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CORRECTION TO "AUTOMATIC BLOCK-LENGTH SELECTION FOR THE DEPENDENT BOOTSTRAP" BY D. POLITIS AND H. WHITE

Andrew Patton,[1] Dimitris N. Politis,[2] and Halbert White[3]

[1]*Department of Economics, University of Oxford, Oxford, UK*
[2]*Department of Mathematics, University of California at San Diego, La Jolla, California, USA*
[3]*Department of Economics, University of California at San Diego, La Jolla, California, USA*

□ *A correction on the optimal block size algorithms of Politis and White (2004) is given following a correction of Lahiri's (1999) theoretical results by Nordman (2008).*

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1. INTRODUCTION

Politis and White (2004) reviewed the problem of (nonparametric) bootstrapping for time series, and presented different block bootstrap methods in a unified way. In addition, results of Lahiri (1999) were reviewed, a corrected bound was suggested on the asymptotic relative efficiency (ARE) of different methods, and practically useful estimators of the optimal block size for the aforementioned block bootstrap methods were proposed.

Recently, however, Nordman (2008) discovered an error in Lahiri's (1999) calculation of the variance associated with the stationary bootstrap of Politis and Romano (1994). Since the theoretical results of Politis and White (2004) were building on Lahiri's (1999) calculations, a correction is in order and given in what follows. Furthermore, the proposed estimators of the optimal block size must be modified, and this results in different

Address correspondence to Dimitris N. Politis, Department of Mathematics, University of California at San Diego, La Jolla, CA, USA; E-mail: dpolitis@ucsd.edu

finite-sample behavior of the bootstrap methods employed with estimated block size.

The corrections are as follows:

1. The correct value for the variance constant D_{SB} defined in Theorem 3.1 of Politis and White (2004) is $D_{SB} = 2g^2(0)$;
2. The above corrected expression for D_{SB} is simple enough so that Lemma 2.1 of Politis and White (2004) is now replaced by the simple statement that:

$$ARE_{CB/SB} := \lim_{N\to\infty} \frac{MSE_{opt,CB}}{MSE_{opt,SB}} = (2/3)^{(2/3)} \simeq 0.7631428, \tag{1}$$

   where $MSE_{opt,CB} := \inf_b MSE(\hat{\sigma}^2_{b,CB})$, and $MSE_{opt,SB} := \inf_b MSE(\hat{\sigma}^2_{b,SB})$;
3. Equations (6) and (7) of Politis and White (2004) still give the optimal (expected) block size for the stationary bootstrap and the optimized large-sample MSE of estimation *as long as the correct expression for D_{SB} is used*;
4. Equation (8) of Politis and White (2004) should be corrected as follows: $\widehat{D}_{SB} = 2\hat{g}^2(0)$;
5. Using the corrected expression for $\widehat{D}_{SB}$, Eq. (9) of Politis and White (2004) gives the estimator of the optimal (expected) block size for the stationary bootstrap, and Theorem 3.2 remains valid as stated.

2. CORRECTED SIMULATION RESULTS

The simulation results of Politis and White (2004) were based on the wrong value of the constant D_{SB} and are, therefore, inaccurate. The simulations were re-run anew, and an additional numerical error in Table 1 of Politis and White (2004) was also captured. The corrected Tables 1–4 are given below; they should replace the respective Tables 1–4 of Politis and White (2004). The corrected MatLab code for the practical implementation of the Politis/White block selection algorithms is available from the Web site http://www.economics.ox.ac.uk/members/andrew.patton/code.html.

For the simulations, 1000 time series were generated of length N from the AR(1) model: $X_t = \rho X_{t-1} + Z_t$, with $\{Z_t\} \sim$ i.i.d. $N(0, 1)$.

What is apparent from the new Tables 2 and 3 is that the Politis/White block selection algorithms work exceptionally well even better to what was previously thought. On average, the estimated block sizes are between 90% and 110% of the respective true optimal block sizes. Furthermore, the reduction in the standard deviation of these estimated block sizes is dramatic when going from $N = 200$ to $N = 800$. This approximate halving of the RMSE with a quadruple sample size was predicted by the theoretical

TABLE 1 Theoretical optimal block sizes $b_{opt,SB}$ and $b_{opt,CB}$; the brackets $[\cdot]$ indicate 'closest integer' to the entry

	$b_{opt,SB}$	$b_{opt,CB}$
$\rho = 0.7,\ N = 200$	11.47	[13.12]
$N = 800$	18.20	[20.83]
$\rho = 0.1,\ N = 200$	2.01	[2.31]
$N = 800$	3.20	[3.66]
$\rho = -0.4,\ N = 200$	5.66	[6.48]
$N = 800$	8.99	[10.23]

TABLE 2 Empirical mean, standard deviation, and root mean squared error (RMSE) of the quantity $\hat{b}_{opt,SB}/b_{opt,SB}$

$\hat{b}_{opt,SB}/b_{opt,SB}$	Mean	St. dev.	RMSE
$\rho = 0.7,\ N = 200$	0.859	0.342	0.370
$N = 800$	0.927	0.244	0.254
$\rho = 0.1,\ N = 200$	0.959	0.943	0.943
$N = 800$	0.881	0.323	0.344
$\rho = -0.4,\ N = 200$	1.062	0.644	0.646
$N = 800$	1.081	0.368	0.377

TABLE 3 Empirical mean, standard deviation, and root mean squared error (RMSE) of the quantity $\hat{b}_{opt,CB}/b_{opt,CB}$

$\hat{b}_{opt,CB}/b_{opt,CB}$	Mean	St. dev.	RMSE
$\rho = 0.7,\ N = 200$	0.896	0.329	0.345
$N = 800$	0.951	0.244	0.249
$\rho = 0.1,\ N = 200$	1.142	0.911	0.922
$N = 800$	1.022	0.336	0.337
$\rho = -0.4,\ N = 200$	1.135	0.638	0.652
$N = 800$	1.128	0.369	0.391

TABLE 4 The true σ^2_∞, and the mean and MSE of its two estimators based on estimated block size; the last column indicates the finite-sample attainable relative efficiency (FARE) of the SB relative to the CB

	σ^2_∞	$E\hat{\sigma}^2_{\hat{b}_{opt,SB}}$	$E\hat{\sigma}^2_{\hat{b}_{opt,CB}}$	$MSE_{\hat{b}_{opt},SB}$	$MSE_{\hat{b}_{opt},CB}$	$FARE_{CB/SB}$
$\rho = 0.7,\ N = 200$	11.111	7.692	8.207	22.220	19.257	0.867
$N = 800$	11.111	9.115	9.460	9.223	7.574	0.821
$\rho = 0.1,\ N = 200$	1.235	1.110	1.142	0.052	0.051	0.983
$N = 800$	1.235	1.143	1.163	0.022	0.018	0.820
$\rho = -0.4,\ N = 200$	0.510	0.611	0.577	0.049	0.031	0.632
$N = 800$	0.510	0.552	0.542	0.010	0.007	0.692

results of Politis and White (2004), but its empirical verification is quite remarkable recalling that these algorithms are totally automatic.

Table 4 reports the performance of the circular and stationary bootstrap methods based on *estimated* block sizes. To do that, the notion of finite-sample "attainable" relative efficiency (FARE) of the SB relative to CB was defined in Politis and White (2004) as $FARE_{CB/SB} := MSE(\hat{\sigma}^2_{\hat{b}_{opt,CB}})/MSE(\hat{\sigma}^2_{\hat{b}_{opt,SB}})$. First note that in the cases of positive dependence, the FAREs are larger than the asymptotic limit of 0.76 from Eq. (1); this is similar to the findings of the old simulation. In the case $\rho = -0.4$, however, the FAREs are smaller than 0.76 but only a somewhat smaller; in this case, the old Table 4 was very misleading (having been constructed based on the wrong formula). In all cases, the tendency of the FAREs to move towards the asymptotic limit of 0.76 when the sample size increases is noted giving credence to the conjecture offered in Section 4 of Politis and White (2004).

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