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Research paper

Political fragmentation versus a unified empire in a Malthusian economy ☆

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ABSTRACT

What are the historical origins of political fragmentation and unification? This study develops a Malthusian growth model with multiple states to explore interstate competition and the endogenous emergence of political fragmentation versus a unified empire. Our model features an agricultural society with citizens and rulers in a Malthusian environment in which the expansion of one state may come at the expense of another state, depending on the intensity of interstate competition captured by the elasticity of the land ratio with respect to the population ratio between states. If this elasticity is less than unity, then multiple states coexist. However, if this elasticity is equal to unity, then a unified empire emerges. Which state becomes the unified empire depends on its military power, agricultural productivity, and its rulers' preference for rent-seeking Leviathan taxation. We also discuss the historical relevance of these theoretical predictions in the Warring States period of ancient China.

> *But where did this political fragmentation come from? Why was Europe decentralized and characterized by competition among relatively small powers, while extensive regions of Asia were controlled by monolithic mega-empires?* Galor (2022, *p. 184).*

1. Introduction

Historically, ancient China had a tendency towards a unified empire, whereas Europe had a tendency towards political fragmentation. Many studies have analyzed the economic implications of the European interstate competition. For example, economic historians, such as North (1981) and Jones (1981), have long argued that political fragmentation and interstate competition in Europe contributed to its economic takeoff in the late eighteenth and early nineteenth century.[1] However, there was also an extended period of political fragmentation in ancient China before a unified empire became the norm with the establishment of the Qin dynasty in 221 BC, which is often known as the first dynasty of Imperial China. Therefore, an interesting question is how ancient China evolved from multiple states to a unified empire.

In this study, we develop a Malthusian growth model with multiple states to explore interstate competition and the endogenous emergence of political fragmentation versus a unified empire in human society. Our model features an agricultural society with

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[1] See also Hume (1741). For more recent work, see Hoffman (2015), Mokyr (2016) and Scheidel (2019).

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citizens and rulers in a Malthusian environment in which the population size of a state is determined by the amount of land it occupies. Following the literature on the macrotechnology of conflict, we adopt a conflict success function for the division of land between competing states, and the share of land occupied by each state is an increasing function in its share of the total population. In other words, a state with a larger population relative to other states is able to capture more land, and its ability to do so partly depends on its military power. Whether or not its population expansion comes at the expense of other states also depends on the intensity of interstate competition, which is captured by the elasticity of the land ratio with respect to the population ratio between states. If this elasticity is less than unity, then multiple states coexist in the steady-state equilibrium. However, if this elasticity is equal to unity, then only one state (i.e., an empire) survives in the long run. Other states disappear because they lose land, which is a crucial input for agricultural production and leads to the eventual collapse of their population due to the lack of food.

Which state becomes the unified empire depends on population growth, which is increasing in the state's level of military power and agricultural productivity and the degree of its citizens' fertility preference but decreasing in the degree of their leisure preference and the degree of the rulers' preference for rent-seeking Leviathan taxation. Furthermore, in the long run, the unified empire has a larger population size than the total population size of multiple states under political fragmentation if the level of military power is the same across states. However, if the state that becomes the unified empire accomplishes this empire formation mainly by having the highest level of military power, then the unified empire may not achieve the largest population size. This is because military power only contributes to population growth via the competition of resources but does not improve the efficiency of the conversion from natural resources to population growth.

This study relates to the economic history literature on political fragmentation and economic development. The beneficial effects of interstate competition on economic development proposed in this literature include the protection of property rights in North (1981), the stimulation of military innovation in Hoffman (2015), the presence of a competitive market for ideas in Mokyr (2016) and the prevention of innovation being blocked by rulers in Scheidel (2019). This study complements the interesting studies in this literature by developing a growth-theoretic framework (i.e., a Malthusian growth model with interstate competition) that captures the transition from political fragmentation to unification that happened in ancient China.

Therefore, this study relates more closely to the growth-theoretic literature on interstate competition and economic growth; see for example, Chaudhry and Garner (2006), Karayalcin (2008), Chu (2010) and Lagerlof (2014). Chaudhry and Garner (2006) use a Schumpeterian growth model to show that the presence of rival states prevents rulers' from blocking innovation and stifling growth. Karayalcin (2008) compares the growth effects of political fragmentation versus unification and explores the effects of tax competition on capital accumulation and economic growth. Chu (2010) considers an AK growth model to explore the effect of military competition in addition to the effect of tax competition on economic growth under political fragmentation and shows that political fragmentation does not necessarily achieve higher economic growth than political unification.[2] Lagerlof (2014) also develops a Malthusian growth model to analyze the scale effect and competition effects on economic growth under fragmentation versus unification but focuses on showing that unification dominates (is dominated by) fragmentation at an early (later) stage of development. The current study considers a related Malthusian growth model and contributes to this literature by exploring the endogenous emergence of political fragmentation versus unification in human society and their consequences on population growth in an agricultural economy.[3] Therefore, this study also relates to studies on the interaction of renewable resources and population growth; see for example, Brander and Taylor (1998), Prskawetz et al. (2003) and de la Croix and Dottori (2008) for their analysis on the population collapse of Easter Island.

This study also relates to a recent study by Fernandez-Villaverde et al. (2023), who develop a quantitative spatial model to evaluate the importance of agricultural productivity and the fractured-land hypothesis, popularized by Diamond (1997), on the origins of political fragmentation in Europe and political unification in China.[4] Our study differs from them by using an analytically tractable Malthusian growth model in which agricultural production, endogenous population growth and interstate resource competition interact to generate a possible transition from political fragmentation to a unified empire.[5] Whether such a transition happens depends on an elasticity parameter that captures the intensity of interstate competition, which could be determined by how fractured land is in a more structural model. Finally, our study also relates to the literature on endogenous country formation pioneered by Alesina and Spolaore (2003, 2005), who explore the determinants of the size and number of states. We use a Malthusian growth model to explore the factors that determine the dynamics of the size of states and whether a unified empire or multiple states emerge in the long run.

The rest of this study is organized as follows. Section 2 discusses the historical context of our theory. Section 3 presents the Malthusian growth model with multiple states. Section 4 explores the dynamics of interstate competition. The final section concludes.

2. Historical context

Before the unification of China by the Qin dynasty in 221 BC, ancient China had an extended period of political fragmentation. During the Warring States period from 481 BC to 221 BC, there were seven major states (including the Qin state) and many minor

[2] Galor (2022), p. 185–186) also argues that the relative growth performance of political fragmentation versus unification depends on the stages of economic development.

[3] Voigtlander and Voth (2013) develop a two-agricultural-sector Malthusian model to show that changes in agricultural production gave rise to delayed marriage and limited fertility in Europe.

[4] See also Ko et al. (2018) who use a Hoteling-style spatial model to show that political unification emerged in China in order to counter the threat of severe unidirectional external invasions.

[5] See also Chu et al. (2022) and Chu and Xu (2024) for theoretical and empirical analyses on the effects of agricultural productivity on the transition of human society from agriculture to industrial production.

states competing with each other.[6] In the final phase (from 247 BC to 221 BC) of the Warring States period, the Qin state became the most powerful state among the Seven Warring States (with the other six states being the Chu state, the Han state, the Qi state, the Wei state, the Yan state, and the Zhao state). In 230 BC, the Qin state unleashed a final set of military campaigns to conquer the other six states one by one, with the first military target being the Han state. In 221 BC, the remaining Qi state surrendered. As a result, the Qin state unified China and established the Qin dynasty. Since then, subsequent dynasties in China had a tendency towards a unified empire, despite some periods of political fragmentation.

Our growth theoretic model predicts that the state with the highest levels of military power and agricultural productivity and the lowest degree of rent-seeking Leviathan taxation would be the one that becomes the unified empire. These predictions are consistent with Qin's unification of China. It is often argued that the irrigation system in the Qin state (e.g., the Dujiangyan and Zhengguo Canal built in 256 BC and 246 BC, respectively) improved the state's agricultural productivity and provided the foundation for its expansionism and unification of China.[7] Furthermore, Shang Yang's political and economic reforms were also often credited to have laid the foundation for Qin's unification of China by weakening the extractive power of landed aristocrats.[8] Specifically, his miliary reform that took away land rights from the Qin nobility and gave them to soldiers with military achievements strengthened the Qin state's military power by providing incentives for Qin citizens to participate in military campaigns to conquer other states.

Our model also predicts that a unified empire does not necessarily emerge. A unified empire emerges only under a high elasticity of the land ratio with respect to the ratio of population between states. If the elasticity is less than unity, then multiple states coexist even in the long run, which corresponds to political fragmentation. This elasticity captures the intensity of interstate competition. A determinant of the intensity of interstate competition is geographic barriers between states. Fernandez-Villaverde et al. (2023) provide empirical support for the importance of the fractured-land hypothesis on the origins of political fragmentation in Europe and political unification in China. Their idea is that the European landscape fractured by high mountains led to a low intensity of interstate competition (which corresponds to a low elasticity of the land ratio with respective to the population ratio between states in our model) and gave rise to political fragmentation in Europe. In contrast, the lack of such high mountains in the Chinese landscape gave rise to a high intensity of interstate competition and a tendency towards political unification in ancient China.

Another determinant of the intensity of interstate competition is governing barriers between states. For example, the policies implemented by the Qin dynasty (on standardizing the units of measurements, currency, length of the axles of carts, legal system and the Chinese script) lowered the governing barriers between different regions of China and also contributed to maintaining a high degree of interstate competition during the intermittent periods of political fragmentation in China. As a result, the subsequent Chinese dynasties had a tendency towards a unified empire.

3. A Malthusian model with interstate competition

Malthus's (1798) insight that population growth is limited by the availability of natural resources, such as land, is at the heart of a thriving strand of research on economic growth. To explore interstate competition, we extend a canonical version of the recent vintage of Malthusian growth models. Specifically, we consider multiple states indexed by $i \in \{1, \dots, m\}$ and capture interstate competition over land with a conflict success function from the literature on the macrotechnology of conflict (Hirshleifer, 2000).[9] In each state there are citizens and rulers. Citizens engage in agricultural production and make fertility decisions, whereas rulers impose a tax on the agricultural output of citizens and consume the tax revenue. We adopt the standard environment of simple models in the Unified Growth Theory (UGT) tradition (see, for example, Ashraf and Galor (2011)):[10] there is no wage or land rental rate but citizens-farmers, who work the land given by the state, keep the after-tax agricultural output of their labor and allocate it to consumption and child rearing. In our conflict-augmented model, we add to this structure a realistic feature: our citizens are also citizens-soldiers who serve in the military. Therefore, the allotment of land that each citizen receives from the state can be interpreted as the payment for military service.

3.1. Agricultural production

In state i, the agricultural output of a citizen who chooses to devote l_t^i units of labor to farming at time t, is

$$y_t^i = \theta^i (l_t^i)^\alpha (z_t^i)^{1-\alpha} = \theta^i (l_t^i)^\alpha \left(\frac{Z_t^i}{N_t^i}\right)^{1-\alpha}. \tag{1}$$

The parameter $\theta^i > 0$ measures agricultural productivity in state i. The parameter $\alpha \in (0, 1)$ measures labor intensity in agriculture and is the same across states for simplicity. The amount of land occupied by state i is Z_t^i and is distributed equally to its citizens such that the land input per citizen is Z_t^i / N_t^i, where N_t^i is the number of citizens in state i at time t.

6 Even during the Spring and Autumn period from 722 BC to 481 BC, there were many *de facto* self-governing states, with the King of the Zhou dynasty merely acting as the figurehead.

7 See for example, Willmott (1989) and Bello (2020).

8 See for example, Kiser and Cai (2003).

9 Chu (2023) also adopts this conflict success function to explore competition between human species in a hunting-gathering economy, but he only shows the dynamics of two interacting populations.

10 See also Strulik and Weisdorf (2008) and Strulik (2017), who extend the unified growth model to explore the effects of agricultural/industrial productivity and contraception, respectively, on the demographic transition and the takeoff to modern growth.

3.2. Endogenous fertility

We consider overlapping generations of citizens. Each citizen lives for two periods. An adult citizen in state i at time t has utility function:

$$u_t^i = \beta^i \ln(1 - l_t^i) + (1 - \gamma^i) \ln c_t^i + \gamma^i \ln n_t^i. \tag{2}$$

The parameter $\beta^i \geq 0$ measures preference for leisure $1 - l_t^i$. The parameter $\gamma^i \in (0, 1)$ measures preference for fertility n_t^i, which denotes the citizen's number of children who then become adults in the next period. The cost of raising children is $\rho^i n_t^i$, where the parameter $\rho^i > 0$ is the cost per child in units of the agricultural good. Adult consumption c_t^i is also in units of the agricultural good.

In the environment of this UGT-style model, the citizen's resource constraint is

$$c_t^i + \rho^i n_t^i = (1 - \tau_t^i) y_t^i, \tag{3}$$

where $\tau_t^i \in (0, 1)$ is the tax rate on farming output imposed by the rulers and also a measure of distortion created by the ruling class. Substituting (1) and (3) into (2), we derive the utility-maximizing choices of fertility, consumption and labor:

$$n_t^i = \frac{\gamma^i (1 - \tau_t^i) y_t^i}{\rho^i}; \tag{4}$$

$$c_t^i = (1 - \gamma^i)(1 - \tau_t^i) y_t^i; \tag{5}$$

$$l_t^i = \frac{\alpha}{\beta^i + \alpha}. \tag{6}$$

These citizen-level decisions have standard properties. To derive the implications for the state's population dynamics we need to aggregate across citizens.

Each adult citizen has n_t^i children, and the number of adult citizens at time t is N_t^i. For simplicity, we assume that the rulers are a negligible fraction of the state's population and do not work in agriculture. Therefore, we can refer to N_t^i as the state's population size and write its law of motion as

$$N_{t+1}^i = n_t^i N_t^i = \frac{\gamma^i (1 - \tau_t^i) y_t^i}{\rho^i} N_t^i. \tag{7}$$

For future use, we also derive the population growth rate:

$$\frac{\Delta N_t^i}{N_t^i} \equiv \frac{N_{t+1}^i - N_t^i}{N_t^i} = \frac{\gamma^i (1 - \tau_t^i) y_t^i}{\rho^i} - 1, \tag{8}$$

which is increasing in the after-tax agricultural output $(1 - \tau_t^i) y_t^i$ and the fertility preference γ^i but decreasing in the fertility cost per child ρ^i.

3.3. Rulers

Each state i is governed by a small group of rulers. The objective function of the generation of rulers at time t is

$$U_t^i = \lambda^i \ln T_t^i + (1 - \lambda^i) \ln N_{t+1}^i, \tag{9}$$

where the parameter $\lambda^i \in (0, 1)$ measures the rulers' preference for rent-seeking Leviathan taxation T_t^i.[11] The tax revenue that they collect for their own consumption is

$$T_t^i = \tau_t^i y_t^i N_t^i. \tag{10}$$

Although the rulers want to collect as much tax revenue as possible, they also care about the future population size N_{t+1}^i of their state. Taking N_t^i as given, the rulers choose τ_t^i to maximize U_t^i. Substituting (1), (6), (7) and (10) into (9), we derive the tax rate chosen by the rulers as

$$\tau_t^i = \lambda^i, \tag{11}$$

which is increasing in the degree of their preference for rent-seeking Leviathan taxation.

[11] All our derivations are robust to a more general objective function $U_t^i = \lambda^i \ln T_t^i + (1 - \lambda^i)[(1 - \sigma^i) \ln N_{t+1}^i + \sigma^i u_t^i]$, where the parameter $\sigma^i \in [0, 1]$ determines the weight the rulers place on the utility of their citizens. Our results are also robust to T_t^i being used for providing a utility-enhancing public good to citizens, so long as this public good is separable in their utility function u_t^i.

3.4. Land competition

Following the literature on the macrotechnology of conflict, we consider a conflict success function for the division of land among competing states; see Hirshleifer (1991, 2000).[12] Specifically, the amount of land occupied by state $i \in \{1, \dots, m\}$ is

$$Z_t^i = \frac{\mu^i (N_t^i)^\phi}{\sum_{j=1}^m \mu^j (N_t^j)^\phi} Z, \tag{12}$$

where the parameter $\mu^i > 0$ represents the military power of state i and the parameter $Z > 0$ is the total amount of land. A larger population N_t^i enables state i to capture more land, unless $\phi = 0$. Furthermore, for a given level of population N_t^i, a larger μ^i also enables state i to capture a larger share of land. The literature refers to the functional form in (12) as the "ratio form" and to $\phi \in [0, 1]$ as the decisiveness parameter.

In our model, the parameter ϕ can be interpreted broadly to capture the intensity of competition between states or the (inverse) degree of fractionalization between states. For example, if $\phi = 0$, the land share of each state is $Z_t^i / Z = \mu^i / \sum_{j=1}^m \mu^j$, which simplifies further to $Z_t^i / Z = 1/m$ under the special case $\mu^i = \mu$ for all $i \in \{1, \dots, m\}$. In this special case, the countries are essentially isolated. As ϕ increases, the land share of state i becomes more and more sensitive to population. More precisely, it becomes more and more sensitive to the bilateral population ratios N_t^j / N_t^i for all $j \in \{1, \dots, m\}$. To see this, we rewrite (12) as

$$\frac{Z_t^i}{Z} = \frac{1}{\sum_{j=1}^m \frac{\mu^j}{\mu^i} \left(\frac{N_t^j}{N_t^i} \right)^\phi},$$

which shows that the state's land share is monotonically decreasing in each N_t^j / N_t^i and in each bilateral ratio of military power μ^j / μ^i. Another way to see this mechanism is to note that (12) allows us to write

$$\frac{Z_t^i}{Z_t^j} = \frac{\mu^i}{\mu^j} \left(\frac{N_t^i}{N_t^j} \right)^\phi,$$

which shows that ϕ is the elasticity of the land ratio between state i and state j with respect to the population ratio between the two states. As we will show, this crucial parameter determines whether a unified empire emerges or not.

4. Political fragmentation versus unification

In this section we discuss our main results. We first derive the general representation of Malthusian population dynamics augmented for conflict. Next, we study the dynamics and uncover conditions that lead to the formation of a unified empire or to the persistence of fragmentation across many states.

4.1. Conflict-augmented Malthusian population dynamics

Substituting (12) into (1), we rewrite the population growth rate in (8) as

$$\frac{\Delta N_t^i}{N_t^i} = \Omega^i \left(\frac{Z_t^i}{N_t^i} \right)^{1-\alpha} - 1 = \Omega^i \left[\frac{\mu^i (N_t^i)^\phi}{\sum_{j=1}^m \mu^j (N_t^j)^\phi} \frac{Z}{N_t^i} \right]^{1-\alpha} - 1, \tag{13}$$

where we define for convenience the composite parameter

$$\Omega^i \equiv \frac{\gamma^i (1 - \lambda^i) \theta^i (l^i)^\alpha}{\rho^i} = \frac{\gamma^i (1 - \lambda^i) \theta^i}{\rho^i} \left(\frac{\alpha}{\beta^i + \alpha} \right)^\alpha .$$

Eq. (13) is the central mechanism in our conflict-augmented model of Malthusian population dynamics.

If we hold land Z_t^i constant, the first equality in (13) describes standard Malthusian population dynamics governed by diminishing returns to labor due to a fixed endowment of land. This provides the interpretation of the composite parameter Ω^i as the summary statistic for several channels: given the state's occupied land per capita, Z_t^i / N_t^i, the transitional growth rate of the state's population is increasing in the state's agricultural productivity θ^i and the citizens' preference for fertility γ^i but decreasing in their preference for leisure β^i (via the labor decision $l^i = \alpha / (\beta^i + \alpha)$) and the rulers' preference for rent-seeking taxation λ^i.

The second equality in (13) augments these dynamics with the conflict success mechanism, which makes the land variable Z_t^i endogenous. Accordingly, the state's transitional population growth rate also depends positively on the states's level of military power μ^i and negatively on the military power μ^j, for $j \neq i$, of the other states. More importantly, the conflict component of our mechanism could make population dynamics interdependent across states. In particular, the long-run population dynamics differs drastically for different values of the decisiveness parameter $\phi \in [0, 1]$. In the following sections, we consider separately the three scenarios: $\phi = 0$, $\phi \in (0, 1)$ and $\phi = 1$.[13]

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[12] The early study by Hirshleifer (1991) considers a static conflict model, whereas subsequent studies by Skaperdas and Syropoulos (1996), Maxwell and Reuveny (2005) and Eggert et al. (2011) embed similar conflict success functions as (12) into dynamic models.

[13] For $\phi > 1$ the dynamics become unstable. The analysis is doable but beyond the scope of this study. We thus leave it to future work.

Before we undertake our main analysis, it is worth highlighting some properties of the structure embedded in (13). First, relative to the basic single-state Malthusian model, fragmentation in our multi-state scheme slows down population growth for a given total land endowment Z simply because, for given state-specific parameters, each state controls only a share of the total land endowment. This means that when we compare groups of states with different historical experiences, fragmentation immediately raises to the top of the list of factors that can explain differences in local (single state) and global (collection of states) population size. Second, for given fragmentation and land endowment Z, conflict provides a mechanism through which each state can relax the Malthusian constraint that it faces. More precisely, territorial expansion allows a state to sustain faster population growth than it can sustain holding constant its land share. Moreover, because conflict is a zero-sum game, the territorial expansion of one state is the territorial contraction of the other states. Therefore, conflict causes the Malthusian constraint faced by the loser states to become tighter. These forces are at the heart of the results that we now derive.

4.2. Independent population dynamics across states

To highlight the importance of interdependence via conflict, we consider first the special case $\phi = 0$ in which the population dynamics of the m states are independent of each other. The expression for population growth in (13) reduces to

$$\frac{\Delta N_t^i}{N_t^i} = \Omega^i \left(\underbrace{\frac{\mu^i}{\sum_{j=1}^m \mu^j}}_{Z^i/Z=\text{ land share}} \frac{Z}{N_t^i} \right)^{1-\alpha} - 1. \tag{14}$$

Here, the share of land occupied by state i, Z_t^i/Z, depends only on the military power parameters μ^i and μ^j for $j \neq i$ and is thus an exogenous constant. Consequently, there is no dynamics of the land share across states and the model features only the basic Malthusian population dynamics within each state.

Given initial population N_0^i in state i, population size N_t^i converges to the unique and stable steady-state value:

$$N^i = \left(\Omega^i\right)^{1/(1-\alpha)} \frac{\mu^i Z}{\sum_{j=1}^m \mu^j}, \tag{15}$$

which is proportional to the amount of land Z^i occupied by state i with factor of proportionality $\left(\Omega^i\right)^{1/(1-\alpha)}$. Consequently, N^i is increasing in the state's level of agricultural productivity θ^i and its citizens' fertility preference γ^i but decreasing in their leisure preference β^i (via the labor decision l^i) and the rulers' preference for rent-seeking taxation λ^i.

Proposition 1 summarizes this subsection's result.

Proposition 1. *Given $\phi = 0$ and $N_0^i > 0$, the population dynamics in any state $i \in \{1, \dots, m\}$ is stable and independent of the population dynamics in other states. In the long run, the population size of each state i converges to its positive steady-state value N^i in (15).*

Proof. Eq. (14) shows that the dynamics of N_t^i is autonomous and globally stable. Then, setting $\Delta N_t^i = 0$ in (14) yields the steady-state value N^i in (15). ■

4.3. Interstate competition and political fragmentation

In the general case $\phi \in (0, 1)$, the population dynamics of the m states in (13) depend on each other. Setting $\Delta N_t^i = 0$ for all $i \in \{1, \dots, m\}$ in (13) yields

$$N^i = \left(\Omega^i\right)^{1/(1-\alpha)} \left[\sum_{j=1}^m \frac{\mu^j}{\mu^i} \left(\frac{N^j}{N^i} \right)^{\phi} \right]^{-1} Z,$$

where N^i and N^j are constant values. Taking the ratio of state i to state j, we obtain

$$\frac{N^i}{N^j} = \left(\frac{\mu^i}{\mu^j} \right)^{1/(1-\phi)} \left(\frac{\Omega^i}{\Omega^j} \right)^{1/[(1-\alpha)(1-\phi)]}. \tag{16}$$

Combining these two expressions and rearranging terms we obtain our main result

$$N^i = \left(\Omega^i\right)^{1/(1-\alpha)} \cdot \underbrace{\frac{\left(\mu^i\right)^{1/(1-\phi)} \left(\Omega^i\right)^{\phi/[(1-\alpha)(1-\phi)]}}{\sum_{j=1}^m \left(\mu^j\right)^{1/(1-\phi)} \left(\Omega^j\right)^{\phi/[(1-\alpha)(1-\phi)]}}}_{Z^i/Z=\text{ land share}} \cdot Z. \tag{17}$$

This solution says again that in the steady state, population is proportional to the amount of land Z^i occupied by state i with factor of proportionality $\left(\Omega^i\right)^{1/(1-\alpha)}$. It is far more general, however, because the state's land share Z^i/Z is the outcome of a dynamic process in which territorial expansion or contraction interacts with Malthusian population dynamics within each state.

As the state's population increases relative to that of any other state, the state captures more and more land from the other states and at no cost deploys its technology and workforce to produce agricultural output. Consequently, it sustains higher fertility and grows an even larger population relative to each one of the other states. Note that, as mentioned earlier, due to the zero-sum nature of the conflict for land, the mechanism has two sides that push in the direction of a growing fertility gap: on one side, there is the acceleration of fertility in the winner state; on the other side, there is the deceleration of fertility in the loser states due to the loss of territory. This positive feedback loop reveals that territorial expansion via zero-sum conflict relaxes the standard Malthusian constraint on population dynamics for the winner while it tightens it for the loser. Therefore, it can potentially result in an unstable process with a winner-takes-it-all outcome.

For $\phi < 1$, however, this does not happen because the concavity in N_t^i of the conflict success function (12) for given N_t^j, $j \neq i$, cannot undo the diminishing returns to labor that characterize agricultural production. Specifically, given initial population N_0^i in state $i \in \{1, \dots, m\}$, the state's population N_t^i converges to the unique and stable steady state N^i in (17). As before, the steady-state population size N^i is increasing in the total amount of land Z. Furthermore, N^i is increasing in the state-specific composite parameter Ω^i and the state's military power μ^i but is now also a decreasing function of the composite parameters Ω^j and the military power parameters μ^j of the other states $j \neq i$. In particular, due to interstate competition, the steady-state population size N^i in state i is decreasing in the other states' level of agricultural productivity θ^j and the degree of their citizens' fertility preference γ^j but increasing in their leisure preference β^j (via the labor decision $l^j = \alpha/(\beta^j + \alpha)$) and the other state rulers' preference for rent-seeking taxation λ^j. More importantly, all m states coexist in the long run despite the potential instability due to interstate competition.

Proposition 2 states and proves formally our main result.

Proposition 2. *Given $\phi \in (0, 1)$ and $N_0^i > 0$, the population dynamics in any state $i \in \{1, \dots, m\}$ is stable and depends on the population dynamics in all other states. In the long run, the population size of each state i converges to its positive steady-state value N^i in (17).*

Proof. First, (17) shows that there is a unique interior (i.e., strictly positive and finite) steady-state population size N^i in all states $i \in \{1, \dots, m\}$. Second, (13) shows that (a) as $N_t^i \to 0$ in any state i, the population growth rate $\Delta N_t^i / N_t^i \to \infty$ in that state i and (b) as $N_t^i \to \infty$ in any state i, the population growth rate $\Delta N_t^i / N_t^i \to -1$ in all states. Since these properties hold for all states $i \in \{1, \dots, m\}$, it follows that no state can converge to $N_t^i \to 0$ or to $N_t^i \to \infty$. Therefore, steady-state population size must be positive and finite in any state i. Third, we rewrite (13) as

$$N_{t+1}^i = \frac{\Omega^i (\mu^i Z)^{1-\alpha}}{\left[\sum_{j=1}^m \mu^j (N_t^j)^\phi\right]^{1-\alpha}} (N_t^i)^{1-(1-\alpha)(1-\phi)}. \tag{18}$$

This expression gives us the key step in the proof. It allows us to write the dynamics of the population ratio between any two states as

$$\frac{N_{t+1}^i}{N_{t+1}^j} = \frac{\Omega^i (\mu^i)^{1-\alpha}}{\Omega^j (\mu^j)^{1-\alpha}} \left(\frac{N_t^i}{N_t^j}\right)^{1-(1-\alpha)(1-\phi)}, \tag{19}$$

where $(1-\alpha)(1-\phi) \in (0, 1)$ implies that N_t^i / N_t^j converges to the steady-state ratio N^i / N^j in (16) from any initial condition $N_0^i / N_0^j \in [0, \infty)$ because the right-hand side is a power function with positive exponent less than 1. Finally, we use the result $N_t^i / N_t^j \to N^i / N^j$ to rewrite (18) as

$$\frac{\Delta N_t^i}{N_t^i} = \Omega^i \left[\underbrace{\frac{\mu^i}{\sum_{j=1}^m \mu^j (N^j / N^i)^\phi}}_{\text{constant}} \frac{Z}{N_t^i}\right]^{1-\alpha} - 1,$$

which shows that the dynamics of N_t^i is stable and converges to the steady state in (17). ■

4.4. Unified empire and political unification

We now consider $\phi = 1$. In this case, (13) simplifies to

$$\frac{\Delta N_t^i}{N_t^i} = \Omega^i \left(\frac{\mu^i N_t^i}{\sum_{j=1}^m \mu^j N_t^j} \frac{Z}{N_t^i}\right)^{1-\alpha} - 1 = \Omega^i \left(\frac{\mu^i}{\sum_{j=1}^m \mu^j \frac{N_t^j}{N_t^i}} \frac{Z}{N_t^i}\right)^{1-\alpha} - 1. \tag{20}$$

More importantly, the dynamics of the population ratio in (19) become

$$\frac{N_{t+1}^i}{N_{t+1}^j} = \frac{\Omega^i (\mu^i)^{1-\alpha}}{\Omega^j (\mu^j)^{1-\alpha}} \frac{N_t^i}{N_t^j}. \tag{21}$$

This linear expression says that for $\Omega^i (\mu^i)^{1-\alpha} / \Omega^j (\mu^j)^{1-\alpha} < 1$, there is a unique and stable steady state, the origin, because that is the only point where the line (21) intersects the 45^0 line. For $\Omega^i (\mu^i)^{1-\alpha} / \Omega^j (\mu^j)^{1-\alpha} > 1$, instead, there is no stable steady state because the line (21) is steeper than the 45^0 line.

Assume for concreteness that state i has parameters such that $\Omega^i(\mu^i)^{1-\alpha} > \Omega^j(\mu^j)^{1-\alpha}$ for all $j \neq i$. Then, for all pairs i and $j \neq i$, the ratio N_t^i/N_t^j diverges to $N_t^i/N_t^j \to \infty$ from any initial condition $N_0^i/N_0^j > 0$. Viewed from the perspective of each state $j \neq i$, the dynamics say that each ratio N_t^j/N_t^i converges to $N_t^j/N_t^i \to 0$ from any initial condition $N_0^j/N_0^i > 0$. More importantly, since N_t^i is always strictly positive, $N_t^j/N_t^i \to 0$ means that $N_t^j \to 0$ so that each state $j \neq i$ gradually disappears. These dynamics follow from the fact that state i always has the highest population growth rate among all states. As state i expands, it seizes land from the other states, which must then reduce their fertility rates while state i increases its own fertility. While population growth eventually slows down in all states, including state i, the fertility gap between state i and all the other states becomes larger and larger. This relative fertility divergence eventually yields that the population sizes $\{N_t^j\}$ for all $j \neq i$ converge to 0 from any initial condition N_0^j, whereas N_t^i converges to

$$N^i = \left(\Omega^i\right)^{1/(1-\alpha)} Z, \tag{22}$$

which is proportional to the total amount of land Z.

Interestingly, although the state that has the largest state-specific composite parameter $\Omega^i(\mu^i)^{1-\alpha}$ emerges as the empire while all other states collapse, N^i is independent of the state's military power μ^i because military power has no direct effect on population growth but only contributes to it via the competition for land with other states. This is a manifestation of the property that the relaxation of a state's Malthusian fertility dynamics due to territorial expansion runs into its own iron law of diminishing returns, namely, because the total land endowment Z is finite, territorial expansion is a zero-sum game that provides only temporary relief from the tyranny of Malthus. Once a state has taken over the world, Malthus resumes its rule uncontested. Finally, the empire's steady-state population is increasing in the level of agricultural productivity θ^i and the citizens' fertility preference γ^i but decreasing in their leisure preference β^i (via the labor decision l^i) and the rulers' preference for rent-seeking taxation λ^i.

Proposition 3 summarizes the main result of this subsection.

Proposition 3. *Given $\phi = 1$ and $N_0^i > 0$, the state with the largest $\Omega^i(\mu^i)^{1-\alpha}$ emerges as the unified empire with the positive steady-state population size in (22) while all other states disappear in the long run. However, the steady-state population size of the unified empire does not depend on its military power μ^i.*

Proof. As argued in the text, assuming that state i has the largest composite parameter $\Omega^i(\mu^i)^{1-\alpha}$, the linearity of the dynamics of the population ratio in (21) yields that all the ratios $N_t^j/N_t^i \to 0$ because $N_t^j \to 0$ as N_t^i remains always strictly positive. Consequently, the growth rate of the population of state i eventually becomes

$$\frac{\Delta N_t^i}{N_t^i} = \Omega^i \left(\frac{\mu^i}{\sum_{j=1}^m \mu^j \frac{N_t^j}{N_t^i}} \frac{Z}{N_t^i} \right)^{1-\alpha} - 1 = \Omega^i \left(\frac{Z}{N_t^i} \right)^{1-\alpha} - 1.$$

This is a stable process that converges to the steady-state value in (22). ■

4.5. Population size under different regimes

In our model economy, interstate competition favors the state with the highest population growth potential (the composite parameter Ω^i that summarizes the Malthusian channels). Therefore, unless the unified empire emerges solely via military power, our conflict-augmented Malthusian mechanism suggests that the unified empire should achieve a larger steady-state population size than the total population size of multiple states under political fragmentation. We now discuss this conjecture by focusing on the specific role that military power plays in our model.

4.5.1. Homogeneous military power

If military power is the same across states (i.e., $\mu^i = \mu$ for all $i \in \{1, \dots, m\}$), an intuitive result emerges; namely, the unified empire's steady-state population size N^i in (22) is greater than the sum $\sum_{i=1}^m N^i$ of the values in (15) or (17) that obtain when unification does not occur. The result is intuitive because the conflict-augmented Malthusian dynamics at the core of our model says that the empire emerges precisely because the state with the largest potential for population growth takes over the entire land endowment Z.

To see the result analytically, note that if $\mu^i = \mu$ for all $i \in \{1, \dots, m\}$, the unified empire is the state with the largest composite parameter Ω^i (i.e., the part $(\mu^i)^{1-\alpha}$ is no longer relevant since it is the same for all states). We denote this value $\Omega^{\max}$. Then, using the steady-state solutions (15) and (17), we write:

$$(\Omega^{\max})^{1/(1-\alpha)} > \underbrace{\sum_{i=1}^m s^i (\Omega^i)^{1/(1-\alpha)}}_{\sum_{i=1}^m N^i \text{ for } \phi \in (0,1)} > \underbrace{\frac{1}{m} \sum_{i=1}^m (\Omega^i)^{1/(1-\alpha)}}_{\sum_{i=1}^m N^i \text{ for } \phi = 0}, \tag{23}$$

where $s^i \equiv (\Omega^i)^{\phi/[(1-\alpha)(1-\phi)]} / \sum_{j=1}^m (\Omega^j)^{\phi/[(1-\alpha)(1-\phi)]}$. This string of inequalities holds because $(\Omega^{\max})^{1/(1-\alpha)}$ is greater than both the weighted average of the $(\Omega^i)^{1/(1-\alpha)}$ for $\phi \in (0, 1)$ and the unweighted average of the $(\Omega^i)^{1/(1-\alpha)}$ for $\phi = 0$.

A related result revealed by (23) is that the total population size under multiple states is larger with interstate competition than without interstate competition because the weight s^i is increasing in Ω^i. This result too is intuitive in our context: with interstate competition, countries with larger Ω^i have larger populations and capture more land, which in turn can support an even larger population size. Therefore, interstate competition is beneficial to global population growth in the long run.

Proposition 4 summarizes the results of this discussion.

Proposition 4. *If the level of military power is the same across states, the steady-state population size of a unified empire is larger than the total steady-state population size of multiple states under political fragmentation.*

Proof. Proven in text. ■

4.5.2. Heterogeneous military power

If military power μ^i is heterogeneous across states, then the above result does not necessarily hold. Recall from Proposition 3 that the state with the largest $\Omega^i(\mu^i)^{1-\alpha}$ becomes the unified empire and recall also that the empire's steady-state population size is increasing in the state-specific composite parameter Ω^i but independent of its military power μ^i; see (22). Therefore, it is possible for a state with strong military power (i.e., a large state-specific μ^i) but weak Malthusian potential (i.e., a small state-specific composite parameter Ω^i) to become the unified empire (e.g., the Mongol Empire). The resulting empire can have a smaller population than under fragmentation because military power only contributes to population growth via land conquest but does not improve the conversion from land to population growth. This result says that military power is conducive to empire formation but not necessarily conducive to population growth.

To see this analytically, we consider an example $m = 2$ and assume $\Omega^1(\mu^1)^{1-\alpha} > \Omega^2(\mu^2)^{1-\alpha}$ and $\Omega^1 < \Omega^2$.[14] Then, state 1 becomes the unified empire with steady-state population size:

$$N^1 = \left(\Omega^1\right)^{1/(1-\alpha)} Z.$$

This value is smaller than the total steady-state population size of the two states under political fragmentation. To see this, we use (17) to write

$$N^1 + N^2 = \left[s^1 \left(\Omega^1\right)^{1/(1-\alpha)} + s^2 \left(\Omega^2\right)^{1/(1-\alpha)}\right] Z,$$

where $s^i \equiv [\mu^i(\Omega^i)^{\phi/(1-\alpha)}]^{1/(1-\phi)} / \sum_{j=1}^{2}[\mu^j(\Omega^j)^{\phi/(1-\alpha)}]^{1/(1-\phi)}$ for $\phi \in [0, 1)$. We then note that

$$\left(\Omega^1\right)^{1/(1-\alpha)} Z < \left[s^1 \left(\Omega^1\right)^{1/(1-\alpha)} + s^2 \left(\Omega^2\right)^{1/(1-\alpha)}\right] Z \Rightarrow \left(1 - s^1\right) \left(\Omega^1\right)^{1/(1-\alpha)} < s^2 \left(\Omega^2\right)^{1/(1-\alpha)}.$$

Since $s^1 + s^2 = 1$, the last inequality reduces to $\Omega^1 < \Omega^2$, which is our starting assumption. Consequently, the unified empire created by the state with strong military power but weak Malthusian potential has smaller population than the sum of the populations of the states that would coexist if the empire could not emerge because the conflict success function featured $\phi \in (0, 1)$ instead of $\phi = 1$.

5. Conclusion

In this study, we have developed a tractable Malthusian growth model with interstate competition and studied analytically the endogenous emergence of political fragmentation versus unification in human society. Whether multiple states or a unified empire emerges in the long-run equilibrium depends on the elasticity of the land ratio with respect to the population ratio between states. This crucial parameter captures the intensity of interstate competition, and it represents in reduced-form real-world characteristics, such as geographic barriers or the ability of military technology to leverage superior numbers, morale and/or organization.

Our growth-theoretic analysis predicts that which state becomes the unified empire depends on factors such as the military power of the state, the productivity of agricultural production and the rent-seeking taxation of rulers. In the case of the Qin state, it is indeed often argued that its irrigation system, which improved agricultural productivity, and its reforms, which limited the extractive power of landed aristocrats and strengthened its military power by redistributing land rights from the Qin nobility to soldiers, were at least partly responsible for its unification of China. We can think of many more similar examples from other regions of the world and periods of history that fit our story but we do not discuss them for reasons of space.

Our model has the advantage of being analytically tractable. Tractability, however, always comes at the cost of abstracting from elements not deemed necessary and the nuance that typically dominates discussions of political fragmentation versus unification. For example, our model does not capture the possible causes, such as natural catastrophes or destructive civil warfare or internal institutional decay, of the multiple collapses of empires and separation of states recorded in human history. These elements could be introduced to our framework as exogenous shocks. One could also endogenize the key parameters, such as military power μ and the elasticity ϕ of the land ratio with respect to the population ratio between states. One could also model the detrimental effects of military conflicts and perform a quantitative analysis on the transition dynamics to explore the importance of initial conditions and convergence speed. We leave all these interesting extensions to future research.

[14] More generally, we can create the appropriate aggregate of the $m-1$ states different from 1 and, for the purpose of this "us vs. them" exercise, compare state 1 to such aggregate.

Declaration of competing interest

I declare that I have no financial or material interests related to the paper.

Data availability

No data was used for the research described in the article.

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