Risk and Return in Equilibrium: The Capital Asset Pricing Model (CAPM)

Introduction

- To operationalize Mean-Variance Analysis we need estimates of expected returns, variances, and covariances
  - ↪ Expected returns are especially hard to estimate
  - ↪ It will be useful to have a theory of what expected returns *should* be
- The Capital Asset Pricing Model (CAPM) provides an *equilibrium* model for *expected returns*
  - ↪ William F. Sharpe, 1990 Nobel Price in Economics
  - ↪ It remains one of the most widely used models in all of finance
  - ↪ It is called an asset *pricing* model, even though it is a model for *expected returns*
  - ↪ The CAPM builds on the Markowitz portfolio problem
  - ↪ The Markowitz portfolio approach remains relevant regardless of whether the *equilibrium* arguments behind the CAPM are correct or not

Equilibrium Pricing

- *Equilibrium* is an economic concept that characterizes a situation where no investor wants to do anything different
  - ↪ How should securities be priced in *equilibrium*?
  - ↪ It must be the case that all assets are bought 100%
  - ↪ For example, if the prices/expected returns that our model comes up with imply that no investor would want to buy IBM, then something is wrong
  - ↪ IBM would be priced too high, or equivalently offer too low an expected rate of return
  - ↪ The price of IBM would have to drop so that in the aggregate investors would want to hold exactly the number of IBM shares outstanding
- So, which prices (risk/return relationships) are feasible in equilibrium?
  - ↪ The CAPM gives an answer based on all investors using Markowitz MV analysis
  - ↪ Many other (potentially better) asset pricing models have been proposed
  - ↪ We will talk about some of these later in the class ...

CAPM Assumptions

- A number of assumptions are necessary to formally derive the CAPM:

  1. No transaction costs or taxes
  2. All assets are tradable and infinitely divisible
  3. No individual can effect security prices (perfect competition)
  4. Investors care only about expected returns and variances
  5. Unlimited short sales, borrowing, and lending
  6. Homogeneous expectations

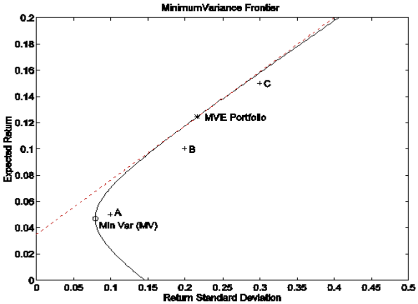
- Taken at “face value” these assumptions are clearly *wrong*, and some even *ridiculous*

  ↪ Some of the assumptions can be relaxed without too much of an effect on the end results, others not

  ↪ Taken together, however, the assumptions have some very powerful implications

MV Analysis and Two Fund Separation

- Assumptions 4, 5 and 6 imply that everyone solves the same passive MV portfolio problem
  - ↪ The tangency portfolio must be the *same* for everybody:



- According to Markowitz, everyone's optimal investment portfolio is therefore comprised of:
  - ↪ The risk-free asset
  - ↪ The *same* tangency portfolio

CAPM and the Market Portfolio

- What is this tangency portfolio?

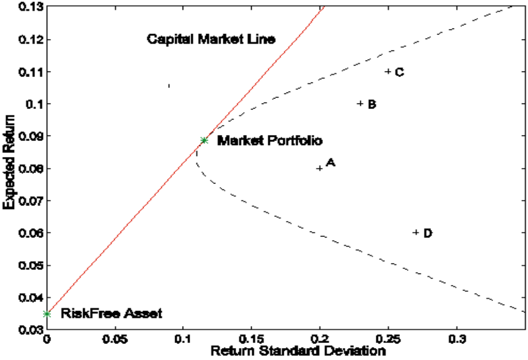
  ↪ Equilibrium theory (market clearing) implies that the tangency portfolio must be equal to the *market portfolio*

  ↪ That is, the average investor *must* want to hold the *market portfolio*

  ↪ The *market*, or total wealth, *portfolio*, should be comprised of *all* risky securities held in exact proportion to their market values

  - In theory this should include *all* risky securities; i.e., not only stocks and bonds, but also real-estate, human capital, etc.
  - This is related to the so-called Roll critique
  - This is also where the assumption that all assets are tradeable comes in

The Capital Market Line



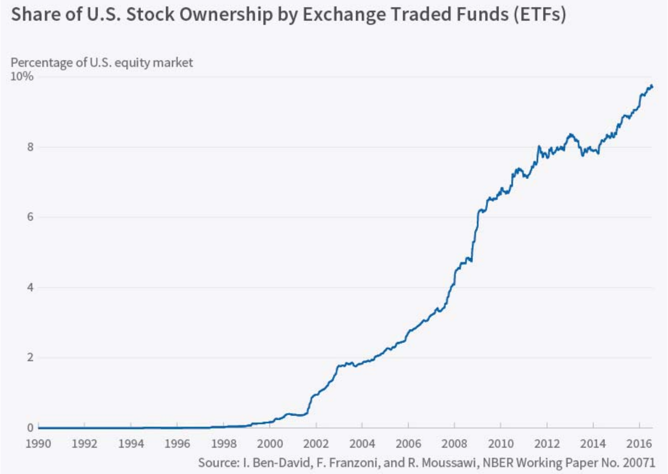
- Every investor faces the same CAL in equilibrium
- This CAL is called the *Capital Market Line* (CML)

- The CML gives the return on all *efficient* portfolios r_e, defined by the CAL with the "market":

$$E(r_e) = r_f + \left(\frac{E(r_m) - r_f}{\sigma_m} \right) \sigma_e$$

  - ↪ This implies that all investors should only hold combinations of the "market" and the risk-free asset
  - ↪ Remember that according to the theory, the "market" portfolio should in include *all* risky securities, not just stocks
  - ↪ Helps explain the increased popularity of index funds and ETFs designed to track a particular "market" index
    - SPY (S&P 500)
    - QQQ (Nasdaq 100)
    - IWM (Russell 2000)

ETFs


Share of U.S. Stock Ownership by Exchange Traded Funds (ETFs)
Percentage of U.S. equity market
10%
8
6
4
2
0
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
2014
2016
Source: I. Ben-David, F. Franzoni, and R. Moussawi, NBER Working Paper No. 20071

The Security Market Line

- The goal of the CAPM is to provide a theory for the expected returns on *all* assets, including *inefficient portfolios* and *individual assets*
  - ↪ But how?

- For investors to want to hold the market portfolio, they should not be able to benefit by deviating from that
  - ↪ What each security adds in terms of risk (variance) must be exactly offset by its reward (expected return)
  - ↪ The ratio of *marginal return* to *marginal variance* (the effect of a small addition) must be the same for all assets
  - ↪ This is the intuition behind the Security Market Line (SML), or the CAPM as it is commonly stated
  - ↪ The marginal return is proportional to the *expected return*
  - ↪ The marginal variance is proportional to the *covariance* with the market portfolio

A Formal Derivation of the SML

- Suppose that you are currently holding the market portfolio, but decide to borrow a *small* additional fraction δ_{GM} at the risk-free rate to invest in GM

$$r_c = r_m - \delta_{GM} \cdot r_f + \delta_{GM} \cdot r_{GM}$$

↪ The expected return and variance on the new portfolio will be:

$$\begin{aligned} E(r_c) &= E(r_m) + \delta_{GM} \cdot (E(r_{GM}) - r_f) \\ \sigma_c^2 &= \sigma_m^2 + \delta_{GM}^2 \cdot \sigma_{GM}^2 + 2 \cdot \delta_{GM} \cdot cov(r_{GM}, r_m) \end{aligned}$$

↪ For small values of δ_{GM}, the changes in the expected return and variance are:

$$\begin{aligned} \Delta E(r_c) &= \delta_{GM} \cdot (E(r_{GM}) - r_f) \\ \Delta \sigma_c^2 &= 2 \cdot \delta_{GM} \cdot cov(r_{GM}, r_m) \end{aligned}$$

- Note, we ignore the δ_{GM}^2 term in the variance equation; if δ_{GM} is small (say 0.01), δ_{GM}^2 is even smaller still (0.0001)

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A Formal Derivation of the SML

- Now suppose that you invest $\delta_{GM} > 0$ more in GM, and invest just enough less $\delta_{IBM} < 0$ in IBM so that your portfolio variance σ_c^2 stays the same
  - ↪ From before, the change in the variance will be:

$$\Delta\sigma_c^2 = 2 \cdot \delta_{GM} \cdot cov(r_{GM}, r_m) + 2 \cdot \delta_{IBM} \cdot cov(r_{IBM}, r_m)$$

  - ↪ Setting this equal to zero (i.e., $\Delta\sigma_c^2 = 0$) and solving:

$$\delta_{IBM} = -\delta_{GM} \left(\frac{cov(r_{GM}, r_m)}{cov(r_{IBM}, r_m)} \right)$$

  - ↪ From before, the change in the expected return for this $\Delta\sigma_c^2 = 0$ portfolio will be:

$$\begin{aligned} \Delta E(r_c) &= \delta_{GM} \cdot (E(r_{GM}) - r_f) + \delta_{IBM} \cdot (E(r_{IBM}) - r_f) \\ &= \delta_{GM} \left[(E(r_{GM}) - r_f) - (E(r_{IBM}) - r_f) \left(\frac{cov(r_{GM}, r_m)}{cov(r_{IBM}, r_m)} \right) \right] \end{aligned}$$

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A Formal Derivation of the SML

- Recall that the CAPM implies that the market portfolio is the tangency portfolio
  - ↪ The market portfolio has the highest Sharpe Ratio of *all* portfolios
  - ↪ Therefore, we cannot increase the expected return relative to the market portfolio, while keeping the variance the same as the variance of the market portfolio
  - ↪ For the previous portfolio constructed to have $\Delta\sigma_c^2 = 0$, it must therefore be the case that $\Delta E(r_c) = 0$, or:

$$\frac{E(r_{GM}) - r_f}{cov(r_{GM}, r_m)} = \frac{E(r_{IBM}) - r_f}{cov(r_{IBM}, r_m)} = \lambda$$

  - ↪ λ represents the ratio of the marginal benefit to the marginal cost of investing a small additional amount in each of the assets
  - ↪ This same relationship must hold for *all* individual assets and *all* portfolios

A Formal Derivation of the SML

- In particular, consider the market portfolio in place of IBM:

$$\frac{E(r_{GM})-r_f}{cov(r_{GM},r_m)}=\frac{E(r_m)-r_f}{cov(r_m,r_m)}=\frac{E(r_m)-r_f}{\sigma_m^2}=\lambda$$

- Consequently:

$$\begin{aligned} E(r_{GM})-r_f &= \frac{E(r_m)-r_f}{\sigma_m^2}cov(r_{GM},r_m) \\ &= (E(r_m)-r_f)\underbrace{\frac{cov(r_{GM},r_m)}{\sigma_m^2}}_{\beta_{GM}} \end{aligned}$$

- This is called the *Security Market Line* (SML)
  - ↪ The SML characterizes the expected returns for *all* individual assets and portfolios as a function of their *betas*

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CAPM Summary

- By definition of the tangent portfolio P^*, it should *not* be possible to achieve a higher return/risk tradeoff (Sharpe Ratio) by combining the tangent portfolio with *any* other asset

- This restriction implies a linear relationship between an asset's expected excess return and its beta with respect to the tangent portfolio P^*:

$$E(r_i) - r_f = (E(r_{P^*}) - r_f) \times \beta_{i,P^*}$$

- The CAPM implies that in *equilibrium*, the tangent portfolio must be equal to the market portfolio, $r_{P^*} = r_m$:

$$E(r_i) - r_f = (E(r_m) - r_f) \times \beta_i$$

$\hookrightarrow$ This says that the reward for bearing risk $E(r_i) - r_f$, must be equal to the amount of risk that is priced, as measured by β_i, times the price of risk, as measured by $E(r_m) - r_f$

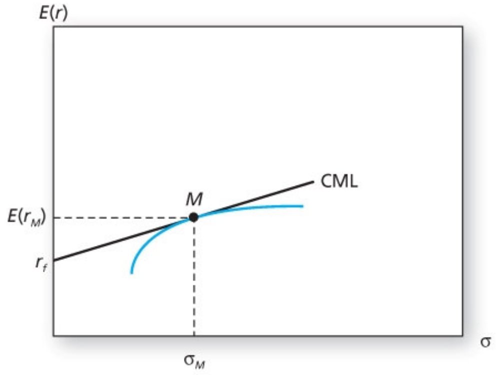

Figure 9.1 The efficient frontier and the capital market line

BKM Recap

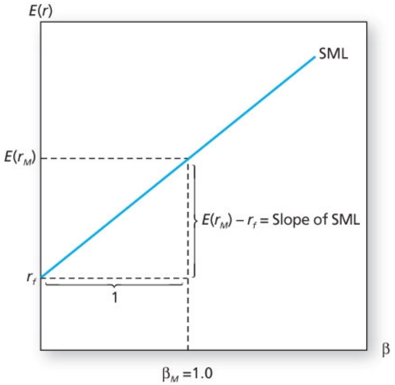

Figure 9.2 The security market line

SML - Example

Asset	$E(r)$	σ
A	8%	20%
B	10%	23%
C	11%	25%
D	6%	27%

Correlations				
Assets	A	B	C	D
A	1.0	0.0	0.3	-0.2
B	0.0	1.0	0.2	-0.2
C	0.3	0.2	1.0	-0.2
D	-0.2	-0.2	-0.2	1.0

- Further assume that $r_f = 3.5\%$
- The resulting MVE portfolio has weights of:

$$w_{MVE} = \begin{bmatrix} 0.2515 \\ 0.3053 \\ 0.2270 \\ 0.2161 \end{bmatrix}$$

- If these were the *only* assets, the CAPM would imply that this is the *market portfolio*

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SML - Example

- Calculate the βs for each of the four assets with respect to this market/tangent portfolio:

$$\beta_i = \frac{cov(r_i, r_m)}{\sigma_m^2}$$

↪ Use the equation for the covariance of a portfolio:

$$\begin{aligned} cov(r_A, r_m) &= cov(r_A, w_A r_A + w_B r_B + w_C r_C + w_D r_D) \\ &= w_A cov(r_A, r_A) + w_B cov(r_A, r_B) \\ &\quad + w_C cov(r_A, r_C) + w_D cov(r_A, r_D) \end{aligned}$$

↪ Use the equation for the variance of a portfolio:

$$\begin{aligned} \sigma_m^2 &= \sum_{i=A}^{D} \sum_{j=A}^{D} w_i w_j cov(r_i, r_j) \\ &= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + w_D^2 \sigma_D^2 + \underbrace{2 w_A w_B cov(r_A, r_B) + \ldots}_{\text{6 covariance terms}} \end{aligned}$$

↪ How do the expected excess returns relate to the resulting βs?

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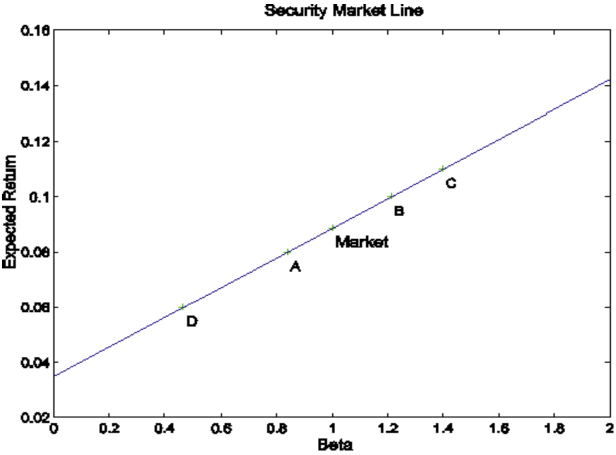
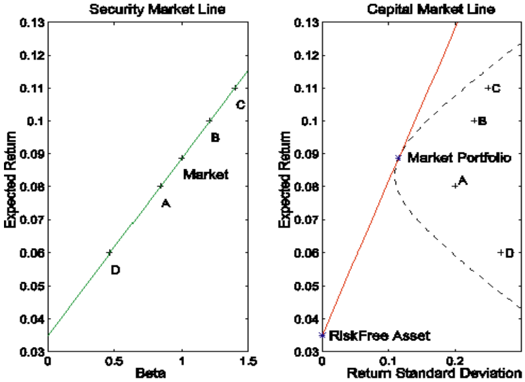
SML - Example



- The expected returns for *all* assets lie on the SML
- What about the CML?

SML vs CML

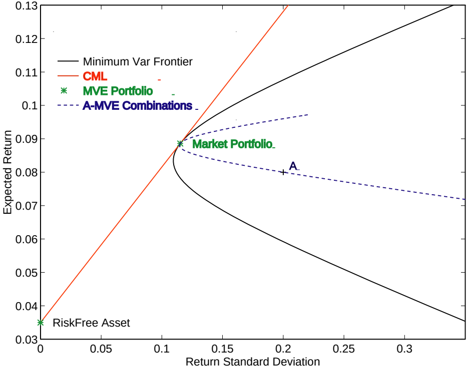


- Every security lies on the SML
- *Only* the market portfolio and the risk-free asset lie on the CML
- SML plots rewards versus *systematic risk*
- CML plots rewards versus *total risk* (systematic $+$ idiosyncratic)

A Graphical Approach to Understanding the CAPM

- Our derivation of the CAPM was based on the idea that it is not possible to improve on the Sharpe Ratio of market portfolio

- Let's look at the previous example and the possible combinations of the MVE portfolio (the market if the CAPM is true) and asset A:



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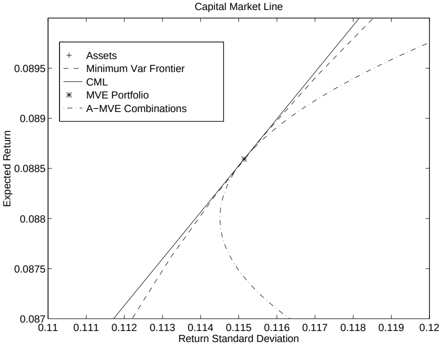
A Graphical Approach to Understanding the CAPM

- Looking at a zoomed in version:



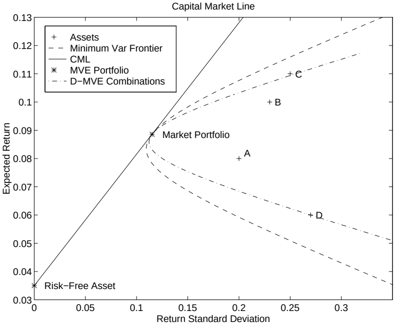
↪ All combinations of A and the MVE portfolio must fall inside the MVE frontier

↪ The marginal return/variance must be the same as for the market

↪ The tangent line must be the same as the CML

A Graphical Approach to Understanding the CAPM

- Now consider asset D:



↪ Exactly the same tangency condition as before

↪ The “combination curves” must be tangent to the CML for every asset

↪ Unless the ratio of marginal return to marginal variance is identical for *all* assets, investors would not want to hold the market portfolio

  - Correspondingly, prices would have to adjust to a new equilibrium

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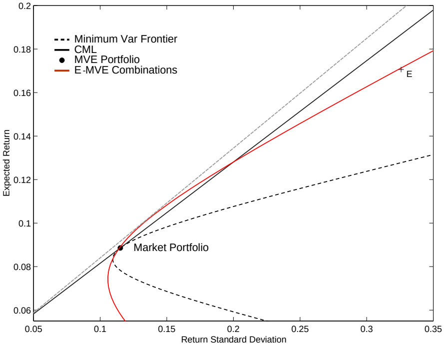
A Graphical Approach to Understanding the CAPM

- To illustrate, suppose that the expected return on asset E is greater than predicted by the CAPM
  - ↪ What is the "alpha" of asset E?
  - ↪ What would the previous graph look like for asset E?



  - ↪ As an investor, how would you take advantage of this situation?

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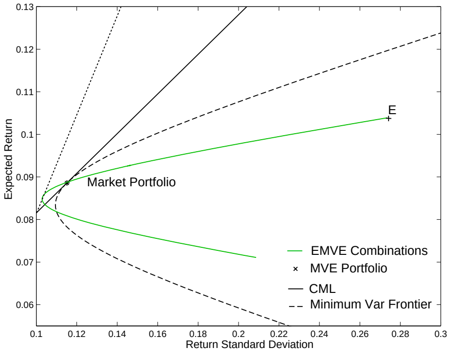
A Graphical Approach to Understanding the CAPM

- Alternatively, suppose that asset E has an expected return that is less than predicted by the CAPM
  - ↪ What is the "alpha" of asset E?
  - ↪ What would the previous graph look like for this new asset E?



- ↪ As an investor, how would you take advantage of this situation?

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CAPM Summary, Again ...

- The CAPM is a theory for what expected returns should be in *equilibrium*
  - ↪ If the CAPM is wrong, we can do better than the market portfolio (assuming that expected return and variance are what we care about)
- Whether the CAPM is supported by the data has been a hotly debated issue over the past forty years
  - ↪ And, as we will talk about later, continues to be so ...
  - ↪ Even if the CAPM isn't literally true, it still provides a very useful benchmark for expected returns to be used in MV analysis, cost of capital, and many other situations
  - ↪ Some of the extensions to the CAPM that we will talk about later also build on the same basic insights and intuition
- The key inputs to implementing the CAPM are the market βs
  - ↪ Let's briefly discuss (again) how to estimate these (more to come later)

Beta is a Regression Coefficient

- Recall the single index model, or time series regression equation, for some asset i:

$$r_{i,t}^e = \alpha_i + \beta_i r_{m,t}^e + \varepsilon_{i,t} \qquad t = 1, 2, ..., T$$

  - $\hookrightarrow$ $\beta_i r_m^e$ gives the part of r_i^e that is "explained" by the market return
  - $\hookrightarrow$ This accounts for the *systematic*, or *market* related risk of the asset
  - $\hookrightarrow$ $\varepsilon_{i,t}$ denotes the random part of r_i^e that is unrelated (uncorrelated) to the return on the market
  - $\hookrightarrow$ This represents the *non-systematic*, or *idiosyncratic*, risk of the asset
  - $\hookrightarrow$ The OLS slope coefficient estimate from this regression provides the empirical counterpart to the CAPM β_i, formally defined by:

$$\beta_i = \frac{cov(r_i^e, r_m^e)}{\sigma_m^2}$$

  - $\hookrightarrow$ What should be the value of α_i if the CAPM is true?

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Systematic vs Idiosyncratic Risk

- This regression effectively decomposes the total variance of asset i as:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon,i}^2$$

  - ↪ Systematic variance: $\beta_i^2 \sigma_m^2$
  - ↪ Non-systematic, or idiosyncratic, variance: $\sigma_{\varepsilon,i}^2$
  - ↪ Why is there no covariance here?

- The CAPM implies that only the systematic part of the risk is "priced"
  - ↪ Why is the non-systematic risk not "priced"?

- The quality/accuracy of the fit of the regression is naturally measured by the R^2:

$$R^2 = \frac{\beta_i^2 \sigma_m^2}{\sigma_i^2} = 1 - \frac{\sigma_{\varepsilon,i}^2}{\sigma_i^2}$$

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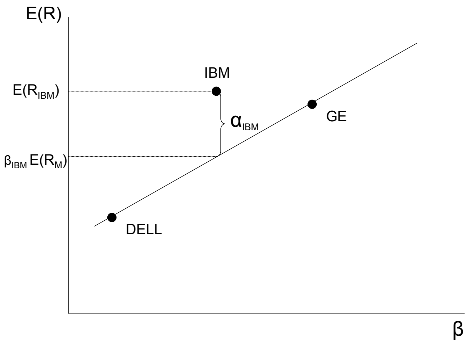
Deviations from the CAPM and α

- α_i denotes the deviation of a security's expected return from the SML

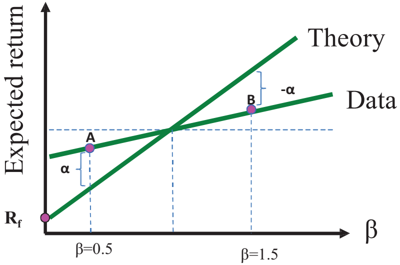
  ↪ Graphically:



  ↪ Is the price of IBM stock too high or too low relative to DELL and GE?

Deviations from the CAPM and α

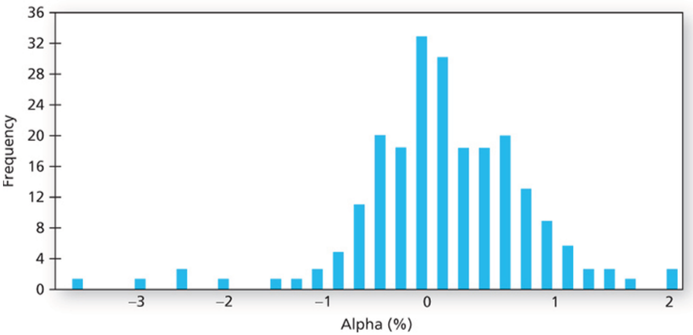
- We will later discuss the empirical evidence pertaining to the CAPM
  - ↪ For individual stocks:



  - ↪ As a long-only mutual fund with a “target” beta of one, how might you take advantage of this situation?
  - ↪ As a long/short hedge fund, how might you take advantage of this situation?
    - ▶ “Betting against beta”

Mutual Fund α's

- So do mutual funds as a whole deliver positive alphas?
  - $\hookrightarrow$ Old regression based estimates, BKM Figure 9.5:



  - $\hookrightarrow$ And there is even a survivorship bias here ...

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Estimating β

- Given the historical returns:

$$r_{i,t}\,,\;\; r_{m,t}\,,\;\; r_{f,t}\,, \qquad t = 1, \ldots, T$$

  ↪ The single index model (“characteristic line regression”) determines beta as the slope coefficient in the regression:

$$r_{i,t} - r_{f,t} \;=\; \alpha_i \,+\, \beta_i(r_{m,t} - r_{f,t}) \,+\, \varepsilon_{i,t} \qquad t = 1, \ldots, T$$

- Alternatively, you may use:

  ↪ The “market model regression”:

$$r_{i,t} - \overline{r_i} \;=\; a_i \,+\, b_i(r_{m,t} - \overline{r_m}) \,+\, \varepsilon_{i,t} \qquad t = 1, \ldots, T$$

  ↪ The simple return regression:

$$r_{i,t} \;=\; a_i \,+\, b_i r_{m,t} \,+\, \varepsilon_{i,t} \qquad t = 1, \ldots, T$$

  ↪ Why is b_i typically still a good estimate for β_i?

Estimating β

- GM, Market and T-Bill returns:

Month	GM Return	Market Return	Monthly T-Bill Rate	Excess GM Return	Excess Market Return
January	6.06	7.89	0.65	5.41	7.24
February	−2.86	1.51	0.58	−3.44	0.93
March	−8.18	0.23	0.62	−8.79	−0.38
April	−7.36	−0.29	0.72	−8.08	−1.01
May	7.76	5.58	0.66	7.10	4.92
June	0.52	1.73	0.55	−0.03	1.18
July	−1.74	−0.21	0.62	−2.36	−0.83
August	−3.00	−0.36	0.55	−3.55	−0.91
September	−0.56	−3.58	0.60	−1.16	−4.18
October	−0.37	4.62	0.65	−1.02	3.97
November	6.93	6.85	0.61	6.32	6.25
December	3.08	4.55	0.65	2.43	3.90
Mean	0.02	2.38	0.62	−0.60	1.75
Std Dev	4.97	3.33	0.05	4.97	3.32

Regression Results	$r_{GM} - r_f = \alpha + \beta(r_M - r_f)$	
	α	β
Estimated coefficient	−2.590	1.1357
Standard error of estimate	(1.547)	(0.309)

Variance of residuals = 12.601
Standard deviation of residuals = 3.550
R-SQR = 0.575

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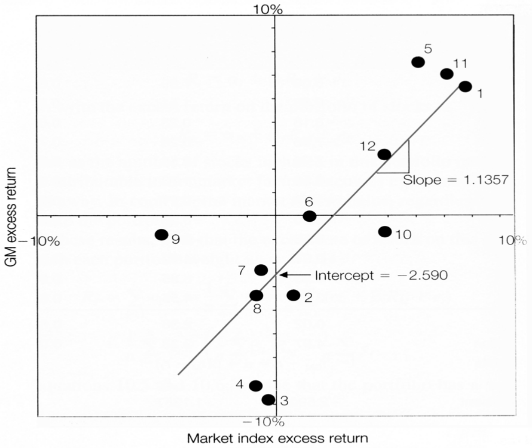
Estimating β

- Graphically:



- How do you read $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{\varepsilon}_{i,t}$ in this plot?

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Estimating β

- Some beta providers also report so-called "adjusted β's":

$$\beta_i^{Adj} \approx 1/3 + (2/3) \cdot \hat{\beta}_i$$

- Why might you want to do that?
  - ↪ Statistical biases
  - ↪ Historically betas tend to mean-revert to one
  - ↪ Why $1/3$ and $2/3$?
  - ↪ Many other more advanced "shrinkage" and β adjustment procedures are used in practice

Estimating β

- In estimating betas we typically rely on a relatively short 5-year *rolling windows* of historical data
  - ↪ Why not 10-years of historical data? Or 25-years?
  - ↪ Time-varying betas

- Possible reasons for time-varying betas
  - ↪ Changes in the firm's leverage
  - ↪ Changes in the firm's operations
  - ↪ Acquisitions and/or expansions into other industries
  - ↪ Changes in the composition of the aggregate market

Estimating β

- Rolling regression β estimates for AT&T:

- GARCH and other more sophisticated statistical procedures explicitly allow for *time-variation* in the βs

- Cross-sectional distribution of βs:

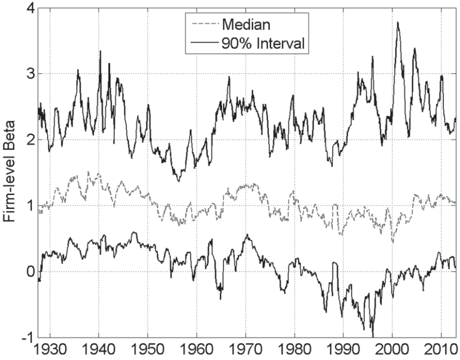


Figure 1. Cross-sectional distribution of firm betas, July 1927 to December 2012. The figure displays statistics for the cross-sectional distribution of firm betas. The dashed line is the median and the solid lines show the 5^{th} and 95^{th} percentiles of firm betas. Firm betas are estimated at the beginning of each month using daily returns over the previous 12 months.

Cederburg and O'Doherty (2016, *Journal of Finance*)

Estimating β

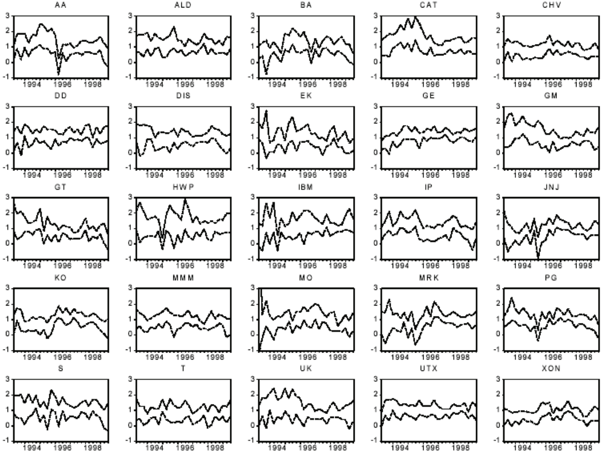
- In estimating betas we typically use monthly data

- To get better estimates, we could use higher frequency weekly, daily, or even intraday data
  - ↪ Many more observations, and in turn more accurate estimates

- But, the use of higher frequency data (especially intraday data) also presents a number of complications
  - ↪ Non-synchronous prices
  - ↪ Bid-ask bounce effects

- Still, for actively traded stocks this is a great *new* way to go ...

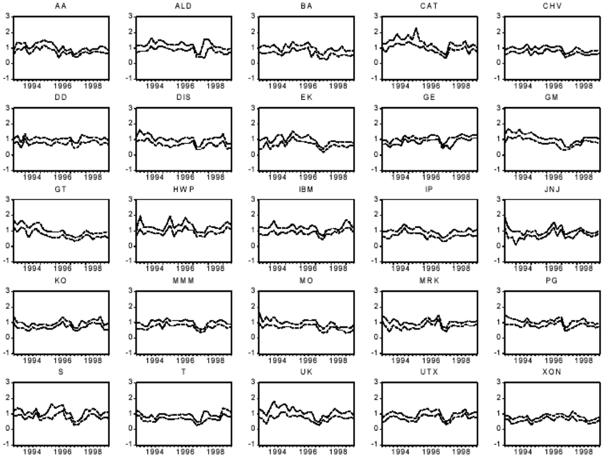
- Confidence intervals for quarterly betas for 25 DJ stocks 1993-1999 based on daily data:



Andersen, Bollerslev, Diebold and Wu (2006, *Advances in Econometrics, Vol.20*)

Estimating β

- Confidence intervals for quarterly betas for 25 DJ stocks 1993-1999 based on 15-minute data:



Andersen, Bollerslev, Diebold and Wu (2006, *Advances in Econometrics, Vol.20*)

Estimating β

- The traditional ways of estimating βs rely on historical returns
- How would you estimate the β for a *new* company without any historical data?
- Standard industry practice is to use "comparables"
  - ↪ Find a similar company, for which you have historical data and use the estimated beta for that company
  - ↪ What if a "comparable" company can not be found?
- Construct a model-predicted beta based on company *characteristics*

Estimating β

- Which characteristics to use in a prediction model for β?
  - ↪ Industry
  - ↪ Firm Size
  - ↪ Financial Leverage
  - ↪ Operating Leverage
  - ↪ Growth / Value
  - ↪ Percentage of revenue from exports
  - ↪ ...

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Estimating β

1. Estimate β_i for a cross-section of companies using historical data

2. Regress the estimated $\hat{\beta}_i$s on various characteristics that supposedly determine the betas:

$$\hat{\beta}_i = a_0 + \gamma_{INDUSTRY_i} + a_1 LEV_i + a_2 SIZE_i + \ldots + u_i$$

   - $\hookrightarrow$ $\gamma_{INDUSTRY_i}$ industry dummy
   - $\hookrightarrow$ LEV_i financial leverage
   - $\hookrightarrow$ $SIZE_i$ market value
   - $\hookrightarrow$ ...

3. The beta for the new company XYZ in the tech industry may then be predicted as:

$$\beta_{xyz} = \hat{a}_0 + \hat{\gamma}_{tech} + \hat{a}_1 LEV_{xyz} + \hat{a}_2 SIZE_{xyz} + \ldots$$

CAPM and Active Portfolio Management

- To create an optimal portfolio we need to estimate the efficient frontier and the best capital allocation line (CAL)

- To do so we need estimates for the expected returns, variances, and covariances for all of the assets

- One approach is to use the sample means, variances and covariances based on past historical returns
  - ↪ We have already seen that there are problems with this approach

- Alternative, we could simply accept the aggregate "opinion" of the market and hold the market portfolio
  - ↪ But, what if you believe that you know better?
  - ↪ The approach we will discuss next incorporates "views" into the CAPM

- Incorporating "views" into the CAPM and Markowitz
  1. Calculate the β's for the securities to be included in the portfolio
  2. Using the β's, calculate the $E(r_i)$s assuming that the CAPM holds exactly
  3. Incorporate your information by carefully "perturbing" the $E(r_i)$s away from the values implied by CAPM
  4. Using these modified estimates, determine the optimal portfolio weights using Markowitz

- If we considered *all* of the assets in the market, and did *not* "perturb" the $E(r_i)$s in step 3, what would be the weights calculated in step 4?

CAPM and Active Portfolio Management

- Start with the characteristic line regression for estimating the β_is:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_{i,t} \qquad t = 1, \ldots, T \tag{1}$$

- Use the CAPM/SML to calculate the expected returns:

$$E(r_i) - r_f = \widehat{\beta}_i \cdot (E(r_m) - r_f) \tag{2}$$

  ↪ The CAPM estimate of $E(r_i) - r_f$ is obtained by imposing $\alpha_i = 0$

  ↪ Note, to get these estimates, we need an estimate for $E(r_m)$

- Note, (1) is about actual *realized* returns, while (2) is about *expected* returns

  ↪ The return given by:

$$\widehat{\alpha}_i + \widehat{\beta}_i(\overline{r_m} - \overline{r_f})$$

  would be *identical* to the historical average excess return

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CAPM and Active Portfolio Management

- We also need *all* of the variances and covariances

- Recall that if we use the single-index model and the betas to estimate the covariances, we only need:

σ_m^2	1	1
β_i	N	100
$\sigma_{\varepsilon,i}^2$	N	100
Total	$2N+1$	201

↪ As we have seen before, this considerably reduces the required number of inputs for large values of N

  - 201 compared to 5,050 for N=100

↪ How do you actually get all of the required $\sigma_{i,j} = Cov(r_i, r_j)$ from these estimates?

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Example

- Estimates with monthly data for GE, IBM, Exxon (XOM), and GM:

	Excess Returns					
	mean	std	alpha	beta	std_{ε}	R^2_{adj}
IBM	3.22%	8.44%	1.31%	1.14	7.13%	28.5%
XOM	1.41%	4.03%	0.42%	0.59	3.28%	33.7%
GM	0.64%	7.34%	-1.06%	1.02	6.14%	30.0%
GE	2.26%	5.86%	0.53%	1.04	4.15%	49.9%
VW-Rf	1.67%	4.02%				
Rf	0.36%	0.05%				

↪ VW denotes the Value-Weighted index of all NYSE, AMEX, and NASDAQ common stocks

↪ Rf denotes the (nominal) one-month T-Bill rate

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- Using the single-index model to calculate the correlations:

$$\rho_{i,j} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j} \qquad \forall i \neq j$$

	IBM	XOM	GM	GE
IBM	1	0.32	0.30	0.39
XOM	0.32	1	0.33	0.42
GM	0.30	0.33	1	0.40
GE	0.39	0.42	0.40	1

Example

- Using these correlations, along with the *sample means* as estimates for the $E(r_i)$s, and the *sample standard deviations* as estimates for the σ_is, gives the following tangency portfolio:

	Weight
IBM	29.6 %
XOM	49.7 %
GM	-21.4 %
GE	42.4 %

$\hookrightarrow$ What do you think about this portfolio?

$\hookrightarrow$ The equilibrium arguments that we used in developing the CAPM indirectly suggest that the market knows something about the future returns that we don't

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Example

- Instead of the sample means, let's use the CAPM,

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f]$$

to calculate the expected returns:

Stock	CAPM $E(r_i^e)$
IBM	1.91 %
XOM	0.99 %
GM	1.70 %
GE	1.73 %

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Example

- With this "equilibrium" set of expected returns, we now get the following portfolio weights:

	weights
IBM	13.6%
XOM	33.3%
GM	16.4%
GE	36.6%

- ↪ Why are the weights so different?
- ↪ Why are these not the market weights?
- ↪ When would these be the actual market weights?
- ↪ Is this the portfolio you would want to hold if you were constrained to only holding these four individual stocks?

Example

- There may be times when you think that the market is wrong along one or more dimensions
  - ↪ A very dangerous assumption ...

- How can you combine your “views” with those of the market?

- For example, suppose that:
  - ↪ You believe the “market” has underestimated the earnings that IBM will announce next month, and that IBM’s return will be 2% higher than market consensus
  - ↪ You have no information on the other three securities that would lead you to believe that they are mispriced
  - ↪ The historical betas and residual standard deviations are all good estimates of their future values

Example

- Changing the expected returns for IBM by +2%, keeping all of the other inputs the same, the new optimal portfolio weights are:

Stock	$E(r_i^e)$	weights
IBM	3.91%	54.1%
XOM	0.99%	17.7%
GM	1.70%	8.7%
GE	1.73%	19.5%

- Compared to the old allocations:

	$E(r_i^e)$	weights
IBM	1.91%	13.6%
XOM	0.99%	33.3%
GM	1.70%	16.4%
GE	1.73%	36.6%

$\hookrightarrow$ What do you make of this new portfolio?

Example

- Alternatively, suppose that you believe the systematic risk of Exxon, or β_{XOM}, is going to increase from 0.59 to 0.8
- You also believe that Exxon's idiosyncratic risk $\sigma_{\varepsilon,XOM}$ will remain the same
- Recalculate *almost* everything using the equations:

$$
\begin{aligned}
E(r_i) &= r_f + \beta_i[E(r_m) - r_f] \\
\sigma_i^2 &= \beta_i^2 \cdot \sigma_m^2 + \sigma_{\varepsilon,i}^2 \\
\rho_{i,j} &= \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j}
\end{aligned}
$$

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Example

- The new correlations are:

	IBM	XOM	GM	GE
IBM	1	0.44	0.30	0.39
XOM	0.44	1	0.45	0.57
GM	0.30	0.45	1	0.40
GE	0.39	0.57	0.40	1

- Compared to the old correlations:

	IBM	XOM	GM	GE
IBM	1	0.32	0.30	0.39
XOM	0.32	1	0.33	0.42
GM	0.30	0.33	1	0.40
GE	0.39	0.42	0.40	1

Example

- If you believe these are the correct new correlations, but that the market still hasn't realized that the β of Exxon has changed, you would want use the old expected returns, resulting in:

	old		new
	$E(r_i^e)$	weight	weight
IBM	1.91%	13.6%	17.0%
XOM	0.99%	33.3%	16.7%
GM	1.70%	16.4%	20.5%
GE	1.73%	36.6%	45.7%

$\hookrightarrow$ How do you explain these new weights?

Example

- If on the other hand, you believe that the market already knows that the β of Exxon has increased, and that the expected return on Exxon is now higher to appropriately compensate for this increased systematic risk, the optimal portfolio becomes:

	old		new	
	$E(r_i^e)$	weight	$E(r^e)$	weight
IBM	1.91%	13.6%	1.91%	9.2%
XOM	0.99%	33.3%	1.34%	54.8%
GM	1.70%	16.4%	1.70%	11.1%
GE	1.73%	36.6%	1.73%	24.8%

$\hookrightarrow$ How do you explain these new weights?

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Example

- Alternatively, suppose that you believe that the market hasn't yet recognize that the systematic risk of Exxon has increased, but that it will soon discover this

  ↪ What will happen as the market finds out?

  ↪ What should you do in this situation?

- Bottom line, optimal investment decisions and trading strategies depend crucially on who knows what when

  ↪ We will return to this in our discussion of "Efficient Markets"

Forming "Views"

- Where is all of this supposedly superior information, or "views," coming from?
  - ↪ It may come from public sources that have not been incorporated into prices yet
  - ↪ Lot's of companies are in the business of gathering and selling all sorts of potentially relevant information, or "views"
  - ↪ It may come from especially diligent research and/or security analysis
  - ↪ Lot's of resources are devoted to obtaining and in turn leveraging superior research and/or security analysis
  - ↪ Or it may come from private, or inside, information ...

Forming “Views” - Example

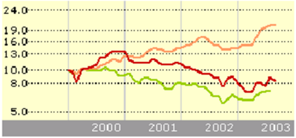
- Old Morningstar report on AIG:

Snapshot

American International Group AIG

Performance more

Growth of $10,000 09-05-03



Total Return%	2000	2001	2002	YTD
Stock	37.0	-19.3	-26.9	2.2
+/- Industry	10.5	-32.9	-25.3	-10.0
+/- S&P 500	46.1	-7.4	-4.9	-15.9

Key Stats

Last Close (09-05-03)	$58.95
Market Cap $Mil	153,793
Sales $Mil	73,501
Morningstar Style Box	Large Core
Industry	Insurance (Property)
Sector	Financial Services
Stock Type	Classic Growth

Morningstar Stock Grades

Growth	A
Profitability	B+
Financial Health	A-

Premium Features

Morningstar Rating ★★★

Business Risk Avg

Fair Value Estimate 65.0

Economic Moat Narrow

Analyst Report Summary

International growth and increased cross-selling should drive expansion for AIG. Read full analyst report

More Premium Features

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Forming "Views" - Example

- Current price: $P_0 = 58.95$
- "View": $V_0 = (1+a)P_0 = 65.00 \rightarrow a = 10.26\%$
- If you were 100% confident in your "view," then AIG's expected return should be higher than the CAPM return by $\alpha = 10.26\%$
  - $\hookrightarrow$ This is a very large number and will likely result in extreme portfolio allocations
- Suppose that you are only "somewhat" confident, say 10%, in your "view"
  - $\hookrightarrow$ Then you might only want to use a value of $\alpha = 10\% \times 10.26\%$ $= 1.26\%$
- This is obviously rather *ad hoc* ...

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Black-Litterman Model

- The Black-Litterman model provides a more systematic framework for combing individual “views” and “market views” in the construction of investment portfolios
  - ↪ The model was originally developed by Goldman Sachs, but similar (and more advanced) models are now widely used

- The model allows you to specify any number of “views” (in the form of expected returns) and corresponding measures of confidence (variances of “views”)
  - ↪ “Market views” are based on the CAPM expected returns
  - ↪ If you have no “views,” you should hold the market portfolio
  - ↪ If your “views” are high variance (not very confident), you should not deviate too much from the market portfolio
  - ↪ If your “views” are low variance (very confident), you should move more aggressively away from the market portfolio

Black-Litterman Model - Example

- International equity returns:

Exhibit 1

Historical Excess Returns

(January 1975 through August 1991)

Total Historical Excess Returns

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currencies	-20.8	3.2	23.3	13.4		12.6	3.0
Bonds CH	15.3	-2.3	42.3	21.4	-4.9	-22.8	-13.1
Equities CH	112.9	117.0	223.0	291.3	130.1	16.7	107.8

Annualized Historical Excess Returns

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currencies	-1.4	0.2	1.3	0.8		0.7	0.2
Bonds CH	0.9	-0.1	2.1	1.2	-0.3	-1.5	-0.8
Equities CH	4.7	4.8	7.3	8.6	5.2	0.9	4.5

Annualized Volatility of Historical Excess Returns

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currencies	12.1	11.7	12.3	11.9		4.7	10.3
Bonds CH	4.5	4.5	6.5	9.9	6.8	7.3	5.5
Equities CH	18.3	22.2	17.8	24.7	16.1	18.3	21.9

Note: Bond and equity excess returns are in U.S. dollars currency hedged (CH). Excess returns on bonds and equities are in excess of the London interbank offered rate (LIBOR), and those on currencies are in excess of the one-month forward rates. Volatilities are expressed as annualized standard deviations.

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Black-Litterman Model - Example

Exhibit 2

Historical Correlations of Excess Returns

(January 1975 through August 1991)

	Germany Equities CH	Germany Bonds CH	Germany Currency	France Equities CH	France Bonds CH	France Currency	Japan Equities CH	Japan Bonds CH	Japan Currency
Germany									
Equities CH	1.00								
Bonds CH	0.28	1.00							
Currency	0.02	0.36	1.00						
France									
Equities CH	0.52	0.17	0.03	1.00					
Bonds CH	0.23	0.46	0.15	0.36	1.00				
Currency	0.03	0.33	0.92	0.08	0.15	1.00			
Japan									
Equities CH	0.37	0.15	0.05	0.42	0.23	0.04	1.00		
Bonds CH	0.10	0.48	0.27	0.11	0.31	0.21	0.35	1.00	
Currency	0.01	0.21	0.62	0.10	0.19	0.62	0.18	0.45	1.00
U.K.									
Equities CH	0.42	0.20	−0.01	0.50	0.21	0.04	0.37	0.09	0.04
Bonds CH	0.14	0.36	0.09	0.20	0.31	0.09	0.20	0.33	0.19
Currency	0.02	0.22	0.66	0.05	0.05	0.66	0.06	0.24	0.54
U.S.									
Equities CH	0.43	0.23	0.03	0.52	0.21	0.06	0.41	0.12	−0.02
Bonds CH	0.17	0.50	0.26	0.10	0.33	0.22	0.11	0.28	0.18
Canada									
Equities CH	0.33	0.16	0.05	0.48	0.04	0.09	0.33	0.02	0.04
Bonds CH	0.13	0.49	0.24	0.10	0.35	0.21	0.14	0.33	0.22
Currency	0.05	0.14	0.11	0.10	0.04	0.10	0.12	0.05	0.06
Australia									
Equities CH	0.34	0.07	−0.00	0.39	0.07	0.05	0.25	−0.02	0.12
Bonds CH	0.24	0.19	0.09	0.04	0.16	0.08	0.12	0.16	0.09
Currency	−0.01	0.05	0.25	0.07	−0.03	0.29	0.05	0.10	0.27

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- Optimal MVE portfolios for $\sigma_p = 10.7\%$:

Exhibit 3

Optimal Portfolios Based on Historical Average Approach

(percent of portfolio value)

Unconstrained

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currency exposure	−78.7	46.5	15.5	28.6		65.0	−5.2
Bonds	30.4	−40.7	40.4	−1.4	54.5	−95.7	−52.5
Equities	4.4	−4.4	15.5	13.3	44.0	−44.2	9.0

With constraints against shorting assets

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currency exposure	−160.0	115.2	18.0	23.7		77.8	−13.8
Bonds	7.6	0.0	88.8	0.0	0.0	0.0	0.0
Equities	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Black-Litterman Model - Example

- Of course, you could just “throw in the towel” and use the CAPM
  - ↪ World CAPM estimates for international stock market returns assuming a “world” market risk premium of 7.15%:

Country	Equity Index Volatility (%)	Equilibrium Portfolio Weight (%)	Equilibrium Expected Returns (%)
Australia	16.0	1.6	3.9
Canada	20.3	2.2	6.9
France	24.8	5.2	8.4
Germany	27.1	5.5	9.0
Japan	21.0	11.6	4.3
UK	20.0	12.4	6.8
USA	18.7	61.5	7.6

	Australia	Canada	France	Germany	Japan	UK
Canada	0.488					
France	0.478	0.664				
Germany	0.515	0.655	0.861			
Japan	0.439	0.310	0.355	0.354		
UK	0.512	0.608	0.783	0.777	0.405	
USA	0.491	0.779	0.668	0.653	0.306	0.652

- But what if you think that you know better?

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Black-Litterman Model - Example

- Suppose you believe that the annual return on the Germany stock market will be 10.7% instead of 9.0% as implied by the CAPM

- Black-Litterman allows you to express this "view" as:

$$q = \pi + \varepsilon, \qquad \varepsilon \sim N(\mu_{ger}, \omega)$$

  - ↪ π refers to the CAPM expected return
  - ↪ $\mu_{ger} = 1.7\%$ represents your "view" about Germany's return
  - ↪ The randomness in ε allows for a margin of error in your "view"
  - ↪ The confidence in your "view" is determined by ω, the variance of your margin of error
  - ↪ The larger ω, the less confident you are in your "view"

- Given your "views" and the CAPM *prior*, the revised "best guess" for the expected returns $\bar{\mu}$, or the *posterior* expected returns, may be calculated using Bayesian statistical techniques

- Formally, $\bar{\mu}$ is going to be a weighted average of the CAPM expected returns Π and the "views" Q:

$$\bar{\mu} = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q]$$

- In the case of one asset with CAPM prior π and one "view" q, this simplifies to:

$$\bar{\mu} = \pi \frac{1/(\sigma^2\tau)}{1/\omega + 1/(\sigma^2\tau)} + q \frac{1/\omega}{1/\omega + 1/(\sigma^2\tau)}$$

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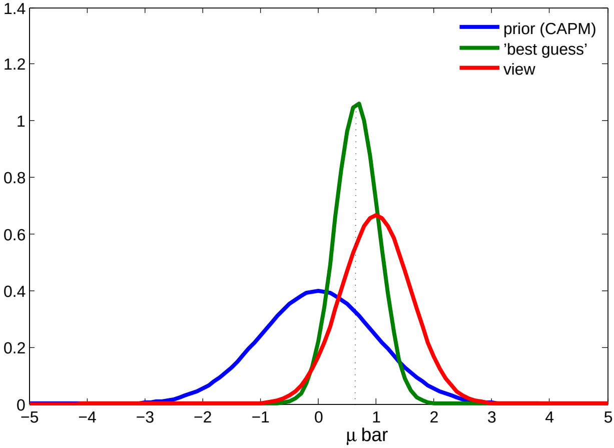
Black-Litterman Model



- The optimal portfolio weights are of the form:

$$w^* = w_{MKT} + P'\Lambda$$

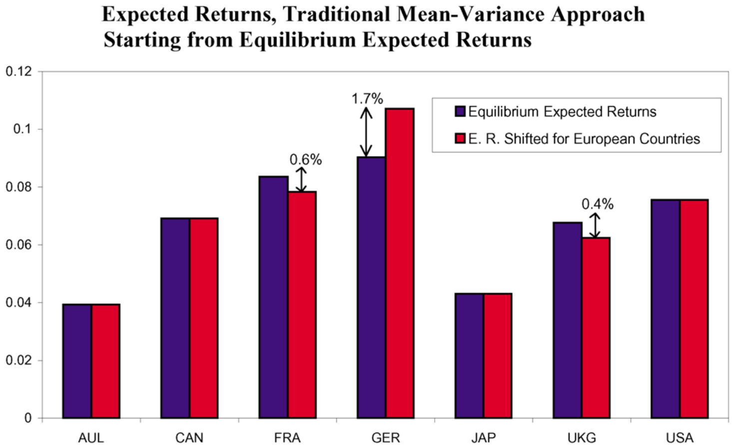
where P represents the investor's "view portfolio," and Λ is a complicated set of weights

  - $\hookrightarrow$ The higher the expected return on a "view" q, the higher the weight attached to that "view"
  - $\hookrightarrow$ The higher the variance of a "view" ω, the lower (in absolute value) the weight attached to that "view"

- If you hold no "view" on an asset, the optimal allocation is the market weight

Black-Litterman Model - Example

- Germany will outperform France and the UK



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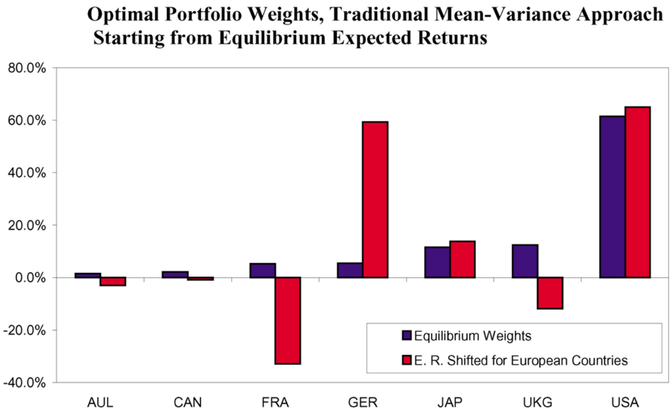
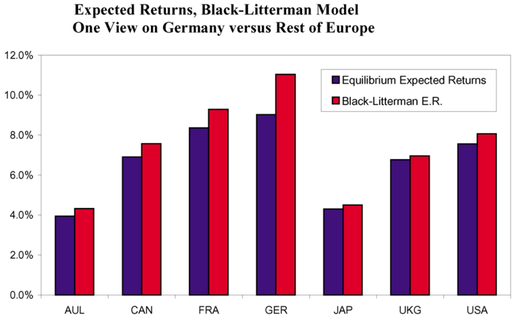
Black-Litterman Model - Example

- Optimal Markowitz allocations:



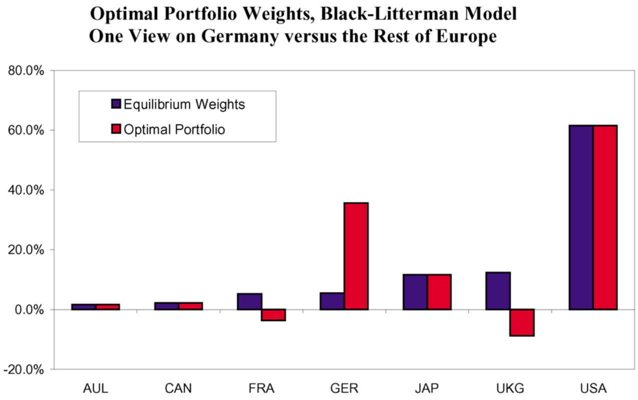
Black-Litterman Model - Example

- The Black-Litterman model internalizes the fact that assets are correlated
  - ↪ The "view" that Germany will outperform should shift $\overline{\mu}$ for other countries
  - ↪ This in turn translates into *implicit* "views" and changes in expected returns for *all* countries



Black-Litterman Model - Example

- The Black-Litterman allocations with “views”:



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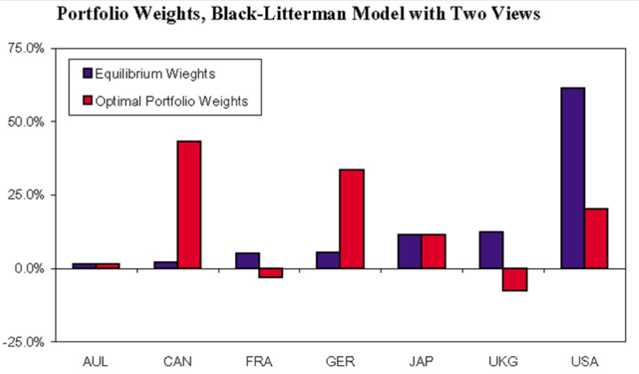
Black-Litterman Model - Example

- Now add the “view” that Canada will outperform the US by 3%:



Black-Litterman versus Markowitz

- Markowitz:
  - ↪ Need to estimate the expected returns for all assets
  - ↪ Small estimation errors can lead to unrealistic portfolio positions
  - ↪ If we change the expected return for one asset, this will change the weights for *all* assets

- Black-Litterman:
  - ↪ The optimal portfolio equals the CAPM market portfolio, plus a weighted average of the portfolios/assets about which the investor has “views”
  - ↪ The investor will only deviate from the market weights for assets about which she has “views”
  - ↪ There is no need for the investor to express “views” about each and every asset

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CAPM Summary

- The CAPM makes predictions about what the expected returns of *all* assets should be in equilibrium under the assumption that all investors base their decisions on the same myopic mean-variance optimization problem

- The key insight that what matters for expected returns is *covariance risk* rather than *variance risk* is of central importance to modern finance
  - ↪ This insight remains true whether the CAPM is true or not

- The CAPM implies that the market portfolio is MVE and that an asset's reward (expected return) should be proportional to the risk (variance) it adds to the market portfolio
  - ↪ This risk-reward relationship is succinctly summarized by the *beta* of an asset
  - ↪ The betas may be estimation by linear regression and other more sophisticated statistical procedures

- Next, we will discuss how well all of this holds empirically