Calculating Price Indexes

In the main text (Section 2.4.2), the different price indexes were explained as based on different weightings of individual price changes. It is not usually convenient, however, to calculate each individual price change and the share of each good in expenditure. It turns out that there are much simpler formulae for the basic indexes.

Laspeyres (or Base-weighted) Index

Number each good (j) from 1 to n. Then p_{jt} is the price and q_{jt} is the quantity of good j in period t. Let $t = 0$ indicate the base period (that is the period for which the expenditure shares would be calculated). A simple formula for the price factor for Laspeyre's index is

(1) $$pf_t^L = \frac{\sum_{j=1}^{n} p_{jt} q_{j0}}{\sum_{j=1}^{n} p_{j0} q_{j0}}.$$

It is easy to show that this formula, in fact, weights the changes in the price of individual goods by their shares in total expenditure in the base period just as in equation section 2.4.2. Notice, first, that the denominator is total expenditure in the base period. (If the basket is all the final goods and services in the economy, then the denominator is base period GDP). Multiplying and dividing by p_{jt} in the numerator and rearranging yields

$$pf_t^L = \frac{\sum_{j=1}^{n} p_{jt} q_{j0}}{\sum_{j=1}^{n} p_{j0} q_{j0}} = \sum_{j=1}^{n} \left[\left(\frac{p_{jt}}{p_{j0}} \right) \left(\frac{q_{j0} p_{j0}}{\sum_{j=1}^{n} p_{j0} q_{j0}} \right) \right].$$

The first term on the right-hand side of (2) is the price factor for the good *j*. The numerator of the second term is expenditure on good *j* in the base period. Consequently, the second term as a whole is the share of expenditure on good *j* in the base period. The price factor is, then, the sum of each price change times its share in expenditure in the base period.

We can check that (1) gives the same result as the more complicated calculation in the text, by substituting in the values from Table 2.4. Take 2001 to be the base year and call tortilla chips good 1 and beer good 2, what is the value of the price factor for 2002?

$$pf_{2002}^L = \frac{\sum_{j=1}^{2} p_{j2002} q_{j2001}}{\sum_{j=1}^{2} p_{j2001} q_{j2001}} = \frac{1.00 \times 5 + 1.25 \times 4}{0.50 \times 5 + 0.75 \times 4} = 1.818,$$

which is the same answer as we calculated in the main text.

Typically, Laspeyres indexes keep the same base for a number of years. Keeping 2001 as the base, what is the price factor for 2003?

$$pf_{2003}^{L} = \frac{\sum_{j=1}^{2} p_{j2003} q_{j2001}}{\sum_{j=1}^{2} p_{j2001} q_{j2001}} = \frac{1.25 \times 5 + 1.40 \times 4}{0.50 \times 5 + 0.75 \times 4} = 2.118 .$$

Using these price factors and taking 2001 to be the reference year (that is, $p_{2001}^{L} = 100$), what is the Laspeyre's index for each year?

$$p_{2001}^{L} = 100$$

$$p_{2002}^{L} = p_{2001}^{L} \times pf_{2002}^{L} = 100 \times 1.818 = 181.8$$

$$p_{2003}^{L} = p_{2001}^{L} \times pf_{2003}^{L} = 100 \times 2.118 = 211.8$$

Paasche (or Current-weighted) Index

There is also a convenient formula for calculating the Paasche index:

(2)
$$pf_{t}^{P} = \frac{\sum_{j=1}^{n} p_{jt} q_{jt}}{\sum_{j=1}^{n} p_{j0} q_{jt}} .$$

Here we want to calculate the ratio of prices in the current period (t) to some earlier period (0). It is easily shown to be the inverse of the sum of the individual price changes weighted by their shares in current expenditure. Notice that the numerator is total expenditure in the current period.

$$pf_t^P = \frac{\sum_{j=1}^{n} p_{j1} q_{j1}}{\sum_{j=1}^{n} p_{j0} q_{j1}} = \sum_{j=1}^{n} \left[\frac{1}{\left(\frac{p_{j0}}{p_{j1}}\right)\left(\frac{p_{j1} q_{j1}}{\sum_{j=1}^{n} p_{j1} q_{j1}}\right)} \right].$$

The first term in the denominator on the right-hand side is the individual price factor for good *j* and the second term is the share of expenditure on good *j* in the current period.

Applying (2) to the data in Table 4 for the first two years in Table 2.4, taking 2002 to be the current year yields

$$pf_{2002}^P = \frac{\sum_{j=1}^{2} p_{j2002} q_{j2002}}{\sum_{j=1}^{2} p_{j2001} q_{j2002}} = \frac{1.00 \times 4 + 1.25 \times 5}{0.50 \times 4 + 0.75 \times 5} = 1.783.$$

Taking 2001 as the reference year, then the values of the price index for each year are

$$p_{2001}^P = 100$$

$$p_{2002}^P = p_{2001}^P \times pf_{2002}^P = 100 \times 1.783 = 178.3$$

Since the Paasche index uses new weights for each current period, the price factors and the values of the price index will different if a later year, say 2003, were taken to be the current period. While Laspeyres indexes are very common, Paasche indexes are used most often as a step in the calculation of chain indexes, where the shifting base is a desired feature.

Real Values Using Chain-weighted Indexes

The construction of chain-weighted indexes from underlying Laspeyres and Paasche indexes, as well as the calculation use of indexes to convert nominal to real values, are described in Section 2.4.2 of the main text. Chain-weighted indexes provide the best method for comparing the levels of a given series at two points in time. Unfortunately, the shifting weights impart some undesired properties to real values calculated using chain-weighted price indexes.

Table 2.F shows nominal GDP and its components, as well as their real counterparts in (chain-weighted)1996 constant dollars for 1991, 1996, and 2001. Notice that using nominal values for any of these years the identity $Y = C + I + G + NEX$ holds exactly. Of course, in 1996 real and nominal values are the same, so that the identity also holds for real values in 1996. But notice that for real values in 1991, $C + I + G + NEX =$ \$6,683.7 > \$6,676.4 = Y. (The student should check that it also fails in 2001.) The general rule that the whole is the sum of its parts fails when chain-weighted indexes are used to calculate real values. In contrast, fixed-weight indexes preserve the rule.

The difference between the real value of GDP and the sum of the real values of its components is reported (as it is in the NIPA tables) as a *residual*. The residual is usually small – often small enough to ignore altogether.

The residual can cause problems. For instance, suppose that we ask, how has the proportion of GDP that is absorbed by government expenditure changed over time? Since both income and expenditure are, in fact, conducted in the current dollars of the day, the correct answer to this question is easily calculated from the nominal (or market) values. The true share for 1991 is Nominal G_{1991}/Nominal Y_{1991} = \$1,235.5/\$5,986.2 = 0.206. Suppose, however, that we tried to calculate this share using (chain-weighted) constant values. Then, the share for 1991 would be Real G_{1991}/Real Y_{1991} = \$1,403.4/\$6,676.4 = 0.210. Table 2.G shows the shares for each of the years in Table 2.F using nominal and real values. In 1996, of course, the shares are the same either way. But in every other year, the real shares are systematically different.

A similar problem occurs in computing the contribution of different components of GDP to the growth rate of GDP (or, equally, the contribution of the components of any series to the growth rate of the whole). Great care must be exercised whenever comparisons are made between different chain-weighted real time series. These calculations become more and more misleading away from the reference year. In computing shares it is best to use the nominal values. To calculate the correct contributions of real components to the growth of the whole is well understood, but somewhat complex. Fortunately, the NIPA includes supplemental tables that make the necessary calculations appropriately. The NIPA also include supplemental tables that use a variety of reference years to facilitate accurate real comparisons.

Table 2.G Shares in GDP Calculated Using Nominal and Chain-weighted Real GDP

	Percentage of Nominal GDP				Percentage of Chain-weighted Real GDP			
	Consumption	**Investment**	**Government Expenditure**	**Net Exports**	**Consumption**	**Investment**	**Government Expenditure**	**Net Exports**
1991	66.3	13.4	20.6	-0.3	66.9	12.4	21.0	-0.2
1996	67.0	15.9	18.2	-1.1	67.0	15.9	18.2	-1.1
2001	69.3	15.7	18.4	-3.5	69.2	17.1	17.8	-4.5

Source: Table 1.

Table 2.F. Nominal and Real (Chain-weighted) GDP and Its Components

	Nominal (billions)					Real (billions chain-weighted 1996 constant dollars)					
	GDP	**Consumption**	**Investment**	**Government Expenditure**	**Net Exports**	**GDP**	**Consumption**	**Investment**	**Government Expenditure**	**Net Exports**	**Residual**
1991	5,986.2	3,971.2	800.2	1,235.5	-20.7	6,676.4	4,466.6	829.5	1,403.4	-15.8	-7.3
1996	7,813.2	5,237.5	1,242.7	1,421.9	-89.0	7,813.2	5,237.5	1,242.7	1,421.9	-89.0	0.0
2001	10,082.2	6,987.0	1,586.0	1,858.0	-348.9	9,214.5	6,377.2	1,574.6	1,640.4	-415.9	22.6

Source: Department of Commerce, Bureau of Economic Analysis.