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Innovation-led growth in a time of debt[☆]

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ABSTRACT

We study the effects of large reductions in government budget deficits (labeled "fiscal consolidations") on firms' entry, innovative investments, productivity and per capita output growth in a model of endogenous technological change. Due to the absence of lump-sum taxes, temporary budget deficits set government debt-output ratios on unsustainable paths. An equilibrium then requires the specification of a date at which the debt-output ratio is stabilized at a constant finite value. We discipline parameters using post-war observations for the U.S. economy. We find that fiscal consolidations produce persistent growth slowdowns, permanently lowering the path of per capita output relative to a benchmark economy in which the fiscal consolidation is achieved with lump-sum taxes. These output losses are sizable. In this sense, government debt is a burden on the economy. Tax-based consolidations produce output losses that are twice as large as those from spending-based consolidations.

1. Introduction

Recent years have seen a resurgence of interest in the question of how large reductions in government budget deficits (labeled "fiscal consolidations") affect aggregate economic outcomes. In many OECD countries, population aging, sluggish output growth, and fiscal stimulus packages implemented in the aftermath of the financial crisis of 2007-09 have set government debts as a percentage of gross domestic product (GDP) on explosive paths, defined as large and rapidly rising debt-to-GDP ratios. Fiscal consolidations are by no means a new phenomenon, rather they have been a recurrent pattern in advanced OECD economies at least since the early 1980s (see Alesina et al., 2019, and references therein).

In this paper, we ask two questions that are central to our understanding of fiscal consolidations:

(i) What types of fiscal consolidations are more harmful in terms of medium- and long-term losses in productivity and per capita output? Tax-based or spending-based?
(ii) How the announcement of a fiscal consolidation at a future date affects the economy today?

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To answer these questions, we develop a general equilibrium model of innovation-led growth with endogenous technological change and government debt. We contribute to the literature on the effects of fiscal consolidations in two ways. First, we study a new set of transmission mechanisms. The literature on this topic invariably uses models in the Neoclassical and New Keynesian tradition in which *technology* is viewed as exogenous and thus invariant to government policy. This literature focuses on mechanisms related to aggregate demand, labor supply and physical capital accumulation, abstracting from innovative investments, market structure, and aggregate productivity – key determinants of the medium- and long-term performance of advanced economies.

Second, but equally important, we let the government debt take center stage. The key policy trade-off is a straightforward implication of the intertemporal budget constraint of the government. As government debt cannot grow forever as a share of income, budget deficits will inevitably be accompanied by a fiscal consolidation at some future date. In an economy *without* lump-sum taxes, such debt policy creates intertemporal distortions of the private sector behavior. Notably, the longer a government delays the reduction of budget deficits (i.e., restore debt sustainability), the larger are the implied distortions when it does. In this sense, government debt is a burden on the economy.

In Sections 2 through 5, we present our model and characterize its main properties. The model is a one sector deterministic growth model in the Schumpeterian tradition that combines product with quality-improving innovation (Peretto, 1998; Dinopoulos and Thompson, 1998; Segerstrom, 1998; Howitt, 1999).[1] Firms' entry and quality-improving investments from incumbents are forward-looking decisions, whose rates of return are determined in equilibrium, jointly with the mass of firms and firm size. The new feature we introduce is government debt, a state variable with a highly *non-linear* dynamics.

In the absence of lump-sum taxes, the dynamics of the government debt-output ratio is inherently unstable. What drives this instability is the property that in infinite-horizon economies with perfect asset markets, the real interest rate must exceeds the growth rate of output along a balanced growth path. Thus, given an initial situation of budget deficit, the debt-output ratio runs an explosive path that is incompatible with a decentralized equilibrium. An equilibrium then requires the specification of the date at which the debt-output ratio is stabilized at some finite value, and the fiscal instruments used to achieve that target. Of course, there are many different time paths of government purchases and tax rates that satisfy the intertemporal government budget constraint. Here, we do not pursue a normative analysis, rather we focus on simple balanced-budget rules.

We consider a simple tax code whereby a government raises revenues by levying a flat-rate tax on labor income.[2] We focus on labor taxes for two main reasons. First, it is well known that labor taxation represents the major source of total tax revenues in OECD countries.[3] Second, historically, fiscal consolidations have relied heavily on increases in labor income taxes. This is perhaps not surprising since labor income is by far the largest tax base available to governments and to a large extent immune to tax avoidance.

In Section 6, we parametrize our model to match salient features of the post-war U.S. economy. We stress that our approach can be readily applied to other countries as well and that we view the application to the United States only as a starting point. The model is consistent with three key observations. First, the personal income tax rates adopted by different countries are generally uncorrelated with their average growth rates (see Easterly and Rebelo, 1993; Stokey and Rebelo, 1995; Mendoza et al., 1997; Jaimovich and Rebelo, 2017). Second, market hours worked and number of firms per capita exhibit no long-run trend (see Bick et al., 2018; Cociuba et al., 2018; Laincz and Peretto, 2006). Third, measures of R&D intensity are correlated with TFP growth (see Madsen, 2008; Ang and Madsen, 2011).

In Section 7, we consider quantitative experiments grouped as tax- and spending-based fiscal consolidations. Specifically, we simulate equilibrium paths of quantities and prices of our calibrated economy under consolidation plans. A "plan" specifies the time path of fiscal variables (i.e., government purchases and tax rates) and the date at which the stabilization of the debt will take place. To gauge the effects of fiscal consolidations, we compare equilibrium paths of two economies. The first – benchmark economy – is on the BGP featuring a balanced-budget rule. The second is on the same BGP but an unexpected increase in government spending generates a budget deficit. At that time, a consolidation plan is announced. The economy runs budget deficits and accumulates debt over a number of years, the government then boosts tax rates or cuts spending to hold the debt-output ratio at its new level forever. We further assume that the private sector has perfect foresights consistently with the observation that fiscal consolidations are usually announced long before they are carried out and implemented over several years (see Alesina et al., 2015; 2018; 2019).

We assume that the two economies have no outstanding debt. While this assumption is by no means necessary to our analysis, it neatly isolates the cumulative distortionary effect of running budget deficits from that of the initial stock of government debt. To be sure, the higher the initial debt, the larger is the increase in tax rates or cut in spending needed to stabilize the debt. In this respect, our results are a conservative estimate of the burden of government debt in actual economies.

[1] See Mansfield (1968), Scherer (1986), Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008), Broda and Weinstein (2010), and Ang and Madsen (2011) for evidence supporting the main ingredients of the theory and its empirical predictions.

[2] In Ferraro and Peretto (2018), we further extend the model with a richer tax code, including a personal income tax, a corporate tax, and a consumption tax.

[3] As of year 2015, taxes on net income (gross income minus allowable tax reliefs), payroll taxes, and social security contributions are on average roughly 50% of total tax revenues in OECD countries. Taxes on corporate profits are 9% of total tax revenues. These figures have been remarkably stable during the period 1965–2015. See OECD (2017) at https://www.oecd-ilibrary.org/content/publication/9789264283183-en.

In the tax-based experiments, the consolidation is carried out with taxes only. At the time of the labor tax rate hike, the labor input falls permanently, leading to a sluggish, permanent reduction in the mass of firms. Output growth decelerates temporarily, permanently lowering the equilibrium path of aggregate output. Market size effects play a key role in the propagation of tax rate hikes to the economy.[4] We find that output losses are sizable. For instance, stabilizing government debt after 10 years of approximately 5% deficit-GDP ratios entails a nearly 2% permanent loss in per capita output, *relative to a benchmark economy in which the fiscal consolidation is achieved with lump-sum taxes*. Output losses raise from 2% to 3% if the initial budget deficit is 10%, instead of 5%, of GDP. To give a metric of how big these output losses are, in the United States, the Great Recession of 2007 has generated an output loss of 10%.[5]

In the case of spending-based consolidations, output losses are half as large as those from tax-based consolidations, insofar as government spending cuts are unanticipated. Hence, consistently with the available empirical evidence, government spending cuts are less harmful than tax rate hikes (see Alesina et al., 2002; Alesina and Ardagna, 2010; Barro and Redlick, 2011; Ramey, 2011; Alesina et al., 2015; 2017; 2018; 2019). If instead the fiscal consolidation is announced beforehand, reductions in government spending lead to sizable output losses, that exceed those generated by tax-based consolidations. For instance, stabilizing the government debt-GDP ratio after 10 years of 5% deficit-GDP ratios produces a 6% permanent loss in per capita output. Output losses become as high as 13% in the case of an initial 10% deficit-GDP ratio.

1.1. Relation to the literature

There is a large and growing empirical literature on the topic. Early work was motivated by the fiscal consolidations occurred in OECD countries since the early 1980s (Giavazzi and Pagano, 1990; Alesina and Perotti, 1995; Perotti, 1996; Alesina and Ardagna, 1998; Alesina et al., 2002). This topic has received renewed attention in the aftermath of the financial crisis of 2007-09 when large and persistent contractions in aggregate economic activity, and fiscal stimulus packages, have set government debt-GDP ratios on explosive paths (Alesina and Ardagna, 2010; 2013; Alesina et al., 2015; Fatás and Summers, 2019).

A “consensus view” has emerged: the composition of the fiscal consolidation matters. Spending-based fiscal consolidations are associated on average with mild and short-lived recessions or no recession at all. Instead, tax-based adjustments have been followed by prolonged reductions in economic activity (see Alesina et al., 2019, for an extensive empirical analysis). Recently, Alesina et al. (2017) show that a standard New Keynesian model matches the consensus view when fiscal shocks are persistent.[6]

What distinguishes our work is twofold. First, we study a new set of transmission mechanisms in the Schumpeterian tradition. While Schumpeterian growth theory has been fruitfully used to address a rich set of questions (see Aghion et al., 2014, for a survey article), the implications of fiscal consolidations in this class of models are largely unexplored. Importantly, our results are broadly consistent with the consensus view as output losses from spending-based consolidations are half as large as those based on tax rate hikes.

Second, we simulate equilibrium paths of our model economy taking in account the nonlinear dynamics of the debt-output ratio. While this might seem a subtle technical point, it bears important implications for the effects of fiscal consolidations. Intuitively, any fiscal consolidation plan must be consistent with the intertemporal budget constraint of the government. Hence, assessing the impact of, say, a tax rate cut cannot be done independently of other fiscal instruments. Note that these issues are routinely bypassed in the literature by invoking lump-sum taxes. Notable exceptions are Dotsey (1994), Davig (2004), Bi et al. (2013), Hansen and İmrohoroğlu (2016). They impose rules according to which tax rates increase when the debt-output ratio reaches a debit limit (akin to reflecting barriers), here we discipline our exercises by limiting the government's ability to borrow over a period of finite length. In our case, the length of the borrowing period is a policy variable. We view these two approaches as highly complementary.

By emphasizing endogenous technical change, our work relates to the literature that uses endogenous growth models to study the effects of fiscal policy (Atkeson and Burstein, 2019; Cozzi and Impullitti, 2010; Jaimovich and Rebelo, 2017; Ortigueira, 1998; Peretto, 2003, 2007; Stokey and Rebelo, 1995; Ferraro, Ghazi, Peretto; Ferraro, Ghazi, Peretto). What distinguishes this paper from prior work, including our own, is the emphasis on government debt dynamics. Notably, we articulate the view that, in the absence of lump-sum taxes, temporary budget deficits impose restrictions on the time path of future fiscal variables for the government intertemporal budget constraint to hold. Understanding how the private sector responds to a policy change requires the specification of a multi-year fiscal plan. In this sense, the far-reaching lesson from our analysis is that government debt matters for the evaluation of government policies – tax policy, spending, and R&D subsidies alike. Indeed, we find that the design of the fiscal plan is a key determinant of its overall impact on the economy, both qualitatively and quantitatively.

[4] See Acemoglu and Linn (2004) and Cerda (2007) for evidence on the link between market size and innovation in the U.S. pharmaceutical industry.

[5] Real GDP in billions of chained 2012 dollars was 15,626.030 in 2007 and 18,050.694 in 2017. Assuming a 2.5% annual growth rate, and absent the Great Recession of 2007, the counterfactual real GDP in 2017 would have been $15,626.030 \times (1 + 2.5\%)^{10}$. The percent real GDP loss can then be calculated as 10%. Data for the annual real GDP in the United States are from the FRED database and available at https://fred.stlouisfed.org/series/GDPC1.

[6] See Leeper et al. (2010), Bi and Traum (2014), and Traum and Yang (2015) for prior work in the New Keynesian tradition on the implications of government debt dynamics.

2. The model economy

We consider an economy without physical capital.[7] In the environment here, the relevant notion of capital is the *stock of knowledge* – a non-rival good that is partially excludable and privately produced within an entity called "firm." At the aggregate level, knowledge is accumulated over time through the creation of new products by entrant firms (horizontal or expanding-variety innovation) and improvements in the quality of existing products by incumbent firms (vertical or quality-improving innovation).

In our model economy, government policy in the form of purchases of market goods and income taxes impinge on equilibrium allocations and prices by altering households' work incentives and firms' incentives to innovate. To keep the analysis as transparent as possible, we introduce into the model a simple tax system whereby the government raises revenues only through a flat-rate tax on labor income. Since we restrict the government's access to lump-sum taxation, the stock of public debt becomes a key determinant of the economy's equilibrium behavior.

2.1. Households

The economy is inhabited by a stand-in household with a continuum of infinitely-lived individuals (population, hereafter). Initial population is normalized to one so that at time t population size is $e^{\lambda t}$, where λ is the exogenous and constant rate of population growth. Each household member is endowed with one unit of time, so that total labor endowment equals population size.

Preferences and budget set Household's preferences are described by

$$U(t) \equiv \int_t^{\infty} e^{-(\rho-\lambda)(s-t)}[\log c(s) + \eta \log(1 - l(s))]ds, \tag{1}$$

where c and l are *per capita* consumption and market hours, respectively, $\rho > \lambda \geq 0$ is the time discount rate and $\eta > 0$ governs the disutility of work. The specification of the log-log per-period utility function in (1) is consistent with a balanced growth path (BGP) where hours per capita are constant and total market hours grow at the rate of population growth.

The household faces an intertemporal budget constraint,

$$\dot{B} + \dot{s}V + C \leq r_B B + sD + (1 - \tau_L)wL + p_F F - T. \tag{2}$$

The constraint in (2) says that household's expenditures must be less than or equal to its after-tax income. Expenditures of the household are total consumption, C, purchases of government bonds, $\dot{B}$, and shares of an "hedge fund," $\dot{s}V$, where s is the number of shares and V is the price per share. The function of the hedge fund is to aggregate equity into an economy-wide portfolio, so that $s \equiv \int_0^N s_i di$ and the (ex-dividend) price per share is $V \equiv (1/N)\int_0^N V_i di$, where s_i and V_i are firm i's number of shares and price per share, respectively, and N is the mass of firms issuing equity.[8] The rate of return to the market portfolio, r_A, is the *average* of the rates of return to firms' equity:

$$r_A \equiv \frac{1}{N}\int_0^N r_{A_i} di = \frac{1}{N}\int_0^N \left(\frac{D_i}{V_i} + \frac{\dot{V}_i}{V_i}\right)di, \tag{3}$$

where D_i is firm i's distributed dividends and $\dot{V}_i \equiv dV_i/dt$ is the change in firm i's stock price V_i over a time interval of infinitesimal length.

Household's income consists of: (i) returns on holding interest-bearing government bonds, $r_B B$; (ii) dividend income, sD; (iii) after-tax labor income, $(1 - \tau_L)wL$; (iv) income generated by the ownership of a fixed factor of production (e.g., land), $p_F F$; (v) lump-sum government transfers, T. (Again, we note here that the gist of our approach is to study the implications of public debt when the government cannot use lump-sum taxes, i.e. $T = 0$ from some specified date onwards.)

Household's problem The household takes the tax rate, τ_L, lump-sum transfers/taxes, T, prices (w, p_F, r_B, r_A), dividend distributions per share, D, and the fixed factor, F, as given. Given the state variables B and s, it chooses sequences of per capita consumption, c, and labor supply, l, to maximize lifetime utility (1) subject to the budged constraint (2). (Standard transversality conditions on equity and bond holdings apply.)

The household's optimal plan yields the standard intratemporal condition describing the static trade-off between consumption and leisure,

$$\frac{\eta c}{1 - l} = (1 - \tau_L)w. \tag{4}$$

In addition, the household's intertemporal saving decisions are governed by an Euler equation describing the dynamic trade-off between current and future consumption,

$$\frac{\dot{c}}{c} = r - \rho, \tag{5}$$

[7] More precisely, there is no capital in the neoclassical sense of a homogenous, durable, intermediate good accumulated through foregone consumption. Instead, in the setting here, there are differentiated, non-durable, intermediate goods produced through foregone consumption. One can think of these goods as capital, albeit with a 100% instantaneous depreciation rate.

[8] In this specification, the hedge fund takes firms' share prices as given and charges no intermediation fees. Alternatively, one could think of a competitive environment where the hedge fund charges fees, but it breaks even in a zero-profit equilibrium.

where $r \equiv r_B = r_A$ is the reservation rate of return to saving. For future reference, we note that an equilibrium where households hold both government bonds and stocks requires rates of return equalization, i.e., $r_A = r_B$ at all times.

2.2. Production and innovation

The production side of the economy consists of a two-tier vertical structure. A final good sector and an upstream intermediate good sector. The final good sector is competitive. A stand-in firm combines labor and intermediate goods with a fixed factor to produce a homogeneous final good. The intermediate good sector is monopolistically competitive, featuring an endogenous market structure: mass of firms and firm size are determined in free-entry equilibrium. Firms in this sector use final goods to produce intermediate goods that are vertically differentiated by "quality." Intermediate good producers are also the innovators; they allocate resources to improve the quality of their own products. Returns to quality-improving investments come in the form of monopoly rents in the imperfectly competitive product market. Investment to improve product quality is the source of long-run growth in income per capita.

2.2.1. Final good producers

A stand-in firm (or final producer) produces a final good Y, that has four uses: (i) private consumption; (ii) input into the production of intermediate goods; (iii) investment in quality-improvements of existing intermediate goods; (iv) investment in the creation of new intermediate goods. The final good is the numeraire, so that $p_Y \equiv 1$.

Final good technology The final producer has access to a production technology that converts X_i units of each existing intermediate input $i \in [0, N]$, L_i units of labor input per intermediate input, and a fixed factor, F, into Y units of final good (or gross output):

$$Y = \int_0^N X_i^\theta \left[Z_i^\alpha Z^{1-\alpha} \left(L_i^\gamma F^{1-\gamma} \right) \right]^{1-\theta} di, \tag{6}$$

where $0 < \theta < 1$, $0 < \alpha < 1$, and $0 < \gamma < 1$ are technological parameters.

First, the production technology (6) features full dilution of labor across intermediate goods, reflecting the property that both labor and intermediate goods are rival inputs. The fixed factor, instead, is non-rival across intermediate goods and labor, thus creating congestion effects whose extent is governed by the parameter γ.

Second, product quality, as indexed by Z_i, is an attribute of the intermediate good i. Notably, higher-quality intermediate goods perform similar functions to those performed by lower-quality goods, but they increase the overall efficiency of the production process, leading to a reduction in unit costs of production. This property is the defining feature of vertical product innovation. Importantly, the term $Z^{1-\alpha}$ captures *knowledge spillovers* whose extent is governed by the parameter α. The contribution of the intermediate good i to output, and so the productivity of the associated labor input L_i, depends on good i's quality as well as an index of the overall quality of existing intermediate goods,

$$Z \equiv \int_0^N \sigma_j Z_j dj, \tag{7}$$

where $0 \leq \sigma_j \leq 1$ weighs individual product's quality (see Peretto and Smulders, 2002).

Aggregating quality across different products into an index of overall quality requires a notion of "distance" in the technological space or, put differently, a measure of the extent to which the knowledge embodied in product j overlaps with that embodied in product $i \neq j$. Here, we adopt the specification $\sigma_j \equiv 1/N$ for all $j \in [0, N]$, thus capturing a notion of *average* technological distance between differentiated goods. When a firm improves the quality of its own differentiated good, it generates *not-excludable* knowledge which spills over into the public domain. Yet, the extent to which this firm-specific knowledge can be used by another firms depends on how far in the technological space firms' products are. The term $1/N$ captures then the idea that as the mass of varieties increases, the average distance between products increases as well, leading to lesser knowledge spillovers across firms.

Final producer's problem The final producer takes quality as given and sets the value marginal product of each intermediate good i equal to its price, p_i, and the value marginal product of labor equal to the wage rate, w. (See Appendix A.1 for more details.)

The demand curve for the intermediate good i is

$$X_i = \left(\frac{\theta}{p_i} \right)^{\frac{1}{1-\theta}} Z_i^\alpha Z^{1-\alpha} L_i^\gamma F^{1-\gamma}. \tag{8}$$

First, it is evident from (8) that product quality shifts the demand for intermediate goods to the right. The higher the intermediate good i's quality, Z_i, the higher the quantity, X_i, demanded by the final good producer. Further, higher overall quality, Z, makes final good producers more efficient, thereby rising the demand for intermediate goods across-the-board. Note that, everything else equal, these two effects reinforce one other and contribute to raise firms' profits in the intermediate good sector, thus enhancing their incentives to engage in quality-improving investment. Second, Eq. (8) also points to the important role of the labor input, i.e. the labor demand shifts intermediate goods' demand to the right.

The downward-sloping labor demand curve is

$$L = \gamma(1-\theta)\frac{Y}{w}, \tag{9}$$

where $L \equiv \int_0^N L_i di$. Given the constant-returns-to-scale (CRS) technology (6) and the final producer's price-taking behavior, compensation of the factors of production equals total value of production. Notably, $\int_0^N p_i X_i di = \theta Y$ goes to intermediate producers, $wL = \int_0^N wL_i di = \gamma(1-\theta)Y$ goes to labor, and $p_F F = (1-\gamma)(1-\theta)Y$ goes to the fixed factor.

2.2.2. Intermediate good producers (innovators)

The intermediate good sector is where innovation takes place. We begin by describing the operations of incumbent firms, then turn to firms' entry.

Incumbents firms An incumbent firm operates a technology that requires one unit of final good per unit of intermediate good produced and the payment of a fixed operating cost, ϕZ, in units of the final good.[9] Firm-specific knowledge accumulates according to the technology

$$\dot{Z}_i = R_i, \tag{10}$$

where $R_i \geq 0$ is firm's quality-improving investment (e.g., R&D), in units of final output. One can think of the technology of knowledge accumulation (10) as the analog of the well-known technology of physical capital accumulation in neoclassical growth theory where the depreciation rate is zero, which formalizes the idea that knowledge is irreversible: what a society has learned cannot be unlearned.

The value of the firm is the present discounted value of dividends (or, equivalently net cash flows),

$$V_i(t) = \int_t^\infty e^{-\bar{r}(t,s)(s-t)} D_i(s) ds, \tag{11}$$

where $\bar{r}(t,s) \equiv 1/(s-t)\int_t^s r(v)dv$ is the average interest rate between t and s, and $D_i \equiv \Pi_i - \phi Z - R_i$ is the dividend distributed to stockholders, where $\Pi_i \equiv (p_i - 1)X_i$ is the firm's gross cash flow (revenues minus variable production costs).

The firm takes average quality, Z, and the interest rate, r, as given and chooses the path of its product's price, p_i, and investment, R_i, to maximize firm's value (11) subject to the demand curve (8) and the technology (10). Product quality Z_i is a state variable of the firm's problem. First, as the firm faces an isoelastic demand curve and the marginal cost of production equals one, the firm sets $p_i = 1/\theta$. Second, the firm undertakes quality-improving innovation up to the point where the shadow value of investment, q_i, is equal to its cost,

$$q_i = 1 \Leftrightarrow R_i > 0. \tag{12}$$

The interpretation of (12) comes from neoclassical investment theory (Jorgenson, 1963; Hall and Jorgenson, 1969; Lucas and Prescott, 1971; Abel, 1977; Hayashi, 1982). Since the firm can freely change the quality of its own product (or, alternatively, the stock of knowledge), it will invest or disinvest until Tobin's q is equal to unity.[10]

Since innovation is implemented *in-house*, its benefits are determined by the marginal profit $\partial\Pi_i/\partial Z_i$ it generates. The opportunity cost of an additional unit of investment is instead the market interest rate, r. Thus, a firm's investment plan is consistent with value maximization if and only if

$$r = \frac{\partial \Pi_i}{\partial Z_i} = \alpha\frac{\Pi_i}{Z_i} \Leftrightarrow R_i > 0. \tag{13}$$

Note that the problem of the incumbent firm features a "dark corner," in the sense that it embeds the possibility that quality-improving innovation shuts down, i.e. $R_i = 0$. An equilibrium with a positive rate of return to innovation is consistent with the individual firm's incentives to innovate if and only if

$$D_i \geq 0 \Leftrightarrow \mu X_i \geq \phi Z, \tag{14}$$

where $\mu \equiv 1/\theta - 1$ is the mark-up over marginal cost charged by the firm. Condition (14) says that $R_i > 0$ insofar as the gross profit rate μX_i exceeds fixed operating costs ϕZ.

Entrant firms Setting up a firm requires the payment of a sunk cost, βX, in units of final output, where X is the average quantity of intermediate goods. Firms enter at the average level of quality, Z. (Note that this simplifying assumption preserves symmetry of the equilibrium.) The economy starts out with a given range of intermediate goods, each supplied by one firm, so that the mass of products equals the mass of firms. Because of the sunk entry cost, new firms do not find profitable to supply an existing good in Bertrand competition with the incumbent monopolist, rather they introduce a new intermediate good that expands product variety. Firms finance entry by issuing equity. A free-entry equilibrium is consistent with the individual firm's incentives to enter the intermediate good sector if and only if

$$V_i \leq \beta X \Leftrightarrow \dot{N} \geq 0, \tag{15}$$

which holds with equality when entry is positive, i.e., $\dot{N} > 0$.

[9] If $\phi = 0$, expanding-variety innovation becomes a source of long-run growth as in Romer (1990). In this case, the model displays the "scale effect," i.e., the growth rate of output along the BGP depends on the level of population.

[10] In technical terms, q is the co-state variable of the firm's optimal control problem. See Appendix A.2 for details.

2.3. Government

The government purchases final goods and finances its spending by collecting taxes and issuing interest-bearing bonds. The stock of public debt, B, evolves over time according to the intertemporal budget constraint,

$$\dot{B} = r_B B + \underbrace{G - \tau_L wL - T}_{\text{primary balance}}, \tag{16}$$

where G is net-of-transfers government spending (or public consumption), $\tau_L wL$ are the revenues from labor income taxation, and T are lump-sum transfers/taxes. For $T > 0$, the government takes resources away from the household, whereas for $T < 0$ it provides resources that add to the household's earned income. In either case, though, note that T is not distortionary in the sense that it does not enter household's conditions for an optimal plan (4)-(5). If $T = 0$, the government cannot use lump-sum taxes or transfers to balance the budget. In this latter case, we will see that the path of public debt, and expectations about future fiscal policy, become key determinants of equilibrium allocations and prices.

We model government spending as an exogenous and constant fraction $0 < g < 1$ of aggregate output Y, so that $G = gY$. In terms of gross output, the government's budget constraint (16) implies that the public debt-to-output ratio evolves over time according to the ordinary differential equation (ODE),

$$\dot{b} = (r_B - g_Y)b + g - \tau_L\left(\frac{wL}{Y}\right) - \chi, \tag{17}$$

where $g_y \equiv \dot{Y}/Y$ is the percentage growth rate of gross output and $\chi \equiv T/Y$ denotes lump-sum taxes-to-output ratio. First, $r_B - g_Y$ is the key equilibrium object that informs stability of the debt-to-output ratio (17). Notably, whether the dynamics of the debt-to-output ratio are stable $r_B < g_Y$ or unstable $r_B > g_Y$ is out of government's control, but determined exclusively by equilibrium forces. Second, the government can affect the path of the debt-to-output ratio by changing fiscal policy, defined by the triplet g, τ_L, and χ.

Public debt stabilization, defined as $\dot{b} = 0$ from the currant date onward, implies the balanced-budget rule,

$$(r_B - g_Y)b + g = \tau_L\left(\frac{wL}{Y}\right) + \chi. \tag{18}$$

For $r_B > g_Y$, the higher the debt at the time of stabilization, the larger the adjustment in government policy required. For future reference, we note that $r_B > g_Y$ will hold along the BGP as required by the no-Ponzi-game condition for the household.

3. Public debt in general equilibrium

We now turn to the general equilibrium of the model. Since the equilibrium is symmetric, henceforth, we will drop the i subscript so that, for example, $X \equiv X_i$ denotes both firm-level and average intermediate good production.[11] We provide a full description of the equilibrium in Appendix A. Here, we present some of the equilibrium conditions we use to study the role of public debt and interpret the results.

3.1. Determinants of labor

We now turn to discuss the *intratemporal* trade-offs driving the determination of labor. In setting the supply of labor, the household equates the marginal rate of substitution (MRS) between consumption and leisure to the effective price of leisure. In the economy here, the consumption good is the numeraire such that the wage represents the relative price of leisure to consumption. A flat-rate labor tax introduces a wedge between the MRS and the wage, thus distorting labor supply decisions.

By rearranging the household's intratemporal condition (4), one obtains an upward-sloping labor supply curve,

$$l = 1 - \frac{\eta c}{(1 - \tau_L)w}. \tag{19}$$

Holding per capita consumption, c, and the wage rate, w, constant, an increase in the labor tax rate, τ_L, reduces labor supply. This is the standard substitution effect. Income effects induced by the flat-rate tax are captured by the presence of per capita consumption, c, on the right-hand side of (19). Everything else constant, higher c leads to lower labor supply. (Given our preferences' specification, the labor supply curve is vertical along the BGP as income and substitution effects offset one other.)

[11] Essentially, two conditions ensure symmetry of equilibrium: (i) firm-specific rate of return to quality innovation is decreasing in its own quality; (ii) entrant firms enter at the average level of quality. The first implies that if one holds constant the mass of firms and starts the model from an asymmetric distribution of firm sizes, then the model converges to a symmetric distribution. The second requirement simply ensures that entrants do not perturb such symmetric distribution. See Peretto (1998) and Peretto (1999) for more discussion.

To provide insight into the determination of equilibrium labor, it is useful to impose market clearing in the labor market, so that $l(t)e^{\lambda t} = L(t)$, for all $t \geq 0$. Household's labor supply (19) and final good producer's labor demand (9) yield per capita labor as a function of the flat-rate labor tax and the aggregate consumption-to-output ratio, $\tilde{c}$:

$$l = \frac{\gamma(1-\theta)(1-\tau_L)}{\gamma(1-\theta)(1-\tau_L)+\eta\tilde{c}}. \tag{20}$$

First, the labor tax rate, τ_L, has a *direct* effect on labor through shifts in the labor supply curve (19). Holding $\tilde{c}$ constant, a higher tax rate leads to lower per capita labor. Second, changes in the labor tax rate may have an *indirect* equilibrium effect through changes in the consumption-to-output ratio. If so, the magnitude of this latter effect depends on the elasticity of per capital labor with respect to the consumption-to-output ratio,

$$\omega(\tilde{c}) \equiv -\frac{d\ln l}{d\ln \tilde{c}} = \frac{\eta\tilde{c}}{\gamma(1-\theta)(1-\tau_L)+\eta\tilde{c}}. \tag{21}$$

In principle, though, the equilibrium response of the consumption-to-output ratio to a change in the labor tax depends on the production side of the economy, government's budget constraint, and the private sector's expectations about future fiscal policy.

3.2. Determinants of product and quality innovation

We now turn to the *intertemporal* trade-offs driving product and quality innovation. In our environment, key to understanding innovation is the equilibrium in the asset market, whereby rates of return are equalized across all available assets (akin to a no-arbitrage condition in portfolio theory), so that

$$r_A = r_B = r, \text{ for all } t \geq 0. \tag{22}$$

Further, product creation by entrants and quality-improving investment by incumbents must yield the same rate of return r for the household to willingly hold stocks issued by both entrant and incumbent firms.

Quality-improving innovation The problem faced by the intermediate good producer is inherently dynamic. In deciding how much to investment, a typical firm trades off the cost of diverting resources from current operating profit with the benefit of rising gross cash flows in the future.

In symmetric equilibrium, the intertemporal optimality condition (13) yields the rate of return to quality innovation,

$$r = \alpha \times \underbrace{\left(\frac{1}{\theta}-1\right)}_{\text{markup}} \times \underbrace{\theta^{\frac{2}{1-\theta}}\left(\frac{l}{x}\right)^{\gamma}}_{\text{“firm size”}}, \tag{23}$$

where l is per capita labor, as determined by (20), and $x \equiv Ne^{-\lambda t}$ is the mass of firms-to-population ratio. We stress that (23) is an equilibrium relationship. First, the higher per capita labor, l, the higher the rate of return to quality-improving innovation (23). This effect highlights the key role of the labor market in shaping firms' incentives to innovate. Higher equilibrium labor signals higher demand for intermediate goods, which leads to heightened firm's profitability and thereby larger "firm size," defined as quality-adjusted production, $X/Z = \theta^{\frac{2}{1-\theta}}(l/x)^{\gamma}$.

Second, the higher the mass of firms per capita, x, the lower the rate of return to quality-improving innovation. This effect captures "business stealing," in the sense that firms' entry reduces the market share of incumbent firms. We note that firm size in (23) is an equilibrium object that is out of the individual firm's control. Thus, condition (23) implies that, in equilibrium, a higher rate of return to stockholders can be achieved only through a larger firm size.

Third, we note that firm's investment, R, a firm's choice variable, does not enter (23). This happens because the firm-level investment decision is a bang-bang problem, so that condition (23) can be interpreted as an aggregate investment "indifference" condition. As a result, changes in the current tax rate or news about future changes mandate adjustment in aggregate investment through an equilibrium response in firm size.

Product innovation Taking logs and time derivatives of the free-entry condition (15) and using firm's value (11) yields the rate of return to firm's entry,

$$r = \frac{D}{V} + \frac{\dot{V}}{V} = \frac{\Pi - \phi Z - R}{\beta X} + \frac{\dot{X}}{X}. \tag{24}$$

Eq. (24) can be rewritten in terms of firm size, $X/Z = \theta^{\frac{2}{1-\theta}}(l/x)^{\gamma}$, and quality-adjusted investment, R/Z,

$$r = \overbrace{\frac{\mu}{\beta} - \underbrace{\frac{\phi}{\beta}\left[\frac{1}{\theta^{\frac{2}{1-\theta}}(l/x)^{\gamma}}\right]}_{\text{static economies of scale}} - \underbrace{\frac{1}{\beta}\left[\frac{1}{\theta^{\frac{2}{1-\theta}}(l/x)^{\gamma}}\right]\frac{R}{Z}}_{\text{cost spreading effect}}}^{\text{dividend–price ratio}} + \overbrace{\frac{R}{Z} + \underbrace{\gamma\left(\frac{\dot{l}}{l}-\frac{\dot{x}}{x}\right)}_{\text{“firmsize” growth}}}^{\text{capital gains}}, \tag{25}$$

where $\mu \equiv (1/\theta - 1)$ is the price markup. First, the rate of return to entry (25) is increasing in firm size $\theta^{\frac{2}{1-\theta}}(l/x)^{\gamma}$. As before, this is an equilibrium relationship. The larger the firm size in equilibrium, the higher the firm's profitability for a given level of investment, leading to a higher dividend-price ratio. A larger firm size raises firm's profitability through two channels: (i) it dilutes fixed operating costs, ϕZ, due to static economies of scale; (ii) it dilutes the cost of quality-improving investment, $p_Y R$, with $p_Y \equiv 1$, due to a cost spreading effect.[12] Second, the rate of return to entry is increasing in firm size *growth*, through an appreciation of the market value of the firm.

Third, the rate of return to entry is increasing in quality-adjusted investment, R/Z, provided that $\theta^{\frac{2}{1-\theta}}(l/x)^{\gamma} > 1/\beta$. In equilibrium, higher investment has two contrasting effects: (i) it diverts resources from dividends, thus reducing the current dividend-price ratio; (ii) it raises future gross cash flows leading to an appreciation of the firm's value.

3.3. Market equilibrium

Market clearing in the labor, asset, and product markets, and consolidating household's (2) and government's budget constraint (16) yields the aggregate resource constraint of the economy,

$$C + G + \mathcal{I} + \mathcal{Q} = Y, \tag{26}$$

where C and G are private consumption and government purchases, respectively, $\mathcal{I}$, is investment (i.e., quality-improving investment, NR, sunk entry costs, $\dot{N}\beta X$, and fixed operating costs, $N\phi Z$), $\mathcal{Q}$ indicates intermediate inputs, NX. Output is either consumed or invested in activities that generate future income and product. (See Appendix A.3 for the derivation of the aggregate resource constraint.)

3.3.1. Saddle-path dynamical system

The market equilibrium of our model economy is fully described by a system of three ODEs in the $(x, \tilde{c})$ space:

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \lambda - \rho - \frac{1 - \theta - g - \tilde{c}}{\beta\theta^2}, \tag{27}$$

$$\frac{\dot{x}}{x} = \frac{\pi(x, \tilde{c})}{\beta\theta^2} + \frac{1 - \theta - g - \tilde{c}}{\beta\theta^2} - \lambda, \tag{28}$$

$$\dot{b} = [r(x, \tilde{c}) - g_Y(x, \tilde{c})]b + g - \gamma(1 - \theta)\tau_L - \chi. \tag{29}$$

In the system (27)–(29), key variables are (i) the profit rate, $\pi \equiv ND/Y$,

$$\pi(x, \tilde{c}) \equiv \theta(1 - \theta)\left[1 - \alpha\left(\frac{\phi + z(x, \tilde{c})}{r(x, \tilde{c})}\right)\right], \tag{30}$$

(ii) the rate of return to quality-improving innovation (23),

$$r(x, \tilde{c}) = \alpha\left(\frac{1}{\theta} - 1\right)\theta^{\frac{2}{1-\theta}} l(\tilde{c})^{\gamma} x^{-\gamma}, \tag{31}$$

(iii) the percentage growth rate of product quality, $z \equiv R/Z = \dot{Z}/Z$,

$$z(x, \tilde{c}) = \left[1 - \frac{\alpha\mu}{\beta r(x, \tilde{c})}\right]^{-1}\left[r(x, \tilde{c}) + \frac{\phi\alpha\mu}{\beta r(x, \tilde{c})} - \frac{\mu}{\beta} + \gamma\omega(\tilde{c})(\rho - \lambda) + \gamma\omega(\tilde{c})\left(\frac{1 - \theta - g - \tilde{c}}{\beta\theta^2}\right)\right], \tag{32}$$

and (iv) the percentage growth rate of gross output,

$$g_Y \equiv \frac{\dot{Y}}{Y} = r(x, \tilde{c}) + \frac{1 - \theta - g - \tilde{c}}{\beta\theta^2}. \tag{33}$$

The equilibrium described by (27)–(29) is *block-recursive*. First, the autonomous ODE (27) determines the path of the consumption-to-output ratio, $\tilde{c}$, independently of the other variables in the system, given the government spending-to-output ratio, g. Second, using (20), we obtain the path of labor per capita, $l(\tilde{c})$, given the labor tax rate, τ_L. Third, given $l(\tilde{c})$, (31) and (32) jointly determine the profit rate $\pi(x, \tilde{c})$ and thereby the path of the mass of firms per capita through the nonlinear ODE (28), given an initial condition $x(0) = x_0$.

Importantly, the no-Ponzi-game (NPG) condition,

$$\lim_{t\to\infty}\left\{\exp\left[-\int_0^t [r(s) - g_Y(s)]ds\right]b(t)\right\} = 0, \tag{34}$$

guarantees that the government's intertemporal budget constraint is satisfied in the sense that the government cannot borrow indefinitely at the rate r or higher. While the ODE (29) and NPG condition (34) are necessarily part of the definition of a market equilibrium, the availability of lump-sum taxes as a fiscal instrument makes, to a large extent, public debt dynamics irrelevant for equilibrium allocations and prices.

[12] Note that quality-improving investment, R, is in units of final output, so that the investment cost for the firm is $p_Y R$. As final output is the numeraire, the price of investment is $p_Y \equiv 1$.

4. Ricardian regime and the irrelevance of public debt

We now turn to study the equilibrium of our model economy in the presence of lump-sum taxes, which we will refer to as "Ricardian" regime. In such a regime, the equilibrium of the private sector is described by the subsystem (27)-(28) independently of the ODE for the public debt-to-output ratio (29). Here we characterize the balanced growth path (BGP) and transitional dynamics of the economy, given the expectation that the government is "solvent," in the sense that the value of outstanding debt equals the present discounted value of future primary surpluses. In doing so, we assume that the government commits to a constant government spending-to-output ratio, g, and labor tax rate, τ_L. Importantly, lump-sum taxes, as a share of output, χ, bear all the adjustment needed to balance the government's budget in present value terms.

4.1. Irrelevance of public debt in the ricardian regime

To determine whether the saddle path characterizing the dynamics of the private sector implied by the subsystem (27)-(28) is indeed the economy's equilibrium path, one has to take in account the dynamics of the debt-to-output ratio, b, as implied by (29).

Public debt dynamics To fix ideas, it is useful to solve the ODE governing the public debt-to-output ratio (29):

$$b(t) = e^{\bar{\nu}(0,t)t}\left\{b(0) + \int_0^t e^{-\bar{\nu}(0,s)s}[g - \gamma(1-\theta)\tau_L - \chi(s)]ds\right\}, \tag{35}$$

where $\bar{\nu}(0,t) \equiv (1/t)\int_0^t [r(s) - g_Y(s)]ds$ is the average effective discount rate and $b(0) \geq 0$ is the initial level of the debt-to-output ratio. Further, along any BGP with a constant r^*, g_Y^*, and χ^*, (35) yields

$$b(t) = e^{(r^*-g_Y^*)t}\left\{b(0) - \frac{[\chi^* + \gamma(1-\theta)\tau_L - g] \times \left[1 - e^{-(r^*-g_Y^*)t}\right]}{r^* - g_Y^*}\right\}. \tag{36}$$

In a market equilibrium, the government, as the household, is subject to a no-Ponzi-game condition, i.e., $\lim_{t\to\infty} e^{-(r^*-g_Y^*)t}b(t) = 0$. As $r^* > g_Y^*$ along any BGP, the no-Ponzi-game condition holds if and only if the term in brackets on the right-hand side of (36) is zero, i.e., if and only if the government follows a balanced-budget rule:

$$\chi^* + \gamma(1-\theta)\tau_L - g = (r^* - g_Y^*)b(0). \tag{37}$$

The key to this rule is that the government can satisfy the intertemporal budget constraint without resorting to distortionary taxes. Public debt, therefore, is neutral because there is no channel linking its time path to the behavior of the private sector. This property of the equilibrium is a manifestation of the "Ricardian equivalence."[13]

Natural debt-limit We note that (37) renders a notion of *"natural debt-limit"* which is imposed by the market equilibrium on the government's ability to issue new debt:

$$b(0) \leq \bar{b} = \frac{\chi^* + \gamma(1-\theta)\tau_L - g}{r^* - g_Y^*}. \tag{38}$$

Since $\chi^* < 1$, $\tau_L < 1$, and $g < 1$, there exists un upper bound on the initial debt-to-output ratio, $\bar{b}$, above which it is *unfeasible* to service the debt. Indeed, as the private sector has perfect foresight about the time path of government policy (g, τ_L, and χ^*), any policy that fails to satisfy the natural debt-limit (38) sets the debt-to-output ratio on an unsustainable path of ever growing debt, a scenario incompatible with a market equilibrium.

4.2. Balanced growth path

The model economy exhibits a balanced growth path (BGP) of the following form. Output of the final consumption good grows at a constant rate $g_Y^* = \lambda + z^*$, where λ and z^* are the constant growth rate of population and product quality, respectively. Output (and so income) per capita and the wage rate both grow at the constant rate, z^*. The allocation of labor to production and the mass of firms both grow at the constant rate of population growth, λ, so that labor per firm, our operational measure of firm size, remains constant over time at the level l^*/x^*. The real interest rate, which is also the rate of return to equity and government bonds, remains constant at $r^* = \rho + z^* > g_Y^*$.

[13] Ricardian equivalence is the proposition that, for a given path of government spending, public deficits and debt are irrelevant for equilibrium allocations and prices (see Barro, 1974). Key to this result is the presence of a government's budget constraint and a no-Ponzi-game condition on the private sector. As the debt-to-output ratio cannot grow forever, the present discounted value of taxes (and other government revenues) must be equal to total expenditures. It is well-known that if one restricts government's access to lump-sum taxes then Ricardian equivalence does no longer hold (see, e.g., Barro, 1989; Seater, 1993).

Steady-state equilibrium Using (27) and (28), $\dot{\tilde{c}}/\tilde{c} = 0$ and $\dot{x}/x = 0$ yields a system of two equations that fully characterize the steady-state equilibrium of the private sector, for a given government spending-to-output ratio, g:

$$\tilde{c}^* = (\rho - \lambda)\beta\theta^2 + 1 - \theta - g, \tag{39}$$

$$\tilde{c}^* = \pi(x^*, \tilde{c}^*) + 1 - \theta - g - \lambda\beta\theta^2. \tag{40}$$

Using the household's Euler Eq. (5), and the property that $g_Y^* \equiv \dot{Y}^*/Y^* = z^* + n^*$ and that the mass of firms grows at the same rate of population, $n^* = \lambda$, yields that along the BGP the real interest rate is $r^* = z^* + \rho$. The aggregate dividend-to-output ratio in (30) can then be rewritten in terms of the steady-state growth rate of quality, z^*, only:

$$\pi(x^*, \tilde{c}^*) = \theta(1-\theta)\left[1 - \alpha\left(\frac{\phi + z^*}{\rho + z^*}\right)\right]. \tag{41}$$

Combining (39) with (40) yields $\pi(x^*, \tilde{c}^*) = \rho\beta\theta^2$, which using (41) uniquely determines the steady-state growth rate of product quality,

$$z^* = \frac{1 - \alpha\phi/\rho - \rho\beta/\mu}{\beta/\mu - (1-\alpha)/\rho}. \tag{42}$$

The growth rate of output per capita, z^*, is independent of the population growth rate, λ. To see how our model economy achieves this outcome, think of the following thought experiment. The baseline economy rests on a BGP in which the mass of firms grows in lockstep with population. Now, think of a counterfactual economy in which the number of firms is held fixed whereas population keeps growing. In this case, average firm size and so the rate of return to quality-improving innovation, as determined by (23), would steadily raise, leading to a path of increasing growth rates of product quality. Next, let the mass of firms be determined in free-entry equilibrium. The rate of return to entry, as determined by (25), raises in response to a larger firm size, pushing firms to enter the market. As the entry process unfolds, average firm size reverts back to the baseline value consistent with the constant steady-state growth rate of product quality z^*.

Long-run effects of fiscal policy Using the autonomous ODE in (27), $\dot{\tilde{c}}/\tilde{c} = 0$ yields a constant consumption-to-output ratio,

$$\tilde{c}^* = (\rho - \lambda)\beta\theta^2 + 1 - \theta - g. \tag{43}$$

On the right-hand side of (43), the term $\beta\theta^2$ is the net worth-to-output ratio, $1 - \theta$ is the household's income-to-output ratio, which equals the labor share of output, $\gamma(1-\theta)$, plus the fixed factor's share of output, $(1-\gamma)(1-\theta)$. Not surprisingly, a higher government spending-to-output ratio, g, leads to an unambiguously lower consumption-to-output ratio, $\tilde{c}^*$. In our environment, government spending is a "pure waste," in the sense that it does not provide utility to the household nor contributes to production. By diverting resources from productive use, government spending generates a negative wealth effect, which crowds out consumption one-to-one.

Note that, for a given labor tax rate, a higher government spending-to-output ratio is associated with more hours worked per capita, as determined by (20). Notably, along the BGP, the elasticity of per capita labor, l^*, with respect to g depends on the elasticity of l^* with respect to $\tilde{c}^*$, $\omega(\tilde{c}^*)$, and the ratio of government spending to consumption, $g/\tilde{c}^*$:

$$\frac{\partial \ln l^*}{\partial \ln g} = \omega(\tilde{c}^*) \times \frac{g}{\tilde{c}^*} = \frac{\eta g}{\gamma(1-\theta)(1-\tau_L) + \eta\tilde{c}^*} > 0. \tag{44}$$

Using the expression for the rate of return to quality-improving innovation (23) and $r^* = \rho + z^*$, yields an *inverse* relationship between the growth rate of product quality, z^*, and the mass of firms per capita,

$$x^* = \left(\frac{\alpha\mu\theta^{\frac{2}{1-\theta}}}{\rho + z^*}\right)^{\frac{1}{\gamma}} l^*. \tag{45}$$

A higher growth rate of product quality is achieved in equilibrium through a reduction in the mass of firms per capita (or, equivalently, a larger firm size).

The endogenous market structure mechanism that sterilizes population growth in the long run, it is also responsible for the *"long-run superneutrality"* of fiscal policy. Labor tax rates and government spending affect the *scale* of the economy, through changes in the consumption-to-output ratio and per capital labor, yet they are irrelevant for the long-run growth rate of the economy.

4.3. Transitional dynamics

Transitional dynamics in the Ricardian regime is saddle-path stable. Moreover, the block-recursive structure of the equilibrium provides an algorithm to compute perfect foresight paths of equilibrium quantities and prices.

1. As the ODE (27) is *unstable*, the consumption-to-output ratio is equal to its steady-state value (43) at all times, i.e., $\tilde{c}(t) = \tilde{c}^*$, for all $t \geq 0$.

2. Using (20), the time path of per capita labor is determined as

$$l(t)=\frac{\gamma(1-\theta)(1-\tau_L)}{\gamma(1-\theta)(1-\tau_L)+\eta\tilde{c}^*},\ \text{for all } t\geq 0. \tag{46}$$

3. Using (31), per capita labor in (46) and the initial condition for the mass of firms per capita $x(0)=x_0$ pin down the real interest rate in the initial period $r(0)=r(x_0,\tilde{c}^*)$. Then, using (32), we obtain quality growth in the initial period as $z(0)=z(x_0,\tilde{c}^*)$. Finally, using (30), we obtain the profit rate as $\pi(0)=\pi(x_0,\tilde{c}^*)$, given $r(x_0,\tilde{c}^*)$ and $z(x_0,\tilde{c}^*)$.
4. Solving the nonlinear ODE (28) provides the entire dynamics of the mass of firm per capita $x(t)$ for $t\geq 0$ given the initial condition x_0.

Short-run effects of fiscal policy To understand how the model economy propagates unexpected and unanticipated changes in government spending and/or labor tax rates—a fiscal policy "shock"—it is useful to keep in mind that the consumption-to-output ratio and per capita labor are jump variables that start the transitional dynamics driven by the state variable mass of firms per capita.

As an example, think of an economy along a BGP with fiscal policy (g, τ_L, and χ^*). Let us shock the economy with, say, a reduction in the labor tax rate so that from the current date onward the new tax rate is $\tau_l' < \tau_L$. Since the government spending-to-output ratio has not changed, the consumption-to-output ratio remains at its steady-state value before the tax shock. Labor per capita, instead, jumps on impact at its higher steady-state level as implied by (20). As the mass of firms per capita is a predetermined variable, the initial jump in per capita labor induces a raise in average firm size, which feeds into a temporary acceleration in firms' entry. Along the new BGP, the economy grows at the same rate before the tax shock.

5. Non-Ricardian regime and the burden of public debt

We now turn to study the equilibrium implications of the model when the government cannot tax income in a lump-sum fashion, which is arguably the empirically relevant case. In the absence of lump-sum taxation, the budget-balance rule (37) implies that either g or τ_L, or both, must *eventually* adjust to guarantee a sustainable time path for the debt-to-output ratio. Before such adjustment takes place, though, the public debt-to-output ratio displays *explosive* dynamics to the extent that $r > g_Y$. We will refer to this scenario as a "non-Ricardian" regime. In such a regime, public debt hits any debt-limit in finite time, thus rendering a notion of "fiscal distress."

To operationalize the idea of a regime in fiscal distress, think of the following scenario. An economy runs under a fiscal policy package (the entire path of fiscal instruments) that is sustainable in the sense of satisfying the government's intertemporal budget constraint, possibly because the government used lump-sum taxes, and there is outstanding debt. We then postulate a permanent change in the environment that makes preexisting fiscal policy unsustainable. A few examples are an unexpected increase in government outlays (e.g., military spending in wartime) and/or a reduction in the rate of population growth (e.g., a reduction in birth rates). In both cases, the steady-state growth rate of output per capita is unchanged, so that one can isolate the source of the debt-sustainability problem from the determinants of long-run growth. Here, we study the implications of alternative strategies ("plans") that the government can implement to address the debt-sustainability problem.

Fiscal consolidation plans The solvency condition (37) is asymptotic (i.e., it holds for $t\to\infty$) and therefore it is not informative about the specific path of public debt and its potential effects on private sector behavior. Here, we relax the focus on this asymptotic condition for government solvency and take instead a more interesting and empirically relevant approach. Specifically, one way to satisfy the government's intertemporal budget constraint and get insight on the dynamics of the debt-to-output ratio and its implications for private sector behavior, is to let tax rates and the public spending-to-output ratio stay the same for a finite period of time and adopt a balanced-budget rule at some future date. The key advantage of this approach is that it makes the debt-output ratio endogenous.

Operationally, we denote $[0, t_b)$ the borrowing period and $[t_b, \infty)$ the no-borrowing period. In the former period, the government follows the initial fiscal package, borrowing to finance the fiscal deficit, and in the latter period, it adopts the policy of a constant debt-to-output ratio, i.e., $\dot{b}=0$ from t_b onward.[14] The approach, therefore, focuses on a notion of *"fiscal consolidation plan"* and makes the date t_b a key component of such a plan.

We consider two alternative scenarios that have received considerable attention in the empirical literature: (i) immediate fiscal stabilization, i.e., $\dot{b}=0$ at $t_b=0$; (ii) delayed fiscal stabilization, i.e., $\dot{b}=0$ at $0<t_b<\infty$. In both cases, *past* fiscal policy matters only to the extent that it determines the government debt-to-output ratio at the time the fiscal stabilization takes place. Importantly, in the case of a delayed stabilization, the current announcement of the future consolidation plan can generate "anticipation effects."

Balanced-budget rule Using (29), and setting $\dot{b}=0$, yields a standard balanced-budget condition that must hold for all $t\geq t_b$,

$$\underbrace{\gamma(1-\theta)\tau_L(x,\tilde{c})}_{\text{tax revenues}}=\underbrace{g+[r(x,\tilde{c})-g_Y(x,\tilde{c})]b(t_b)}_{\text{government outlays}}, \tag{47}$$

[14] To be precise, in the "no-borrowing" period, government debt grows at the same rate of output growth. The quantity of new debt issued by the government is just enough to keep the debt-to-output ratio constant.

where $b(t_b)$ is the public debt-to-output ratio at the time debt stabilization takes place. Using (33), and the property that the consumption-to-output ratio is always at its steady-state value (39), one obtains a simplified balanced-budget rule,

$$\gamma(1-\theta)\tau_L = g + (\rho - \lambda)b(t_b), \tag{48}$$

where the tax rate hike and/or government spending cut needed to implement $\dot{b} = 0$ pins down $b(t_b)$.

5.1. Immediate debt stabilization

Here we study the scenario in which the government aims to stabilize the debt-to-output ratio, immediately, i.e. from the current date $t = t_b = 0$ onwards. To achieve this goal, the government must follow a balanced-budget rule according to which the labor tax rate and/or government spending *endogenously* adjust to guarantee that tax revenues equal government outlays (including interest payments) for all $t \geq 0$. In this case, there are no anticipation effects; the government surprises the private sector with a once-and-for-all change in fiscal policy.

Proposition 1 (Tax-based immediate stabilization)**.** *Consider the case of an immediate debt stabilization implemented with a tax rate hike, keeping the government spending-to-output ratio constant at g for all t. The path of the tax rate is $\tau_L(t) = \tau_L$ for $t < 0$ and $\tau_L(t) = \tilde{\tau}_L > \tau_L$ for all $t \geq 0$, as implied by the balanced-budget rule (48),*

$$\tilde{\tau}_L = \frac{g + (\rho - \lambda)b_0}{\gamma(1-\theta)}. \tag{49}$$

As g does not change, the steady-state consumption-to-output ratio remains the same, such that $\tilde{c}(t) = \tilde{c}^*$ for all $t \geq 0$. Per capita labor falls from $l(\tilde{c}^*, \tau_L)$ to $l(\tilde{c}^*, \tilde{\tau}_L) < l(\tilde{c}^*, \tau_L)$. Along the transition, the mass of per capita firms gradually falls towards its new steady-state level as determined by (45). Steady-state growth rate of per capita output remains unchanged.

Proposition 1 indicates that labor supply is a key channel through which a tax-based debt consolidation impact the economy. Critically, though, general equilibrium forces are key to understand its overall effect. The permanent reduction in per capita labor stifles incentives to firms' entry, leading to a growth deceleration relative to the BGP.

Proposition 2 (Spending-based immediate stabilization)**.** *Consider the case of an immediate debt stabilization implemented with government spending cuts, keeping the tax rate constant at τ_L for all t. The path of the government spending-to-output ratio is $g(t) = g$ for $t < 0$ and $g(t) = \tilde{g} < g$ for all $t \geq 0$, as implied by the balanced-budget rule (48),*

$$\tilde{g} = \gamma(1-\theta)\tau_L - (\rho - \lambda)b_0. \tag{50}$$

Due to the reduction in the government spending-to-output ratio, the steady-state consumption-to-output ratio unambiguously raises from $\tilde{c}^*$ to its new steady-state value $\tilde{c}^{**} = (\rho - \lambda)\beta\theta^2 + 1 - \theta - \tilde{g} > \tilde{c}^*$, such that $\tilde{c}(t) = \tilde{c}^{**}$ for all t. Per capital labor unambiguously falls from $l(\tilde{c}^*, \tau_L)$ to $l(\tilde{c}^{**}, \tau_L) < l(\tilde{c}^*, \tau_L)$ and so does the mass of per capita firms that gradually falls towards its lower steady-state level as determined by (45). Steady-state growth rate of per capita output remains unchanged.

Proposition 2 assumes that the fiscal adjustment comes about government spending cuts only. However, one might argue that a government may implement a consolidation with a spending cut that is large enough to allow for a permanent reduction in tax rates. In this case, the response of per capita labor depends on the change in the tax rate implied by the balanced-budget rule (48). Notably, using (20) and (50), one obtains that

$$l(\tilde{c}^{**}) \geq l(c^*), \text{ if and only if } \frac{\tilde{c}^{**}}{1 - \tilde{\tau}_L(\tilde{g}, b_0)} \leq \frac{\tilde{c}^*}{1 - \tau_L}. \tag{51}$$

Whether the condition (51) is satisfied ultimately depends on the size of the spending cut and the public debt-to-output ratio at the time of the stabilization.

5.2. Delayed debt stabilization

Should a government stabilize the level of the debt now or later on? Intuitively, the longer the government waits to consolidate, the higher the required tax increase or spending cut when it does. This logic is, to a large extent, a straightforward implication of the dynamic budget constraint of the government. Importantly, in the absence of lump-sum taxation, governments' choice to delay stabilization may imply a higher level of distortions, thus having first-order effects on welfare.

Here, we use the notion of a borrowing window $[0, t_b)$ proposed earlier to study two intertwined issues: (i) the implications of delaying fiscal adjustment; (ii) endogeneity of public debt. As we consider a market equilibrium in which the government satisfies the NPG condition (34), the unstable dynamics of the public debt-to-output ratio implies that eventually the government *must* adopt a "credible" fiscal plan, i.e., a path of government spending and tax rates consistent with a constant debt-output ratio from some future date onwards.

We consider a scenario in which the government runs an "unsustainable" fiscal policy in the sense that the public debt-output ratio is steadily rising as implied by the unstable ODE (29). We assume that the government maintains fiscal policy unchanged for $t < t_b$. At $t = 0$ it commits to a balanced-budget rule for all $t \geq t_b$, so that $\dot{b} = 0$ from t_b onwards. Importantly, the private sector has *perfect foresight* so it correctly forecasts the path of fiscal instruments required to balance the budget on a period-by-period basis for all $t \geq t_b$.

To study how the economy responds to the announcement of a delayed stabilization, it is important to keep in mind that the announcement itself may generate real effects at the time of the announcement due to anticipation effects. The private sector knows that fiscal policy will change at some future date, so that its behavior may change now before the new fiscal policy plan is implemented. Yet, while this is true in principle, the extent to which it applies in our model economy critically depends on whether the announced fiscal plan changes the steady-state level of the consumption-to-output ratio.

Proposition 3 (Tax-based delayed stabilization)**.** *Consider the case of a delayed debt stabilization implemented with a tax rate hike, keeping the spending-to-output ratio constant at g for all t. The path of the tax rate is $\tau_L(t) = \tau_L$ for $t < t_b$ and $\tau_L(t) = \tilde{\tau}_L > \tau_L$ for all $t \geq t_b$, where $\tilde{\tau}_L$ is the tax rate implied by the balanced-budget rule (48),*

$$\tilde{\tau}_L = \frac{g + (\rho - \lambda) b(t_b)}{\gamma(1-\theta)}, \tag{52}$$

and $b(t_b)$ is the debt-to-output ratio at the time debt stabilization takes place (obtained by solving the ODE (29) forward over the time interval $[0, t_b]$),

$$b(t_b) = e^{(\rho-\lambda)t_b} b_0 + \frac{e^{(\rho-\lambda)t_b}[g - \gamma(1-\theta)\tau_L] \times \left[1 - e^{-(\rho-\lambda)t_b}\right]}{\rho - \lambda}. \tag{53}$$

As g does not change, the steady-state consumption-to-output ratio remains the same, such that $\tilde{c}(t) = \tilde{c}^*$ for all $t \geq 0$. The path of per capita labor is $l(t) = l(\tilde{c}^*, \tau_L)$ for $t < t_b$ and $l(t) = l(\tilde{c}^*, \tilde{\tau}_L)$ for all $t \geq t_b$, with $l(\tilde{c}^*, \tilde{\tau}_L) < l(\tilde{c}^*, \tau_L)$. Along the transition, the mass of per capita firms gradually falls towards its new steady-state level as determined by (45). Steady-state growth rate of per capita output remains unchanged.

Note that, not surprisingly, $b(t_b)$ in (53) is increasing in the level of the debt inherited from the past, b_0, the primary deficit, $g - \gamma(1-\theta)\tau_L > 0$, in the borrowing period, and the policy variable t_b that parametrizes the extent of delay in debt stabilization.

Proposition 4 (Spending-based delayed stabilization)**.** *Consider the case of a delayed debt stabilization implemented with government spending cuts, keeping the tax rate constant at τ_L for all t. The path of the spending-to-output ratio is $g(t) = g$ for $t < t_b$ and $g(t) = \tilde{g} < g$ for all $t \geq t_b$, where $\tilde{g}$ is the government spending-to-output ratio implied by the balanced-budget rule (48),*

$$\tilde{g} = \gamma(1-\theta)\tau_L - (\rho - \lambda) b(t_b). \tag{54}$$

Due to the reduction in the government spending-to-output ratio, the steady-state consumption-to-output ratio unambiguously raises from $\tilde{c}^*$ to its new steady-state value $\tilde{c}^{**} = (\rho - \lambda)\beta\theta^2 + 1 - \theta - \tilde{g} > \tilde{c}^*$. In response to the news about the future path of government spending, $\tilde{c}(t)$ jumps on an unstable trajectory connecting with the new saddle path associated with the no-borrowing period at $t = t_b$. The path of the consumption-to-output ratio is

$$\tilde{c}(t) = \begin{cases} \dfrac{\tilde{c}^*}{\left(\frac{\tilde{c}^* - \tilde{c}_0}{\tilde{c}_0}\right) e^{\left(\tilde{c}^*/\beta\theta^2\right)t} + 1} & \text{for} \quad t < t_b \\ \tilde{c}^{**} & \text{for} \quad t \geq t_b, \end{cases} \tag{55}$$

where $\tilde{c}_0$ is calculated such that $\tilde{c}(t_b) = \tilde{c}^{**}$ (see Appendix A.5 for details on derivations). The path of per capita labor is $l(t) = l(\tilde{c}(t)) < l(\tilde{c}^*)$ for $t < t_b$ and $l(t) = l(\tilde{c}^{**}) < l(\tilde{c}^*)$ for all $t \geq t_b$. Along the transition, the mass of per capita firms gradually falls towards its new steady-state level as determined by (45). Steady-state growth rate of per capita output remains unchanged.

Proposition 4 highlights the role played by the expectations of the private sector about the future path of fiscal instruments. In the tradition of Sargent and Wallace (1973), we will use the unstable ODE (27) to compute the perfect foresight path of the consumption-to-output ratio from the date of the announcement $t = 0$ forwards.

6. Taking the model to the data

Here we parametrize the model based on the post-war observations for the U.S. economy, and comment on some basic model predictions on the long-run behavior of observable variables.

6.1. Model parametrization

Before conducting the experiments of interest, we are to assign values to the parameters describing preferences, technology, and government policy variables. Following a long tradition in growth theory, the model is calibrated based on the long-run properties of the U.S. economy. We stress that while none of the parameters has a one-to-one relationship to a specific moment, it is instructive to describe the calibration as a few distinct steps. Parameter values and targeted moments are summarized in Table 1.

Table 1
Parameter values.

Parameter	Description	Value	Target
A. Preferences & technology			
ρ	Time discount rate	0.020	Real interest rate (4%)
η	Disutility of work	1.458	Time spent at work (27.7%)
λ	Population growth rate	0.010	Population growth (1%)
θ	Elas. substitution int. goods	0.500	Int. goods share of output (50%)
γ	Labor congestion	0.650	Labor share of GDP (65%)
α	Knowledge spillovers	0.123	Per capita GDP growth (2%)
β	Sunk entry cost	6.596	Number firms per capita (2.2%)
ϕ	Fixed operating cost	0.262	Profit share of GDP (6.6%)
B. Government policy			
τ_L	Labor tax rate	0.290	Average AMTR
g	Gov't spending-to-output	0.104	Gov't spending-to-GDP (20.7%)

Notes: See Section 6 for further details on the parametrization of the model.

Preferences Along the BGP, the real interest rate is $r^* = z^* + \rho$. Given a target of 2% growth rate of output per capita, the time discount rate ρ is set equal to 0.02, which yields a steady-state real interest rate of 4% (see, e.g., McGrattan and Prescott, 2003; Gomme et al., 2011). We set η equal to 1.458 so that, along the BGP, the stand-in household spends 27.7% of the available time at work (see, e.g., McGrattan and Prescott, 2017). Finally, $\lambda = 0.01$ yields a 1% population growth rate, consistently with the U.S. experience for 1951–2016.

Technology In the model, the parameter that governs the elasticity of substitution between intermediate goods in production pins down the intermediate goods share of gross output. We then set $\theta = 0.5$ to reproduce the intermediate goods share observed in U.S. data of about one-half (see Jones, 2011). The labor share of GDP is pinned down by the congestion parameter γ, that we then set equal to 0.65 (see Koh et al., 2016). Finally, we *jointly* calibrate three parameters (α, β, and ϕ) to match three moments in the U.S. data: (i) the average growth rate of real GDP per capita of 2% for 1948–2017; (ii) number of firms per capita of 0.022 for 1977–2015 (data on the number of firms are from the Business Dynamics Statistics and available at https://www.census.gov/ces/dataproducts/bds/data_firm.html); (iii) the corporate profits-GDP ratio of 6.6% for 1947–2017. In Section 6.2, below, we further comment on the implications that these calibrated technology parameters (θ, γ, α, β, and ϕ) have vis-à-vis some untargeted moments.

Government policy In the baseline calibration, the labor tax rate τ_L is set to 29%, which is the time-series average of the average marginal tax rate (AMTR), which is the sum of the average marginal individual income tax rate (AMIITR) and the average marginal payroll tax rate (AMPTR), for 1946–2012, as constructed by Barro and Redlick (2011). One can think of $1 - \tau_L$ as the overall distortion on labor supply. In the model, the government spending-to-GDP ratio equals 20.7%, which is the average in the data for 1946–2017.

6.2. Properties of the calibrated economy

The model gives sharp predictions on the long-run (steady state) equilibrium relationship between observable variables, that are to a large extent borne by the data.

Tax rates and long-run growth In the model, income tax rates are neutral with respect to the steady-state growth rate of output per capita. This property of the equilibrium is consistent with the large literature on the (lack of) long-run growth effects of fiscal policy (see, e.g., Easterly and Rebelo, 1993; Mendoza et al., 1994; Stokey and Rebelo, 1995; Mendoza et al., 1997; Jaimovich and Rebelo, 2017). Yet, tax rates affect the long-run *level* of variables. Of course, how large these level effects are, critically depends on the parametrization of the model. We quantify the magnitude of these effects in Section 7.1.

Labor input and firms per capita In the data, market hours worked and the number of firms per capita exhibit no long-run trend, displaying a correlation of 0.9 over the 30-year period 1977–2007.[15] This is evident from panels A and B of Fig. 1. In the model, the positive comovement between the labor input and the mass of firms per capita is, in fact, the key channel through which government policies propagate in the economy.

To fix ideas, consider a BGP along which a balanced-budget rule holds and expected to hold indefinitely, so that

$$\chi + \gamma(1-\theta)\tau_L = g + [r(x^*, \tilde{c}^*) - g_Y(x^*, \tilde{c}^*)]b. \tag{56}$$

In panels A and B of Fig. 2, we set $b = 0$ at all times, progressively increase the labor tax rate, and let the lump-sum transfer χ balance the budget on a period-by-period basis, keeping the government spending-to-output ratio constant at its baseline value. This exercise isolates the long-run effects of distortionary labor taxation. First, increases in tax rates unambiguously lower equilibrium labor per capita. This is the standard direct effect on labor supply of proportional tax rates; the relative price of leisure raises so that the household works less intensively. Second, by lowering the labor input, the increase in tax rates trigger general equilibrium forces that lead to a permanent reduction in the mass of firms per

[15] The correlation drops to 0.73 for the period 1977–2015, when one includes the Great Recession of 2007.

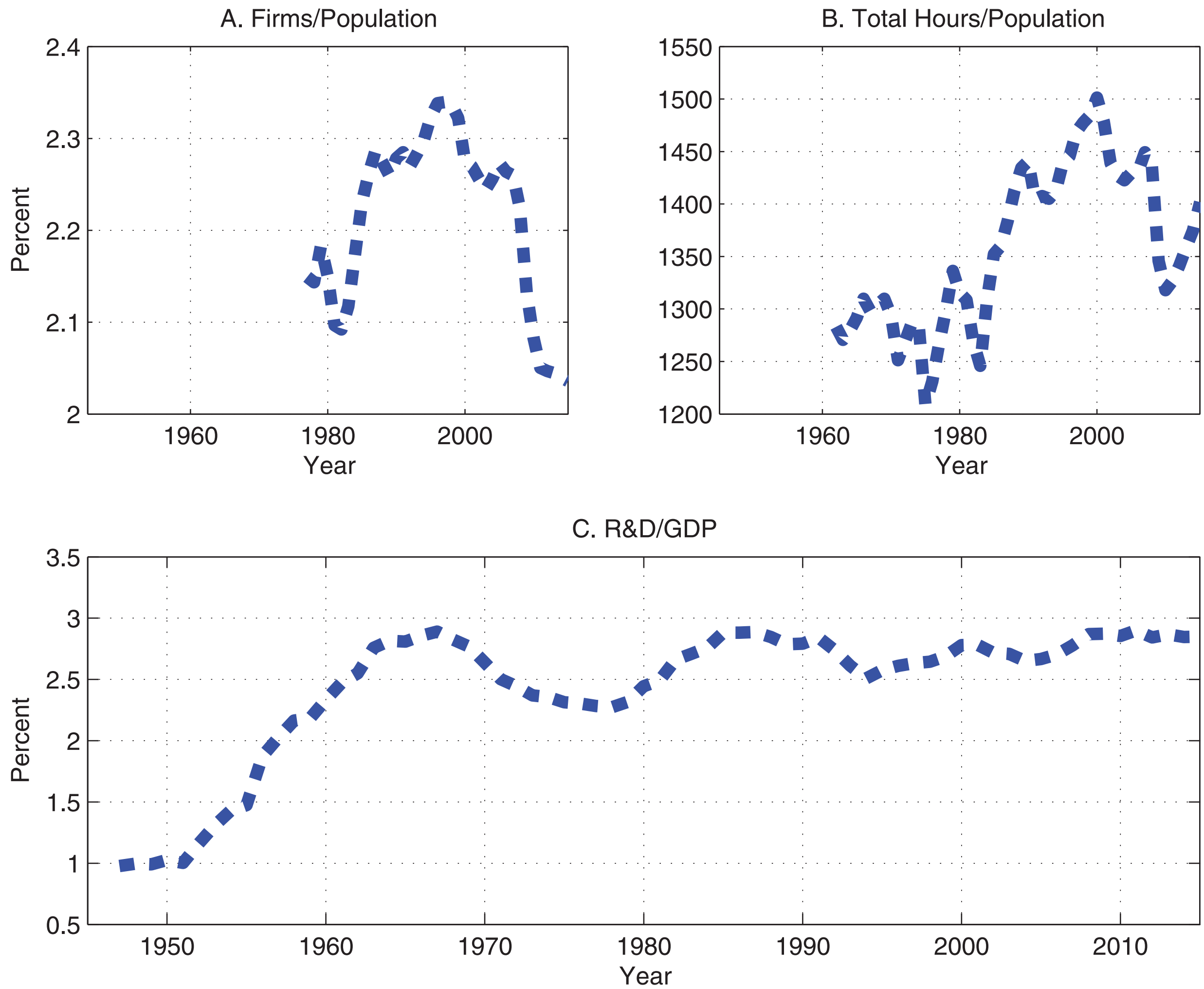

Fig. 1. Long-Run ratios in the United States.

capita. This indirect effect works through changes in the market size. In this sense, one can view it as the core mechanism of Schumpeterian growth.

Importantly, both variables move in lockstep so that their long-run ratio is invariant to the level of taxation. Any change in labor per capita induced by a tax rate change is perfectly offset by an equal percent change in the number of active firms per capita. Note that though this is true along the BGP, in transition dynamics, per capita labor jumps on impact in response to changes in the tax rate, whereas the mass of firms per capita adjusts sluggishly over time. We stress that the endogenous adjustment of the mass of firms per capita is the mechanism that neutralizes the long-run growth effects of income taxation. If one prevented such an adjustment, small changes in tax rates would have implausible large effects on the long-run growth rate of the economy. As pointed out by Stokey and Rebelo (1995), and many others thereafter, a model economy featuring a link between income tax rates and long-run growth rates of income per capita would be at odds with the post-war U.S. experience.

Strength of work incentives In panels C and D, we set $b = \chi = 0$ at all times, and let the income tax rate increase to offset the increases in g. In this experiment, there are two opposing effects at play. First, an increase in the government spending-to-output ratio reduces the consumption-to-output ratio, leading to an increase in labor (negative "income effect"). Such a positive relationship between government purchases and the labor input is a prediction shared by standard RBC and NK theories alike, and a robust finding in the SVARs literature (see Ramey, 2016, for a survey). We quantify the output effects of permanent cuts in government spending in Section 7.2.

Second, the increase in the income tax rate required to balance the government budget, distorts labor supply decisions such that per capita labor falls (negative "work incentive effect"). In our calibrated economy, the incentive effect dominates, so that the balanced-budget government spending multiplier is negative: labor per capita falls in response to an increase in g. Importantly, the model property that the work incentive effect dominates the income effect lines up nicely with the empirical evidence (see Burnside et al., 2004; Ohanian et al., 2008; Barro and Redlick, 2011). Also, note that the relative

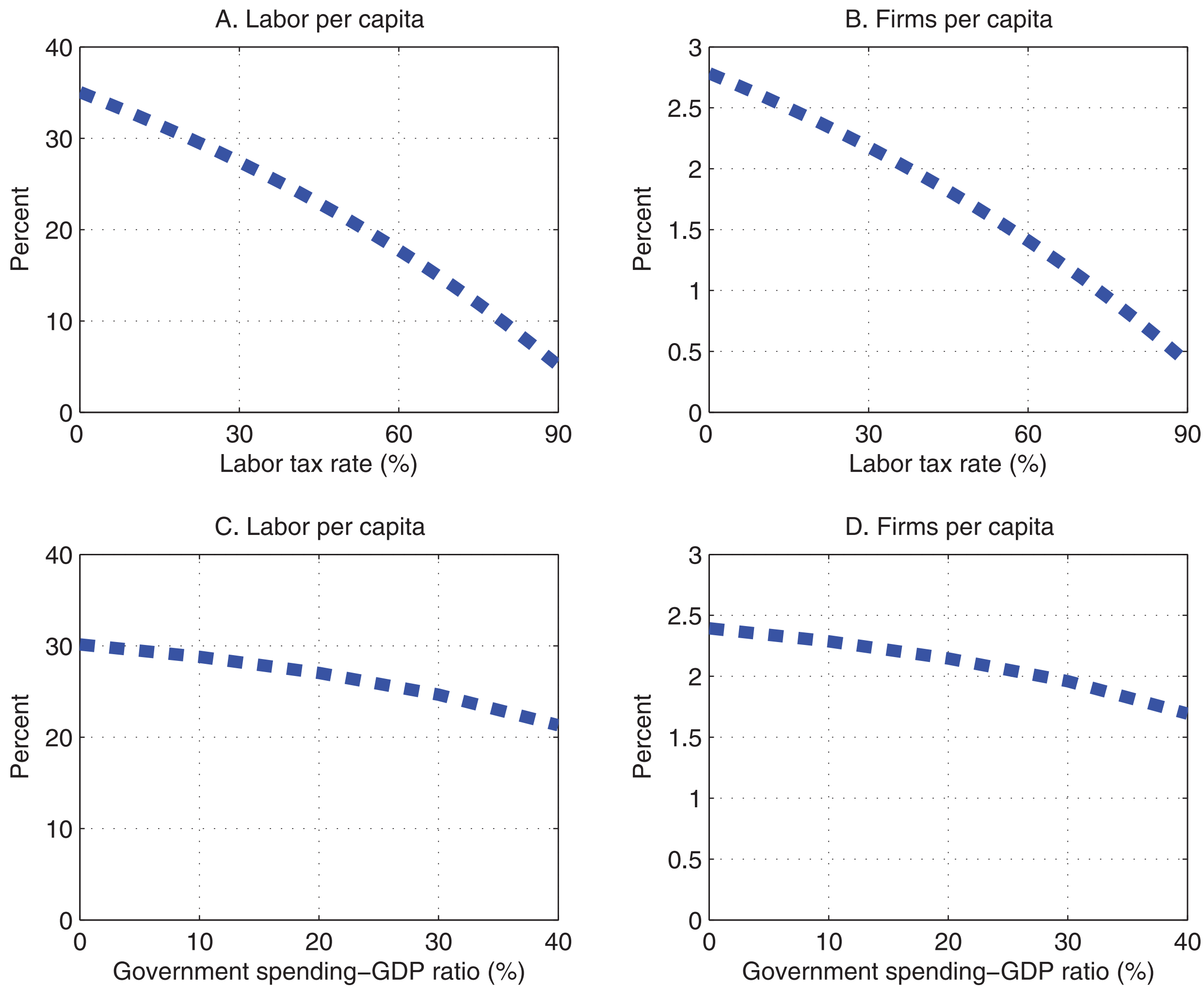

Fig. 2. Long-Run Properties of the Calibrated Economy. *Notes*: In panels A and B, we set $b = 0$ and let the lump-sum transfer χ balance the budget, keeping the government spending-to-output ratio, g, constant at its baseline value. In panels C and D, we set $b = \chi = 0$ and let the tax rate balance the budget. See Section 6 for further details on the parametrization of the model.

strength of work incentives versus income effects determines the overall output effect of fiscal stabilizations based on a *mix* of government spending cuts and tax rate cuts. We quantify the magnitude of these effects in Section 7.3.

Labor elasticity In the calibrated model, the equilibrium labor elasticity with respect to a permanent increase in the labor income tax rate is -0.3, which lines up nicely with the available empirical estimates on the intensive margin of aggregate hours worked (see Chetty et al., 2013, for a survey of empirical estimates). Disciplining the labor elasticity is important for our measurement, as the magnitude of the labor response to tax changes is a key element of the transmission mechanism of fiscal consolidations.

Net worth-GDP ratio For lack of data, we measure the output share of entry costs indirectly, using our calibrated model. In the model, the share of entry costs in output, $\dot{N}\beta X/Y$, equals the asset-output ratio, $NV/Y = \beta\theta^2$, times the growth rate of the mass of firms, $n \equiv \dot{N}/N$:

$$\frac{\dot{N}\beta X}{Y} = \beta\theta^2 \times n. \tag{57}$$

Along the BGP of our calibrated economy, $n = \lambda = 1\%$ per year. Note that in the data, the number of U.S. firms has grown on average at 1% a year over the period 1977–2015. The parameters β and θ pin down then the model counterpart of the net worth-GDP ratio in the data. In the model, the asset-GDP ratio is $NV/\mathrm{GDP} = \beta\theta^2/1 - \theta = 3.3$, that lines up nicely with the average households' net worth-GDP ratio of 3.7 in the United States for 1951–2012 (data on net worth-GDP ratio are from the FRED database and available at https://fred.stlouisfed.org/graph/?g=dGy). Note that the net worth-GDP ratio is an untargeted moment. Based on our measurement, entry costs are 3.3% of GDP.

R&D-GDP ratio In the data, the average R&D-GDP ratio is 2.5% for 1947–2017 (data on R&D are from the FRED database and available at https://fred.stlouisfed.org/series/Y694RC1Q027SBEA). In addition, as evident from panel C of Fig. 1, the ratio

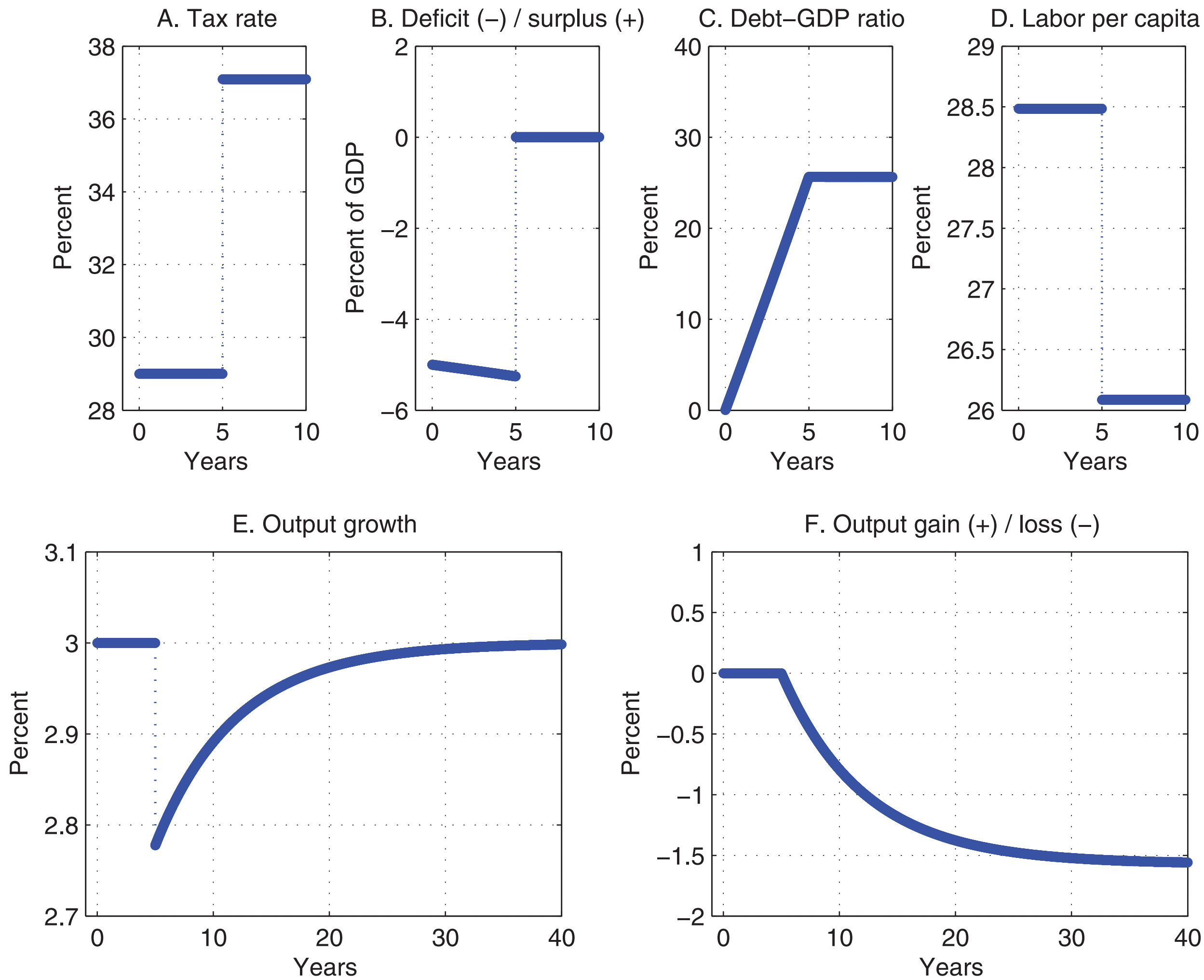

Fig. 3. Tax-based consolidation. *Notes*: 5% initial deficit-GDP ratio; 5-year plan. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

fluctuates around a stable level as early as the early-1960s. In the model, the share of R&D in GDP, NR/GDP, along the BGP, is constant and equal to

$$\frac{NR}{Y} \times \frac{Y}{\text{GDP}} = \frac{z}{\theta^{\frac{2\theta}{1-\theta}} (l/x)^{\gamma}} \times \frac{1}{1-\theta}. \tag{58}$$

Given our calibrated parameters, and the values for product quality growth, $z \equiv \dot{Z}/Z = 2\%$, per capita labor, $l = 27.7\%$, and number of firms per capita, $x = 2.2\%$, the expression in (58) yields an R&D-GDP ratio of 3%, that is remarkably close to the untargeted 2.5% in U.S. data. (Note that the average R&D-GDP ratio equals 2.7% for 1960–2017, which is the period when the U.S. economy fluctuates around a stable R&D-GDP ratio.)

7. Policy experiments

Here we conduct a host of experiments aimed at quantifying the effects of different fiscal consolidation plans on aggregate quantities.

Measurement of output losses/gains We measure losses or gains in aggregate output as follows. First, we take as a benchmark an economy along a BGP featuring a balanced budget. Second, we simulate equilibrium paths for a counterfactual economy in which an increase in government spending generates budget deficits. After a certain number of years of accumulating debt, the government stabilizes the debt-to-output ratio at its new level forever via an increase in taxes or a reduction in government spending. We measure output losses/gains relative to the level of output that would have prevailed along the BGP without budget deficits.

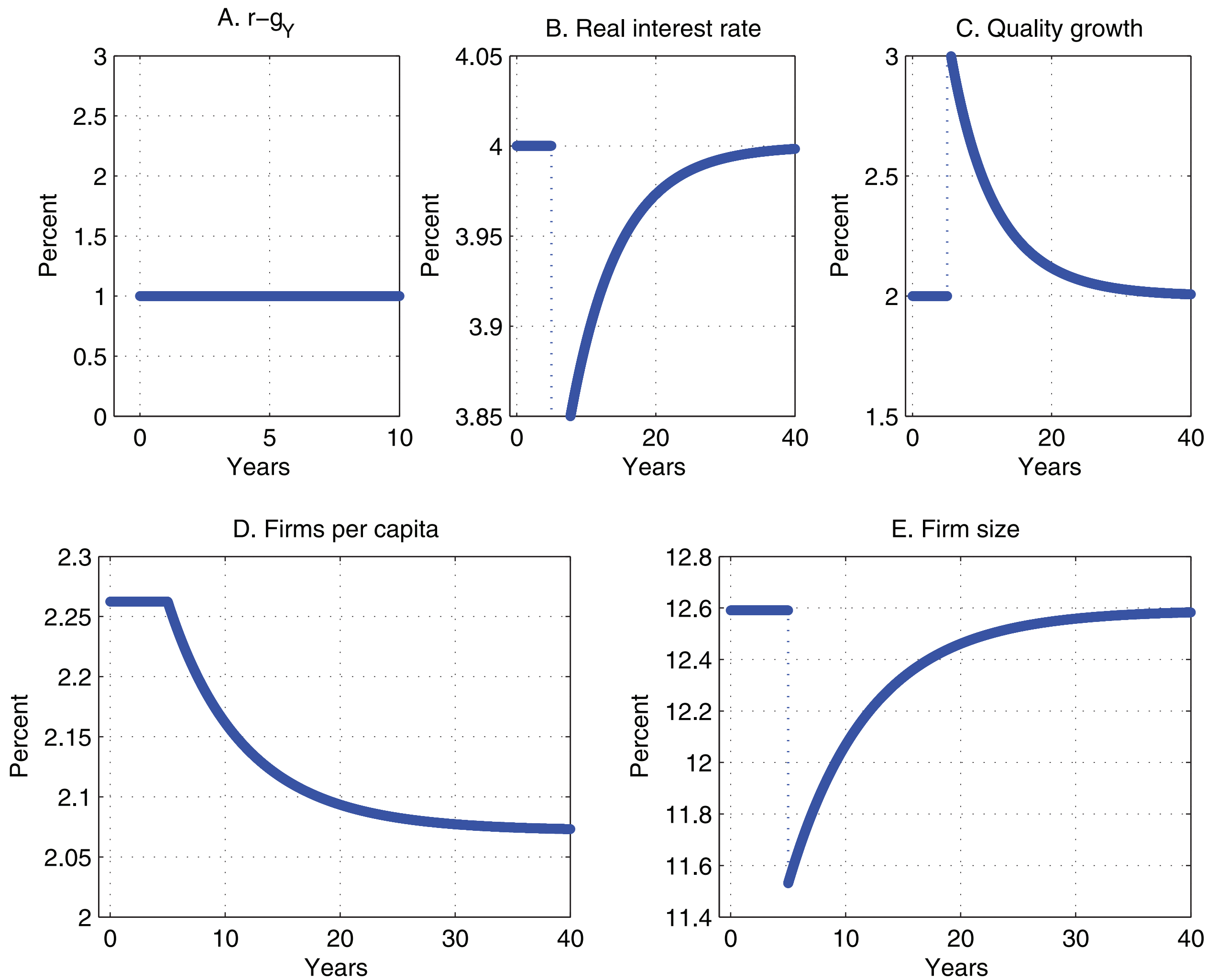

Fig. 4. Tax-based consolidation. *Notes*: 5% initial deficit-GDP ratio; 5-year plan. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

Table 2
Long-term gains (+) andlosses (−) from Fiscal consolidations.

	5% deficit 5-yrs delay	5% deficit 10-yrs delay	10% deficit 5-yrs delay	10% deficit 10-yrs delay
A. Tax rate hike				
Labor	−8.80%	−9.28%	−18.85%	−19.96%
Consumption	−1.57%	−1.65%	−3.16%	−3.32%
Output	−1.57%	−1.65%	−3.16%	−3.32%
Quality	+7.23%	+7.63%	+15.69%	+16.64%
B. Unanticipated spending cut				
Labor	−4.61%	−4.87%	−9.44%	−9.95%
Consumption	+5.55%	+5.86%	+11.52%	+12.14%
Output	−0.84%	−0.89%	−1.67%	−1.76%
Quality	+3.77%	+3.98%	+7.77%	+8.19%
C. Anticipated spending cut				
Labor	−4.61%	−4.87%	−9.44%	−9.95%
Consumption	+1.51%	+0.36%	+3.34%	+0.93%
Output	−4.88%	−6.38%	−9.86%	−12.96%
Quality	−0.27%	−1.51%	−0.42%	−3.01%

Notes: The table reports percent differences between the level of a variable in the counterfactual economy along the new BGP, after the fiscal consolidation, and the level that would have prevailed along the old BGP. In all experiments, the initial debt-GDP ratio is zero. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

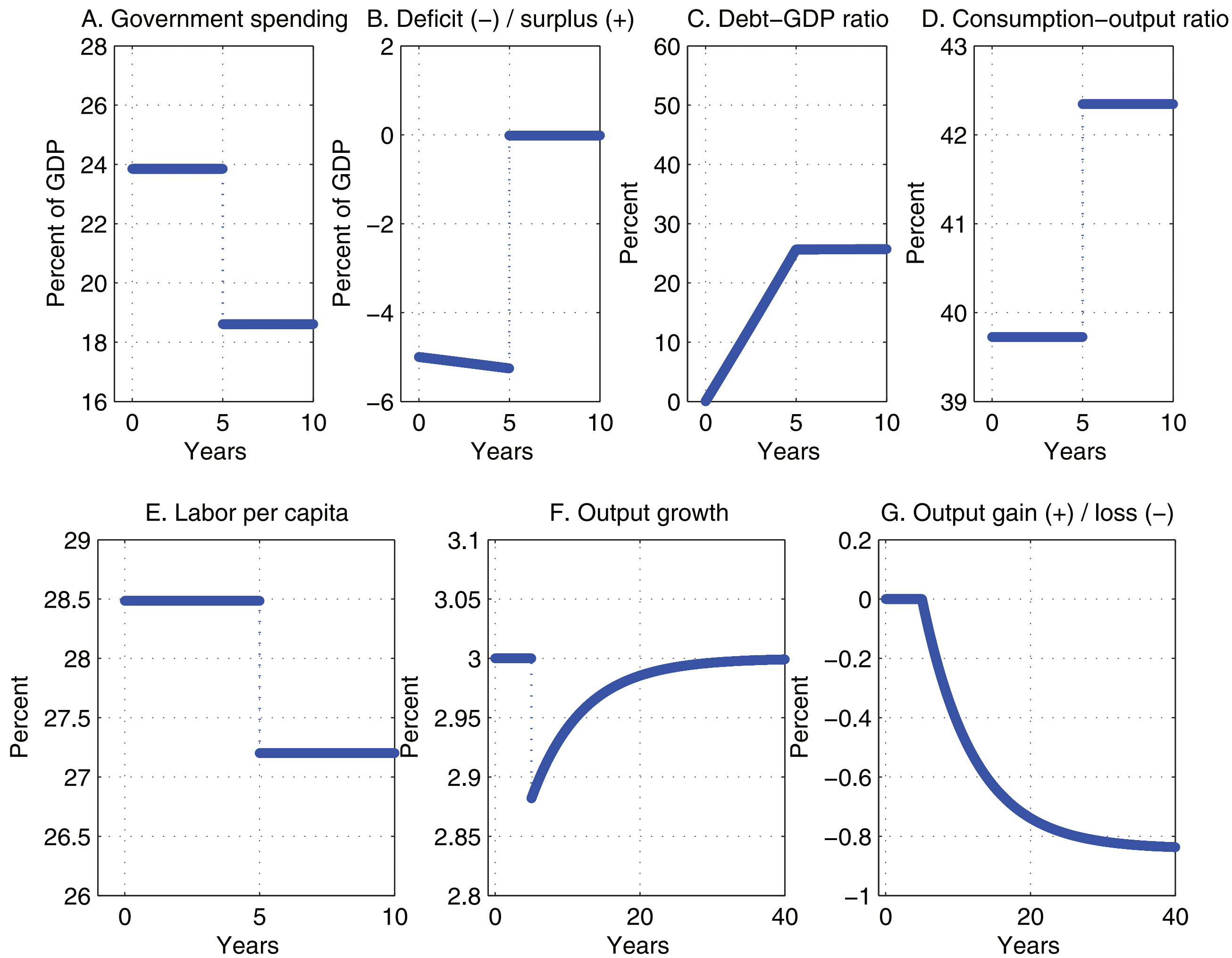

Fig. 5. Unanticipated spending-based consolidation. *Notes*: 5% initial deficit-GDP ratio; 5-year plan. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

In Table 2, we report the *long-run* elasticity of aggregate output per capita relative to trend with respect to a permanent change in government policy (either a tax rate hike or a spending cut):

$$\log y'_t - \log y_\infty \approx \log Z'_t - \log Z_\infty + \log x'_t - \log x_\infty, \text{ for } t = 100. \tag{59}$$

Note that the approximation in (59) is exact as $t \to \infty$, i.e. when output per capita in the counterfactual economy settles on the new BGP, y'_∞.[16] Along this new BGP,

$$\log x'_\infty - \log x_\infty = \log l'_\infty - \log l_\infty, \tag{60}$$

as the long-run, average firm size is invariant to government policy.

The *short-run* output elasticity with respect to the same permanent change in government policy is

$$\begin{aligned}\log y'_t - \log y_\infty &= \log Z'_t - \log Z_\infty + \log x'_t - \log x_\infty \\ &\quad + \gamma\left[\log l'_t - \log l_\infty - \left(\log x'_t - \log x_\infty\right)\right], \text{ for } t \geq 0,\end{aligned} \tag{61}$$

where the term in square brackets on the right-hand side of (61) vanishes as the economy settles on the new BGP.

Figs. 3, , , , ,–8 show the short-run elasticities of aggregate output per capita and transition dynamics of key variables from the old to the new BGP for three experiments: (i) tax-based consolidation; (ii) *unanticipated* spending-based consolidation; (iii) *anticipated* spending-based consolidation. In all three experiments, the initial budget deficit is 5% of GDP, and the stabilization occurs after 5 years. (Transition dynamics when initial budget deficits are 10% of GDP and/or the stabilization takes place after 10 years are similar, qualitatively.)

[16] To check the accuracy of the approximation in (59), we simulated time paths for $t > 100$ and found that the measured long-run output losses were in fact indistinguishable.

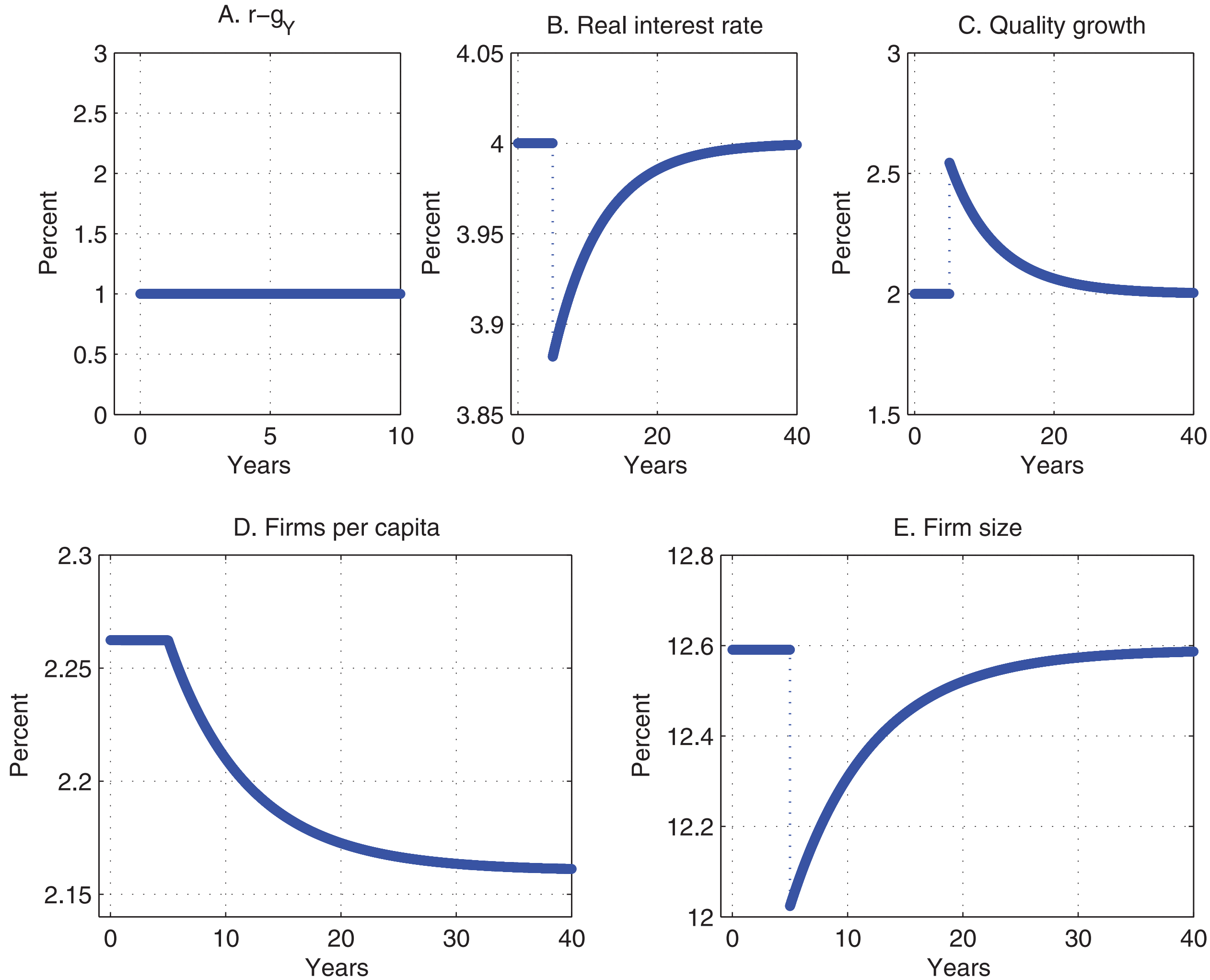

Fig. 6. Unanticipated spending-based consolidation. *Notes*: 5% initial deficit-GDP ratio; 5-year plan. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

Computing equilibria To compute the equilibrium time paths of prices and quantities in the counterfactual economy, we must numerically solve the system of three ODEs in (27)–(29). To allow convergence to a BGP, we assume that the government policy variables remain constant from the stabilization date $t_b \geq 0$ onwards. Further, as lump-sum taxes are not available, we use an iterative algorithm that alters a particular fiscal instrument until the balanced-budget rule (47) is satisfied. For example, in the tax-based experiments, we first compute a candidate equilibrium for an arbitrary tax rate, then check whether the balanced-budget rule is satisfied. Usually, one finds that in the candidate equilibrium lump-sum taxes are needed. If the government budget would have a deficit in present value terms, without lump-sum taxes, then we increase the tax rate and iterate until we find the tax rate that makes the balanced budget rule hold from t_b onwards.[17]

In the case of spending-based stabilizations, we use the same procedure altering the government spending-to-output ratio, g, and keeping the tax rate constant at its baseline value. Note that when we implement experiments in which both fiscal instruments are changed at the same time, one has to restrict the class of policies considered since there are many different combinations of government spending and tax rates that satisfy the government budget constraint.

7.1. *Tax-based fiscal consolidations*

A benchmark economy runs along the BGP with no outstanding government debt and a balanced budget. In a counterfactual economy, an unanticipated increase in government spending produces a budget deficit. We let the economy accumulate debt for $t_b \in \{5, 10\}$ years and boost taxes to keep the debt-to-GDP ratio at its new level forever, keeping the government spending-GDP ratio unchanged. Note that the government commits to this plan at $t = 0$, so that the private sector

[17] Braun (1994) and McGrattan (1994) use a similar algorithm to compute the equilibrium of an RBC model with distortionary taxation.

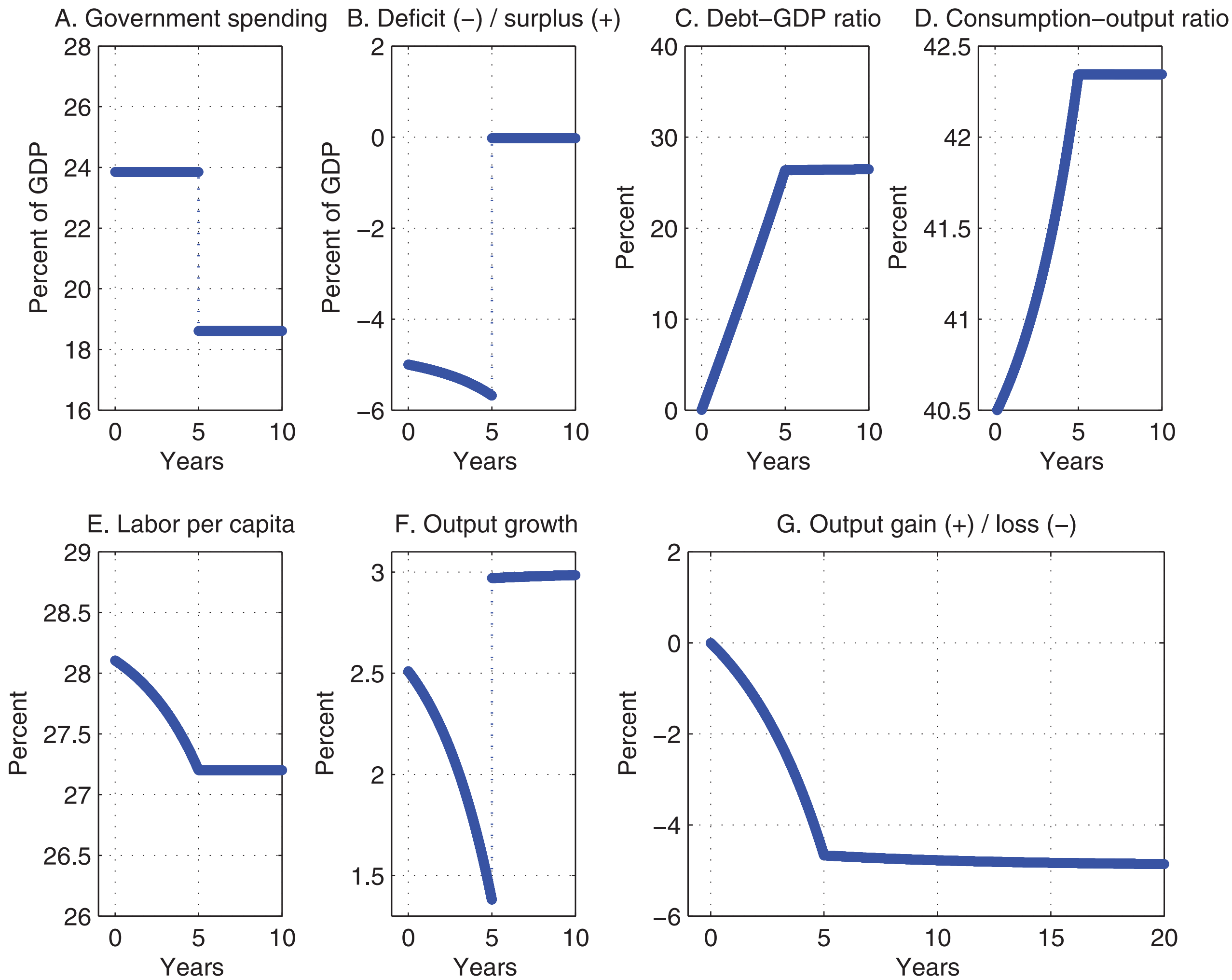

Fig. 7. Anticipated spending-based consolidation. *Notes*: 5% initial deficit-GDP ratio; 5-year plan. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

anticipates that at year t_b the tax rate will go up to balance the budget.[18] We consider two different counterfactual economies; one has an initial 5% deficit-GDP ratio, the other has a 10% deficit-GDP ratio. In the postwar U.S. experience, sustained budget deficits in the order of 5% and 10% of GDP have occurred during the eighties and in the aftermath of the Great Recession of 2007.

Long-run output losses Stabilizing the government debt-GDP ratio at its higher level after 5 years requires a 8 percentage points (p.p.) increase in the tax rate. In equilibrium, such an increase in distortions implies a 1.57% permanent loss in aggregate output. Per capita labor falls by nearly 9%. (Note that in this experiment the consumption-to-output ratio does not change, so that output losses equal consumption losses.)

If one stabilizes the debt-GDP ratio after 10 instead of 5 years, the output loss raises only slightly from 1.57% to 1.65%, pointing to a small cost of delaying fiscal adjustment. Though surprising, the relatively small output costs of delaying the debt stabilization are an immediate implication of the government budget constraint. In our economy, $r - g_Y = 1\%$, so that the cost in terms of output of delaying the fiscal stabilization from the current to the next period is outweighed by the size of the budget deficit. We stress, however, that for any alternative, empirically plausible parametrization of $r - g_Y$, the cost of delaying fiscal adjustment is bound to be small, given the observed real returns on government debt and output growth rates in the United States over the postwar period (see Seater, 1981; Hall and Sargent, 2011).

The size of the initial budget deficit makes a big difference in our calculations. With an initial budget deficit-GDP ratio of 10%, a fiscal stabilization implemented after 5 years generates an output loss of 3.16%, which is twice as large as that with an initial 5% deficit-GDP ratio. The permanent output loss raises only slightly to 3.32%, if the stabilization occurs after 10

[18] In the tax-based experiments, there are no "anticipation effects," in the sense that current variables do not respond to the announcement of the future tax hike. This happens as, in our environment, the consumption-to-output ratio remains at its steady-state value insofar as the government spending-to-output ratio does not change.

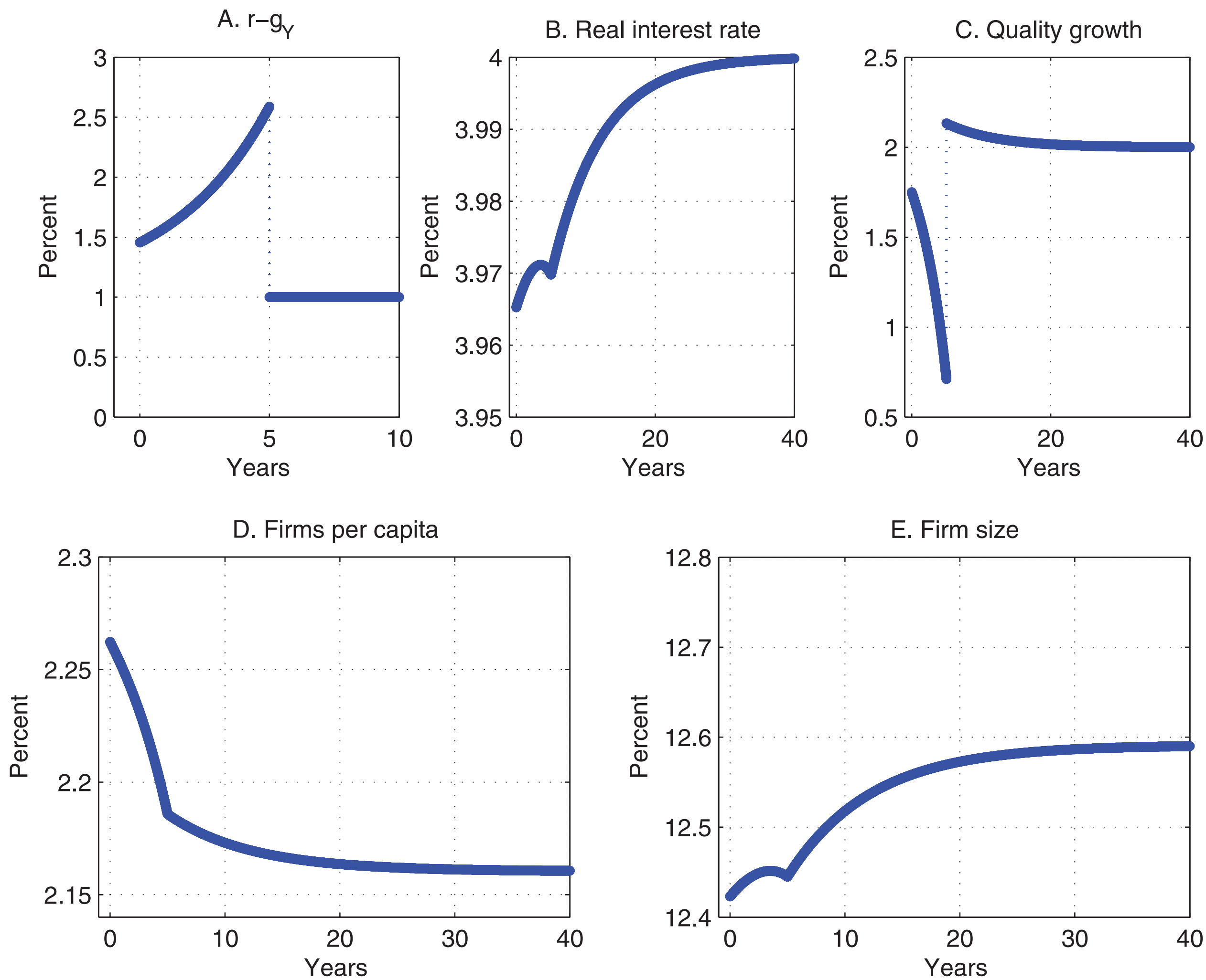

Fig. 8. Anticipated spending-based consolidation. *Notes*: 5% initial deficit-GDP ratio; 5-year plan. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

years. In this case, per capita labor falls by nearly 20%. We stress that the magnitude of these responses is comparable to the drops in hours worked and output observed during a typical U.S. recession. Our measurement thus points to quantitatively important recessionary effects of tax-based fiscal consolidations.

Short-run growth slowdown In all tax-based experiments, transition dynamics starts with the permanent fall in labor per capita, in response to the permanent increase in the tax rate. The transition towards the new BGP continues with the sluggish reduction in the mass of firms per capita towards a permanently lower level. Output growth experiences a temporary, but persistent deceleration. It takes nearly 20 years for the economy to reach the new BGP.

Note that big changes in income tax rates are associated with small changes in growth rates. The output growth rate falls by 0.2 p.p. on impact, and reverts back to its 3% long-run level over time. Yet, these small changes in growth rates cumulate to sizable level effects. Quantitatively, the recessionary effect of the drop in per capita labor is magnified by the endogenous fall in the mass of firms per capita. Product quality growth, instead, temporarily accelerates, thus dampening the adverse output effect of the tax hike.

7.2. *Spending-based fiscal consolidations*

For the spending-based experiments, anticipation effects turn out to be important, both qualitatively and quantitatively. To formalize the idea of an unanticipated spending cut, we assume that the private sector expects the government to balance the budget at t_b with lump-sum taxes, however, when date t_b comes, the government cuts spending to ensure a balanced budget from t_b onwards. In this precise sense, the reduction in government spending is a “surprise.” In the case of an anticipated spending cut, instead, the private sector correctly anticipates a permanent reduction in government spending. In this latter case, the consumption-to-output ratio responds today in anticipation of the future change in the government spending-to-output ratio, according to the system in (55).

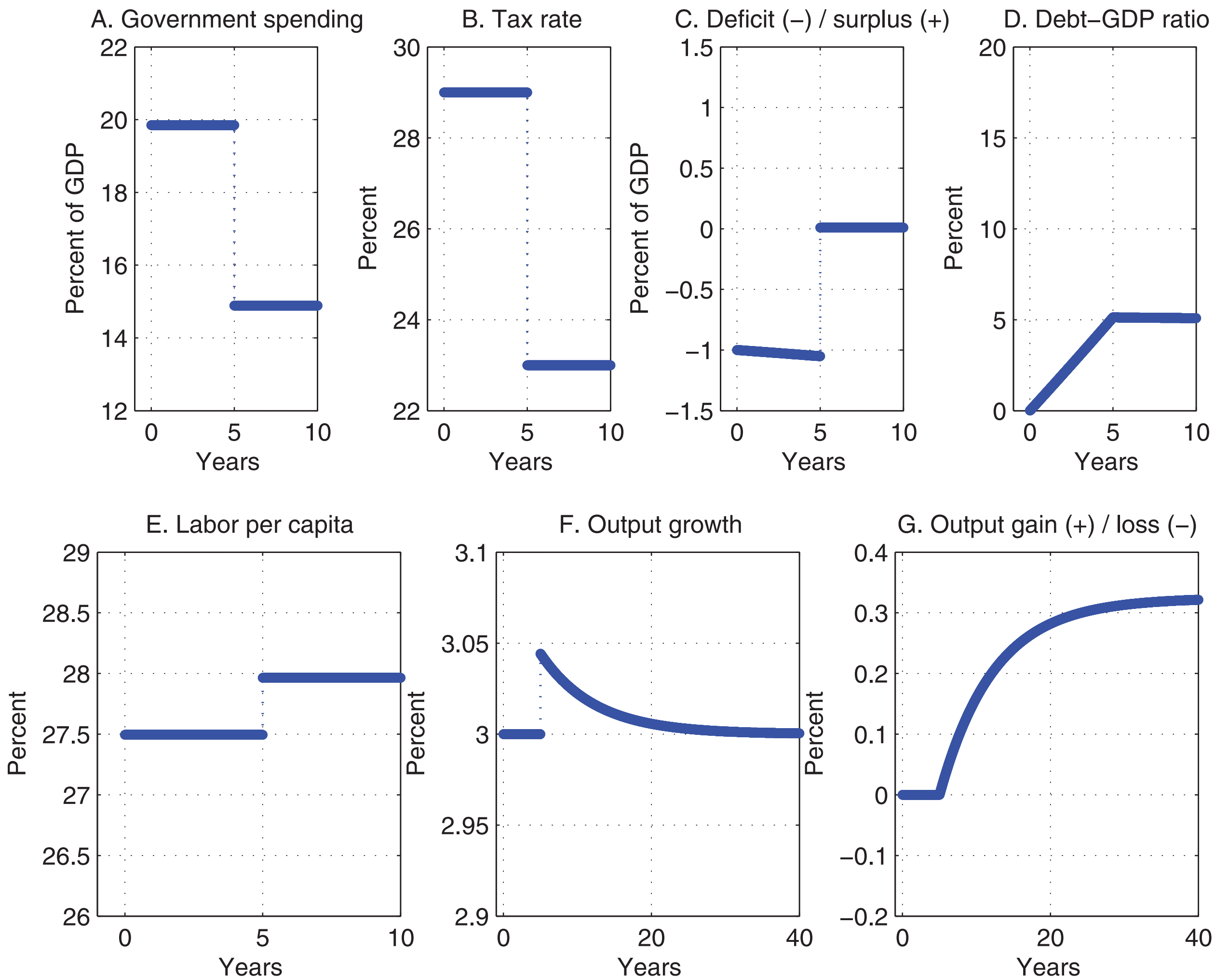

Fig. 9. Unanticipated Spending-Based Consolidation - 6 p.p. Tax Rate Cut. *Notes*: 1% initial deficit-GDP ratio; 5-year plan; 6 p.p. tax rate cut. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

Unanticipated spending cuts When the government surprises the private sector with an unanticipated cut in government purchases, output losses are considerably smaller than those originated from tax-based consolidations, approximately half as large across-the-board. Importantly, though, the economy experiences permanent gains in aggregate consumption. These gains are sizable: nearly 6% in the experiment with a 5% initial deficit-GDP ratio and as high as 12% when the initial budget deficit is 10% of GDP.

Transition dynamics in response to the unanticipated cut in government purchases are qualitatively similar to those in the tax-based experiments. At the time of the stabilization, the consumption-to-output ratio jumps up to its new steady-state level. Per capita labor falls. The economy experiences an output growth slowdown, relative to the BGP, fueled by the permanent fall in the labor input, and the sluggish reduction in the number of firms per capita. Along this transition, output growth and quality growth co-move negatively. As before, the temporary acceleration in quality growth partly offsets the contractionary output effect from the permanent reduction in labor and firms per capita.

Anticipated spending cuts When the government announces spending cuts beforehand, the output losses are substantially larger than those generated by unanticipated spending cuts. As a result, consumption gains are much smaller. What drives these large differences in the output cost of stabilization is the response of product quality growth. In the case of a tax-based consolidation and unanticipated spending cuts, quality growth temporarily accelerates, partly offsetting the permanent fall in firms' entry. By contrast, when spending cuts are anticipated, product quality growth decelerates, thus magnifying the overall effect of the fiscal stabilization. Output losses can be as high as 13% when the government awaits 10 years to stabilize the debt-GDP ratio resulting from an initial 10% deficit-GDP ratio.

7.3. Expansionary austerity

So far we have argued that fiscal consolidations inevitably produce losses in aggregate output. Here, we ask the question: Can government spending cuts lead to an expansion in output per capita? We will refer to this possibility as "expansionary

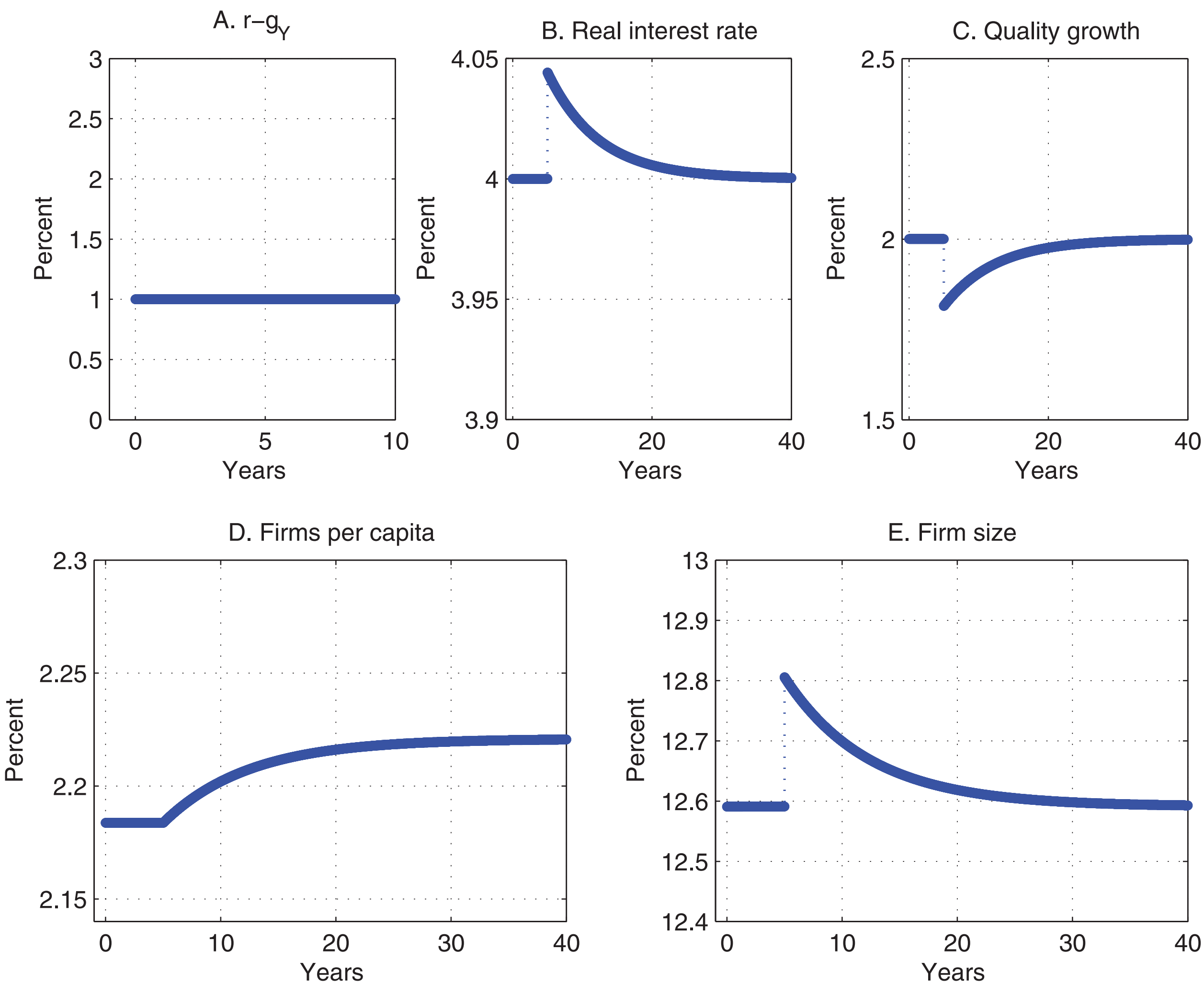

Fig. 10. Unanticipated Spending-Based Consolidation - 6 p.p. Tax Rate Cut. *Notes*: 1% initial deficit-GDP ratio; 5-year plan; 6 p.p. tax rate cut. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

Table 3
Long-term gains (+) and losses (−) fromspending-based consolidations - 6 p.p. tax rate cut.

	1% deficit 5-yrs delay	5% deficit 5-yrs delay	10% deficit 5-yrs delay
A. Unanticipated spending cut			
Labor	+1.69%	−1.99%	−6.81%
Consumption	+6.10%	+10.52%	+16.46%
Output	+0.33%	−0.37%	−1.23%
Quality	−1.36%	+1.62%	+5.58%
B. Anticipated spending cut			
Labor	+1.69%	−1.99%	−6.81%
Consumption	+2.40%	+3.63%	+5.45%
Output	−3.37%	−7.26%	−12.23%
Quality	−5.06%	−5.27%	−5.42%

Notes: The table reports percent differences between the level of a variable in the counterfactual economy along the new BGP, after the fiscal consolidation, and the level that would have prevailed along the old BGP. In all experiments, the initial debt-GDP ratio is zero; 6 p.p. tax rate cut. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

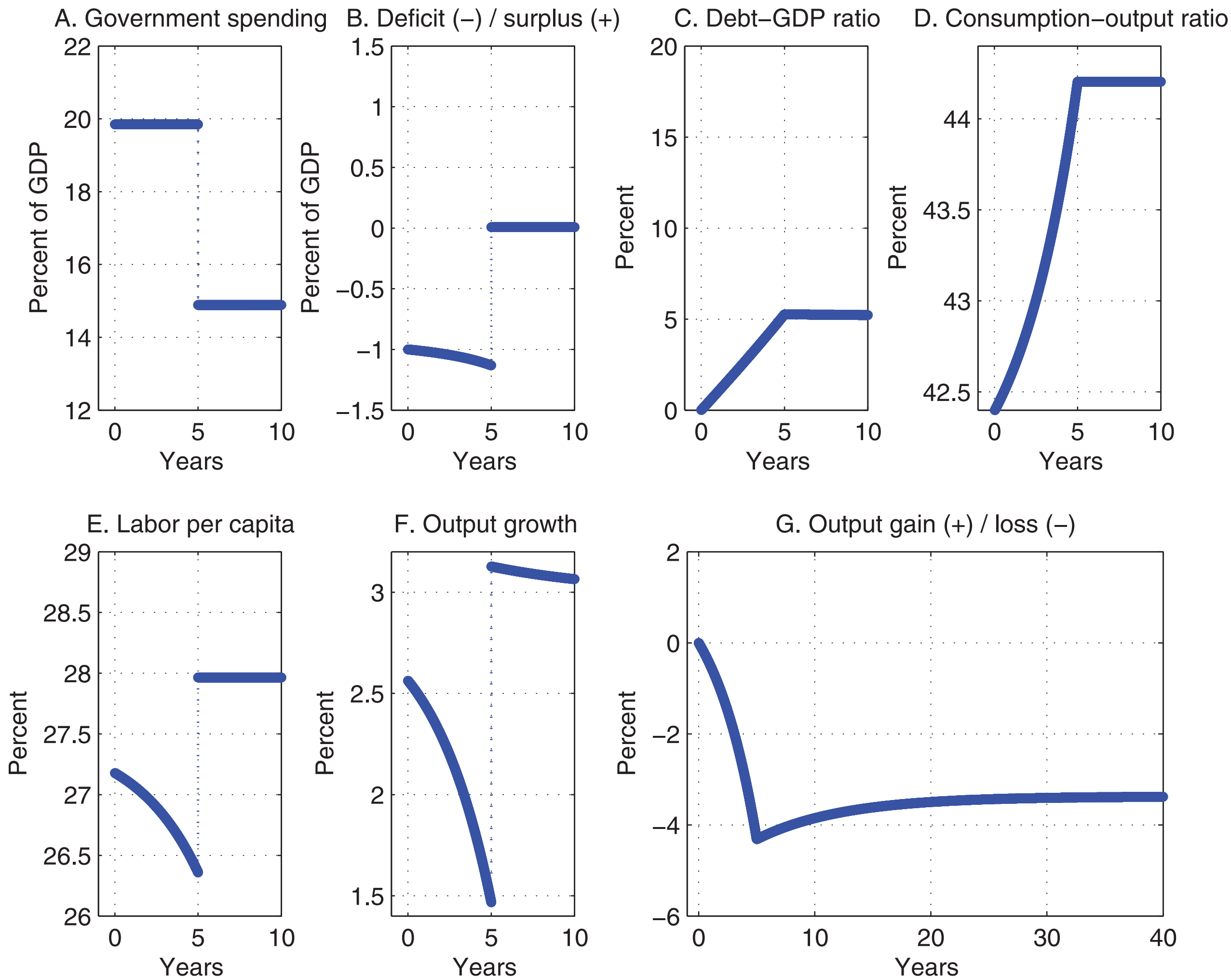

Fig. 11. Anticipated Spending-Based Consolidation - 6 p.p. Tax Rate Cut. *Notes*: 1% initial deficit-GDP ratio; 5-year plan; 6 p.p. tax rate cut. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

austerity." Our model suggests that expansionary austerity is possible to the extent that government spending cuts are accompanied by permanent reductions in distortionary income taxes. Overall, we find the case for expansionary austerity to be rather limited.

Table 3 summarizes the results. In all experiments, debt stabilization is implemented, with a 5-year delay, via a permanent reduction in government spending, accompanied by a 6 p.p. cut in the income tax rate. We consider three different initial deficit-GDP ratios: 1%, 5%, and 10%. Figs. 9, , ,–12 show the simulated time paths of the counterfactual economy for an initial 1% deficit-GDP ratio. (Transition dynamics for 5% and 10% deficit-GDP ratios are similar, qualitatively.)

We find only one case of expansionary austerity. With an initial deficit of 1% of GDP, an unanticipated spending-based consolidation (and a 6 p.p. tax rate cut) leads to a growth acceleration, with a 0.3% long-run permanent gain in output. In all other experiments, though, spending-based consolidations lead to aggregate output losses, but sizable gains in aggregate consumption.

8. Conclusion

In this paper, we study the effects of fiscal consolidations in a general equilibrium model of innovation-led growth. In the model, as arguably in actual economies, the government cannot use lump-sum taxes, such that temporary budget deficits inevitably call for higher distortionary tax rates and/or lower government spending in the future. By generating persistent growth slowdowns, fiscal consolidations permanently lower the path of per capita output. Consistently with the available empirical evidence, fiscal consolidations based on unanticipated government spending cuts are considerably less harmful than consolidations based on income tax rate hikes. However, announcements of future fiscal consolidations based on spending cuts induce sizable contractions in economic activity that are substantially larger than those under tax rate hikes.

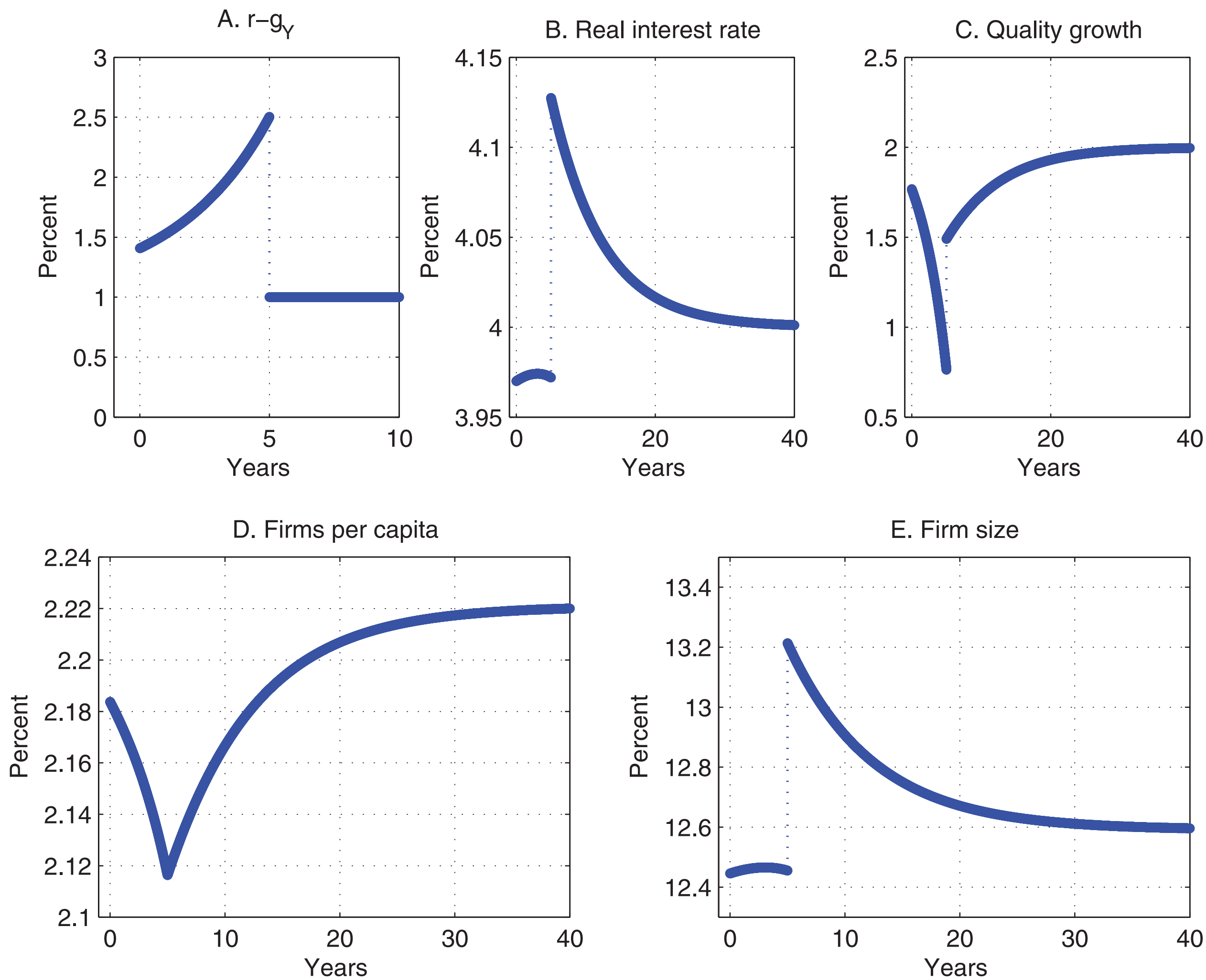

Fig. 12. Anticipated Spending-based consolidation - 6 p.p. tax rate cut. *Notes*: 1% initial deficit-GDP ratio; 5-year plan; 6 p.p. tax rate cut. Equilibrium time paths of the counterfactual economy are simulated by numerically solving the system of ODEs (27)–(29).

Our results indicate that the burden of government debt is large even in economies in which the difference between the real interest rate and the growth rate of output is small. While the additional costs of delaying a fiscal consolidation by one year are negligible, the cumulative effect of prolonged budget deficits is substantial.

Appendix A. Model derivations

A1. Problem of final producers

The problem of the final producer is

$$\max_{X_i, L_i, F} \int_0^N X_i^\theta \left[Z_i^\alpha Z^{1-\alpha} \left(L_i^\gamma F^{1-\gamma} \right) \right]^{1-\theta} di - p_i \int_0^N X_i di - w \int_0^N L_i di - p_F F. \tag{A.1}$$

The first-order conditions (FOCs) are:

$$\theta X_i^{\theta-1} \left[Z_i^\alpha Z^{1-\alpha} \left(L_i^\gamma F^{1-\gamma} \right) \right]^{1-\theta} = p_i, \tag{A.2}$$

$$(1-\theta) X_i^\theta \left[Z_i^\alpha Z^{1-\alpha} \left(L_i^\gamma F^{1-\gamma} \right) \right]^{-\theta} Z_i^\alpha Z^{1-\alpha} F^{1-\gamma} \gamma L_i^{\gamma-1} = w, \tag{A.3}$$

$$(1-\theta) X_i^\theta \left[Z_i^\alpha Z^{1-\alpha} \left(L_i^\gamma F^{1-\gamma} \right) \right]^{-\theta} Z_i^\alpha Z^{1-\alpha} L_i^\gamma (1-\gamma) F^{-\gamma} = p_F. \tag{A.4}$$

Rearranging Eqs. (A.2)–(A.4) yields factor demands:

$$X_i = \left(\frac{\theta}{p_i}\right)^{\frac{1}{1-\theta}} Z_i^{\alpha} Z^{1-\alpha} L_i^{\gamma} F^{1-\gamma}, \tag{A.5}$$

$$L_i = \gamma(1-\theta)\frac{p_i X_i}{\theta w}, \tag{A.6}$$

$$F = (1-\gamma)(1-\theta)\frac{p_i X_i}{\theta p_F}. \tag{A.7}$$

A2. Problem of intermediate producers

The current value Hamiltonian is

$$H_i = \Pi_i - \phi Z - R_i + q_i R_i. \tag{A.8}$$

The FOCs are:

$$\frac{\partial H_i}{\partial R_i} = -1 + q_i = 0 \quad \rightarrow \quad q_i = 1, \tag{A.9}$$

$$\frac{\partial H_i}{\partial Z_i} = \frac{\partial \Pi_i}{\partial Z_i} = r q_i - \frac{\dot{q}_i}{q_i}. \tag{A.10}$$

Using (A.9), Eq. (A.10) yields:

$$\frac{\partial \Pi_i}{\partial Z_i} = r. \tag{A.11}$$

Next, using $\Pi_i = (p_i - 1)X_i$,

$$\frac{\partial \Pi_i}{\partial Z_i} = \left(\frac{1}{\theta} - 1\right)\frac{\partial X_i}{\partial Z_i} = \alpha\left(\frac{1}{\theta} - 1\right)\frac{X_i}{Z_i} = \alpha\left(\frac{\Pi_i}{Z_i}\right). \tag{A.12}$$

Finally, using Eqs. (A.11) and (A.12) yields:

$$r = \alpha\left(\frac{\Pi_i}{Z_i}\right). \tag{A.13}$$

A3. Aggregate resource constraint

The government budget constraint reads:

$$\dot{B} = rB + G - \tau_L wL - T, \tag{A.14}$$

$$= rB + gY - \tau_L \gamma(1-\theta)Y - T. \tag{A.15}$$

The household's budget constraint reads:

$$\dot{B} + \dot{s}V + C = rB + sD + (1-\tau_L)wL + p_F F - T, \tag{A.16}$$

where

$$s = \int_0^N s_i di, \tag{A.17}$$

$$V = (1/N)\int_0^N V_i di, \tag{A.18}$$

$$D = (1/N)\int_0^N D_i di. \tag{A.19}$$

Using $s_i = 1$, for all i, yields:

$$\dot{B} + \dot{N}V + C = rB + sD + wL - \tau_L wL + p_F F - T. \tag{A.20}$$

Next, using the government budget constraint in (A.15), Eq. (A.16) yields:

$$\dot{N}V + C + G = ND + wL + p_F F, \tag{A.21}$$

$$= ND + \gamma(1-\theta)Y + (1-\gamma)(1-\theta)Y, \tag{A.22}$$

$$= ND + (1-\theta)Y. \tag{A.23}$$

Next, using the free-entry condition $V = \beta X$ and $D = (p-1)X - \phi Z - R$, Eq. (A.23) yields:

$$C + G + \dot{N}\beta X - ND + \theta Y = Y, \tag{A.24}$$

$$C + G + \dot{N}\beta X - N(pX - X - \phi Z - R) + \theta Y = Y, \tag{A.25}$$

$$C + G + \dot{N}\beta X + NR + N\phi Z + NX = Y, \tag{A.26}$$

$$C + G + \mathcal{I} + \mathcal{Q} = Y, \tag{A.27}$$

where $\mathcal{I} \equiv \dot{N}\beta X + NR + N\phi Z$ and $\mathcal{Q} \equiv NX$.

A4. Steps towards the dynamical system

We describe the derivation of the dynamical system in three steps.

Step 1 Let us start with Eq. (A.23),

$$\dot{N}V = ND + (1-\theta)Y - C - G, \tag{A.28}$$

$$\dot{N}V = N\Pi - N\phi Z - NR + (1-\theta)Y - C - gY, \tag{A.29}$$

$$= N\Pi - N\phi Z - NR + (1-\theta-g)Y - C. \tag{A.30}$$

Multiplying left- and right-hand side of Eq. (A.30) by NV yields:

$$\frac{\dot{N}}{N} = \frac{\Pi}{V} - \frac{\phi Z}{V} - \frac{R}{V} + \frac{(1-\theta-g)Y - C}{NV}, \tag{A.31}$$

$$= \frac{\Pi}{V} - \frac{\phi Z}{V} - \frac{R}{V} + \frac{(1-\theta-g)Y - C}{NV} \times \frac{Y}{Y}, \tag{A.32}$$

$$= \frac{\Pi - \phi Z - R}{\beta X} + \frac{1-\theta-g-\tilde{c}}{NV/Y}, \tag{A.33}$$

where we used the free-entry condition $V = \beta X$ and $\tilde{c} \equiv C/Y$. Using the relationship $NX = \theta^2 Y$, Eq. (A.33) yields:

$$\frac{\dot{N}}{N} = \frac{N(\Pi - \phi Z - R)/Y}{\beta\theta^2} + \frac{1-\theta-g-\tilde{c}}{\beta\theta^2}. \tag{A.34}$$

Equation (A.34) can be rewritten in a more compact form as

$$n = \frac{\pi}{\beta\theta^2} + \frac{1-\theta-g-\tilde{c}}{\beta\theta^2}, \tag{A.35}$$

where $n \equiv \dot{N}/N$ and $\pi \equiv ND/Y$. After some manipulations,

$$\pi = \theta(1-\theta)\left[1 - \frac{(\phi+z)\alpha}{r}\right], \tag{A.36}$$

where $z \equiv \dot{Z}/Z$. Finally, using $x \equiv N/e^{\lambda t}$ and $\dot{x}/x = \dot{N}/N - \lambda$ yields:

$$\frac{\dot{x}}{x} = \frac{\pi}{\beta\theta^2} + \frac{1-\theta-g-\tilde{c}}{\beta\theta^2} - \lambda. \tag{A.37}$$

Step 2 Let us start with Eq. (A.33),

$$\frac{\dot{N}}{N} = \frac{D}{V} + \frac{1-\theta-g-\tilde{c}}{NV/Y}. \tag{A.38}$$

Using the relationship $D/V = r - \dot{V}/V$, Eq. (A.38) yields:

$$\frac{\dot{N}}{N} + \frac{\dot{V}}{V} = r + \frac{1-\theta-g-\tilde{c}}{NV/Y}, \tag{A.39}$$

$$= r + \frac{1-\theta-g-\tilde{c}}{\beta\theta^2}. \tag{A.40}$$

Next, using the free-entry condition $V = \beta X$ and $\dot{V} = \beta\dot{X}$, Eq. (A.40) yields:

$$\frac{\dot{N}}{N} + \frac{\dot{X}}{X} = r + \frac{1-\theta-g-\tilde{c}}{\beta\theta^2}. \tag{A.41}$$

Next, using the relationship $NX = \theta^2 Y$, Eq. (A.41) yields:

$$\frac{\dot{Y}}{Y} = r + \frac{1-\theta-g-\tilde{c}}{\beta\theta^2}. \tag{A.42}$$

Finally, let us consider the Euler equation,

$$\frac{\dot{C}}{C} = r - \rho + \lambda. \tag{A.43}$$

Using Eqs. (A.42), (A.43) yields:

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} = \lambda - \rho - \frac{1-\theta-g-\tilde{c}}{\beta\theta^2}. \tag{A.44}$$

Step 3 Using the reduced-form production function, $Y = \theta^{\frac{2\theta}{1-\theta}} Z N^{1-\gamma} L^{\gamma}$,

$$\frac{\dot{Y}}{Y} = \frac{\dot{Z}}{Z} + (1-\gamma)\frac{\dot{N}}{N} + \gamma\frac{\dot{L}}{L}, \tag{A.45}$$

$$= \frac{\dot{Z}}{Z} + \frac{\dot{N}}{N} - \gamma\left(\frac{\dot{N}}{N} - \frac{\dot{L}}{L}\right), \tag{A.46}$$

so that

$$\frac{\dot{Z}}{Z} = \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} + \gamma\left(\frac{\dot{N}}{N} - \frac{\dot{L}}{L}\right), \tag{A.47}$$

$$= \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} + \gamma\left(\frac{\dot{N}}{N} - \frac{\dot{l}}{l} - \lambda\right), \tag{A.48}$$

where

$$\frac{\dot{l}}{l} = -\frac{\eta\tilde{c}}{(1-\tau_L)\gamma(1-\theta)+\eta\tilde{c}} \times \left(\frac{\dot{\tilde{c}}}{\tilde{c}}\right). \tag{A.49}$$

Using (A.49), Eq. (A.48) yields:

$$\frac{\dot{Z}}{Z} = \frac{\dot{Y}}{Y} + (\gamma-1)\left(\frac{\dot{N}}{N}\right) + \gamma\omega(\tilde{c})\left(\frac{\dot{\tilde{c}}}{\tilde{c}}\right) - \gamma\lambda, \tag{A.50}$$

with

$$\omega(\tilde{c}) \equiv \frac{\eta\tilde{c}}{(1-\tau_L)\gamma(1-\theta)+\eta\tilde{c}}. \tag{A.51}$$

Finally, using (A.42), (A.44), and (A.35), and some manipulations, Eq. (A.50) yields:

$$\frac{\dot{Z}}{Z} = \left[1 - \frac{\alpha\mu}{\beta r}\right]^{-1}\left[r + \frac{\phi\alpha\mu}{\beta r} - \frac{\mu}{\beta} + \gamma\omega(\tilde{c})(\rho-\lambda) + \gamma\omega(\tilde{c})\left(\frac{1-\theta-g-\tilde{c}}{\beta\theta^2}\right)\right]. \tag{A.52}$$

A5. Solution to the Bernoulli ODE

The unstable ODE for the consumption-to-output ratio can be rewritten as

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = -\frac{\tilde{c}^*}{\beta\theta^2} + \frac{\tilde{c}}{\beta\theta^2}, \tag{A.53}$$

where $\tilde{c}^* = (\rho - \lambda)\beta\theta^2 + 1 - \theta - g$ is the steady-state consumption-to-output ratio. Note that (A.53) is a Bernoulli differential equation:

$$\dot{\tilde{c}} = q_1\tilde{c} + q_2\tilde{c}^2, \tag{A.54}$$

where $q_1 \equiv -\tilde{c}^*/\beta\theta^2 < 0$ and $q_2 \equiv 1/\beta\theta^2 > 0$. As the solution to the Bernoulli Eq. (A.54) is well-known, here, we sketch only the key steps for its derivation. First, let's define $\nu \equiv \tilde{c}^{-1}$ so that $\dot{\nu} = -\dot{\tilde{c}}/\tilde{c}^2$. Second, through a change of variable, (A.54) reduces to a linear first-order ODE,

$$\dot{\nu} = -q_1\nu - q_2, \tag{A.55}$$

whose solution is

$$\nu = (\nu_0 - \nu^*)e^{-q_1 t} + \nu^*, \tag{A.56}$$

where $\nu_0 \equiv 1/\tilde{c}_0$ and $\nu^* \equiv 1/\tilde{c}^*$. Third, using $\nu \equiv \tilde{c}^{-1}$, (A.56) yields the solution to (A.53) in terms of the consumption-to-output ratio,

$$\tilde{c} = \frac{\tilde{c}^*}{\left(\frac{\tilde{c}^*}{\tilde{c}_0} - 1\right)e^{(\tilde{c}^*/\beta\theta^2)t} + 1}. \tag{A.57}$$

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.euroecorev.2019.103350.

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