

MODELLING THE COHERENCE IN SHORT-RUN NOMINAL EXCHANGE RATES: A MULTIVARIATE GENERALIZED ARCH MODEL

Tim Bollerslev*

Abstract—A multivariate time series model with time varying conditional variances and covariances, but constant conditional correlations is proposed. In a multivariate regression framework, the model is readily interpreted as an extension of the Seemingly Unrelated Regression (SUR) model allowing for heteroskedasticity. Parameterizing each of the conditional variances as a univariate Generalized Autoregressive Conditional Heteroskedastic (GARCH) process, the descriptive validity of the model is illustrated for a set of five nominal European U.S. dollar exchange rates following the inception of the European Monetary System (EMS). When compared to the pre-EMS free float period, the comovements between the currencies are found to be significantly higher over the later period.

I. Introduction

FOLLOWING the limited success of the “snake,” the European Monetary System (EMS) became effective March 13, 1979, with the intention of reducing the intracurrency variability by increased policy coordination among the member countries.¹ The system, which still is effective, consists of eight of the member countries in the European Economic Community (EEC), with the notable exception of Great Britain. In the present paper a comparison of the coherence in the short-run nominal exchange rate movements for the pre-EMS and EMS periods is intended. However, the well-documented conditional heteroskedasticity in short-run exchange rate movements, cf. Hodrick and Srivastava (1984) and Mussa (1979), renders traditional homoskedastic econometric models invalid and complicates such a comparison.

With this observation in mind, a simple multivariate conditional heteroskedastic time series model is proposed. The model has time varying conditional variances and covariances, but con-

stant conditional correlations. This structure greatly simplifies the estimation and inference procedures. Also, in a multivariate regression framework the model is readily interpreted as an extension of Seemingly Unrelated Regressions (SUR) allowing for conditional and/or unconditional heteroskedasticity. Furthermore, the assumption of constant correlations allows for obvious between period comparisons.

The plan for the rest of the paper is as follows. In the next section the new multivariate heteroskedastic time series model is formally developed, and Maximum Likelihood (ML) estimation of the model is briefly discussed. Model specification and testing are illustrated in section III, where model estimates for a set of five nominal European U.S. dollar exchange rates over the EMS period, March 1979–August 1985, are presented. Assuming the weekly differences of the logarithm of the spot rates to be approximately uncorrelated through time, along with a Generalized Autoregressive Conditional Heteroskedastic (GARCH) structure for each of the conditional variances, cf. Engle (1982) and Bollerslev (1986), a series of diagnostic tests indicate a reasonably good fit for the full five dimensional model. When compared to the estimates over the pre-EMS period, July 1973–March 1979, given in section IV, it is found that although the variabilities for all of the five currencies were higher over the EMS period, the comovements as measured by the conditional correlations, were also higher over the later period. Interestingly, this increase in the coherence was more pronounced for the EMS than for the non-EMS currencies. Finally, section V gives a few concluding remarks and some suggestions for future research.

II. Econometric Methodology

As mentioned above, the heteroskedastic nature of short-run exchange rate dynamics is already well documented. Thus, in order to model the coherence among several exchange rates, a

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* Northwestern University.

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¹ The snake existed from April 1972 until the inception of the EMS.

multivariate heteroskedastic time series model is called for. In this section a particularly simple parameterization of the conditional heteroskedasticity is suggested. Since this new econometric model applies in a much wider context, the discussion below will be somewhat general.

Let y_t denote the $N \times 1$ time-series vector of interest with time varying conditional covariance matrix H_t , i.e.,

$$y_t = E(y_t | \psi_{t-1}) + \epsilon_t \tag{1}$$

$$\text{Var}(\epsilon_t | \psi_{t-1}) = H_t,$$

where ψ_{t-1} is the σ -field generated by all the available information up through time $t - 1$, and H_t is almost surely (a.s.) positive definite for all t . Note, the formulation in (1) allows for both conditional and/or unconditional heteroskedasticity.

Also, let h_{ijt} denote the ij^{th} element in H_t , and y_{it} and ϵ_{it} the i^{th} element in y_t and ϵ_t , respectively. Then a natural scale invariant measure of the coherence between y_{it} and y_{jt} evaluated at time $t - 1$ is given by the conditional correlation $\rho_{ijt} = h_{ijt} / \sqrt{(h_{iit}h_{jjt})}$, where $-1 \leq \rho_{ijt} \leq 1$ a.s. for all t . Of course, in general, this measure of coherence will be time varying as H_t varies through time. However, in some applications the time varying conditional covariances might be taken as proportional to the square root of the product of the corresponding two conditional variances,

$$h_{ijt} = \rho_{ij}(h_{iit}h_{jjt})^{1/2},$$

$$j = 1, \dots, N, \quad i = j + 1, \dots, N, \tag{2}$$

leaving the conditional correlations constant through time. Needless to say, as for any other parameterization of the conditional heteroskedasticity, the validity of the assumption in (2) remains an empirical question. However, as illustrated below, the assumption in (2) seems reasonable for the five exchange rates over each of the two separate subperiods studied in this paper.

An appealing feature of the model with constant conditional correlations relates directly to the simplified estimation and inference procedures. To that end, rewrite each of the conditional variances as,

$$h_{iit} \equiv \omega_i \sigma_{it}^2, \quad i = 1, \dots, N, \tag{3}$$

with ω_i a positive time invariant scalar and $\sigma_{it}^2 > 0$

a.s. for all t . Note, the decomposition in (3) is only unique up to scale. Given (2) and (3), the full conditional covariance matrix, H_t , may be partitioned as

$$H_t = D_t \Gamma D_t, \tag{4}$$

where D_t denotes the $N \times N$ stochastic diagonal matrix with elements $\sigma_{1t}, \dots, \sigma_{Nt}$ and Γ is an $N \times N$ time invariant matrix with typical element $\rho_{ij} \sqrt{(\omega_i \omega_j)}$. It follows now, that H_t will be a.s. positive definite for all t if and only if each of the N conditional variances are well defined and Γ is positive definite. Compared to many alternative parameterizations for the time varying covariance matrix, these conditions are very easy to impose and verify; see, for instance, Baba, Engle, Kraft, and Kroner (1989) for a discussion of the similar conditions for the multivariate linear GARCH(p, q) model.

Assuming conditional normality, the log likelihood function for the general heteroskedastic model in (1) becomes, apart from some initial conditions,

$$L(\theta) = -\frac{TN}{2} \log 2\pi$$

$$- \frac{1}{2} \sum_{t=1}^T (\log |H_t| + \epsilon_t' H_t^{-1} \epsilon_t) \tag{5}$$

where θ denotes all the unknown parameters in ϵ_t and H_t . Under standard regularity conditions the ML estimate for θ is asymptotically normal and traditional inference procedures are immediately available.² However, as the evaluation of the likelihood function in (5) requires one $N \times N$ matrix inversion for each time period the maximization of $L(\theta)$ by iterative methods can be quite costly even for moderately sized T and N .

The assumption in (2) reduces this computational complexity enormously. By direct substitu-

² Following Bollerslev and Wooldridge (1989) and Weiss (1986), if the model correctly specifies the first two conditional moments but the conditional normality assumption is violated, under suitable regularity conditions the quasi-maximum likelihood estimates obtained from (5) will still be consistent and asymptotically normal, but the usual standard errors have to be modified.

tion,

$$\begin{aligned} L(\theta) &= -\frac{TN}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^T \log |D_t \Gamma D_t| \\ &\quad - \frac{1}{2} \sum_{t=1}^T \epsilon_t' (D_t \Gamma D_t)^{-1} \epsilon_t \\ &= -\frac{TN}{2} \log 2\pi - \frac{T}{2} \log |\Gamma| - \sum_{t=1}^T \log |D_t| \\ &\quad - \frac{1}{2} \sum_{t=1}^T \tilde{\epsilon}_t' \Gamma^{-1} \tilde{\epsilon}_t \end{aligned} \quad (6)$$

where $\tilde{\epsilon}_t = D_t^{-1} \epsilon_t$ denotes the $N \times 1$ vector of standardized residuals. Note, except for the third term, a Jacobian term arising from the transformation from ϵ_t to $\tilde{\epsilon}_t$, the likelihood function in (6) is equivalent to the likelihood function for $\tilde{\epsilon}_t$ conditionally normal with time invariant covariance matrix Γ .

Of course, the likelihood function in (6) is still highly nonlinear in the parameters, and just as for the general heteroskedastic likelihood function in (5) an iterative maximization technique is called for. Nevertheless, when compared to (5), (6) is much easier to evaluate and requires only one $N \times N$ matrix inversion as opposed to T inversions in (5).³ Note also, that $\log |D_t|$ is just equal to the sum of $\log \sigma_{1t}, \dots, \log \sigma_{Nt}$.

Furthermore, it follows from the SUR analogue, that conditional on $\tilde{\epsilon}_t, t = 1, \dots, T$, the ML estimate of Γ is given by the sample analogue, $\hat{\Gamma} = T^{-1} \sum_t \tilde{\epsilon}_t \tilde{\epsilon}_t'$, which is a.s. positive definite by construction. Therefore, by the invariance principle the ML estimate for each of the conditional correlations must solve,

$$\hat{\rho}_{ij} = \frac{\sum_t \tilde{\epsilon}_{it} \tilde{\epsilon}_{jt}}{\left(\sum_t \tilde{\epsilon}_{jt}^2 \right)^{1/2} \left(\sum_t \tilde{\epsilon}_{it}^2 \right)^{1/2}},$$

and the $1/2N(N+1)$ parameters in Γ can be concentrated out of the likelihood function simplifying the computations even further,

$$\begin{aligned} L(\theta) &= -\frac{TN}{2} (1 + \log 2\pi - \log T) \\ &\quad - \sum_{t=1}^T \log |D_t| - \frac{T}{2} \log \left| \sum_{t=1}^T \tilde{\epsilon}_t \tilde{\epsilon}_t' \right|. \end{aligned} \quad (7)$$

³ In the model presented below a total of 30 parameters are estimated using numerical derivatives. With 333 observations, the number of matrix inversions for each iteration is reduced from 10,323 to only 31.

Note, however, that the information matrix is not block diagonal between the parameters in D_t and Γ , so in order to obtain an estimate of the asymptotic covariance matrix by standard ML techniques, the derivatives of the full likelihood function in (6) is called for.

The approach taken below is to use the Berndt, Hall, Hall, and Hausman (1974) algorithm along with numerical first order derivatives; for a more detailed discussion see, e.g., Bollerslev, Engle, and Wooldridge (1988).

III. EMS Estimates

The data are interbank closing spot prices on Wednesdays taken from the International Monetary Markets Yearbook for the German mark (DM), the French franc (FF), the Italian lira (IL), the Swiss franc (SF), and the British pound (BP) versus the U.S. dollar. The full data set covers the floating rate period from July 1973 until August 1985.⁴ However, below we shall concentrate on the results for the EMS period starting the third week of March 1979 until the second week of August 1985, for a total of 333 observations.

Not surprisingly, for none of the five weekly exchange rates is the martingale property rejected, i.e.,

$$y_{it} = 100 \cdot \log(s_{it}/s_{it-1}) = \mu_i + \epsilon_{it},$$

where s_{it} refers to the i^{th} spot rate, $i = \text{DM, FF, IL, SF, BP}$. Formally, the Phillips and Perron (1988) $Z(\phi_3)$ test statistic for a unit root and no deterministic time trend in the univariate time series representation for $\log s_{it}$ ranges from 1.999 for the FF to 3.937 for the BP, all of which are below the 10% critical value equal to 5.34.⁵ The Ljung and Box (1978) portmanteau test for up to 20th order serial correlation in ϵ_{it} takes its maximum value of 23.203 for the FF, which is far below 31.410, the 95% fractile in the asymptotic $\chi^2(20)$ distribution. These findings are very much in line with previous results reported in the literature for other currencies and time periods, see, e.g., Baillie and Bollerslev (1989), Diebold and Nerlove (1989), and Meese and Singleton (1982).

⁴ The data were kindly provided by Frank Diebold, and have been analyzed from a different perspective in the papers by Diebold and Pauly (1988) and Diebold and Nerlove (1989).

⁵ In calculating the test statistic a Newey and West (1987) type variance estimator with a truncation lag equal to ten was employed. Almost identical results were obtained for other values of the truncation lag.

Even though the deviations from the simple random walk model are approximately uncorrelated, the ϵ_{it} 's are clearly not independent through time. As noted by Mussa (1979), there is a tendency for large (small) residuals to be followed by other large (small) residuals, but of unpredictable sign. For instance, the Ljung-Box test for up to 20th order serial correlation in ϵ_{BPt}^2 takes the value 113.020, which is very extreme in the corresponding asymptotic $\chi^2(20)$ distribution under the null hypothesis of independence over time. However, the ARCH(q) model developed in Engle (1982) explicitly allows for this type of temporal dependence by parameterizing the conditional variance as a linear function of the past q squared residuals. In many applications, a more parsimonious representation is often obtained by the GARCH(p, q) model, cf. Bollerslev (1986, 1987), where

$$\begin{aligned} \text{Var}_t(\epsilon_{it}) &= h_{iit} \\ &= \omega_i + \sum_{j=1}^q \alpha_{ij} \epsilon_{it-j}^2 + \sum_{j=1}^p \beta_{ij} h_{iit-j}. \end{aligned}$$

The GARCH model can be seen as a univariate ARMA model for conditional second moments, and the selection of the orders p and q may be addressed by traditional time series techniques applied to ϵ_{it}^2 . The descriptive validity of the univariate ARCH and GARCH models in characterizing short-run exchange rate dynamics have already been well documented; see, for instance, Baillie and Bollerslev (1989), Bollerslev (1987), Diebold and Nerlove (1989), Diebold and Pauly (1988), Domowitz and Hakkio (1985), Engle and Bollerslev (1986), Hsieh (1988, 1989), McCurdy and Morgan (1987, 1988), and Milhøj (1987), among many others.

Thus, guided by this preliminary univariate analysis for each of the five currencies, we shall here assume a GARCH(1, 1) structure for the conditional variances, but allow for non-zero constant conditional correlations across countries, as discussed in section II,

$$\begin{aligned} y_{it} &= \mu_i + \epsilon_{it} \\ h_{iit} &= \omega_i + \alpha_{i1} \epsilon_{it-1}^2 + \beta_{i1} h_{iit-1} \\ &\quad i, j = DM, \dots, BP \quad i \neq j \quad (8) \\ h_{ijt} &= \rho_{ij} (h_{iit} h_{jjt})^{1/2}. \end{aligned}$$

The ML estimates for the model in (8) obtained under the additional assumption of condi-

tional normality, are given in table 1. Of course, the assumption of conditional normality should merely be viewed as an approximation in the present context. Given the EMS bounds, the intra-EMS currency rates will be truncated dependent variables. However, the robustness results developed in Bollerslev and Wooldridge (1989) and Weiss (1986) may be invoked.⁶

Turning to the results, the large appreciation of the U.S. dollar over this period is apparent for all the currencies. The DM, the SF, and the BP all depreciated by roughly the same percentage, whereas the FF and the IL weakened by considerably more.

It is interesting to note that all the parameters in the time varying conditional variances are individually significant as the usual 5% level. Indeed, the Likelihood Ratio (LR) test statistic for absence of conditional heteroskedasticity, i.e., $\alpha_{i1} = \beta_{i1} = 0$ for all i , equals 117.028 the value of an asymptotic $\chi^2(10)$ distribution under the null hypothesis, rejecting the homoskedastic SUR model at any reasonable level.

The estimates for all the ten conditional correlations are also highly significant. The LR test statistic for $\rho_{ij} = 0 \quad i \neq j$ equals 1911.078, which asymptotically under the null hypothesis of independence across countries should be the realization of a $\chi^2(10)$ distribution. As the exchange rate movements, to a large extent, depend on a common set of international economic variables, unobservable on a weekly basis, this overwhelming rejection is hardly surprising.

In order to assess the general descriptive validity of the model in (8) a series of misspecification tests were also performed. In particular, the Lagrange Multiplier (LM) test statistic for AR(1), or equivalently MA(1), disturbances in each of the five equations takes the value 8.459 corresponding to the 0.867 fractile in the asymptotic $\chi^2(5)$ distribution. Also, the Ljung-Box portmanteau

⁶ Along these lines, it is interesting to note, that whereas daily exchange rate data show a considerable amount of leptokurtosis even after accounting for ARCH effects, cf. Bollerslev (1987) and Hsieh (1988, 1989), the assumption of conditional normality appears much more reasonable with weekly data. In the present context the coefficient of kurtosis from the five univariate GARCH(1, 1) models equal 3.308, 5.363, 3.471, 3.045, and 3.755, respectively, and only for the FF is the assumption of conditional normality obviously violated. These findings are in accordance with the results reported in Baillie and Bollerslev (1989).

tests for up to 20th order serial correlation in the standardized residuals, $\hat{\epsilon}_{it}/\sqrt{\hat{h}_{it}}$, range from 15.540 for the BP to 24.993 for the SF, all of which are less than the 95% fractile.

Turning to the tests for the conditional second moments, the GARCH(1, 1) specification for each of the five conditional variances also seems reasonable. The LM test statistic for GARCH(1, 2) equals 7.669, the 0.825 fractile in the $\chi^2(5)$ distribution. Similarly, the Ljung-Box tests for up to 20th order serial correlation in the squared standardized residuals, $\hat{\epsilon}_{it}^2\hat{h}_{it}^{-1}$, take the values 27.612, 6.133, 26.318, 22.276, 22.942 for $i = \text{DM, FF, IL, SF, BP}$, respectively, and all are insignificant at the usual 5% level.⁷

Under the assumption of constant conditional correlations, the cross products of the standardized residuals, $\hat{\epsilon}_{it}\hat{\epsilon}_{jt}/\sqrt{(\hat{h}_{it}\hat{h}_{jt})}$, $i \neq j$, should also be serially uncorrelated. It therefore lends further support to the model that the portmanteau tests for these cross products range from a low of 16.681 for the FF and the SF to a high of only 28.062 for the DM and the IL. It ought to be emphasized that the model predicts the product of the standardized cross-country residuals are uncorrelated, and not the product of the raw residuals, $\hat{\epsilon}_{it}\hat{\epsilon}_{jt}$, as would be implied by a model with constant conditional covariances. For instance, the 20th order Ljung-Box test for the cross product of the raw residuals for the SF and the BP equals 101.496, compared to only 23.700 for the standardized residuals.

A further set of residual-based diagnostic type tests for the validity of the specification in (8) was also calculated; for a general discussion of residual-based diagnostics see Pagan and Hall (1983) and Domowitz and Hakkio (1985). From (1), if the model is correctly specified $E(\epsilon_{it}\epsilon_{jt}|\psi_{t-1}) = h_{ijt}$. Thus, for $i = j$ the tests are constructed by regressing $(\hat{\epsilon}_{it}^2\hat{h}_{it}^{-1} - 1)$ on \hat{h}_{it}^{-1} and $\hat{\epsilon}_{it-1}^2\hat{h}_{it}^{-1}, \dots, \hat{\epsilon}_{it-5}^2\hat{h}_{it}^{-1}$ and testing whether the estimated coefficients are significantly different from zero by a conventional F -test. The test statistics take the values 1.793, 0.366, 1.369, 1.350,

and 0.572, respectively, none of which exceeds 2.13, the 95% critical value for the $F_{6,326}$ distribution. For $i \neq j$ $(\hat{\epsilon}_{it}\hat{\epsilon}_{jt}\hat{h}_{ijt}^{-1} - 1)$ is regressed on

$$\hat{h}_{ijt}^{-1}, \hat{\epsilon}_{it-1}^2\hat{h}_{ijt}^{-1}, \hat{\epsilon}_{jt-1}^2\hat{h}_{ijt}^{-1}$$

and $\hat{\epsilon}_{it-1}\hat{\epsilon}_{jt-1}\hat{h}_{ijt}^{-1}, \dots, \hat{\epsilon}_{it-5}\hat{\epsilon}_{jt-5}\hat{h}_{ijt}^{-1}$. Again, all the asymptotic $F_{8,324}$ test statistics fall below the 95% critical value of 1.97, the largest being the test for the IL and the BP equal to 1.797.

Summing up, the tests discussed above do not present any serious evidence against the multivariate GARCH(1, 1) model in (8) with constant conditional correlations as a simple and parsimonious description of the short-run dynamics for the five weekly European U.S. dollar exchange rates over the EMS period analyzed here.

IV. Pre-EMS Comparisons

In this section, the EMS results presented above are compared to the estimates obtained for the same model (8) using data over the pre-EMS period July 1973 until March 1979, i.e., 299 weekly observations. We shall not discuss these estimates given in table 2 in any great detail. It is certainly possible that a better model might be constructed for the pre-EMS period, but the statistical evidence against the model is not serious, and for comparison purposes we shall retain the same specification over both periods.

When comparing tables 1 and 2 it is immediately seen that all the conditional correlations are significantly higher for the EMS period, and that this increase in the conditional correlations is not restricted to the EMS currencies only. The seven conditional correlations for the BP and the SF, both of which are not included in the EMS, all increased by several asymptotic standard errors. In fact the largest absolute increase in the conditional correlations occurred for the IL and the SF. However, whereas the BP and the IL exhibit very similar correlations with the other currencies over the pre-EMS period, the increase in the conditional correlations are much more pronounced for the IL. The conditional correlations for the BP in table 1 are by far the lowest among all the currencies. Also, the increase in the conditional correlations for the DM were highest with the FF and the IL, namely 0.325 and 0.461, compared to 0.203 and 0.231 with the SF and the BP. Thus, even though the general level

⁷ As noted by Cumby and Huizinga (1988), only when the regressors are strictly exogenous as here is it generally safe to ignore that the residuals have been estimated when evaluating portmanteau tests for serial correlation in the mean. However, the estimation error is of minor order when testing for serial correlation in the squared residuals; see McLeod and Li (1983).

TABLE 1.—EMS ESTIMATES

<i>i</i>	DM	FF	IL	SF	BP
μ_i	.174 (.079)	.255 (.095)	.298 (.070)	.180 (.083)	.171 (.080)
ω_i	.120 (.085)	.027 (.020)	.340 (.112)	.055 (.041)	.110 (.071)
$\alpha_{i,1}$.036 (.085)	.017 (.008)	.152 (.038)	.061 (.015)	.086 (.033)
$\beta_{i,1}$.919 (.047)	.975 (.011)	.685 (.081)	.930 (.023)	.879 (.047)
ρ_{DMi}	1.000 (—)	—	—	—	—
ρ_{FFi}	.932 (.008)	1.000 (—)	—	—	—
ρ_{ILi}	.886 (.013)	.876 (.017)	1.000 (—)	—	—
ρ_{SFi}	.917 (.010)	.866 (.019)	.816 (.021)	1.000 (—)	—
ρ_{BPi}	.674 (.032)	.676 (.034)	.622 (.040)	.635 (.035)	1.000 (—)

Note: Asymptotic standard errors are in parentheses.

TABLE 2.—PRE-EMS ESTIMATES

<i>i</i>	DM	FF	IL	SF	BP
μ_i	-.073 (.060)	.013 (.076)	.020 (.045)	-.114 (.076)	.070 (.075)
ω_i	.043 (.014)	.210 (.051)	.028 (.005)	.056 (.019)	.226 (.082)
$\alpha_{i,1}$.042 (.011)	.151 (.039)	.527 (.060)	.127 (.026)	.112 (.053)
$\beta_{i,1}$.921 (.019)	.699 (.053)	.626 (.022)	.851 (.016)	.675 (.117)
ρ_{DMi}	1.000 (—)	—	—	—	—
ρ_{FFi}	.607 (.041)	1.000 (—)	—	—	—
ρ_{ILi}	.425 (.043)	.441 (.046)	1.000 (—)	—	—
ρ_{SFi}	.714 (.029)	.517 (.049)	.336 (.063)	1.000 (—)	—
ρ_{BPi}	.443 (.050)	.488 (.046)	.345 (.055)	.305 (.069)	1.000 (—)

Note: See table 1.

of coherence among the five exchange rates were higher over the later period, there is some evidence that the policy coordination among the EMS member countries might indeed have led to an additional increase for the EMS currencies.⁸

It is also interesting to note that the estimates of the unconditional variances, $\hat{\omega}_i(1 - \hat{\alpha}_{i,1} -$

⁸ Of course, as pointed out by a referee, the increased coherence of the IL with both EMS and non-EMS countries may simply reflect increased discipline in Italian monetary and fiscal policy in response to a common set of economic variables, rather than heightened policy coordination between the EMS member countries.

$\hat{\beta}_{i,1})^{-1}$, and the corresponding sample analogues, $T^{-1}\sum_i \hat{\epsilon}_{it}^2$, were greater over the EMS period for all the five currencies.⁹ Therefore, at the same time that the first six years of the EMS were characterized by greater short-run comovements among the major European currencies vis-à-vis the U.S. dollar relative to the preceding six years, the volatilities were also greater for the EMS period. This increase in short-run exchange rate volatility could be related to the well-documented increase in the volatility of U.S. interest rates following the change of operating procedures by the U.S. Federal Reserve Bank in the first week of October 1979.¹⁰

Along these lines it might be interesting to test whether the break actually occurred in March 1979 or at some other time; e.g., October 1979. Techniques from the switching regression literature could be helpful in that respect; see, e.g., Goldfeld and Quandt (1973) and Holbert (1982). We shall not formally pursue this idea here. However, it is interesting to note that, when the model in (8) is estimated with data for the July 1973–October 1979 and October 1979–August 1985 periods, the maximized value of the two log likelihood functions are -2134.305 and -1887.265, respectively. This compares to the maximized log likelihoods for the EMS and pre-EMS estimates in tables 1 and 2 of -1969.969 and -2032.052, respectively. Thus, two times the difference in the sum of these two pairs of non-nested log likelihood functions equals 39.098, suggesting that a break corresponding to the inception of the EMS is more likely than a break in the first week of October 1979. Also, a formal LR test for parameter constancy over the entire 12 year period versus a break in March 1979 takes the value 388.110, which is highly significant in the $\chi^2(30)$ distribution.

It ought to be emphasized that the above findings are not specific to the U.S. dollar rates. When the same two multivariate models were estimated for the exchange rates for the five European currencies vis-à-vis the Japanese yen

⁹ The only exception is the IL, where the sample variance is higher for the EMS period, but the estimate for the earlier period implies that the unconditional variance does not exist as $\hat{\alpha}_{IL,1} + \hat{\beta}_{IL,1} > 1$. Nonetheless, the conditional moments from the model are still well defined.

¹⁰ This new policy of targeting money supply was abandoned three years later in 1982, in favor of a less stylized monetary policy.

the qualitative results were unaltered. Interpreting the U.S. dollar/Japanese yen exchange rate as a simple set of weights, this robustness of the result to the choice of "numeraire currency" should not be too surprising.

V. Concluding Remarks

Several other interesting questions in empirical economics and finance naturally fall within the same econometric framework as discussed here. For instance, according to most modern asset pricing theories, the return on an asset is determined by its future covariance with a benchmark portfolio or some marginal rate of substitution. A failure to take account of any temporal variation in the conditional second moments when econometrically implementing such models may lead to erroneous inference. The multivariate ARCH methodology provides a parametric approach for analyzing these issues. For some recent evidence on empirical tests and implementation of domestic asset pricing models and in modelling the term structure of interest rates using ARCH models see Bollerslev, Engle and Wooldridge (1988), Engle, Ng and Rothschild (1990), and the references therein.

Compared to the linear diagonal GARCH model estimated in Bollerslev, Engle and Wooldridge (1988), the latent factor ARCH model in Diebold and Nerlove (1989), or the factor GARCH model in Engle, Ng, and Rothschild (1990), the parameterization proposed here with time varying conditional covariances but constant conditional correlations represents a major reduction in terms of computational complexity. The conditions to ensure that the time varying covariance matrices are positive definite and the model well defined are also very easy to impose and verify. Of course, as for any form of the heteroskedasticity, the validity of the model remains an empirical question. Interestingly, however, the parameterization and estimation methods suggested here have already found use in other applications by Baillie and Bollerslev (1990), Kroner and Claessens (1989), McCurdy and Morgan (1989), Ng (1989), and Schwert and Seguin (1990) among others. Also, independently Cecchetti, Cumby and Figlewski (1988) in calculating the optimal futures hedge, used non-linear least squares estimates from a simple bivariate

ARCH type model for one-month Treasury bills and twenty-year Treasury bonds with the assumption of a constant correlation pre-imposed.

Finally it is worth stressing, that the various ARCH and GARCH parameterizations suggested in the literature represent nothing but a convenient statistical tool for summarizing the time series dependence observed in the data. Further theoretical work on explaining the temporal variation in the conditional variances and covariances, providing economic justification for the ARCH class of models remains an extremely important topic for future research.

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