\tau_w)w_t l_t(h) + \iota_t}{(r_t - \lambda)\,a_t + (1-\tau_w)w_t l_t + \iota_t}, \tag{47}$$

Similarly, we measure after-tax income inequality by the standard deviation of income share $s_{I,t}^{A}(h)$ defined as $\sigma_{I,t}^{A} \equiv \sqrt{\int_0^1 \left[s_{I,t}^{A}(h) - 1\right]^2 dh}$. We use the parameter values from Table 1 and simulate the effect of labor income tax τ_w on after-tax income inequality $\sigma_{I,t}^{A}$. Figure 11 and 12 respectively simulate the instantaneous and transitional effects of tax policy on after-tax income inequality $\sigma_{I,t}^{A}$. Figure 11 shows that labor income tax τ_w has a positive instantaneous effect on after-tax income inequality $\sigma_{I,t}^{A}$ for all $\omega \in \{0.2, 1, 1.5\}$, and the magnitude is about the same in all cases. Interestingly, this effect is different from gross income inequality. Intuitively, after-tax income inequality consists of asset income and after-tax wage income. Compared to asset income, wage income is relatively equal. An increase in the labor income tax rate reduces the importance of wage income (the relatively equal component) relative to asset income (the relatively unequal component), thereby increasing income inequality. Figure 12 shows that labor income tax τ_w increases after-tax income inequality $\sigma^A_{I,t}$; however, in the case of $\omega \in \{1, 1.5\}$, this positive effect becomes larger over time as the growth rate g_t and the real interest rate r_t rise and return to their initial steady-state values. In the case of $\omega = 0.2$, labor income tax τ_w also increases income inequality $\sigma_{I,t}$; however, this positive effect becomes smaller over time as the growth rate g_t and the real interest rate r_t fall and return to their initial steady-state values.

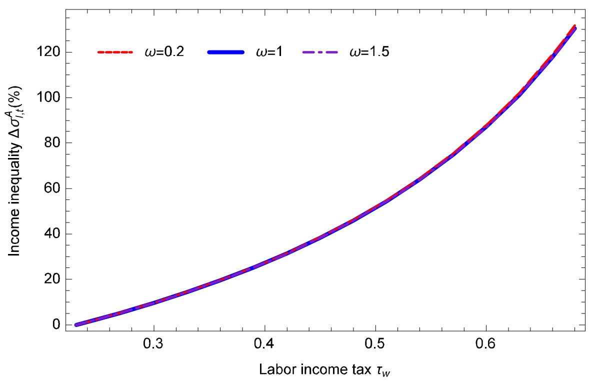

Figure 11: Instantaneous effect of τ_w on $\sigma^A_{I,t}$

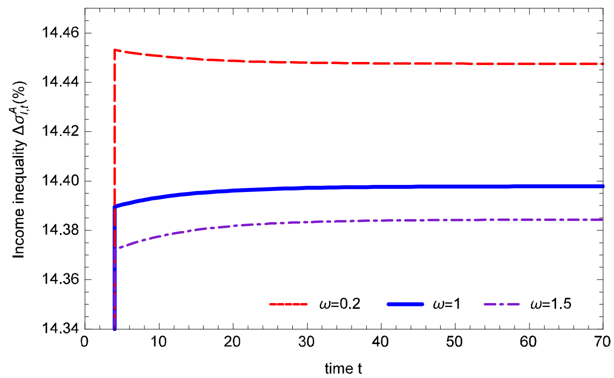

Figure 12: Transition dynamics of $\sigma^A_{I,t}$

We now perform a robustness check by considering a lower value of $\delta = 0.5$. If we define the bottom 50% households in terms of wealth as poor households, then they own 2.3% of total wealth, which corresponds to a value of 0.95 for ε. We recalibrate the rest of the parameters to aggregate data of the US economy. Table 2 summarizes the benchmark parameter values.

Table 2: Benchmark parameter values ($\delta = 0.5$)

ω	ρ	μ	α	σ	θ	τ_w	λ	ϕ	β	η	γ	δ	ε
0.20	0.03	1.20	0.33	0.50	0.35	0.23	0.016	0.086	2.709	0.395	-0.011	0.50	0.954
1.00	0.03	1.20	0.33	0.50	0.35	0.23	0.016	0.086	2.709	2.000	-0.011	0.50	0.954
1.50	0.03	1.20	0.33	0.50	0.35	0.23	0.016	0.086	2.709	2.289	-0.011	0.50	0.954

Figure 13 and 14, respectively, simulate the instantaneous effects of tax policy on average employment l^* and the growth rate g_t of output per capita for the case of $\delta = 0.5$. They show that the employment and growth effects of labor income tax τ_w follow the same pattern as before for $\omega \in \{1, 1.5\}$. As for $\omega = 0.2$, labor income tax τ_w continues to have a positive effect on employment l^* and the growth rate g_t if the increase in the labor income tax rate τ_w is sufficiently large. However, for a small increase in τ_w, we see a slightly negative effect on employment l^* and the growth rate g_t because the lower share δ of poor households reduces the influence of their labor supply $l^*(p)$ on employment l^*, strengthening the negative effect channel of τ_w on employment l^* and the growth rate g_t. Figure 15 and 16, respectively, simulate the instantaneous effects of tax policy on income inequality $\sigma_{I,t}$ and consumption inequality $\sigma_{c,t}$ for the case of $\delta = 0.5$. They show that the inequality effects of labor income τ_w also follow the same pattern as before for all $\omega \in \{0.2, 1, 1.5\}$. Specifically, increasing the labor income tax rate τ_w from 0.23 to 0.33 reduces income inequality $\sigma_{I,t}$ by 2.72% in the case of $\omega = 0.2$, as compared to 8.39% in the case of $\omega = 1$.

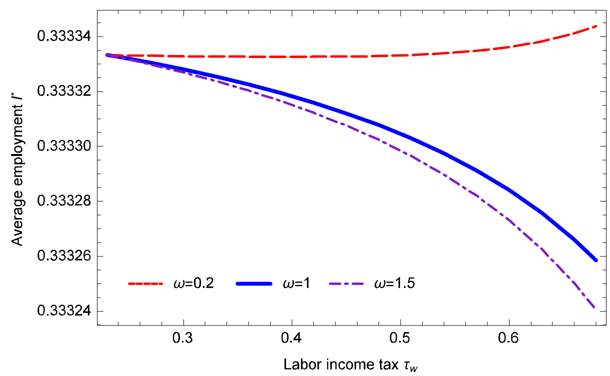

Figure 13: Instantaneous effect of τ_w on l^* ($\delta = 0.5$)

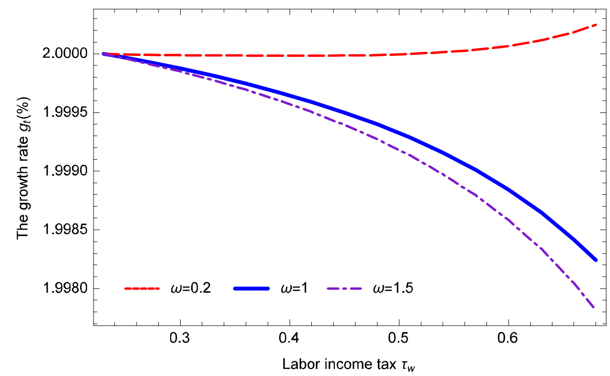

Figure 14: Instantaneous effect of τ_w on g_t ($\delta = 0.5$)

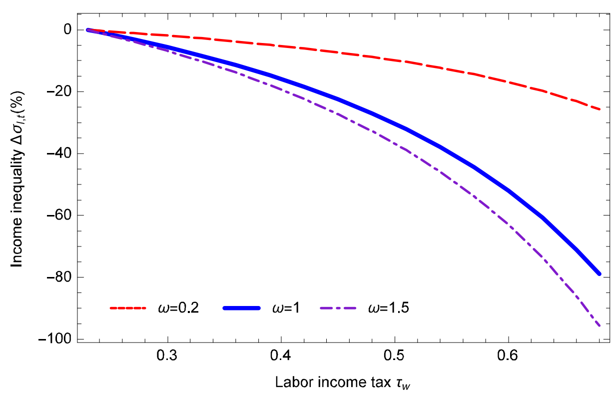

Figure 15: Instantaneous effect of τ_w on $\sigma_{I,t}$ ($\delta = 0.5$)

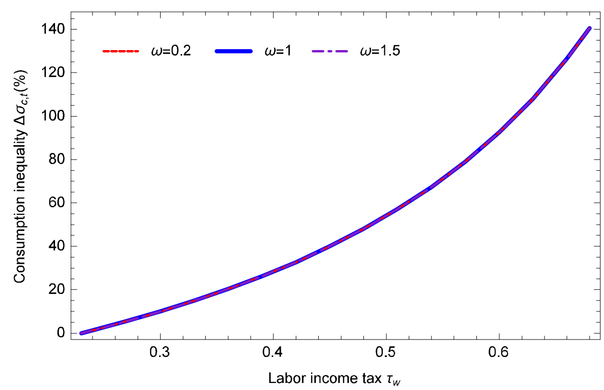

Figure 16: Instantaneous effect of τ_w on $\sigma_{c,t}$ ($\delta = 0.5$)

Figure 17 simulates the transition dynamic effects of tax policy on the growth rate g_t of output per capita for the case of $\delta = 0.5$. They show that the effect of labor income tax τ_w follows the same pattern as before for $\omega \in \{1, 1.5\}$. However, for $\omega = 0.2$, the transitional effect of τ_w on the growth rate g_t of output per capita becomes negative due to the decrease in average employment l^*. Figure 18 and 19, respectively, simulate the transition dynamic effects of tax policy on income inequality $\sigma_{I,t}$ and consumption inequality $\sigma_{c,t}$ for the case of $\delta = 0.5$. Figure 18 shows that labor income tax τ_w reduces income inequality $\sigma_{I,t}$ and this negative effect becomes smaller over time as the growth rate g_t and the real interest rate r_t rise and return to their initial steady-state values for all $\omega \in \{0.2, 1, 1.5\}$. Figure 19 shows that the consumption inequality effect of τ_w also follows the same pattern as before for all $\omega \in \{0.2, 1, 1.5\}$.

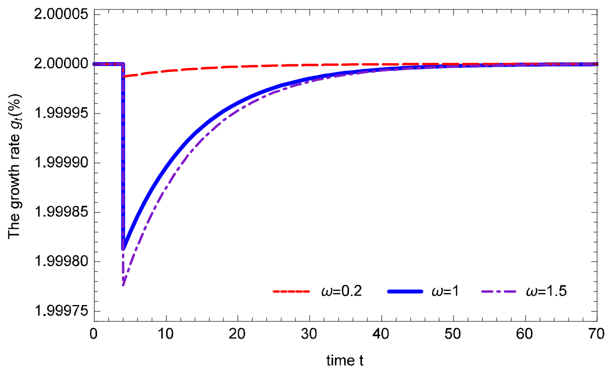

Figure 17: Transition dynamics of g_t
($\delta = 0.5$)

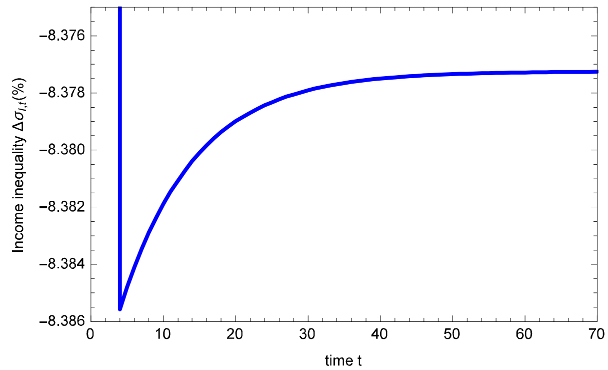

Figure 18a: Transition dynamics of $\sigma_{I,t}$
($\omega = 1, \delta = 0.5$)

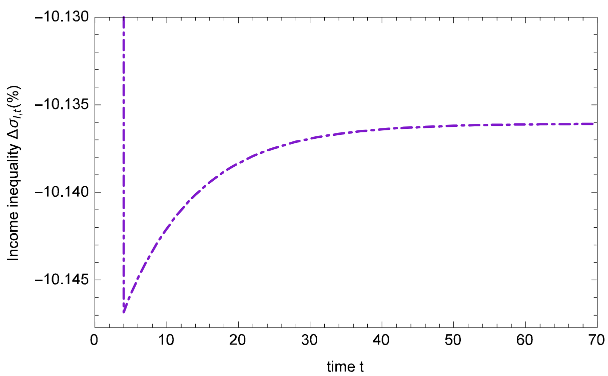

Figure 18b: Transition dynamics of $\sigma_{I,t}$
($\omega = 1.5, \delta = 0.5$)

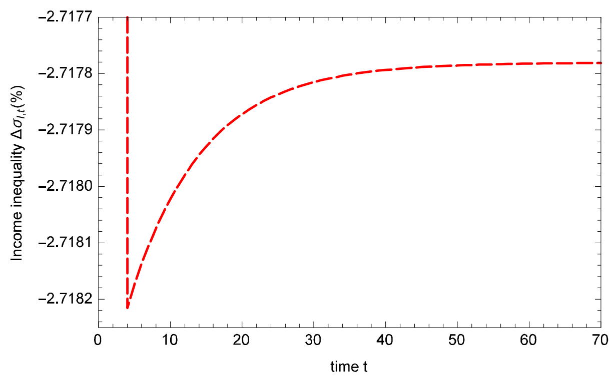

Figure 18c: Transition dynamics of $\sigma_{I,t}$
($\omega = 0.2, \delta = 0.5$)

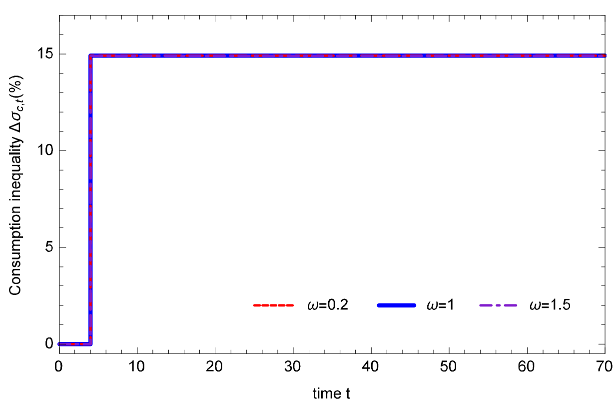

Figure 19: Transition dynamics of $\sigma_{c,t}$
($\delta = 0.5$)

Figure 20 and 21, respectively, simulate the instantaneous and transitional effects of tax policy on after-tax income inequality $\sigma^A_{I,t}$ for the case of $\delta = 0.5$. Figure 20 shows that the after-tax income inequality effect of τ_w follows the same pattern as before for all $\omega \in \{0.2, 1, 1.5\}$. Figure 21 shows that labor income tax τ_w increases after-tax income inequality $\sigma^A_{I,t}$ and this positive effect becomes larger over time as the growth rate g_t and the real interest rate r_t rise and return to their initial steady-state values for all $\omega \in \{0.2, 1, 1.5\}$.

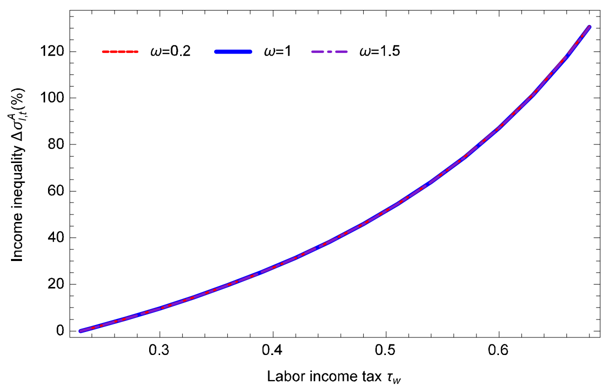

Figure 20: Instantaneous effect of τ_w on $\sigma^A_{I,t}$ ($\delta = 0.5$)

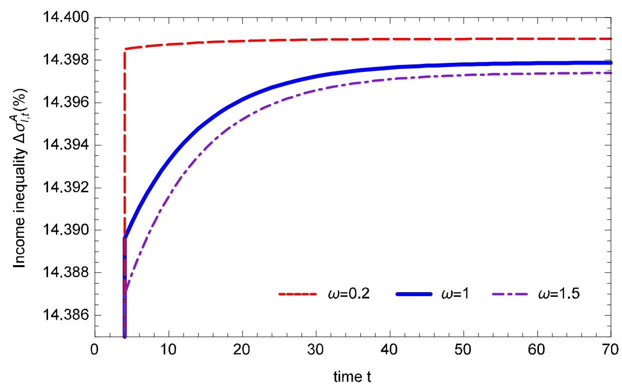

Figure 21: Transition dynamics of $\sigma^A_{I,t}$ ($\delta = 0.5$)

5 Conclusion

In this study, we have developed a Schumpeterian growth model with wealth heterogeneity to explore how taxation affects economic growth and income inequality. A novelty of our analysis is that our model features iso-elastic utility on leisure under which the change in consumption dispersion across heterogeneous households can give rise to a surprising positive effect of labor income tax on employment and economic growth. This positive effect of labor income tax on economic growth in turn causes the negative effect of labor income tax on income inequality to become smaller quantitatively. Therefore, household heterogeneity not only influences how tax policy affects the aggregate economy but also how it affects the income distribution. Our Schumpeterian growth model provides a tractable framework to illustrate the complete dynamic effects of labor income tax on economic growth and income inequality under heterogeneous households. Although we have considered a simple wealth distribution to illustrate our results and their intuition as clearly as possible, we can also extend our analysis to a more general wealth distribution to examine their robustness. We leave this extension to future research.

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Appendix A

Proof of Lemma 1. The current-value Hamiltonian of the monopolistic firm in industry i is

$$H_t(i) = \Pi_t(i) - R_t(i) + \vartheta_t(i)\dot{Z}_t(i) + \kappa_t(i)\left[\mu - P_t(i)\right], \tag{A1}$$

where $\vartheta_t(i)$ is the costate variable on (9) and $\kappa_t(i)$ is the multiplier on $P_t(i) \leq \mu$. Substituting (8)-(10) into (A1), we derive

$$\frac{\partial H_t(i)}{\partial P_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial P_t(i)} = \kappa_t(i), \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial R_t(i)} = 0 \Rightarrow \vartheta_t(i) = 1, \tag{A3}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha\left\{\left[P_t(i) - 1\right]\left[\frac{\theta}{P_t(i)}\right]^{1/(1-\theta)}\frac{L_{y,t}}{N_t^{1-\sigma}} - \phi\right\} Z_t^{\alpha-1}(i)\, Z_t^{1-\alpha} = r_t\vartheta_t(i) - \dot{\vartheta}_t(i). \tag{A4}$$

Based on (A2), we obtain the following results. If $P_t(i) < \mu$, then we have $P_t(i) = 1/\theta$ because $\kappa_t(i) = 0$ in this case. If the constraint on $P_t(i)$ is binding, then we have $P_t(i) = \mu$ because $\kappa_t(i) > 0$ in this case. Here we assume $\mu < 1/\theta$, which implies $P_t(i) = \mu$ as shown in (12). In addition, we substitute (A3), (12) and (14) into (A4) and impose symmetry to derive (15). ■

Proof of Lemma 2. Substituting $P_t(i) = \mu$ into $\theta Y_t = \int_0^{N_t} P_t(i)X_t(i)di$ and using symmetry yield $\theta Y_t = \mu N_t X_t$. Combining (17) and $a_t L_t = N_t V_t$ and using $\theta Y_t = \mu N_t X_t$, we obtain

$$a_t = (\theta/\mu)\,\beta y_t. \tag{A5}$$

Differentiating (A5) with respect to t yields

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{a}_t}{a_t} = (r_t - \lambda) + \frac{(1-\tau_w)\, w_t l_t}{a_t} - \frac{c_t}{a_t} + \frac{\iota_t}{a_t}, \tag{A6}$$

where the second equality uses (2) with $a_t \equiv \int_0^1 a_t(h)dh$, $l_t \equiv \int_0^1 l_t(h)dh$ and $c_t \equiv \int_0^1 c_t(h)dh$. We then manipulate (A6) using (4), (7) and (A5) to derive

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{y}_t}{y_t} = \frac{\mu}{\theta\beta}\left\{\frac{c_t}{y_t} - \left[(1-\tau_w)(1-\theta) + \gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)\right]\right\}, \tag{A7}$$

which also uses $L_{y,t} = l_t L_t$ and $\iota_t = \gamma y_t$. Given (A7), the dynamics of c_t/y_t is characterized by saddle-point stability such that c_t/y_t jumps to the unique steady-state value $(c/y)^*$ in (24). ■

Proof of Lemma 3. Taking the log of (14) and differentiating it with respect to t yield

$$\frac{\dot{x}_t}{x_t} = \lambda - (1-\sigma)\, n_t. \tag{A8}$$

Combining (4) and (21) with $\dot{c}_t/c_t = \dot{y}_t/y_t$ from Lemma 2, we derive $r_t = \sigma n_t + z_t + \rho + \dot{l}_t/l_t$. Substituting this condition and $r_t = r_t^e$ into (18) yields

$$n_t = \frac{1}{\beta}\left(\mu - 1 - \frac{\phi + z_t}{x_t l^*}\right) + \lambda - \rho, \tag{A9}$$

where we have used (A8) and $\dot{l}_t/l_t = 0$ due to employment l^* being stationary. Given $r_t = r_t^q$, we use (15) and $r_t = \sigma n_t + z_t + \rho + \dot{l}_t/l_t$ to obtain

$$z_t = \alpha \left[(\mu - 1) x_t l^* - \phi \right] - \rho - \sigma n_t, \tag{A10}$$

where we have used $\dot{l}_t/l_t = 0$. We substitute (A10) into (A9) to show that n_t is given by

$$n_t = \frac{1}{\beta - \frac{\sigma}{x_t l^*}} \left\{ (1-\alpha)(\mu - 1) + \beta(\lambda - \rho) - \frac{(1-\alpha)\phi - \rho}{x_t l^*} \right\}, \tag{A11}$$

where we follow Peretto (2015) to approximate $\sigma/(x_t l^*) \approx 0$ and substitute the approximated version of (A11) into (A8) to derive (29). ∎

Proof of Proposition 1. Using (3), (4) and (23), we prove that $s_{c,t}(h) = s_c^*(h)$ always holds for all time $t > 0$. We substitute this condition into (34) to obtain

$$\dot{s}_{a,t}(h) = \frac{c_t - (1-\tau_w) w_t l_t - \gamma y_t}{a_t} s_{a,t}(h) - \frac{s_c^*(h) c_t - (1-\tau_w) w_t l_t(h) - \gamma y_t}{a_t}. \tag{A12}$$

Lemma 2 shows that $\{a_t, c_t, y_t, w_t\}$ all grow at the same rate g_t at any point in time. Given this condition, we combine (4) and (32) to derive

$$\frac{c_t - (1-\tau_w) w_t l_t - \gamma y_t}{a_t} = \rho - \lambda > 0, \tag{A13}$$

which shows that the coefficient on $s_{a,t}(h)$ is positive. Substituting (A13) into (A12) yields

$$\dot{s}_{a,t}(h) = (\rho - \lambda)\left[s_{a,t}(h) - 1 \right] - \frac{c_t}{a_t}\left[s_c^*(h) - 1 \right] + \frac{w_t}{a_t}(1-\tau_w)\left[l_t(h) - l_t \right]. \tag{A14}$$

From (A5), the consumption-wealth ratio $c_t/a_t = (c/a)^* = \mu (c/y)^* / (\theta\beta)$ is stationary due to the stationary consumption-output ratio $c_t/y_t = (c/y)^*$. Given $s_{c,t}(h) = s_c^*(h)$ and $\Delta_{c,t} = \Delta_c^*$, Lemma 2 implies $l_t(h) = l^*(h)$ and $l_t = l^*$ are stationary. As for the wage-wealth ratio w_t/a_t, we use $w_t l_t = (1-\theta) y_t$ from (7) and combine it with (A5) to obtain a stationary wage-wealth ratio $w_t/a_t = (w/a)^* = \mu(1-\theta)/(\theta\beta l^*)$. Substituting these conditions into (A14) yields

$$\dot{s}_{a,t}(h) = (\rho - \lambda)\left[s_{a,t}(h) - 1 \right] - \left(\frac{c}{a}\right)^* \left[s_c^*(h) - 1 \right] + \left(\frac{w}{a}\right)^* (1-\tau_w)\left[l^*(h) - l^* \right], \tag{A15}$$

which becomes (35) given $(c/a)^* = \mu (c/y)^* / (\theta\beta)$ and $(w/a)^* = \mu(1-\theta)/(\theta\beta l^*)$. Then, $\rho - \lambda > 0$ implies that $\dot{s}_{a,t}(h) = 0$ for all time t because $s_{a,t}(h)$ is a pre-determined variable. ∎

Proof of Proposition 2. For $\omega = 1$, the employment function of household h and the average employment function are respectively

$$l^*(h) = 1 - \left(\frac{c}{w}\right)^* \frac{\eta s_c^*(h)}{1-\tau_w}, \tag{A16}$$

$$l^* = 1 - \left(\frac{c}{w}\right)^* \frac{\eta}{1-\tau_w}, \tag{A17}$$

where we have used $c_t(h) = s_c^*(h)c_t$ and $c_t/w_t = (c/w)^*$ from Lemma 2. Substituting (A16) into (A15) and imposing $\dot{s}_{a,t}(h) = 0$ yield

$$(\rho - \lambda)\left[s_{a,0}(h) - 1\right] = \left(\frac{c}{a}\right)^* \left[(1+\eta)\, s_c^*(h) - 1\right] - \left(\frac{w}{a}\right)^* (1-\tau_w)(1-l^*), \tag{A18}$$

which also uses $a_t(h) = s_{a,t}(h)a_t = s_{a,0}(h)a_t$. Using (A17), we rearrange (A18) to obtain

$$s_c^*(h) = 1 - \frac{(\rho - \lambda)\left[1 - s_{a,0}(h)\right]}{\left(\frac{c}{a}\right)^* (1+\eta)}, \tag{A19}$$

where $(c/a)^* = \mu\,(c/y)^*/(\theta\beta)$. Using (A16), (A17) and $\iota_t = \gamma y_t$, we re-express (42) as

$$s_{I,t}(h) - 1 = \frac{\left(\frac{a}{w}\right)^* (r_t - \lambda)\left[s_{a,0}(h) - 1\right] - \left(\frac{c}{w}\right)^* \frac{\eta}{1-\tau_w}\left[s_c^*(h) - 1\right]}{1 + \left(\frac{a}{w}\right)^* (r_t - \lambda) - \left(\frac{c}{w}\right)^* \frac{\eta}{1-\tau_w} + \left(\frac{y}{w}\right)^* \gamma}. \tag{A20}$$

Substituting (A19) into (A20) yields the standard deviation of income share $s_{I,t}(h)$ given by

$$\sigma_{I,t} \equiv \sqrt{\int_0^1 \left[s_{I,t}(h) - 1\right]^2 dh} = \frac{\left(\frac{a}{w}\right)^* (r_t - \lambda) - \left(\frac{a}{w}\right)^* \frac{\eta}{1+\eta}\frac{\rho-\lambda}{1-\tau_w}}{1 + \left(\frac{a}{w}\right)^* (r_t - \lambda) - \left(\frac{c}{w}\right)^* \frac{\eta}{1-\tau_w} + \left(\frac{y}{w}\right)^* \gamma}\sigma_{a,0}, \tag{A21}$$

where

$$\left(\frac{c}{w}\right)^* = \left(\frac{c}{y}\right)^* \frac{l^*}{1-\theta} = \frac{(1-\tau_w)(1-\theta) + \gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)}{(1-\theta)(1+\eta) + \frac{\eta}{1-\tau_w}\left[\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)\right]}, \tag{A22}$$

$$\left(\frac{a}{w}\right)^* = \left(\frac{\theta\beta}{\mu}\right) \frac{l^*}{1-\theta} = \frac{\frac{\theta\beta}{\mu}}{(1-\theta)(1+\eta) + \frac{\eta}{1-\tau_w}\left[\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)\right]}, \tag{A23}$$

$$\left(\frac{y}{w}\right)^* = \frac{l^*}{1-\theta} = \frac{1}{(1-\theta)(1+\eta) + \frac{\eta}{1-\tau_w}\left[\gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)\right]}. \tag{A24}$$

Given (A21), we use $r_t = g_t + \rho$ from (25) to obtain (43). ■

Proof of Proposition 3. Using (A19) yields the standard deviation of $s_{c,t}(h)$ given by

$$\sigma_c \equiv \sqrt{\int_0^1 \left[s_c^*(h) - 1\right]^2 dh} = \frac{(\rho - \lambda)}{\left(\frac{c}{a}\right)^* (1+\eta)}\sigma_{a,0}, \tag{A25}$$

where $(c/a)^* = \mu\,(c/y)^*/(\theta\beta)$. Given (A25), we use $(c/y)^*$ from (24) to derive (44). ■

Proof of Proposition 7. The consumption share of a poor household is given by

$$s_c^*(p) = 1 + \frac{\frac{\mu(1-\tau_w)(1-\theta)}{\beta\theta}\frac{l^*(p)-l^*}{l^*} - \varepsilon(\rho - \lambda)}{\frac{\mu}{\beta\theta}\left[(1-\tau_w)(1-\theta) + \gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)\right]}. \tag{A26}$$

Given (A26), the employment level of a poor household is implicitly determined by

$$l^{*}(p)=1-\left[\frac{\eta l^{*}\,(c/y)^{*}\,s_{c}^{*}(p)}{(1-\tau_{w})(1-\theta)}\right]^{\omega}=1-\left\{\eta l^{*}\frac{\gamma+\frac{\theta\beta}{\mu}\,(\rho-\lambda)\,(1-\varepsilon)}{(1-\tau_{w})(1-\theta)}+\eta l^{*}(p)\right\}^{\omega}. \tag{A27}$$

The consumption share of a rich household is given by

$$s_{c}^{*}(r)=1+\frac{\frac{\mu(1-\tau_{w})(1-\theta)}{\beta\theta}\,\frac{l^{*}(r)-l^{*}}{l^{*}}+\frac{\varepsilon\delta}{1-\delta}\,(\rho-\lambda)}{\frac{\mu}{\beta\theta}\left[(1-\tau_{w})\,(1-\theta)+\gamma+\frac{\theta\beta}{\mu}\,(\rho-\lambda)\right]}. \tag{A28}$$

Given (A28), the employment level of a rich household is implicitly determined by

$$l^{*}(r)=1-\left[\frac{\eta l^{*}\,(c/y)^{*}\,s_{c}^{*}(r)}{(1-\tau_{w})(1-\theta)}\right]^{\omega}=1-\left\{\eta l^{*}\frac{\gamma+\frac{\theta\beta}{\mu}\,(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)}{(1-\tau_{w})(1-\theta)}+\eta l^{*}(r)\right\}^{\omega}, \tag{A29}$$

where the average level of employment is given by

$$l^{*}=\delta l^{*}(p)+(1-\delta)\,l^{*}(r). \tag{A30}$$

Substituting (A30) into (A27) and (A29) yields a system of two equations with two unknowns $\{l^{*}(p),l^{*}(r)\}$. Comparing (A27) and (A29), we can easily derive $l^{*}(p)>l^{*}(r)$. Substituting this result into (A30) yields $l^{*}(p)>l^{*}>l^{*}(r)$.

Now, we explore the effect of labor income tax τ_{w} on the average level of employment l^{*}. If $\omega=1$, then one can express (A30) as

$$l^{*}=\frac{1}{1+\eta\left\{1+\frac{1}{(1-\tau_{w})(1-\theta)}\left[\gamma+\frac{\theta\beta}{\mu}\,(\rho-\lambda)\right]\right\}}, \tag{A31}$$

which is decreasing in τ_{w} given $\gamma+\frac{\theta\beta}{\mu}\,(\rho-\lambda)>0$. If $\omega\neq1$, then we can rearrange (A27) and (A29) as follows:

$$[1-l^{*}(p)]^{\frac{1}{\omega}}=\eta l^{*}\left\{\frac{\gamma+\frac{\theta\beta}{\mu}\,(\rho-\lambda)\,(1-\varepsilon)}{(1-\tau_{w})(1-\theta)}\right\}+\eta l^{*}(p), \tag{A32}$$

$$[1-l^{*}(r)]^{\frac{1}{\omega}}=\eta l^{*}\left\{\frac{\gamma+\frac{\theta\beta}{\mu}\,(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)}{(1-\tau_{w})(1-\theta)}\right\}+\eta l^{*}(r). \tag{A33}$$

Substituting (A30) into (A32) and (A33) and total differentiating the resulting expressions yield

$$\begin{bmatrix} a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix} dl^{*}(p)\\ dl^{*}(r)\end{bmatrix}=\begin{bmatrix} a_{13}\\ a_{23}\end{bmatrix},$$

where

$$
\begin{aligned}
a_{11} &= -\left\{\frac{1}{\omega}\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}+\eta\left[1+\delta\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)}{(1-\tau_w)(1-\theta)}\right]\right\}, \\
a_{12} &= -\eta(1-\delta)\left[\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)}{(1-\tau_w)(1-\theta)}\right],\ \ a_{21}=-\eta\delta\left[\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)}{(1-\tau_w)(1-\theta)}\right], \\
a_{22} &= -\left\{\frac{1}{\omega}\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}+\eta\left[1+(1-\delta)\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)}{(1-\tau_w)(1-\theta)}\right]\right\}, \\
a_{13} &= \eta(1-\theta)\,l^*\left\{\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)}{\left[(1-\tau_w)(1-\theta)\right]^2}\right\}d\tau_w,\ \ a_{23}=\eta(1-\theta)\,l^*\left\{\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)}{\left[(1-\tau_w)(1-\theta)\right]^2}\right\}d\tau_w.
\end{aligned}
$$

Note the following properties: (a) $\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)>0$ (or <0); (b) $a_{11}<0$ is due $\left[1-l^*(p)\right]^{1/\omega}>0$ from (A32). We express the labor solutions for a poor household and a rich household in implicit function form, which are respectively $l^*(p)=f(\tau_w)$ and $l^*(r)=\vartheta(\tau_w)$. The effects of labor income tax τ_w on a poor household $l^*(p)$ and a rich household $l^*(r)$ are:

$$
f_{\tau_w}\equiv\frac{dl^*(p)}{d\tau_w}=-\frac{\eta(1-\theta)\,l^*}{\Phi}\left\{\eta+\frac{\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}}{\omega}\right\}\underbrace{\left\{\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)}{\left[(1-\tau_w)(1-\theta)\right]^2}\right\}}_{+/-}, \tag{A34}
$$

$$
\vartheta_{\tau_w}\equiv\frac{dl^*(r)}{d\tau_w}=-\frac{\eta(1-\theta)\,l^*}{\Phi}\left\{\eta+\frac{\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}}{\omega}\right\}\left\{\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)}{\left[(1-\tau_w)(1-\theta)\right]^2}\right\}<0, \tag{A35}
$$

where $\Phi>0$ can be expressed as follows:

$$
\begin{aligned}
\Phi \equiv{}& \left(\frac{1}{\omega}\right)^2\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}+\frac{\eta}{\omega}\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}\left[1+\delta\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)}{(1-\tau_w)(1-\theta)}\right] \\
&+\frac{\eta}{\omega}\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}\left[1+(1-\delta)\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)}{(1-\tau_w)(1-\theta)}\right]+\eta^2\left[1+\frac{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)}{(1-\tau_w)(1-\theta)}\right].
\end{aligned}
$$

Equation (A34) shows that $l^*(p)$ is decreasing (increasing) in τ_w if $\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)>(<)0$. Equation (A35) shows that $l^*(r)$ is decreasing in τ_w. As for the effect of τ_w on l^*, we take the total differentials of l^* from (A30) and substitute (A34) and (A35) into the resulting expression to obtain

$$
\frac{dl^*}{d\tau_w}=-\frac{\eta(1-\theta)\,l^*}{\Phi\left[(1-\tau_w)(1-\theta)\right]^2}\left\{\begin{array}{c}\delta\left[\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)\right]\left\{\eta+\frac{1}{\omega}\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}\right\} \\ +(1-\delta)\left[\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)\right]\left\{\eta+\frac{1}{\omega}\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}\right\}\end{array}\right\}. \tag{A36}
$$

We consider three scenarios for (A36):

(a) We suppose $\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon) > 0$. Then, l^* is decreasing in τ_w. In other words, the effect of τ_w on l^* regardless of whether $\omega < 1$ or $\omega > 1$ in this case.

(b) We suppose $\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon) < 0$ and $\omega > 1$. Then, l^* is still decreasing in τ_w because substituting $l^*(p) > l^*(r)$ into (A36) yields

$$\frac{dl^*}{d\tau_w} < -\frac{\eta(1-\theta)l^*}{\Phi\left[(1-\tau_w)(1-\theta)\right]^2}\left\{\left[\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)\right]\left\{\eta + \frac{1}{\omega}\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}\right\}\right\} < 0.$$

(c) We suppose $\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon) < 0$ and $\omega < 1$. Then, the effect of τ_w on l^* is ambiguous. That is, we substitute (A32) and (A33) into (A36) and use (A30) to find that $dl^*/d\tau_w > (<)0$ holds if the following inequality holds:

$$\frac{\left[l^* - l^*(r)\right] - \frac{[1-l^*(r)][\omega-(\omega-1)l^*(p)]\left[\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\right]}{\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)}}{l^*(1-l^*)} < (>)\frac{\gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)}{(1-\tau_w)(1-\theta)}. \tag{A37}$$

Recall that $l^*(p) > l^* > l^*(r)$ and $\delta \in (0,1)$. Given that the right-hand side of (A37) is monotonically increasing in δ, $dl^*/d\tau_w > (<)0$ becomes more likely to hold as δ increases (decreases). Specifically, we can show that the inequality $< (>)$ in (A37) must hold as $\delta > \tilde{\delta}$ ($\delta < \hat{\delta}$), where both $\tilde{\delta}$ and $\hat{\delta}$ are the threshold values. This result implies $dl^*/d\tau_w > (<)0$ for sufficiently large δ (small δ). As $\delta \in (\hat{\delta}, \tilde{\delta})$, the effect of τ_w on l^* becomes non-monotonic.

In the rest of this proof, we explore the effect of labor income tax τ_w on the consumption of poor households $s_c^*(p)$. We first multiply both sides of (A26) by l^* to yield

$$s_c^*(p)l^* = l^* \underbrace{\frac{(1-\tau_w)(1-\theta)\frac{l^*(p)}{l^*} + \gamma + \frac{\beta\theta}{\mu}(\rho-\lambda)(1-\varepsilon)}{(1-\tau_w)(1-\theta) + \gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)}}_{+\text{ due to } s_c^*(p)l^*>0}. \tag{A38}$$

Differentiating (A26) with respect to τ_w yields

$$\frac{ds_c^*(p)}{d\tau_w} = \frac{(1-\theta)\left\{-\left[l^*(p)-l^*\right] + (1-\tau_w)(1-\delta)\left[\frac{l^*(r)}{l^*}\frac{dl^*(p)}{d\tau_w} - \frac{l^*(p)}{l^*}\frac{dl^*(r)}{d\tau_w}\right] + \left[s_c^*(p)-1\right]l^*\right\}}{l^*\left[(1-\tau_w)(1-\theta) + \gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)\right]}, \tag{A39}$$

where we have used (A30). Substituting (A34) and (A35) into (A39) and then using (A38) and the value of Φ, we perform a few steps of mathematical manipulation to derive

$$\frac{ds_c^*(p)}{d\tau_w} = \frac{\Omega + \Theta}{\Phi(1-\tau_w)l^*\left[(1-\tau_w)(1-\theta) + \gamma + \frac{\theta\beta}{\mu}(\rho-\lambda)\right]}, \tag{A40}$$

where

$$\Omega \equiv -(1-\tau_w)(1-\theta)\left(\frac{1}{\omega}\right)^2\left[l^*(p) - s_c^*(p)l^*\right]\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}, \tag{A41}$$

$$\Theta \equiv \frac{-\left[1-l^*(p)\right]\left[\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\left(1+\frac{\varepsilon\delta}{1-\delta}\right)\right]+\left[1-l^*(r)\right]\left[\gamma+\frac{\beta\theta}{\mu}(\rho-\lambda)(1-\varepsilon)\right]}{\omega\left[(1-\tau_w)(1-\theta)+\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\right] / \left\{(1-\delta)(1-\tau_w)(1-\theta)\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}\right\}}. \tag{A42}$$

Based on $l^*(p) > l^*$ and $s_c^*(p) < 1$, we derive $l^*(p) - s_c^*(p)l^* > 0$ implying that $\Omega < 0$ holds. Since we don't know whether the value of Θ are positive or negative, we consider three scenarios for (A40):

(a) We suppose $\gamma + \frac{\beta\theta}{\mu}(\rho-\lambda)(1-\varepsilon) < 0$. Then, we obtain $\Theta < 0$ implying that $s_c^*(p)$ is decreasing in τ_w.

(b) We suppose $\gamma + \frac{\beta\theta}{\mu}(\rho-\lambda)(1-\varepsilon) > 0$ and $\omega < 1$. Then, $s_c^*(p)$ is still decreasing in τ_w because we can easily derive that $\Theta < 0$ still holds in this case by using (A32) and (A33).

(c) We suppose $\gamma + \frac{\beta\theta}{\mu}(\rho-\lambda)(1-\varepsilon) > 0$ and $\omega > 1$. In this case, we combine (A41) and (A42) and substitute (A32) and (A33) into the resulting expression to obtain

$$\Omega+\Theta = \frac{\left(1-\frac{1}{\omega}\right)\left\{l^*(p)\left[\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\right]-l^*\left[\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)(1-\varepsilon)\right]\right\}-\frac{\theta\beta\varepsilon}{\mu}(\rho-\lambda)}{\omega\left[(1-\tau_w)(1-\theta)+\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\right] / \left\{(1-\tau_w)(1-\theta)\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}\right\}}, \tag{A43}$$

where we have used (A30) and (A38). From (A32) and (A33), we derive

$$\left[1-l^*(p)\right]^{\frac{1}{\omega}} < \left[1-l^*(r)\right]^{\frac{1}{\omega}} \Longrightarrow l^*(p) - l^* < \frac{l^*\frac{\theta\beta\varepsilon}{\mu}(\rho-\lambda)}{(1-\tau_w)(1-\theta)}. \tag{A44}$$

By removing $1/\omega$ from the numerator of (A43) and substituting (A44) into the resulting expression yield

$$\Omega+\Theta < \frac{l^*\underbrace{\left[(1-\tau_w)(1-\theta)+\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\right]}_{=(c/y)^*}-(1-\tau_w)(1-\theta)}{\omega\left[(1-\tau_w)(1-\theta)+\gamma+\frac{\theta\beta}{\mu}(\rho-\lambda)\right] / \left\{\frac{\theta\beta\varepsilon}{\mu}(\rho-\lambda)\left[1-l^*(p)\right]^{\frac{1-\omega}{\omega}}\left[1-l^*(r)\right]^{\frac{1-\omega}{\omega}}\right\}}. \tag{A45}$$

Additionally, we use (A29) and $s_c^*(r) > 1$ to obtain

$$l^*(r) > 0 \Longrightarrow \frac{\eta l^*(c/y)^* s_c^*(r)}{(1-\tau_w)(1-\theta)} < 1 \Longrightarrow l^*(c/y)^* < (1-\tau_w)(1-\theta) \text{ if } \eta \geq 1. \tag{A46}$$

Given (A46), we can easily derive that if $\eta \geq 1$, then $\Omega + \Theta < 0$ holds implying that $s_c^*(p)$ is decreasing in τ_w. Moreover, (A43) shows that the value of $\Omega + \Theta$ is increasing in $l^*(h)$ for a given l^* and $h \in \{p, r\}$.[21] From (A32) and (A33), we derive that $l^*(h)$ is decreasing in η. As

[21] It is useful to note that Φ from (A40) is also a function of $l^*(h)$.

$\eta = 0$, we can obtain the upper value $l^*(h) = 1$ from (5) and substitute it into (A26) to yield

$$s_c^*(p) = 1 - \frac{\varepsilon(\rho - \lambda)}{\frac{\mu}{\beta\theta}\left[(1 - \tau_w)(1 - \theta) + \gamma + \frac{\theta\beta}{\mu}(\rho - \lambda)\right]}.$$

Equation (A47) shows that $s_c^*(p)$ is still decreasing in τ_w. As a result, from the results of three scenarios, it can be seen that the overall effect of τ_w on $s_c^*(p)$ would be negative. ■