

Realized Semi(co)variation: Signs That All Volatilities are Not Created Equal*

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Abstract

I provide a selective review of recent developments in financial econometrics related to measuring, modeling, forecasting, and pricing “good” and “bad” volatilities based on realized variation type measures constructed from high-frequency intraday data. An especially appealing feature of the different measures concerns the ease with which they may be calculated empirically, merely involving cross-products of signed, or thresholded, high-frequency returns. I begin by considering univariate semivariation measures, followed by multivariate semicovariation and semibeta measures, before briefly discussing even richer partial (co)variation measures. I focus my discussion on practical uses of the measures emphasizing what I consider to be the most noteworthy empirical findings to date pertaining to volatility forecasting and asset pricing.

Key words: cross-sectional return variation, downside risk, high-frequency data, jumps and co-jumps, partial variation, realized variation, return predictability, semibeta, semi(co)variation, volatility forecasting

JEL classification: C22, C51, C53, C58, G11, G12

It is difficult to pinpoint the origin of the research field that we now refer to as financial econometrics. What is clear, however, is that the rapid growth of the field over the past three decades has in no small part been fueled by the development of new procedures and

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empirical findings related to the measurement, modeling, forecasting, and pricing of time-varying financial market volatility. The importance of volatility for the field as a whole is also underscored by the SoFiE logo, which prominently features the time series plot of a financial asset price subject to volatility clustering. Looking at the price path featured in the logo, it is evident that “large changes tend to be followed by large changes—of either sign—and small changes tend to be followed by small changes” (Mandelbrot, 1963).

The initial research on time-varying volatility in financial markets, and the importance thereof, were primarily based on parametric GARCH (Engle, 1982; Bollerslev, 1986) and stochastic volatility type models (Taylor, 1982). However, the advent of high-frequency intraday data for a host of different assets and instruments, starting in the early 2000s, spurred somewhat of a paradigm shift, and many of the most influential developments over the past two decades have involved so-called realized volatility measures constructed from high-frequency intraday data (Andersen and Bollerslev, 1998; Andersen et al., 2001b; Barndorff-Nielsen, and Shephard, 2002). Importantly, this approach also helped to embed the econometric analyses of return volatility within the vast probability and statistics literatures on Itô semimartingales and the theory of quadratic variation, thereby affording a rigorous justification for the realized volatility concept based on no-arbitrage assumptions and theoretical in-fill asymptotic arguments relying on the idea of ever finer sampled returns over fixed time intervals.¹ The quadratic variation estimated by the original realized volatility measures is effectively blind to the signs of the underlying returns. On the other hand, numerous studies, dating back to Roy (1952) and Markowitz (1959), have argued that investors primarily care about negative returns and downside risks.² Correspondingly, the basic mean–variance tradeoff arguments that underlie many popular asset pricing models and predictions, the traditional CAPM included, should instead be based on the downside portion of the variation only (Hogan and Warren, 1972; Bawa and Lindenberg, 1977). An extensive body of research in behavioral finance, supported by experimental evidence and more formal theoretical arguments rooted in prospect theory and loss aversion (Kahneman and Tversky, 1979), also suggest that up and downside risks are not treated the same by investors.³ The ubiquitous Value-at-Risk (VaR) and Expected Shortfall measures used for assessing the risk of an investment portfolio, and the Sortino ratio (Sortino and van der Meer, 1991) sometimes employed in lieu of the traditional Sharpe ratio for portfolio performance evaluation, further echo this asymmetric treatment of gains and losses.⁴

- 1 The introductory chapter to the collection of seminal volatility papers in Andersen and Bollerslev (2018) provides a more in depth discussion of the key new concepts and ideas that have helped shape these developments.
- 2 The Merriam-Webster dictionary also explicitly defines “risk” as the “possibility of loss or injury,” as exemplified by “the chance that an investment (such as a stock or commodity) will lose value.”
- 3 Scenarios in which upside gains and downside losses are not symmetric arise in many other economic situations and prediction problems; for additional discussion, see, for example, Christoffersen and Diebold (1997), Patton and Timmermann (2007), and Babii et al. (2021).
- 4 A growing recent literature has also sought to relate these measures of downside risks to macroeconomic outcomes; see, for example, Giglio, Kelly, and Pruitt (2016), Adrian, Boyarchenko, and Giannone (2019), and Carriero, Clark, and Marcellino (2020). Some of the ideas and new empirical measures that I discuss below could possibly be exploited in that context as well.

Motivated by this line of reasoning and the idea that “good” and “bad” volatilities are not necessarily created equal, [Barndorff-Nielsen, Kinnebrock, and Shephard \(2010\)](#) first proposed decomposing the original realized volatility measure into separate up and downside realized semivariation measures based on the summation of the squared positive and negative high-frequency returns, respectively. As I will discuss below, this simple decomposition has in turn resulted in a number of new and interesting empirical findings pertaining to both volatility forecasting and the differential pricing of the up and downside realized semivariation measures. Anticipating some of the key findings, higher values of downside (upside) realized semivariance for the aggregate market portfolio appear to be associated with higher (lower) future aggregate market volatility, higher (lower) future aggregate market return, while differences in the up minus downside semivariances for individual stocks appear to be priced positively in the cross-section.

Of course, most issues in asset pricing finance are inherently multivariate in nature, entailing non-diversifiable risks and the covariation among multiple assets and/or the covariation with specific systematic risk factor(s), or benchmark portfolios. Extending the realized semivariance concept to a multivariate setting, [Bollerslev et al. \(2020\)](#) first proposed an analogous decomposition of the standard realized covariance matrix into *four* additive realized semicovariance components defined by the sum of the cross-products of the signed pairs of high-frequency returns. In parallel to the findings for the realized semivariances, this “look inside” of the quadratic covariation has similarly been used in the constructions of improved covariance matrix forecasts.

The semicovariances have also been used in the definition of so-called realized semibetas ([Bollerslev, Patton, and Quaadvlieg, 2021a](#)) and in turn more accurate cross-sectional asset price predictions. Consistent with the implications from a downside CAPM and the results based on separately estimated up and downside betas ([Ang, Chen, and Xing, 2006](#)), only the two semibetas associated with downside market risk appear to be priced. However, counter to the implications from a conventional downside CAPM model, the risk premiums for two separate downside semibetas associated with “good” and “bad” asset-specific covariations seemingly differ.

The choice of a zero threshold underlying the definitions of the realized univariate semivariation, multivariate semicovariation, and semibeta measures is often motivated by the idea that investors value gains and losses asymmetrically, and therefore also price and process “good” and “bad” volatility differently. However, other economic considerations naturally dictate different definitions of “good” and “bad” returns based on some fixed non-zero threshold, or some benchmark return. From a purely statistical perspective, the choice of a zero threshold is also somewhat arbitrary, and a non-zero threshold and/or multiple thresholds could in principle be used in the definition of more refined decompositions of the quadratic variation. I will briefly discuss recent results based on this idea and corresponding so-called realized partial (co)variation measures ([Bollerslev et al., 2021](#)).

Other measures of asymmetries and non-linear dependencies based on high-frequency data, including measures of coskewness and cokurtosis (e.g., [Neuberger, 2012](#); [Amaya et al., 2015](#)), have, of course, been proposed and analyzed empirically in the literature.⁵

5 The SoFiE Presidential Addresses by [Engle \(2011\)](#) and [Ghysels \(2014\)](#) also both explicitly highlight the importance of allowing for asymmetries and skewness in the calculation of VaR and other downside risk measures.

However, many of these measures involving higher order moments can be difficult to accurately estimate in practice. By comparison, the realized semi(co)variation measures, constructed from sums of squares and cross-products of signed high-frequency returns, afford a simple “look inside” the quadratic variation.

Realized bipower variation measures (Barndorff-Nielsen and Shephard, 2004b), specifically designed to be robust to jumps, afford another such look. Along these lines, several studies have argued for the importance of separately considering the quadratic variation stemming from price discontinuities, or jumps and cojumps. I will not discuss this extensive literature on estimating and testing for jumps and co-jumps based on high-frequency-data at any great length here (see, e.g., Bollerslev, Law, and Tauchen, 2008; Lee and Mykland, 2008; Jacod and Todorov, 2009; Mancini and Gobbi, 2012; Ait-Sahalia and Xiu, 2016; Li, Todorov, and Tauchen, 2017, among others). However, as I will discuss below, appropriately defined differences between semi(co)variances also consistently estimate jumps and co-jumps.

Closely related to the above-mentioned studies on jumps, there is a recent and rapidly growing literature on the pricing of downside tail, or crash risk (including, e.g., Bollerslev and Todorov, 2011; Kelly and Jiang, 2014; Cremers, Halling, and Weinbaum, 2015; Bollerslev, Li, and Todorov, 2016; Chabi-Yo, Ruenzi, and Weigert, 2018; Lu and Murray, 2019; Orłowski, Schneider, and Trojani, 2020, among others). However, in contrast to the realized measures constructed solely from high-frequency data that I focus on here, all of these studies rely on additional information gleaned from options prices for inferring risk-neutral distributions, and assessing tail dependencies and the pricing thereof.

Directly paralleling the decompositions of realized volatilities into separate up and downside variation measures, options implied volatilities may similarly be decomposed into “good” and “bad” variation through the use of options with different strikes (see, e.g., Andersen, Bondarenko, and Gonzalez-Perez, 2015). Going one step further, the variance risk premium, defined as the difference between the risk-neutral and the actual expected return variation, may also be separated into up and downside variance risk premiums.⁶ Consistent with the findings discussed below that most of the volatility persistence can be traced to “bad” volatility, most of the return predictability inherent in the variance risk premium (as originally documented by Bollerslev, Tauchen, and Zhou, 2009) seemingly stems from negative jumps and the downside portion of the premium (see, e.g., Andersen, Fusari, and Todorov, 2015; Bollerslev, Todorov, and Xu, 2015; Feunou, Jahan-Parvar, and Okou, 2018; Kilic and Shaliastovich, 2019).

To help focus the paper, I will not discuss any of these results based on options-implied up and downside variation measures any further here. To be clear, however, I think there is much to be learned from comparing and contrasting the high-frequency-based realized semi(co)variation measures to the corresponding “good” and “bad” variation measures implied from options prices. Not only in terms of return predictability, but also in terms of changes in market-wide perceptions of risks and risk aversion and the underlying economic mechanisms at work (see, e.g., Bekaert and Engstrom, 2017; Bekaert, Engstrom, and Xu, 2021; Feunou et al., 2020).

6 Further extending this idea, Chabi-Yo and Loundis (2021) provides an options-based approach for calculating conditional up and downside risk premiums for arbitrary moments.

The plan for the rest of the paper is as follows. I begin with a discussion of the original univariate realized semivariation measures in Section 1, followed by multivariate semico-variation measures in Section 2, before briefly considering newly proposed partial (co)variation measures in Section 3. I will concentrate my discussion on intuition and what I consider to be the most important new empirical insights to date pertaining to volatility forecasting and asset pricing, leaving more technical details and formal theoretical arguments aside.

1 Semivariation Measures

To formally set out the basic ideas, let p_τ denote the time- τ logarithmic price for some asset, with the underlying price process originating at time 0. Consistent with the absence of arbitrage, assume that the price process may be described by an Itô semimartingale of the form,

$$p_\tau = \int_0^\tau \mu_s ds + \int_0^\tau \sigma_s dW_s + J_\tau, \quad \tau \geq 0, \tag{1}$$

where μ_s corresponds to the drift, σ_s defines the diffusive volatility processes, W_s is a standard Brownian motion, and J_τ denotes a finite activity pure jump process. For concreteness, I will refer to the unit time interval $[t, t + 1]$ as a day, with the realized measures being defined at the daily frequency.⁷ For ease of notation, I will assume that high-frequency intraday prices $p_t, p_{t+1/K}, \dots, p_{t+1}$ are observed at $K + 1$ equidistant times over the day, with the corresponding logarithmic discrete-time return over the k th time interval denoted by $r_{t,k} \equiv p_{t+k/K} - p_{t+(k-1)/K}$. It follows then from the theory of quadratic variation for semimartingales that for $K \rightarrow \infty$ (see, e.g., the general discussion in [Ait-Sahalia and Jacod, 2014](#)),⁸

$$RV_t \equiv \sum_{k=1}^K r_{t,k}^2 \xrightarrow{\mathbb{P}} \int_{t-1}^t \sigma_s^2 ds + \sum_{t-1 \leq \tau \leq t} (\Delta J_\tau)^2, \tag{2}$$

where ΔJ_τ captures the jump in the price if a jump occurred at time τ , and otherwise equals zero. In other words, for increasingly finer sampled intraday prices, the realized daily variance, defined as the summation of the within-day high-frequency squared returns, converges uniformly in probability to the sum of the “continuous” price variation plus the squared “jump” price increments over the day.⁹

- 7 Most of the empirical results in the literature, the illustrations highlighted below included, have also been based on daily realized measures constructed from high-frequency intraday data. However, the same measures could, of course, be defined over other non-trivial time intervals, such as a week or a month.
- 8 The seminal insight underlying this result and the approximation of the quadratic variation process of a semimartingale by its approximate quadratic variation, aka the realized volatility in the present context, is according to [Jacod and Protter \(2012\)](#) attributable to [Meyer \(1967\)](#).
- 9 The assumption of equidistant prices and intraday returns spanning the same $1/K$ time interval is not critical for this result, as long as the span of the longest intraday return-interval converges to zero; see, for example, the references and discussion of alternative sampling schemes in [Hansen and Lunde \(2006\)](#).

Data limitations and market microstructure complications that disrupt the martingale property of the price path over very fine time intervals invariably put an upper bound on the practical choice of K , and thus render the continuous limit unattainable in practice. The “volatility signature plot” (Andersen et al., 2000b) provides an oft-used informal diagnostic tool to help gauge the sampling frequency at which market microstructure “noise” and scarcity of observations start to overwhelm the signal, and in turn bias the realized volatility estimates.¹⁰ I will not discuss this deliberate approach for choosing K in practice any further here, nor will I discuss any of the other more advanced estimators and adjustment procedures that have been proposed in the literature to more efficiently estimate the quadratic variation in the presence of “noise.” Suffice it to say, that the same practical complications pertain to the semivariation measures that I will discuss next, and that some of these same “noise robust” procedures might fruitfully be applied in that context as well.¹¹ Meanwhile, the comprehensive empirical analysis in Liu, Patton, and Sheppard (2015) comparing more than 400 alternative estimators across numerous asset classes suggests that in practice it is difficult to beat a simple sub-sampled RV estimator based on 5-min returns.

The realized variation measure in Equation (2) does not differentiate between “good” and “bad” volatility. In an effort to do so, the realized up and down semivariance measures first proposed by Barndorff-Nielsen, Kinnebrock, and Sheppard (2010) (BNKS henceforth) separate the total realized variation into two components associated with the positive and negative high-frequency returns, say $r_{t,k}^+$ and $r_{t,k}^-$, respectively,¹²

$$RV_t^+ \equiv \sum_{k=1}^K (r_{t,k}^+)^2, \quad RV_t^- \equiv \sum_{k=1}^K (r_{t,k}^-)^2. \quad (3)$$

It follows trivially that $RV_t = RV_t^+ + RV_t^-$ for all values of K . Meanwhile, maintaining the basic Itô semimartingale setup and assumptions in Equation (1), in which the order of the drift is dominated by the diffusive and jump components in the in-fill asymptotic limits, BNKS show that the separately defined positive and negative semivariance measures converge uniformly in probability to one-half of the integrated variation plus the sum of the squared positive and negative price jumps, respectively. Hence, to a first-order asymptotic

- 10 Corsi et al. (2001) and Oomen (2003) have independently proposed a similar procedure for determining an appropriate sampling frequency.
- 11 The most prominent ways in which to accommodate “noisy” and scarce high-frequency prices, without resorting to coarser sampling frequencies and discarding potentially valuable information, include the two-scale approach of Zhang, Mykland, and Ait-Sahalia (2005), the kernel-based approach of Barndorff-Nielsen et al. (2008), and the pre-averaging techniques of Jacod et al. (2009).
- 12 While the basic realized volatility concept, the semivariation measures included, rely on the notion of ever finer sampled returns over a fixed time interval and asymptotically vanishing drifts, as typically implemented empirically in the form of daily measures constructed from intraday data, similarly defined measures based on more coarsely sampled data, say monthly measures constructed from daily returns, are often employed in the finance literature. In this situation, the impact of price drifts may not be as immaterial. Accordingly the in-fill asymptotic arguments discussed below may also provide a poorer guide. To help alleviate those concerns, one could rely on mean-adjusted positive and negative returns.

approximation, the difference between the semivariances is entirely determined by the difference in the signed squared jumps,

$$SJ_t \equiv RV_t^+ - RV_t^- \stackrel{\mathbb{P}}{\rightarrow} \sum_{t-1 \leq \tau \leq t} (\Delta J_\tau^+)^2 - (\Delta J_\tau^-)^2, \tag{4}$$

where ΔJ_τ^+ (ΔJ_τ^-) denotes any positive (negative) price jump occurring at time τ . A more refined second-order asymptotic theory would allow for further differentiation between RV_t^+ and RV_t^- arising from a non-zero price drift and/or a non-zero correlation between the innovations to the price and stochastic volatility processes, also commonly referred to as a “leverage effect.” However, this second-order asymptotic theory involves bias terms that are difficult to quantify, rendering it of limited practical use (for additional details see BNKS, Kinnebrock and Podolskij, 2008).¹³ I will briefly return to this issue in my discussion of the realized semicovariance measures in Section 2.

In addition to “looking inside” the semivariation measures to try and identify where differences in RV_t^+ and RV_t^- might be coming from, it would also be interesting to investigate potential time-of-day effects. For instance, it is possible that two different intraday price paths that manifest in the same daily RV_t^+ and RV_t^- , say one up-down and one down-up, may have different implications for investor’s risk perception and end-of-day investment and portfolio rebalancing decisions (see, e.g., the experimental evidence and related discussion in Grosshans and Zeisberger, 2018). As such, semivariation measured over shorter interday time intervals may afford additional useful information.

Meanwhile, to empirically illustrate the daily semivariation measures, Figure 1 shows the logarithmic prices for the S&P 500 SPY ETF at 5-min intervals on four different days in 2020. The first panel shows March 3. At 10:00 am that day the Federal Open Market Committee (FOMC) issued a press release (outside its regularly scheduled announcement cycle) intended to install confidence and ensure that it was closely monitoring the evolving pandemic¹⁴:

The fundamentals of the U.S. economy remain strong. However, the coronavirus poses evolving risks to economic activity. In light of these risks and in support of achieving its maximum employment and price stability goals, the Federal Open Market Committee decided today to lower the target range for the federal funds rate by 1/2 percentage point, to 1 to 1-1/4 percent. The Committee is closely monitoring developments and their implications for the economic outlook and will use its tools and act as appropriate to support the economy.

As the figure shows, the market initially interpreted the statement by the FOMC very positively, resulting in a 2% jump in the value of the index. However, prices gradually drifted down over most of the remaining part of the trading day, and as a result RV_t^+ and RV_t^- ended up fairly close for the day (52.85% and 45.71%, respectively,

- 13 The Online Appendix to Bollerslev et al. (2020) also provides a more general second-order multivariate convergence result, which encompasses the convergence of the realized semivariances as a special “diagonal” case.
- 14 The effect of FOMC announcements on asset prices has been the subject of a growing recent literature; see, for example, Lucca and Moench (2015); Bollerslev, Li, and Xue (2018); and Cieslak, Morse, and Vissing-Jorgensen (2019), and the many references therein. I will not discuss this literature here.

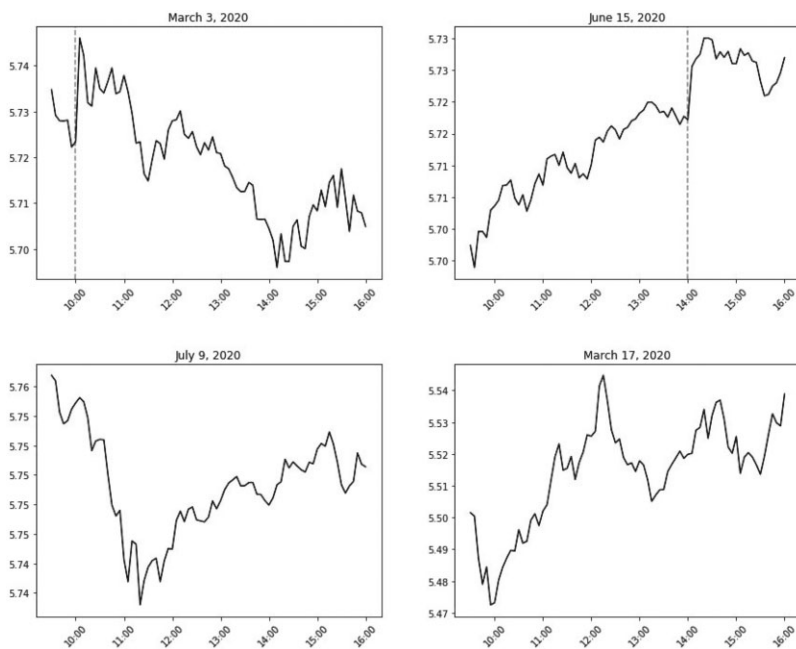


Figure 1 Intraday S&P 500 index prices. The figure shows the logarithmic prices for the S&P 500 SPY ETF at 5-min intervals for four different days in 2020.

in annualized volatility unit, implying a total daily realized volatility of $69.88\% = (52.85\%^2 + 45.71\%^2)^{1/2}$.¹⁵ In contrast, on June 15, as shown in the second top panel, the S&P 500 steadily rose over most of the trading day. The FED's long-planned program to help facilitate lending to smaller businesses, which finally launched on that day, may in part account for this increase. At 2:00 pm on that same day, the FED further announced that it would begin purchasing individual corporate bonds to inject liquidity into the economy, resulting in an apparent jump in the price at that exact time. Correspondingly, RV_t^+ far exceeded RV_t^- for the day (equaling 24.28% and 15.01%, respectively). By comparison, on July 9, as shown in the first panel in the bottom row, RV_t^+ was less than RV_t^- (equaling 11.63% and 16.08%, respectively). That day began with reports of sharply increasing coronavirus cases in many parts of the United States, while a subsequent report of falling unemployment insurance claims may have helped alleviate some of the worst fears about the adverse economic consequences of the pandemic. The final March 17 panel shows another day with exceptionally high overall RV_t (78.18% for the day), yet almost identical RV_t^+ and RV_t^- (equal to 54.99% and 55.58%, respectively). The previous day, March 16, was one of the historically worst days for the market, with the S&P 500 falling by more than 12% (and also one of the highest daily RV_t 's on record at 83.71%). The strong positive pre-noon trend on March 17 obviously helped recover some of those losses.

15 Guided by the aforementioned findings in Liu, Patton, and Sheppard (2015), the realized measures reported throughout are based on 5-min returns sub-sampled and averaged at a 1-min frequency.

Looking closer at the price paths for the four deliberately chosen days in [Figure 1](#), the prolonged intraday return persistence evident for all of the days arguably goes beyond the notion of gradual jumps put forth by [Barndorff-Nielsen et al. \(2009\)](#) (see also [Li et al., 2017](#)). Nor can the identical signed successive price changes be explained by short-lived bursts in volatility, as put forth by [Christensen, Oomen, and Podolskij \(2014\)](#) (see also [Bajgrowicz, Scaillet, and Treccani, 2015](#)). Instead, the observed price paths point to more sustained violations of the basic Itô semimartingale assumption and short-lived price drifts, possibly associated with more difficult to interpret “soft” news, as also recently discussed by [Bollerslev et al. \(2020\)](#).¹⁶ The occasional occurrence of such periods of extreme return persistence, or drift-bursts, and the importance thereof for realized volatility estimation, have also been emphasized by [Laurent and Shi \(2020\)](#) and [Christensen, Oomen, and Renò \(2021\)](#). Further along these lines, [Andersen et al. \(2021c\)](#) and [Laurent, Renò, and Shi \(2021\)](#) have both proposed a new family of realized volatility estimators for the integrated volatility in [Equation \(2\)](#) based on a difference-in-difference type approach explicitly designed to negate the impact of temporary price drifts. It would be interesting to formally extend and apply these ideas to the “robust” estimation of semivariation type measures. For less liquid assets, price staleness, or zero returns, as recently emphasized by [Bandi et al. \(2020\)](#), will also need to be properly accounted for in the estimation.

1.1 Semivariance-Based Volatility Forecasting

The vast empirical literature on GARCH and other parametric stochastic volatility models suggests that equity return volatility tends to increase more following negative return shocks than equally sized positive return shocks. Following [Black \(1976\)](#) and [Christie \(1982\)](#), this return-volatility asymmetry is commonly referred to as the “leverage effect.” This common terminology notwithstanding, financial leverage has long since been discredited as the primary explanation for the observed asymmetries, as the effect is too large empirically to be explained solely by changes in leverage. Additionally, the asymmetries also tend to be much stronger for aggregate equity index portfolio returns than for individual stock returns, further discrediting an all leveraged-based explanation.¹⁷

Expanding on this theme, BNKS in their original empirical investigations of the semivariance measures report that the inclusion of RV_t^- in a daily asymmetric GJR-GARCH model ([Glosten, Jagannathan, and Runkle, 1993](#)),¹⁸

$$h_{t+1} = \omega + \alpha r_t^2 + \beta h_t + \delta r_t^2 I(r_t < 0) + \gamma RV_t^-, \quad (5)$$

typically renders δ and the traditional daily leverage effect term insignificant. The downside realized semivariance RV_t^- is generally also more informative and drives out the significance of the total realized variance RV_t when both are included in the conditional variance

- 16 Relatedly, the recent parametric error correction type model developed by [Andersen et al. \(2021a\)](#) explicitly seeks to link the arrival of new information and price discovery to short-lived price drifts and local mispricing.
- 17 For additional discussion and more recent estimates of the leverage effect based on high-frequency data see [Bollerslev, Litvinova, and Tauchen \(2006\)](#); [Aït-Sahalia, Fan, and Li \(2013\)](#); [Corsi and Renò \(2012\)](#); and [Kalnina and Xiu \(2017\)](#).
- 18 Following [Zakoïan \(1994\)](#), the GJR-GARCH models are also sometimes referred to as a threshold TGARCH model.

equation. Multiplicative Error Models (Engle, 2002), high-frequency-based volatility models (Shephard and Sheppard, 2010), or augmented realized GARCH models (Hansen, Huang, and Shek, 2012) explicitly characterizing the dynamic dependencies in the realized semivariation measures, would allow for a more thorough exploration of these features, and the idea that investors process and interpret “good” versus “bad” news differently (see also the “good” versus “bad” environment GARCH type of models proposed by Bekaert, Engstrom, and Ermolov (2015)).

Instead of the GARCH-based approach in Equation (5), building on the unified framework of Andersen et al. (2003), most realized volatility-based forecasting procedures now tend to rely on simple-to-implement reduced form time series models estimated directly on the realized measures. The heterogeneous autoregressive (HAR) model of Corsi (2009), in which the future realized volatility depends linearly on past realized volatilities over different horizons, has arguably emerged as the most popular such model. This particular formulation is now also routinely used as *the* benchmark model for volatility forecast comparisons (see, e.g., Bollerslev et al., 2018).

The HAR model also provides an especially convenient framework for incorporating the realized semivariation measures (and other explanatory variables) into the construction of volatility forecasts. In particular, following Patton and Sheppard (2015), the basic HAR model for forecasting the daily realized volatility as a function of the lagged daily, weekly, and monthly realized volatilities, is readily extended to a semivariance HAR (SHAR) model,

$$RV_{t+1} = \phi_0 + \phi_D^+ RV_t^+ + \phi_D^- RV_t^- + \phi_W RV_{t:t-4} + \phi_M RV_{t:t-22} + \varepsilon_{t+1}, \quad (6)$$

where $RV_{t:t-4}$ and $RV_{t:t-22}$ refer to the weekly and monthly realized volatilities defined by the summation of the daily volatilities over the past 5 and 22 days, respectively. For $\phi_D^+ = \phi_D^-$, the model obviously reduces to a conventional symmetric HAR model. To help alleviate concerns of heteroscedasticity, the HAR and SHAR models are also sometimes estimated in logarithmic form.

Meanwhile, estimating the SHAR model in Equation (6) with daily realized volatilities for the S&P 500 SPY ETF from January 2, 2002, to December 31, 2020, for a total of 4532 observations, the estimate for ϕ_D^- equals 1.127, with a heteroscedasticity robust standard error of 0.360, while the estimate for ϕ_D^+ equals -0.320 , with a standard error of 0.373. In other words, short-run changes in aggregate market volatility are primarily driven by “bad” volatility. As such, differentiating between “good” and “bad” volatility also results in more accurate volatility forecasts, with the R^2 from the SHAR model equal to 0.624, compared with an R^2 of 0.594 for the basic HAR model.¹⁹ In addition to corroborating Patton and Sheppard (2015), these results also align with the earlier empirical findings of Chen and Ghysels (2011), and the more recent analyses in Audrino and Hu (2016) and Baillie et al. (2019). These same qualitative findings carry over to individual stock return volatilities, although consistent with the “leverage effect” manifesting more strongly at the aggregate

19 Further restricting $\phi_D^+ = -\phi_D^-$, corresponding to a HAR model with the signed jump variation in place of RV_t , the estimated coefficient for SJ_t equals -0.483 , with a standard error of 0.331. Along these lines, the recent text-based analysis in Baker et al. (2021) also suggests that stock market jumps triggered by monetary policy news, which tend to be positive on average, typically lead to lower future volatility.

market level, the differences in the estimates for ϕ_D^- and ϕ_D^+ tend to be more muted for individual stocks.²⁰

It would be interesting to further explore the economic mechanisms underlying these differences. It is possible that “bad” news tend to be released more slowly than “good” news, thereby resulting in a stronger degree of persistence and predictability. Relatedly, it may be that negative news simply receive more and prolonged coverage by the financial news media than positive news. Another possibility is that investors, or market makers, tend reduce their risk exposures after negative shocks, whether voluntarily or by decree, which then through a gradual process of portfolio rebalancing induces higher volatility persistence. In any event, regardless of the underlying mechanism, as I will discuss next, these differences in the way in which “good” and “bad” volatilities impact future changes in total volatility also naturally translate into differences in the way in which the semivariation measures are priced, both in the time-series and cross-sectional dimensions.

1.2 Semivariance-Based Asset Pricing

The “leverage effect” and the apparent asymmetry in the relationship between aggregate equity index return and volatility discussed above may alternatively be interpreted as indirect evidence for a risk-based volatility feedback effect. As elucidated by Pindyck (1982) and French, Schwert, and Stambaugh (1987),²¹ if expected returns and expected volatility are indeed positively related, as in Merton (1973), such a relationship should in turn induce a *negative* relation between realized returns and *unexpected* volatility; see also Campbell and Hentschel (1992). Meanwhile, estimates based on GARCH-in-mean models, or simple regressions of returns on lagged measures of volatility and/or surprises therein, often fail to establish a significant risk-return tradeoff relationship, and sometimes even suggest that expected returns and volatility are negatively related (see, e.g., Bekaert and Wu, 2000; Ghysels, Santa-Clara, and Valkanov, 2005; Bollerslev, Litvinova, and Tauchen, 2006; Rossi and Timmermann, 2015; Theodossiou and Savva, 2016; Hong and Linton, 2020, for empirical evidence and reviews of this extensive literature). However, if investors rationally price downside risk more dearly than upside potential, the risk-return tradeoff might naturally manifest differently in “good” versus “bad” volatility measures. The empirical results discussed in the previous section that improved volatility forecasts may be obtained by differentiating between the influence of past “good” and “bad” volatilities also indirectly supports this conjecture.

To illustrate, Table 1 reports the results from a set of simple return predictability regressions, in which I regress daily, weekly, and monthly S&P 500 returns on a constant and \sqrt{RV} , and a constant and $\sqrt{RV^+}$ and $\sqrt{RV^-}$. For simplicity, and ease of comparisons, I define the weekly (monthly) returns as the sum of 5 (21) daily returns divided by 5 (21). The sample spans the same 2002–2020 time period analyzed above, for a total of 4783 daily

- 20 As a case in point, the average estimates for ϕ_D^- and ϕ_D^+ over the identical 2002–2020 sample period for the “bad covid” stocks and FAANG stocks discussed further in Section 2, equal to 0.621 and -0.008 , respectively, with most of the ϕ_D^- s being strongly statistically significant, and most of the ϕ_D^+ s insignificant.
- 21 Robins and Smith (2021) provides a fresh look and empirical re-evaluation of the widely cited French, Schwert, and Stambaugh (1987) paper with recent data and modern econometric techniques.

Table 1 S&P 500 return predictability regressions

	Daily	Weekly	Monthly
\sqrt{RV}	0.038 (0.056) [0.675]	0.003 (0.049) [0.055]	0.003 (0.030) [0.111]
$\sqrt{RV^+}$	-0.246 (0.236) [-1.041]	-0.100 (0.081) [-1.227]	-0.051 (0.036) [-1.439]
$\sqrt{RV^-}$	0.303 (0.248) [1.223]	0.106 (0.085) [1.246]	0.057 (0.027) [2.145]

The table reports the results from simple return predictability regressions of daily, weekly (5 days), and monthly (21 days) S&P 500 SPY returns on a constant and RV , and a constant and RV^+ and RV^- . The sample spans 2002–2020, for a total of 4783 daily observations. All of the regressions are estimated at a daily frequency with overlapping returns. Newey–West standard errors accounting for the serial correlation in the errors induced by the overlap are reported in parentheses. The square brackets report alternative Hodrick (1992) t -statistics for testing the null of no predictability.

return observations. All of the regressions are estimated at a daily frequency with overlapping weekly and monthly returns. In addition to Newey–West standard errors accounting for the serial correlation induced by the overlap (given in parentheses), following Hodrick (1992) I also report t -statistics (in square brackets) for testing the null hypothesis of no predictability based on the rearranged return regressions without any overlap.

Looking at the results, there is a clear pattern in the point estimates. In line with the extant empirical literature referred to above, the estimated regression coefficients for \sqrt{RV} are all small and insignificant, underscoring the difficulties in empirically establishing a traditional risk-return tradeoff relationship. However, differentiating between “good” and “bad” volatility, the estimated regression coefficients for $\sqrt{RV^+}$ are all negative, albeit not statistically significant at conventional levels, while the coefficients associated with $\sqrt{RV^-}$ are all positive, and also statistically significant at the longer monthly return horizon.²²

These simple regression-based results also mirror those of the earlier unpublished study by Breckenfelder and Tédongap (2012), and the time-series regressions reported therein in which the up minus down realized semivariance measure for the aggregate market portfolio negatively predicts future market returns. The results are also broadly consistent with Feunou, Jahan-Parvar, and Tédongap (2013), and the empirically significant risk-return tradeoff relationship for the model-based relative downside risk measure established therein.²³ The equilibrium asset pricing model in Farago and Tédongap (2018), based on a representative investor with generalized disappointment

- 22 The degree of return predictability afforded by any one of the regressions is invariably very limited, with R^2 s close to zero. As such, the “economic significance” of the estimates should be interpreted with some caution.
- 23 The results also echo the earlier empirical findings of Bali, Demirtas, and Levy (2009), and a positive tradeoff between expected aggregate market returns and VaR, with VaR interpreted as a measure of downside risk.

aversion, provides a possible rationale for this differential pricing of “good” versus “bad” volatility at the aggregate market level.

Motivated in part by these empirical findings for the market portfolio, [Bollerslev, Li, and Zhao \(2020\)](#) look instead at the role of “good” versus “bad” volatility at the individual stock level. Focusing on the stock-specific differences in RV^+ and RV^- —and the signed jump variation defined in [Equation \(4\)](#) scaled by total RV , they find that stocks with relatively higher (scaled) SJ earn systematically *lower* future returns than stocks with relatively lower (scaled) SJ . Also, these cross-sectional differences cannot be explained by any of the standard controls and/or systematic risk factors traditionally used for explaining the variation in individual stock returns. Nor can the differences be explained by high-frequency-based realized skewness and/or kurtosis measures ([Amaya et al., 2015](#)), reinforcing that the semivariation measures are not simply capturing previously documented skewed or fat tailed distributional deviations from normality. Further expanding on this, [Yu, Mizrach, and Swanson \(2020\)](#) find that SJ -based sorts restricted to “smaller sized” jumps result in even larger and more significant cross-sectional return differences.

It is possible that a systematic risk factor explicitly related to downside aggregate market volatility, along the lines of the aforementioned study by [Farago and Tédongap \(2018\)](#), could help explain why investors value individual stocks with relatively high SJ more dearly than comparable stocks with low SJ , although that has yet to be established. An alternative, and in my opinion more likely, explanation is that the cross-sectional differences in future returns associated with differences in firm level “good” versus “bad” volatility may be linked to behavioral biases and investors’ overreaction to “bad” news coupled with limits to arbitrage, along the lines of [Shleifer and Vishny \(1997\)](#). Corroborating this thesis, the differences in returns for stocks with high versus low relative SJ appear stronger for smaller firm stocks, stocks with higher overall volatility, and less liquid stocks, all of which arguably pose greater arbitrage risks. I will return to that same theme in my discussion of the empirical results pertaining to the pricing of the semicovariation measures introduced next.

2 Multivariate Semicovariation Measures

The univariate realized semivariation measures and empirical results discussed in the previous section were based on the decomposition of the total variation for a given asset into “good” versus “bad” volatility for that particular asset. However, most empirical questions and hypotheses in asset pricing finance are inherently multivariate in nature, entailing measures of the covariation among multiple assets and considerations of systematic non-diversifiable risks. Fortunately, the traditional realized volatility measure in [Equation \(2\)](#) is readily extended to a realized covariance measure by considering the sum of cross-products of vectors of high-frequency intraday returns ([Andersen et al., 2003](#); [Barndorff-Nielsen and Shephard, 2004a](#)). Following [Bollerslev et al. \(2020\)](#) (henceforth BLPQ) the realized semivariances defined in [Equation \(3\)](#) may similarly be extended to a multivariate setting and a decomposition of the realized covariance matrix into *four* unique additive realized semicovariance components determined by the signs of the underlying high-frequency returns.

To fix ideas, consider the multivariate extension of the generic Itô semimartingale in Equation (1) to a d -dimensional vector log-price process,

$$\mathbf{p}_\tau = \mathbf{p}_0 + \int_0^\tau \boldsymbol{\mu}_s ds + \int_0^\tau \boldsymbol{\sigma}_s d\mathbf{W}_s + \mathbf{J}_\tau, \quad \tau \geq 0, \quad (7)$$

where $\boldsymbol{\mu}_s$ is a \mathbb{R}^d -valued drift process, \mathbf{W}_s is a d -dimensional standard Brownian motion, $\boldsymbol{\sigma}_s$ is a $d \times d$ dimensional stochastic volatility matrix, and \mathbf{J}_τ is a finitely active pure-jump process.²⁴ For notational simplicity, and in parallel to the discussion and definitions in Section 1, I will assume that intraday prices for all of the d assets are available at K equally spaced times over the trading day $[t, t + 1]$. Then, in a direct parallel to the seminal result in Equation (2),

$$\mathbf{RCOV}_t \equiv \sum_{k=1}^K \mathbf{r}_{t,k} \mathbf{r}'_{t,k} \xrightarrow{\mathbb{P}} \int_{t-1}^t \boldsymbol{\sigma}_\tau \boldsymbol{\sigma}'_\tau + \sum_{t-1 \leq \tau \leq t} \mathbf{J}_\tau \mathbf{J}'_\tau, \quad (8)$$

where $\mathbf{r}_{t,k} = \mathbf{p}_{t+k/K} - \mathbf{p}_{t+(k-1)/K}$ denotes the logarithmic discrete-time return vector for the k th time interval on day t . Now, denote the corresponding vectors of signed positive and negative high-frequency returns by $\mathbf{r}_{t,k}^+$ and $\mathbf{r}_{t,k}^-$, respectively.²⁵ Mirroring the decomposition of the realized variance into two semivariances in Equation (3), the four realized semicovariance matrices are then simply defined by,

$$\mathbf{P}_t \equiv \sum_{k=1}^K \mathbf{r}_{t,k}^+ \mathbf{r}_{t,k}^{+\prime}, \quad \mathbf{N}_t \equiv \sum_{k=1}^K \mathbf{r}_{t,k}^- \mathbf{r}_{t,k}^{-\prime}, \quad \mathbf{M}_t^+ \equiv \mathbf{M}_t^{-\prime} \equiv \sum_{k=1}^K \mathbf{r}_{t,k}^+ \mathbf{r}_{t,k}^{-\prime}. \quad (9)$$

Note that by definition $\mathbf{RCOV}_t \equiv \mathbf{P}_t + \mathbf{N}_t + \mathbf{M}_t^+ + \mathbf{M}_t^-$.

The two “concordant” realized semicovariance matrices (\mathbf{P}_t and \mathbf{N}_t) comprise the positive and negative realized semivariances on their diagonals together with scalar realized covariances constructed from identical-signed positive or negative high-frequency returns on their off-diagonals. The “discordant” realized semicovariance matrices (\mathbf{M}_t^+ and \mathbf{M}_t^-) have zeros along their diagonals and scalar realized covariances constructed from opposite-signed returns on their off-diagonals. Since \mathbf{P}_t and \mathbf{N}_t are defined as sums of vector outer-products, these matrices are both positive semidefinite, while \mathbf{M}_t^+ (and \mathbf{M}_t^-) is indefinite. In situations where the ordering of the assets is arbitrary, the two discordant matrices are naturally combined into a single “mixed” matrix $\mathbf{M}_t \equiv \mathbf{M}_t^+ + \mathbf{M}_t^-$. Note that while the realized variance of a portfolio may be calculated from the realized covariance matrix of the assets included in the portfolio based on the identity $\text{RV}_t \equiv \mathbf{w}' \mathbf{RCOV}_t \mathbf{w}$, where \mathbf{w} refers to the vector of portfolio weights, the up and down semivariance measures for a portfolio are not simply equal to portfolio weighted averages of the semicovariances.

- 24 This general setup, which also underlies the asymptotic theory in BLPQ, explicitly allows for multivariate “leverage effects” in the form of dependence between changes in the price and changes in volatility, as well as stochastic volatility-of-volatility, volatility jumps and price-volatility co-jumps.
- 25 Formally, $\mathbf{r}_{t,k}^+ \equiv \mathbf{r}_{t,k} \odot \mathbf{I}_{t,k}^+$ and $\mathbf{r}_{t,k}^- \equiv \mathbf{r}_{t,k} \odot \mathbf{I}_{t,k}^-$, where \odot denotes the Hadamard (element-by-element) product, and $\mathbf{I}_{t,k}^+ \equiv [1_{\{r_{t,k,1} > 0\}}, \dots, 1_{\{r_{t,k,N} > 0\}}]'$ and $\mathbf{I}_{t,k}^- \equiv [1_{\{r_{t,k,1} \leq 0\}}, \dots, 1_{\{r_{t,k,N} \leq 0\}}]'$, respectively.

In particular, $RV_t^+ \neq \mathbf{w}'(\mathbf{P}_t + \mathbf{M}_t^+)\mathbf{w}$ and $RV_t^- \neq \mathbf{w}'(\mathbf{N}_t + \mathbf{M}_t^-)\mathbf{w}$.²⁶ Instead, the realized semicovariance measures afford an alternative asset-specific “look inside” the quadratic covariation.

To help further intuit the measures, consider the stylized setting in which the logarithmic price process in Equation (7) follows a continuous semi-martingale with no drift ($\mu_s \equiv 0$), no jumps ($J_\tau \equiv 0$), constant unit volatility for all of the assets, and instantaneous correlation ρ among all of the different pairs of assets, in which case

$$RCOV_t \xrightarrow{\mathbb{P}} \mathbf{I}_d + (\mathbf{J}_d - \mathbf{I}_d)\rho,$$

where \mathbf{I}_d denotes the $d \times d$ identity matrix and \mathbf{J}_d is a $d \times d$ matrix of ones. In this situation,

$$\mathbf{P}_t, \mathbf{N}_t \xrightarrow{\mathbb{P}} \frac{1}{2}\mathbf{I}_d + (\mathbf{J}_d - \mathbf{I}_d) \times \frac{\sqrt{1 - \rho^2} + \rho \arccos(-\rho)}{2\pi},$$

while

$$\mathbf{M}_t^+, \mathbf{M}_t^- \xrightarrow{\mathbb{P}} (\mathbf{J}_d - \mathbf{I}_d) \times \frac{\rho \arccos \rho - \sqrt{1 - \rho^2}}{2\pi}.$$

As these limiting values make clear, the more strongly correlated the returns and the larger the value of ρ , the larger (respectively, smaller) and closer to 1/2 (respectively, 0) are the limiting values of the off-diagonal elements in the two concordant (resp. discordant) semicovariances. Accordingly, one might naturally expect the relative importance of the concordant semicovariances to increase during times of financial crises and “market stress,” periods that are typically accompanied by higher overall asset return correlations.

The above stylized diffusive setting does not allow for any differences between \mathbf{P}_t and \mathbf{N}_t (or \mathbf{M}_t^+ and \mathbf{M}_t^-). Nor does it provide for any distinction between the limiting values of RV_t^+ and RV_t^- for portfolios constructed from the d assets. Meanwhile, in line with the discussion pertaining to the realized semivariances in the previous section, and as illustrated further below, empirically on days with important “directional news,” \mathbf{P}_t and \mathbf{N}_t can differ quite dramatically. The more advanced limit theory developed in BLPQ identifies three distinct channels through which such differences can occur, namely directional “co-jumps,” a type of “co-drifting,” and a specific form of “dynamic leverage effect.” In parallel to the results for the semivariances in Equation (4), the first of these channels manifests directly in the first-order asymptotics, with the difference between the probability limits of \mathbf{P}_t and \mathbf{N}_t being entirely determined by identically signed co-jumps. That is,

$$\mathbf{P}_t - \mathbf{N}_t \xrightarrow{\mathbb{P}} \sum_{t-1 \leq \tau \leq t} \Delta J_\tau^+ \Delta J_\tau^{+'} - \Delta J_\tau^- \Delta J_\tau^{-'}, \tag{10}$$

where ΔJ_τ^+ (respectively, ΔJ_τ^-) refers to the d -dimensional vector of time- τ positive (respectively, negative) jumps in each of the d assets. As shown in BLPQ, a feasible central limit theorem further permits the formulation of a formal test (termed the JCSD test) for significant differences in \mathbf{P}_t and \mathbf{N}_t due to co-jumps. By comparison, co-drifting and dynamic leverage effects, both of which can cause the diffusive components of \mathbf{P}_t and \mathbf{N}_t to differ,

26 This also implies that the portfolio weights for the standard minimum variance portfolio will generally not minimize RV_t^- . To minimize the “bad” volatility, one could rely on the approximate procedures recently developed by Rigamonti et al. (2021).

formally manifest in second-order bias terms in a non-central limit theorem.²⁷ To aid in interpreting the magnitude of these diffusive differences, BLPQ provides an additional inference tool (termed the DCSD detection scheme), which under more restrictive regularity conditions may be justified as an asymptotically valid test.

To empirically illustrate these features, Figure 2 shows the within-day 5-min logarithmic prices (normalized to zero at the beginning of the day) on June 15, 2020, and July 9, 2020, for two separate groups of stocks: “bad covid” stocks and the FAANG group of stocks. The FAANG group of stocks comprises the five tech giants: Facebook (FB), Amazon (AMZN), Apple (AAPL), Netflix (NFLX), and Alphabet (GOOG). These five stocks currently account for close to 20% of the total market capitalization of the S&P 500 index. My definition of the “bad covid” group of stocks follows [Bollerslev, Patton, and Zhang \(2021\)](#), who rely on hierarchical clustering methods together with a novel cross-validation approach for determining the optimal number of clusters and cluster groupings for the S&P 100 individual stocks based on their daily realized semicorrelations. The “bad covid” cluster of stocks that I consider here first arose on January 31, 2020, coincident with the World Health Organization first declared the coronavirus outbreak a health emergency of international concern.²⁸ The cluster consists of: Boeing (BA), Occidental Petroleum (OXY), Raytheon (RTX), Schlumberger (SLG), and Simon Property Group (SPG). These companies obviously range quite widely in terms of their main lines of business, and do not line up with conventional industry type classifications. Instead, the grouping reflects commonalities in the way in which the prices of the different stocks respond to new information, as seen through the lens of the semicovariation measures.

Looking first at June 15, the price paths for the “bad covid” stocks fairly closely mirror the intraday price path for the S&P 500 SPY ETF on that same day shown in Figure 1. There is a mostly monotonic increase in the prices for all of the stocks over the earlier part of day, along with an apparent jump at 2:00 pm when the FED announced its intention to inject additional liquidity into the economy. By comparison, the price paths for the FAANG stocks on that same day, shown in the second top panel, evidence much more muted appreciations. Looking at the bottom two panels for July 9, which as previously noted saw a sharp rise in new reported coronavirus cases across the United States, the price paths again show clear within-cluster similarities and across cluster differences. All of the “bad covid” stocks performed very poorly losing more than 4% on that day, while the FAANG stocks as a group ended up almost flat for the day.

In line with these observations, the relative importance of the different semicovariance components also differ quite markedly within and across the two clusters of stocks on each of the two different days. For instance, while the average P_{ij} normalized by $(RV_i \cdot RV_j)^{1/2}$ for all of the (i, j) pairs of “bad covid” stocks equals 0.528 on June 15, the similarly normalized N_{ij} and M_{ij} components only amount to 0.279 and -0.028 , respectively, implying

- 27 These terms also set the analysis pertaining to realized semivariances and semicovariances apart from most other high-frequency econometrics and related in-fill asymptotics, which typically on standard central limit theorem type arguments; see, for example, [Aït-Sahalia and Jacod \(2014\)](#).
- 28 In contrast, when clustering the S&P 100 stocks based on their standard realized correlations, a similar “bad covid” cluster of stocks did not arise until March 18, 2020.

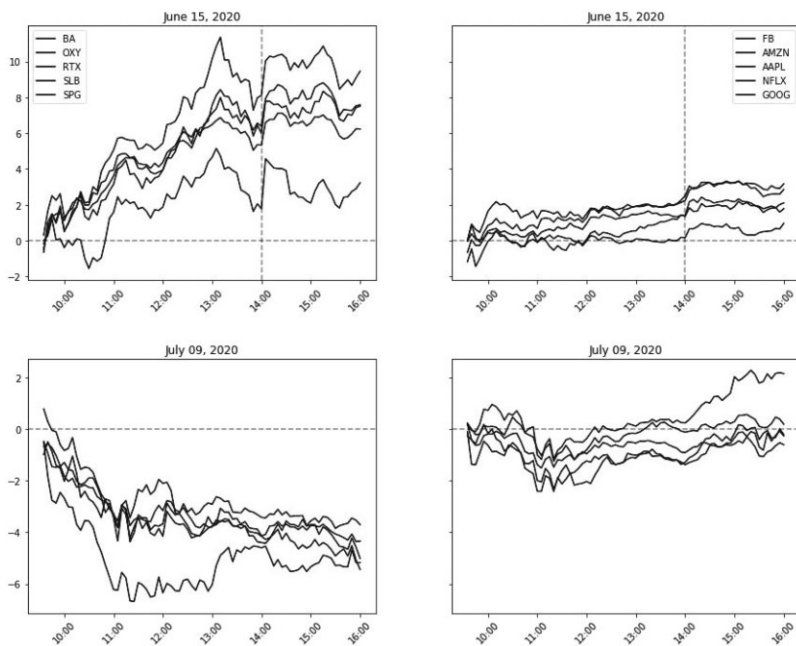


Figure 2 Intraday individual equity prices. The figure shows the intraday (normalized to zero at the beginning of the day) logarithmic prices at 5-min intervals on June 15, 2020 (top panels) and July 9 (bottom panels) for each of the “bad covid” stocks (left panels) and FAANG stocks (right panels), as defined in the main text.

a total within-cluster average realized correlation of 0.779 for the day.²⁹ By comparison, the average normalized N_{ij} for the cluster of “bad covid” stocks equals 0.442 on July 9, compared with 0.235 and -0.097 for the normalized P_{ij} and M_{ij} components. The mixed semicovariance components, of course, tend to play a relatively more important role for the between cluster correlations. As a case in point, looking at all of the pairwise combinations of “bad covid” stocks and FAANG stocks on July 9, the average normalized M_{ij} equals -0.134 , while the normalized P_{ij} and N_{ij} equal 0.197 and 0.365, respectively.

Putting the discussion pertaining to Figure 2 further into perspective, recall that under the standard Itô semimartingale assumption, to a first-order asymptotic approximation differences in the concordant (discordant) semicovariance components are entirely driven by co-jumps. Empirically, many large-sized jumps tend to be readily associated with precisely timed news announcements, or “sharp” news. However, aside from the fairly minor-sized jumps for most of the stocks evident at 2:00 pm on June 15 in response to the FED’s announcement at that time, co-jumps do not seem to account for the differences in the realized semicovariation measures for either of the 2 days depicted in Figure 2. Instead, echoing the discussion pertaining to the price paths for the S&P 500 SPY ETF in Figure 1, both of

29 This particular normalization ensures that the so-defined three realized semicorrelations add up to the standard realized correlation. Following Bollerslev, Patton, and Quaevliog (2020), other normalizations based on the realized semivariances could be employed in the definition of alternative realized semicorrelation type measures.

the days shown in Figure 2 seem to be characterized by “soft” and more difficult to interpret news, resulting in distinctly different intraday return persistence for the two different clusters of stocks, and in turn varying importance of the three different semicovariance components.³⁰

Further highlighting these differences, Table 2 shows the average values of the previously defined daily realized semicorrelations for all of 2020 for the 10 unique pairs of stocks in the “bad covid” and FAANG clusters, along with the 25 unique “bad covid” versus FAANG stock pairs. As expected, the within-cluster averages of the concordant ρ^P s and ρ^N s are very close, as are the corresponding cross-cluster averages. Meanwhile, underscoring the different high-frequency dynamic features of the stocks in each of the two clusters, the average value of the discordant ρ^M s for the between cluster pairs of stocks is much larger (in an absolute value sense) than the two averages for the within-cluster stock pairs. I turn next to a discussion of how these differences in the relative importance of the semicovariance components across stocks and through time may be used in the construction of improved volatility forecasts.

2.1 Semicovariance-Based Volatility Forecasting

A number of different GARCH and stochastic volatility type models have been proposed in the literature to account for return-volatility asymmetries in multivariate settings (e.g., Kroner and Ng, 1998; McAleer, Hoti, and Chan, 2009; Francq and Zakoian, 2012). To illustrate, let \mathbf{r}_t^+ and \mathbf{r}_t^- denote the vector of signed daily positive and negative returns, respectively. A straightforward multivariate generalization of the aforementioned univariate GJR-GARCH model, in which the conditional covariance matrix \mathbf{H}_{t+1} responds asymmetrically to cross-products of lagged returns depending on the signs of the returns, may then be expressed as,³¹

$$\mathbf{H}_{t+1} = \Omega + \alpha_P \mathbf{r}_t^+ \mathbf{r}_t^{+'} + \alpha_N \mathbf{r}_t^- \mathbf{r}_t^{-'} + \alpha_M (\mathbf{r}_t^+ \mathbf{r}_t^{-'} + \mathbf{r}_t^- \mathbf{r}_t^{+'}) + \beta \mathbf{H}_t.$$

Directly paralleling the univariate models discussed in Section 1.1, the cross-products of the daily lagged return vectors in this multivariate model may naturally be replaced by their respective realized semicovariance components,

$$\mathbf{H}_{t+1} = \Omega + \alpha_P \mathbf{P}_t + \alpha_N \mathbf{N}_t + \alpha_M \mathbf{M}_t + \beta \mathbf{H}_t. \quad (11)$$

For $\alpha_P = \alpha_N = \alpha_M$ this obviously collapses to a symmetric multivariate realized GARCH model. However, by allowing α_P , α_N , and α_M to differ, the realized semicovariance-based model in Equation (11) allows for more refined and potentially more informative intraday “continuous” as opposed to daily threshold-based multivariate “leverage effects.”

The estimation results in Bollerslev, Patton, and Quaedvlieg (2020) for a cross-section of individual stocks support this idea, and point to significant improvements in overall model fit by allowing the impact of the lagged semicovariance components to differ. In line

- 30 Relatedly, Jiang, Li, and Wang (2021) have recently documented the existence of pervasive underreaction to various types of firm-specific news and strong intraday individual stock price drifts following large (in an absolute sense) “news-driven” returns.
- 31 For illustrative purposes, I assume the α s and β to be scalar, but richer non-scalar parameterizations, and models in which the impact of $\mathbf{r}_t^+ \mathbf{r}_t^{-'}$ and $\mathbf{r}_t^- \mathbf{r}_t^{+'}$ are not necessarily the same, could, of course, be, and has been, entertained empirically.

Table 2 Average daily realized semicorrelations

	ρ	ρ^P	ρ^N	ρ^M
“Bad Covid”	0.422	0.268	0.290	-0.139
FAANG	0.525	0.315	0.315	-0.103
“Bad Covid” vs. FAANG	0.148	0.196	0.206	-0.254

The table shows the average daily realized semicorrelations for 2020 for the 10 unique pairs of stocks in the “Bad Covid” and FAANG clusters of stocks, respectively, along with the 25 unique pairs of cross-group correlations. The three realized semicorrelations for stock pair (i, j) are defined as P_{ij} , N_{ij} and M_{ij} divided by $(RV_i \cdot RV_j)^{1/2}$.

with existing empirical evidence pertaining to more traditional multivariate asymmetric GARCH models, the estimates for α_N for the realized models in Equation (11) are typically larger than the estimates for both α_P and α_M , implying that most of the co-persistence may be traced to common “bad” intraday news. As such, the asymmetric realized semicovariance-based models generally also produce more accurate covariance matrix forecasts compared with the forecasts from symmetric multivariate realized GARCH models that restrict all of the α 's to be the same.

Multivariate GARCH models, their realized versions included, can be challenging to implement empirically, especially in large dimensions. Alternatively, and in direct parallel to the univariate semivariance-based HAR models discussed in Section 1.1, the realized semicovariance measures may similarly be used in the formulation of simple-to-implement multivariate HAR type forecasting models. To illustrate the basic idea, consider the bivariate case and the 3D HAR model originally estimated in BLPQ, in which each of the scalar semicovariance components are allowed to depend on its own daily, weekly, and monthly lags, as well as the lags of the other two components. Specifically, for asset pairs (i, j) ,

$$\begin{bmatrix} P_{t+1,ij} \\ N_{t+1,ij} \\ M_{t+1,ij} \end{bmatrix} = \begin{bmatrix} \phi_P \\ \phi_N \\ \phi_M \end{bmatrix} + \Phi_D \begin{bmatrix} P_{t,ij} \\ N_{t,ij} \\ M_{t,ij} \end{bmatrix} + \Phi_W \begin{bmatrix} P_{t:t-4,ij} \\ N_{t:t-4,ij} \\ M_{t:t-4,ij} \end{bmatrix} + \Phi_M \begin{bmatrix} P_{t:t-21,ij} \\ N_{t:t-21,ij} \\ M_{t:t-21,ij} \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1,ij}^P \\ \epsilon_{t+1,ij}^N \\ \epsilon_{t+1,ij}^M \end{bmatrix},$$

where Φ_D , Φ_W , and Φ_M are 3×3 parameter matrices. Restricting the Φ matrices to be scalar, and forcing the intercepts to be the same ($\phi_P = \phi_N = \phi_M$), the above formulation obviously collapses to a standard univariate HAR model for $RCOV_{ij} \equiv P_{ij} + N_{ij} + M_{ij}$. However, the estimates for a sample of individual stocks reported in BLPQ reveal highly significant differences in the freely estimated parameters, with the dynamic dependencies in both P_{ij} and N_{ij} driven almost exclusively by the lagged N_{ij} terms, while the mixed semicovariances M_{ij} appear to be mostly driven by their own lags. Digging deeper, the semicovariance-based models generally also assign greater weights to the daily lagged measures and thus respond faster to new information, compared with traditional multivariate HAR models for $RCOV_{ij}$. Of course, to ensure that the forecasts for a full covariance matrix based on these element-wise forecasts for the individual covariances are positive definite, additional parameter restrictions, and/or regularization will have to be imposed.

These gains from the use of the realized semicovariance measures for covariance matrix forecasting extend to the forecasts of portfolio variances. Consider the decomposition of

the realized variance of a portfolio with portfolio weights w into its portfolio specific semi-covariance components,

$$RV_t \equiv w'RCOV_t w = w'P_t w + w'N_t w + w'M_t w.$$

As previously noted, these portfolio semicovariance measures differ from the up and down semivariance measures defined in Equation (3), and in contrast to the latter, which only require high-frequency returns on the portfolio itself, the semicovariance measures require high-frequency returns for *all* of the assets included in the portfolio. BLPQ again find univariate HAR models for realized portfolio variances that exploit these portfolio specific semicovariance measures to be both faster, in the sense of assigning larger weights to the lagged daily realized measures, and more persistent, in the sense of shocks decaying at a slower rate, than conventional HAR and SHAR models based on lagged portfolio realized variances and semivariances only.

I will next discuss various findings which suggest that the separate semicovariance components underlying this covariance matrix and portfolio variance forecasting results are not priced the same either.

2.2 Semicovariance-Based Asset Pricing

The basic premise that investors only care about downside systematic risk(s) effectively implies that only “bad” covolatility should be priced. In the case of aggregate market risk, only the covariation with negative market returns ought to carry a risk premium, as in the downside version of the CAPM (Ang, Chen, and Xing, 2006). Meanwhile, market frictions and/or behavioral biases may cause assets with identical downside covariation to be priced differently. In particular, consider two assets with the same total downside covariation. If one of the two assets covaries more strongly with the market when the market is performing poorly, thereby exacerbating the systematic downside risk, it may naturally be expected to carry a *higher* overall risk premium than the other asset, which covaries less strongly with the market when the market is performing poorly.

To succinctly illustrate this idea, consider Figure 3 adapted from Hogan and Warren (1974). If investors do not care about “good” volatility, the covariation stemming from the two states where the market returns are positive (P and M^+) should not earn any risk premium. In contrast, any covariation associated with joint negative market and individual asset returns (N) should be positively compensated, while the mixed covariation stemming from negative market returns and positive individual asset returns (M^-) ought to carry a negative, and in an absolute value sense lower, risk premium. The decomposition of the conventional CAPM beta into four realized semibetas proposed by Bollerslev, Patton, and Quaedvlieg (2021a) is directly motivated by these considerations.

Specifically, omitting the time t subscript for notational convenience, and relying on the same element-wise notation as above, the four realized semibetas for asset i with respect to the market portfolio f are simply defined by,

$$\beta_i^P \equiv \frac{P_{fi}}{RV_f}, \quad \beta_i^N \equiv \frac{N_{fi}}{RV_f}, \quad \beta_i^{M^+} \equiv \frac{-M_{fi}^+}{RV_f}, \quad \beta_i^{M^-} \equiv \frac{-M_{fi}^-}{RV_f}, \quad (12)$$

where RV_f refers to the realized variance of the return on the market. The negative signs for the two discordant semibetas ensure that all of the realized semibetas are non-negative by

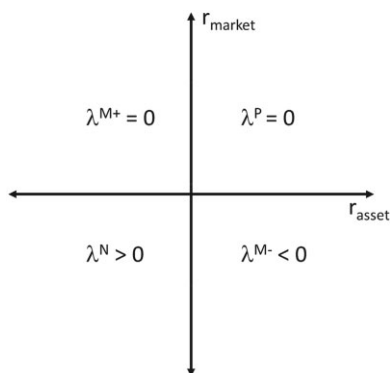


Figure 3 Semicovariance pricing. The figure, adapted from Hogan and Warren (1974), shows the signs of the risk premiums (λ 's) associated with the four semicovariance components implied by a mean semivariance pricing framework.

definition. If the four semibetas move proportionally with each other, as would be the case in a Gaussian world, they would convey no additional information over and above the conventional realized beta, $\beta \equiv \beta^P + \beta^N - \beta^{M+} - \beta^{M-}$ (Barndorff-Nielsen and Shephard, 2004a; Andersen et al., 2006). As such, the four semibetas would also necessarily be priced the same.

To illustrate, consider the same stylized diffusive setting with no drift, no jumps, and constant volatility discussed in Section 2, for which $\text{RCOV}_t \xrightarrow{\mathbb{P}} \mathbf{I}_d + (\mathbf{J}_d - \mathbf{I}_d)\rho$. Further denote the probability limit of the standard realized beta by b . It follows that in this situation,

$$\beta^P, \beta^N \xrightarrow{\mathbb{P}} \frac{b}{2\pi} \left(\rho^{-1} \sqrt{1 - \rho^2} + \arccos(-\rho) \right),$$

while

$$\beta^{M+}, \beta^{M-} \xrightarrow{\mathbb{P}} \frac{b}{2\pi} \left(\rho^{-1} \sqrt{1 - \rho^2} - \arccos(\rho) \right).$$

Hence, while the relative contribution of the concordant versus discordant semibetas to the standard beta obviously depends on the strength of the correlation between the asset and the market, each of the two concordant (discordant) semibetas consistently estimate the same quantity. In a non-diffusive setting, or non-Gaussian world, however, the probability limits for *all* of the semibetas may formally differ. In accordance with the economic intuition conveyed by Figure 3, they may also be priced differently.

The empirical results reported in Bollerslev, Patton, and Quaedvlieg (2021a) support this conjecture. Only β^N and β^{M-} appear to be priced in the cross-section of individual stocks, in the sense that higher values of β^N and $-\beta^{M-}$ both tend to be associated with higher realized returns. Meanwhile, the estimated risk premium for β^N across a variety of specifications and different samples of stocks and time periods is typically around double that of the estimated premium for $-\beta^{M-}$, possibly related to a “betting against beta” type story (Frazzini and Pedersen, 2014). The hypothesis that the two risk premiums are

numerically the same is also easily rejected statistically.³² By comparison the estimated risk premium for the traditional realized β is even less than the premium for $-\beta^{M^-}$.

On the face of it, the different risk premiums for β^N and $-\beta^{M^-}$ may seem puzzling. In a frictionless financial market the sign of the covariation with the market can costless be changed through short positions, so that in order to prevent arbitrage opportunities the two risk premiums ought to be the same. The downside version of the CAPM also effectively combines the “good” and “bad” downside semibetas into a single downside beta $\beta^D \equiv \beta^N - \beta^{M^-}$, with a single risk premium. However, as argued by [Pontiff \(1996\)](#) and [Shleifer and Vishny \(1997\)](#), with legal constraints and charters impeding many institutional investors from short-selling, and many individual investors simply reluctant to sell short, this may result in limits-to-arbitrage and accompanying arbitrage risks (see also [Hong and Sraer, 2016](#)). These arbitrage risks may in turn induce a wedge between the pricing of the N and $-M^-$ semicovariation components. Consistent with this reasoning, [Bollerslev, Patton, and Quaadvlieg \(2021a\)](#) further find that the hypothesis of identical risk premiums for β^N and $-\beta^{M^-}$ is more strongly rejected for stocks with higher idiosyncratic volatility, a commonly used proxy for greater impediment to price-correcting arbitrage (see, e.g., [Stambaugh, Yu, and Yuan, 2015](#)).

I will not pursue this line of reasoning and the pricing of the different semibetas any further here. Instead, to merely illustrate the practical calculation of the semibetas, [Table 3](#) reports the averages of the daily realized semibetas for all of 2020 for the same five “bad covid” and five FAANG stocks discussed in Section 2. All of the betas are calculated with respect to the S&P 500 market portfolio. As the table shows, the averages of the semibetas for the “bad covid” stocks all exceed those for the FAANG stocks, indicative of more pronounced non-normal dependencies. It would be interesting to further explore the economic mechanisms behind these differences, and what explain the variation in semibetas across stocks and time more generally. The model in [Bolorforoosh et al. \(2020\)](#), explicitly allowing for time-varying betas and beta risk, may prove useful in thinking about these issues.³³

In addition to the averages of the individual stocks semibetas, [Table 3](#) also reports the averages of the 2020 daily realized semibetas for the two equally weighted portfolios comprise the five “bad covid” and five FAANG stocks, respectively. While the traditional realized beta of a portfolio, and the up and downside portfolio betas, may be calculated as the portfolio weighted averages of the respective realized betas for the individual stocks included in the portfolio, the semibetas of a portfolio are not simply equal to the portfolio weighted averages of the individual semibetas. Rather, consistent with the idea that portfolio formation mute the impact of firm-specific jumps and other idiosyncratic risks, thereby rendering the portfolio returns closer to the returns on the market portfolio compared with the returns on the individual stocks included in the portfolio, the values of the two

32 The significant premium (respectively, discount) for lower-tail (respectively, upper-tail) asymmetric dependence estimated by [Alcock and Hatherley \(2017\)](#) and [Alcock and Sinagl \(2020\)](#) also indirectly supports these findings. The findings are also generally in line with [Schneider, Wagner, and Zechner \(2020\)](#), and the tendency for assets with positive (respectively, negative) coskewness to offer lower (respectively, higher) returns than predicted by the traditional CAPM.

33 Relatedly, the framework developed in [Buraschi, Porchia, and Trojani \(2010\)](#) for intertemporal portfolio choice with stochastic second moments may be helpful in thinking about optimal asset allocation decisions and how to hedge against time-varying semicovariances and semibetas.

Table 3 Average daily realized semibetas

	β	β^P	β^N	β^{M^+}	β^{M^-}
Individual stocks					
“Bad Covid”	1.230	0.863	0.874	0.271	0.237
FAANG	1.185	0.681	0.690	0.096	0.089
Portfolios					
“Bad Covid”	1.233	0.740	0.769	0.146	0.130
FAANG	1.185	0.622	0.637	0.037	0.036

The top panel shows 2020 average daily realized semibetas with respect to the S&P 500 for the five stocks included the “Bad Covid” and FAANG clusters of stocks, respectively. The bottom panel shows the average daily realized semibetas for 2020 for equally weighted portfolios comprise the five stocks in each of the two clusters.

portfolio discordant semibetas are both closer to zero than their individual stock averages. Correspondingly, the two concordant portfolio semibetas are also both smaller than the averages of the concordant semibetas for the individual stocks. This phenomenon, of course, is not unique to the semibetas. Many other non-linear features, some of which have previously been associated with cross-sectional differences in returns, are similarly diminished through the effects of portfolio diversification.

In addition to their use for more accurate *ex-ante* return predictions, the realized semibetas could also help shed new light on *ex-post* investment return performance. There is an extensive literature, dating back to the early work by [Treyner and Mazuy \(1966\)](#) and [Merton and Henriksson \(1981\)](#), devoted to the question of whether mutual funds and other investment vehicles are able to “time” the market. This question is typically answered empirically by comparing what effectively amounts to estimates of the up and downside betas of a fund, with good timing ability manifest by $\beta^D < \beta^U$ and/or changes in β^U (β^D) positively (negatively) correlated with the performance of the market. Unfortunately, the returns for many funds are only available at relatively coarse monthly or quarterly frequencies, hindering direct estimation of dynamically varying fund betas at the horizons over which the funds might actively be changing their market exposures. Alternatively, the up and downside betas of a fund may be estimated as the portfolio weighted averages of the up and downside betas of the individual asset included in the fund’s portfolio, thereby allowing for the calculation of time-varying beta estimates at the same frequency over which fund holdings are available (see, e.g., [Bodnaruk, Chokaev, and Simonov, 2019](#)). Further decomposing these estimates into separate “good” and “bad” up and downside fund betas,

$$\beta_F^U = \sum_{k=1}^d w_i(\beta_i^P - \beta_i^{M^+}) \equiv \beta_F^P - \beta_F^{M^+}, \quad \beta_F^D = \sum_{k=1}^d w_i(\beta_i^N - \beta_i^{M^-}) \equiv \beta_F^N - \beta_F^{M^-},$$

may afford additional insights into where the fund performance is coming from.³⁴ In particular, it follows that if $\beta_F^P > \beta_F^N$ and $\beta_F^{M^-} > \beta_F^{M^+}$, then $\beta_F^D < \beta_F^U$, which is traditionally

34 Note, that while β_F^U and β_F^D are identical to the up and downside fund betas that would obtain with high-frequency fund returns, if such returns were available, β_F^P , β_F^N , $\beta_F^{M^+}$, and $\beta_F^{M^-}$ do not directly match the four fund semibetas that would be calculated directly with high-frequency fund returns.

considered to be indicative of good overall market timing. Appropriately timed variation in a fund's semibetas, may, of course, also be commensurate with superior overall market timing ability.³⁵ Counter to this, however, [Bandi and Renò \(2021\)](#) report that many hedge funds seemingly have larger betas when market returns are especially low, or what effectively amounts to larger “betas in the tails.”

The semibetas discussed above are all rooted in the standard one-factor CAPM. However, the same basic idea readily extends to multi-factor pricing models based on factors for which high-frequency returns are available, whether in the form of ETFs or other actively traded financial instruments, or in the form of brute force constructed high-frequency factor returns.³⁶ With a single factor, or approximately uncorrelated factors, realized semiloading may naturally be defined by the same expressions as in [Equation \(12\)](#), with the relevant factor in place of the market factor f . With multiple non-trivially correlated factors different normalizations may be called for. In parallel to the semibeta pricing results discussed above, the law-of-one-price coupled with limits-to-arbitrage may again impose certain restrictions and/or bounds on the values of the risk premiums associated with the relevant semiloading.

Along these lines, [Ait-Sahalia, Jacod, and Xiu \(2021\)](#) reports that allowing for separate risk premiums for the continuous and jump components of the Fama–French risk factors significantly enhances the explanatory power of second-stage Fama–MacBeth cross-sectional return regressions compared with the fit afforded by conventional models that price the two components the same.³⁷ Relatedly, [Massacci, Sarno, and Trapani \(2021\)](#) find that allowing for different factor structures in endogenously determined up and downside regimes afford a superior fit compared with traditional factor models that do not condition on the state of the economy. It would be interesting to further explore the interplay between these findings and the semiloading idea proposed here, both empirically and theoretically. The framework developed by [Engle and Mistry \(2014\)](#) for analyzing asymmetric volatility and skewness in factor returns within the context of an intertemporal capital asset pricing model may prove useful in guiding such investigations.

3 Partial (Co)Variation Measures

The zero-threshold that underlies the realized semi(co)variation measures, and related volatility forecasting and pricing results, discussed above is firmly rooted in the idea that investors process and price up and downside risks differently. From a purely statistical

- 35 Further expanding on this theme and the previous analysis in [Artavanis, Eksi, and Kadlec \(2019\)](#), “good” and “bad” up and downside fund betas could potentially also help explain mutual fund flows.
- 36 Intraday high-frequency versions of the Fama–French size and value factors were first constructed by [Bollerslev and Zhang \(2003\)](#), while high-frequency versions of all the five Fama–French factors and the Carhart momentum factor have been put together by [Ait-Sahalia, Kalnina, and Xiu \(2020\)](#). High-frequency versions of the more than one hundred factors defined in [Jensen, Kelly, and Pedersen \(2021\)](#) have also recently been explored by [Aletti \(2021\)](#).
- 37 The earlier study by [Bollerslev, Li, and Todorov \(2016\)](#) similarly finds that continuous and jump CAPM betas are not priced the same. The much more extensive empirical analysis of the full “factor zoo” in [Aletti \(2021\)](#) also further corroborates this differential pricing.

perspective, however, the choice of a zero threshold may seem somewhat arbitrary. More elaborate GARCH and other parametric volatility forecasting models involving multiple non-zero thresholds have also been entertained in the literature (see, e.g., Medeiros and Veiga, 2009; Cai and Stander, 2019, and the many references therein). Following Bollerslev et al. (2021), the high-frequency-based realized semi(co)variation measures may similarly be extended to so-called partial (co)variation measures, by allowing for a non-zero threshold and/or multiple thresholds.

To fix ideas, let $f_g(\cdot)$, for $g = 1, 2, \dots, G$, denote a partition of the real line into G non-overlapping intervals, so that the k th intraday return may be expressed as $r_{t,k} \equiv f_1(r_{t,k}) + f_2(r_{t,k}) + \dots + f_G(r_{t,k})$. The basic realized variation measure in Equation (2) may then alternatively be expressed as the sum of the corresponding G partial variation measures $PV_t^{(g)}$ implicitly defined by,

$$RV_t = \sum_{k=1}^K f_1(r_{t,k})^2 + \dots + f_G(r_{t,k})^2 \equiv \sum_{g=1}^G PV_t^{(g)}. \tag{13}$$

The realized semivariation measures in Equation (3) obviously obtained as special cases by setting $G=2$, and $f_1(r_{t,k}) \equiv r_{t,k} \cdot I(r_{t,k} < 0)$ and $f_2(r_{t,k}) \equiv r_{t,k} \cdot I(r_{t,k} > 0)$, respectively. But, other thresholds may be used in the definition of more refined partial realized variation measures.

Multivariate realized partial covariation measures may be defined analogously by partitioning the vectors of high-frequency returns. Specifically, let $\mathbf{r}_{t,k} = f_1(\mathbf{r}_{t,k}) + f_2(\mathbf{r}_{t,k}) + \dots + f_G(\mathbf{r}_{t,k})$ denote an exact decomposition of the k th intradaily return vector into G components based on the partition functions $f_g(\mathbf{x}) = \mathbf{x}^o \mathbf{1}\{\mathbf{c}_g < \mathbf{x} \leq \mathbf{c}_{g+1}\}$, where the thresholds are monotonically increasing $\mathbf{c}_{g-1} \leq \mathbf{c}_g$, with $\mathbf{c}_1 = -\infty$ and $\mathbf{c}_{G+1} = \infty$. The resulting G^2 realized partial covariation measures $PCOV_t^{(g,g')}$ are then implicitly defined by,

$$\begin{aligned} RCOV_t &\equiv \sum_{k=1}^K \mathbf{r}_{t,k} \mathbf{r}'_{t,k} \\ &= \sum_{k=1}^K f_1(\mathbf{r}_{t,k}) f_1(\mathbf{r}_{t,k})' + f_1(\mathbf{r}_{t,k}) f_2(\mathbf{r}_{t,k})' + \dots + f_G(\mathbf{r}_{t,k}) f_G(\mathbf{r}_{t,k})' \\ &= \sum_{g=1}^G \sum_{g'=1}^G PCOV_t^{(g,g')}. \end{aligned} \tag{14}$$

The realized semicovariance measures defined in Equation (9) again obtain as special cases for a single threshold at zero. In situations when the ordering of the assets is arbitrary, mirroring the combination of the two discordant semicovariance matrices into a single “mixed” matrix ($\mathbf{M}_t \equiv \mathbf{M}_t^+ + \mathbf{M}_t^-$), all of the matched pairs of “mixed” partial covariance matrices may similarly be combined (i.e., $PCOV_t^{(g,g')} + PCOV_t^{(g',g)}$ for $g \neq g'$), resulting in “only” $G(G + 1)/2$, as opposed to G^2 , partial covariance matrices in total.

Rather than relying on time-invariant thresholds in the definition of the partition functions $f_g(\cdot)$, one might also naturally consider time-varying thresholds based on the volatility-standardized intraday returns and the quantiles of said distributions. Doing so will help avoid certain partitions becoming especially thin or densely populated during extended time periods of high or low volatility. The conventional zero threshold, of course, is typically very close to the median of both the raw and the standardized high-frequency

return distributions, and as such the semi(co)variation measures still obtain as special cases of the so-defined partial (co)variation measures.

3.1 Partial (Co)variance-Based Volatility Forecasting

The freedom to choose the number and location of the thresholds in the definition of the realized partial (co)variance measures affords a great deal of added flexibility compared to the semi(co)variance measures discussed in Sections 1 and 2.³⁸ Meanwhile, faced with the oft-observed empirical tradeoff between better in-sample fits of more complicated models versus better out-of-sample forecast performance of simpler models, it is not clear whether the use of the richer partial (co)variation measures will necessarily result in superior volatility forecasting models compared to the semi(co)variance-based forecasting models discussed in Sections 1.1 and 2.1.

The empirical analyses in [Bollerslev et al. \(2021\)](#) shed a first light on this question. Using the same sample of stocks and same univariate and multivariate HAR structures analyzed in BLPQ, the results suggest that it is difficult, although not impossible, to improve upon the fixed threshold at zero. At the same time, however, when considering only a single threshold, or $G = 2$, zero clearly emerges as the “hero.” Allowing for multiple thresholds, $G = 3$ and partial (co)variance-based models with one threshold close to zero and another threshold in the left tail of the standardized intraday return distributions typically emerge as the best performing forecasting models. In other words, “good” and “bad” (co)volatility and negative (co)jumps all manifest differently in terms of their dynamic dependencies.

The partial covariances defined in [Equation \(14\)](#) rely on simple threshold-based cutoffs and rectangular-shaped partitions of the total covariation. Alternative partitions based on ellipsoids, or other geometric shapes, possibly centered at non-zero coordinates, could be employed in the definition of alternative classes of partial covariation measures. It is possible that some of these alternative decompositions may be used in the construction of even better multivariate volatility forecasting models. Given the vast set of decompositions and related models to potentially consider, I would envision ideas and techniques adapted from machine learning to be very helpful in terms of disciplining or regularizing the estimation of such models and further exploring this question.

3.2 Partial (Co)variance-Based Asset Pricing

There is a rapidly growing recent literature on the use of machine learning techniques in economics and finance. A common finding for many of these studies in the area of asset pricing finance concerns the importance of allowing for non-linearities and interactions among predictor variables (e.g., [Freyberger, Neuhierl, and Weber, 2020](#); [Gu, Kelly, and Xiu, 2020](#)). Relatedly, and in parallel to the use of the semicovariances for asset pricing in the form of the semibetas or semi-factor-loadings discussed in Section 2.2, the partial covariances may similarly be used in the definition of partial betas or partial-factor-loadings capturing different parts of the systematic risk exposures. This again is reminiscent of the aforementioned studies by [Bollerslev, Li, and Todorov \(2016\)](#); [Ait-Sahalia, Jacod, and Xiu \(2021\)](#); and [Aletti \(2021\)](#), and the idea that the continuous and jump components of the systematic risk factors may be priced differently. However, other partitions of the systematic

38 The freedom to choose the number and location of the thresholds also pose formidable theoretical challenges in establishing the in-fill asymptotic distributions of the partial (co)variation measures.

risks different from continuous versus jump variation, or “good” versus “bad” covolatility, may possibly result in even better asset price predictions. I am currently exploring this idea in joint ongoing work (Bollerslev, Patton, and Quaedvlieg, 2021b), in which we seek to estimate a full “risk premium surface.”³⁹ Following the discussion above, the law-of-one-price together with considerations of arbitrage again impose certain restrictions on the surface. The random field regressions combined with sieve approximations that we rely on for the estimation affords an especially convenient framework for incorporating such restrictions.

4 Conclusion

Financial markets are inherently forward-looking. The dynamics of financial asset prices thus encodes potentially valuable information about investors’ expectations and beliefs about future state variables, as well as preferences and attitudes toward different types of risks. In particular, as I have argued here, there are both sound economic reasons and ample empirical evidence to support the thesis that “good” and “bad” volatilities are not created equal. “Looking inside” the quadratic return variation through the lens of newly developed simple-to-implement high-frequency-based realized semi(co)variation measures, it is clear that “bad” (co)volatility is both more informative about future (co)volatility and priced more dearly by investors than “good” (co)volatility. In addition to these conclusions, gleaned from the uses of realized semi(co)variance measures in simple reduced form volatility forecasting models and regression-based return predictions, there are many other intriguing questions still left to be explored in regards to the theoretical properties and wider empirical applications of the new semi and partial (co)variation measures. I look forward to seeing the fruits from continued financial econometrics research efforts devoted to these ideas.

Supplementary Data

Supplementary data are available at <https://www.datahostingsite.com>.

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39 Our partition of the factor space underlying this estimation bears some resemblance to the asset pricing trees and related set of test assets developed by Bryzgalova, Pelger, and Zhu (2020).

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