Supplemental Appendix to:

Realized semibetas: Disentangling “good” and “bad’ downside risks

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- Section S1 provides details on the hypothetical asset return distributions depicted in Figure 1 of the main paper.
- Section S2 provides more detailed estimation results pertaining to our discussion of arbitrage risk in the main part of the paper.
- Section S3 illustrates the unconditional distributions of the daily realized (semi)betas and their autocorrelation functions.
- Section S4 provides additional information pertaining to the asymmetric dependencies in the daily realized semibetas.
- Section S5 provides information about the betting on semibeta portfolios' industry allocations.
- Section S6 contains additional descriptives regarding the betting on and against semibeta portfolios, including exposures to the betting-against-beta (BAB) factor and portfolio downside semivariances.
- Section S7 reports conditional alpha estimates of the betting on and against semibeta portfolios.
- Section S8 discusses and evaluates the performance of alternative portfolios designed to jointly capture the risk premia for the two downside semibetas.
- Section S9 provides additional results regarding alternative partial adjustment schemes to account for transaction costs.

S1. Hypothetical Return Distributions

Figure 1 of the main paper presents iso-probability contour plots of hypothetical bivariate distributions for the "market" factor and four individual assets, all with CAPM beta equal to one. All of the joint distributions employ N(0,1) marginal distributions for the market and each of the individual assets. Using values for market volatility and average firm volatility from the data used in Section 4 of the paper, 15.08% and 34.33% respectively, a beta of unity implies a linear correlation of 0.44, and this value is used in all four panels.

The upper-left panel (Asset A) uses a Normal copula with parameter ρ=0.44, leading to a bivariate Normal distribution with the familiar elliptical contours. The upper-right and lower-left panels (Assets B and C) use a Clayton copula (see Nelson, 2006) and a "rotated Clayton" copula (see Patton, 2004), with parameter κ=0.8, which combined with $N(0,1)$ marginal distributions leads to linear correlation of 0.44. The lower-right panel (Asset D) uses an asymmetric (or non-exchangeable) Clayton copula (see Section 15.2 of McNeil, Frey and Embrechts, 2015) with parameters κ=4.25 and α=[0.9,0.5], which combined with $N(0,1)$ marginal distributions similarly results in a linear correlation of 0.44.

McNeil, Frey, R., Embrechts, P., 2015. *Quantitative Risk Management*, Revised Edition. Princeton University Press.

Nelson, R.B., 2006. *An Introduction to Copulas.* 2nd ed., Springer Series in Statistics.

Patton, A.J., 2004. On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. *Journal of Financial Econometrics* 2, 130-168.

S2. More Detailed Arbitrage Risk Results

Table S.1: **Monthly Semibeta Pricing and Arbitrage Risk** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from monthly Fama-MacBeth cross-sectional predictive regressions for firms with below and above median arbitrage risk, proxied by Idiosyncratic Volatility (IVOL) and Turnover (TO). The monthly semibetas are calculated from daily data. The control variables are measured prior to the daily returns, as detailed in Appendix A. The estimates are based on all of the common, non-penny, stocks in the CRSP data base from January 1963 to December 2019.

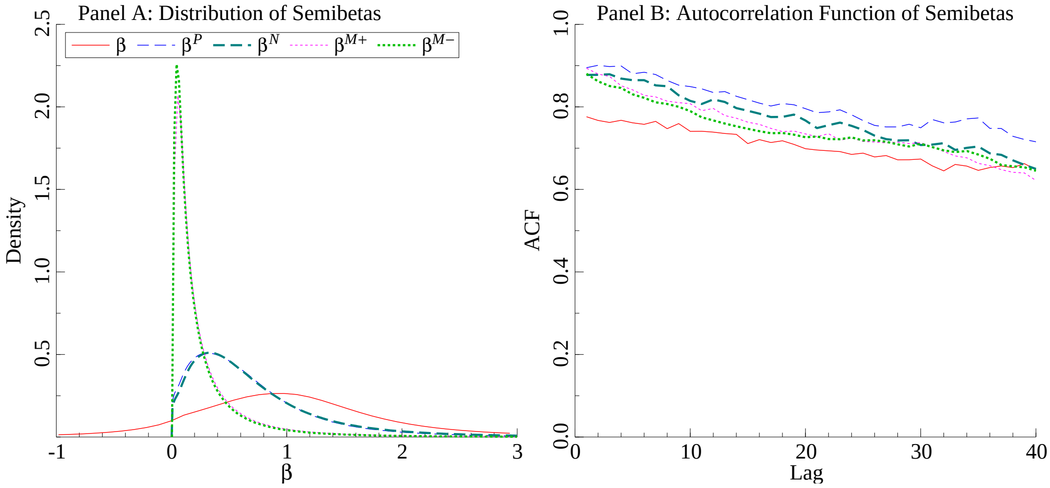
	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\delta^{\mathcal{M}-}$	$\delta^{\mathcal{M}+}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\delta^{\mathcal{M}-}$	$\delta^{\mathcal{M}+}$	R^2
	Below Median				Above Median				
IVOL	8.23	3.62	3.84	1.56	15.08	-7.03	10.98	-0.41	6.64
	3.83	2.03	1.02	0.42	4.79	-3.07	2.59	-0.07	
TO	7.49	5.31	5.60	0.17	15.51	-7.16	8.38	5.14	7.11
	3.50	2.79	1.22	0.04	4.63	-3.49	2.30	1.05	

Table S.2: **Daily Semibeta Pricing and Arbitrage Risk** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional predictive regressions for firms with below and above median arbitrage risk, proxied by Idiosyncratic Volatility (IVOL) and Turnover (TO). The daily semibetas are calculated from fifteen-minute intraday data. The control variables are measured prior to the daily returns, as detailed in Appendix A. The estimates are based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2019 sample.

	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\delta^{\mathcal{M}-}$	$\delta^{\mathcal{M}+}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\delta^{\mathcal{M}-}$	$\delta^{\mathcal{M}+}$	R^2
	Below Median				Above Median				
IVOL	17.58	1.06	2.58	-2.90	15.00	-2.39	23.93	8.75	6.65
	5.17	0.37	0.54	-0.55	4.25	-0.85	3.82	1.45	
TO	18.74	-11.21	-3.89	-7.95	15.61	2.79	8.04	-4.09	6.89
	5.59	-4.24	-0.79	-1.39	4.45	0.92	2.18	-0.72	

S3. Daily Betas and Semibetas

Figure S.1: **Unconditional Distributions and Autocorrelations.** Panel A displays kernel density estimates of the unconditional distribution of the daily realized beta and semibetas averaged across time and stocks. Panel B reports the average autocorrelation functions for the daily realized beta and semibetas averaged across stocks. The sample consists of all of the S&P 500 constituent stocks from January 1993 to December 2019.



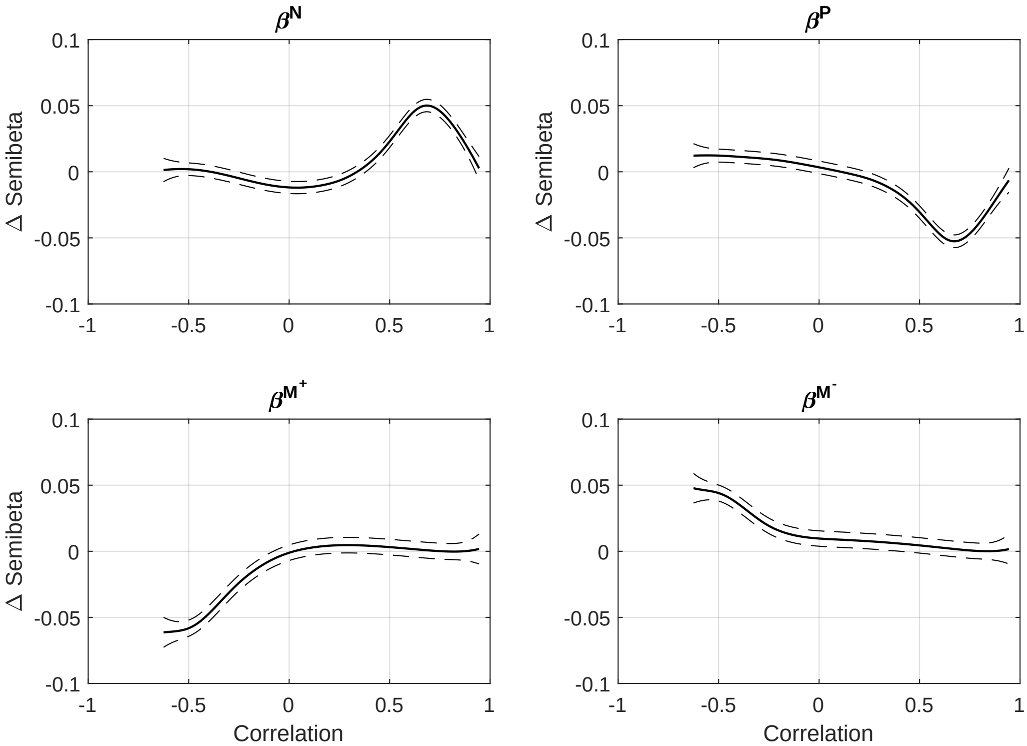
S4. Asymmetric Dependencies in Daily Semibetas

To help gauge whether the semibetas convey potentially useful additional information about asymmetric dependencies, it is instructive to compare the realized semibeta estimates to the limiting values that would obtain if the individual stock and aggregate market returns were jointly Normally distributed.

To that end, Figure S.2 reports the differences between the observed realized semibetas and the theoretical values that would hold under joint Normality, with positive values indicating greater correlation than under Normality. To facilitate the interpretation and more clearly highlight the differences, we report the results as a function of the daily realized correlations, averaged across all of the stocks and days in the sample. More specifically, for each day and stock in the sample, we standardize all of the intraday returns to have unit daily variance. We then compute the daily realized covariance (correlation) and the four semibetas, averaging the estimates within correlation bins of width 0.01. Finally, we use a spline to smooth the differences from their implied Gaussian values.

The top panel reveals that realized negative semibetas are generally higher than they would be if returns were jointly Normally distributed, particularly for relatively highly correlated assets (e.g., for correlations between 0.4 and 0.9), where the confidence interval clearly excludes zero. Similarly, the realized positive semibetas are lower than would be expected under joint Normality. For the discordant semibetas, we find a similar story: β^{M-} is significantly larger (in magnitude) than would be expected under joint Normality, particularly for negatively correlated assets, while the opposite is true for β^{M+}. Taken together, these findings are consistent with the stylized fact that asset return dependence is stronger in downturns than upturns.

Figure S.2: **Asymmetric Dependencies** The figure plots the deviations of the daily realized semibetas from their Gaussian limits as a function of the daily correlations between the individual stocks and the market, along with pointwise 99% confidence intervals. The estimates are averaged across all of the S&P 500 constituent stocks and days in the January 1993 to December 2019 sample.

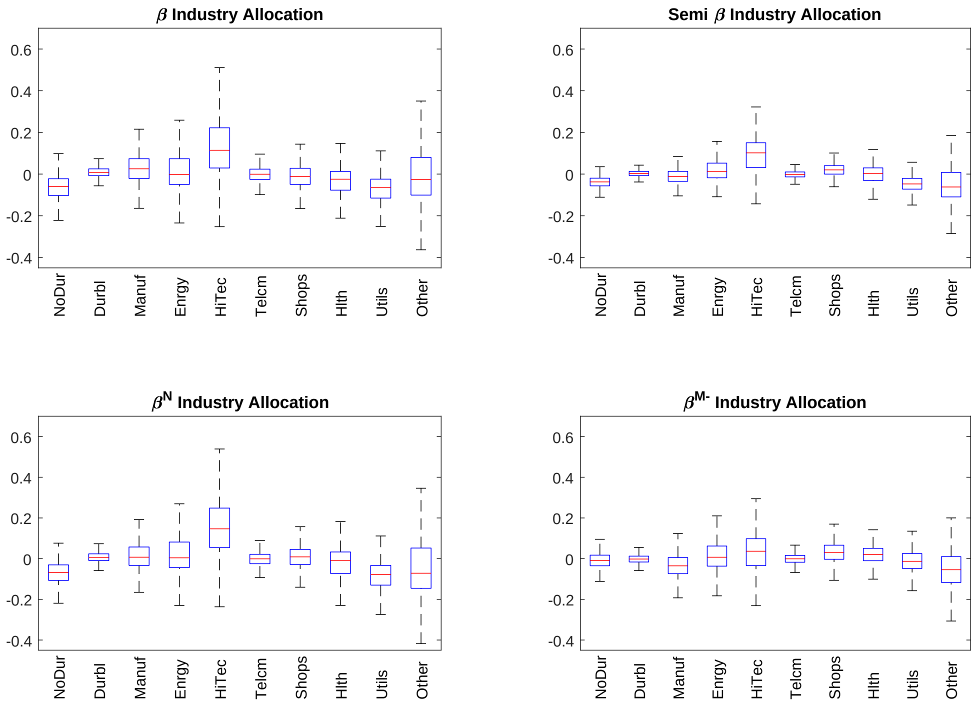


S5. Industry Concentrations of Semibeta Portfolios

In an effort to help better understand the composition of the different portfolios, we compute their allocations across industries according to Kenneth French's 10-Industry classification scheme based on SIC codes. Since we consider long-short portfolios, the portfolio weights add up to zero. However, they need not sum to zero within different industries.

Figure S.3 presents the resulting box-plots of the distributions. The top row reveals that the semibeta portfolio industry allocations are more concentrated around zero than those of the traditional beta portfolio, suggesting greater diversification along that dimension. The bottom row of the figure shows that the industry concentrations of the β and $\beta^{\mathcal{N}}$ portfolios are similar, while the $\beta^{\mathcal{M}^-}$ portfolios are noticeably different. The combination of the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ portfolios in turn generates comparatively low industry allocations for the semibeta portfolio, as shown in the upper-right panel.

Figure S.3: **Industry Concentration:** The figure presents box-plots of the time-series variation in industry allocation of the β, Semi β, $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ long-short portfolios, based on Kenneth French's 10-Industry classification.



S6. Robustness of Risk Exposures

This section provides factor exposures and alphas for the 3-factor Fama and French model plus the Betting against Beta (BAB) factor. Since our portfolios bet *on* $\beta^{\mathcal{N}}$ and *against* $\beta^{\mathcal{M}^-}$, BAB factor loadings are typically negative, resulting in higher alphas compared to the FFC4 and FF5 factor models considered in the main paper. We also report semi standard deviations, computed based on the sample mean of the squared demeaned negative daily returns, and corresponding semi Sharpe ratios.

Table S.3: **Betting On and Against Semibetas.** This table provides additional information on the portfolios studied in Table 7 in the main paper. The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced daily. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FF3 + BAB factor model, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β	Semi β	β^N	β^{M-}
Avg ret	5.62	8.17	10.02	5.56
Std dev	15.37	8.86	15.78	7.80
Sharpe	0.37	0.92	0.63	0.71
Semi Std dev	15.46	9.23	16.21	8.11
Semi Sharpe	0.36	0.88	0.62	0.69
α	7.02	9.04	11.21	6.09
	3.96	8.42	6.26	3.88
β_{MKT}	0.43	0.21	0.45	-0.02
	56.68	46.54	58.87	-3.49
β_{SMB}	0.14	0.23	0.26	0.21
	11.42	31.23	20.86	19.07
β_{HML}	0.08	0.06	0.03	0.09
	7.01	8.67	2.86	8.53
β_{BAB}	-0.53	-0.29	-0.52	-0.07
	-43.50	-39.93	-42.13	-6.45
R^2	62.99	59.84	64.90	8.24

Table S.4: **Betting On and Against Semibetas with Transaction Costs.** This table provides additional information on the portfolios studied in Table 11 in the main paper. The top panel reports annualized descriptive statistics for the semibeta portfolios. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FF3 + BAB factor model, along with the corresponding alphas in annualized percentage terms. All of the portfolios are self-financing based on value-weighted long-short positions determined by the combined semibeta strategy rebalanced monthly. T-cost refers to the roundtrip transaction costs. The left panel is identical to the second panel in Table 10 and fully adjusted portfolio weights. The right three panels report the results based on partially-adjusted portfolio weights, as discussed in the main text. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2019 sample.

T-cost Adjustment	0% Full	0% Partial	0.5% Partial	1.0% Partial
Avg ret	3.44	3.75	3.46	3.17
Std dev	8.10	7.32	7.32	7.32
Sharpe	0.42	0.51	0.47	0.43
Semi Std dev	8.45	7.76	7.76	7.75
Semi Sharpe	0.41	0.48	0.45	0.41
α	4.38	4.32	4.03	3.74
	4.00	5.16	4.81	4.46
β_{MKT}	0.16	0.17	0.17	0.17
	34.23	47.26	47.27	47.27
β_{SMB}	0.25	0.24	0.24	0.24
	33.08	41.36	41.36	41.34
β_{HML}	0.01	-0.10	-0.10	-0.10
	1.09	-18.26	-18.24	-18.22
β_{BAB}	-0.25	-0.19	-0.19	-0.19
	-33.36	-33.51	-33.51	-33.50
R^2	51.08	56.67	56.65	56.60

S7. Conditional Alphas

To guard against potential biases in the unconditional alphas arising from temporal variation in conditional betas (see, e.g., Jagannathan and Wang (1996) and Lewellen and Nagel (2006)), we calculate conditional alphas following the approach of Cederburgh and O'Doherty (2016) (cf. Section II.B).

Specifically, we cumulate the daily portfolio returns to a quarterly window of 60 days, indexed by τ, and run regressions of the form:

$$r_\tau = \alpha + \sum_{k=1}^{K} (\lambda_{0,k} + \lambda'_{1,k} Z_{\tau-1}) f_{k,\tau} + \epsilon_\tau,$$

where $f_{k,\tau}$ denote quarterly factors stemming from the FFC4 or FF5 model, and $Z_{\tau-1}$ denotes a set of instruments measured before the return window.[1] In parallel to Cederburgh and O'Doherty (2016), we consider four different sets of instruments Z_τ.

First, we compute 'Lagged-Component' (LC) betas estimated on 3- and 36-month windows. These are calculated as the portfolio-weighted average of the lagged beta estimates for the constituent firms included in the portfolio. The constituent betas are estimated with daily data, where we require at least 36 and 450 daily observations to compute the 3- and 36-month betas, respectively. In addition, we use the default spread (DS) and dividend yield (DY). The default spread is computed as the difference between Moody's BAA and AAA-rated bonds, available from the Federal Reserve Bank of St.Louis' website. For the dividend yield, we compute the quarterly cumulated difference between the CRSP value-weighted return including and excluding dividends and distributions.

The resulting average conditional alpha estimates for the different portfolios and various combinations of instruments are summarized in Table S.5. The complete regression results are reported in Tables S.6 to S.8. Consistent with the unconditional alpha estimates reported in the main part of the paper, we find that the conditional alphas for

[1] The quarterly factors were constructed from daily data retrieved from Kenneth French's website, by cumulating the long- and short sides of each factor separately and then computing the difference.

the semibeta portfolios are significantly positive across all of the different specifications, while the conditional alphas for the standard β portfolio are always insignificant.[2] The magnitudes of the average conditional alphas for the Semi-β and $\beta^{\mathcal{N}}$ portfolios are also very similar to the unconditional alpha estimates reported in Table 7, while those for the $\beta^{\mathcal{M}^-}$ portfolio are marginally lower.

Table S.5: **Conditional Alphas - Summary.** The return regression is given by $r_\tau = \alpha + \sum_{k=1}^{K}(\lambda_{0,k} + \lambda'_{1,k}Z_{\tau-1})f_{k,\tau} + \epsilon_\tau$, where $f_{k,\tau}$ denote quarterly factors stemming from the four-factor model of Fama-French-Carhart (1997) and the five-factor model of Fama and French (1993, 2015), and $Z_{\tau-1}$ is a set of instruments. Specification I uses 3- and 36- month lagged component betas as instruments. Specification II uses Dividend Yield and Default Spread as instruments. Specification III uses all four instruments. We report estimates of α along with Newey-West robust t-statistics for both the FFC4 and FF5 factor models. The alpha is reported in annualized percentage terms.

	FFC4				FF5			
	β	Semi β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^-}$	β	Semi β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^-}$
I	3.46	4.77	6.75	1.36	3.12	5.49	8.48	2.34
	1.34	2.53	2.51	0.89	1.07	2.78	2.69	1.18
II	3.71	6.19	8.70	3.76	2.77	5.42	7.22	3.78
	1.41	3.35	3.26	1.93	1.04	3.23	2.50	2.01
III	4.08	5.68	7.36	3.19	3.10	4.89	7.28	3.57
	1.33	3.56	3.29	1.95	1.22	2.91	2.82	2.05

[2]The only exception is the $\beta^{\mathcal{M}^-}$ portfolio when we only use the LC betas as instruments, in which case the alpha is positive but statistically insignificant.

Table S.6: **Conditional Alphas - Specification I.** The return regression is given by $r_\tau = \alpha + \sum_{k=1}^{K}(\lambda_{0,k} + \lambda'_{1,k} Z_{\tau-1}) f_{k,\tau} + \epsilon_\tau$, where $f_{k,\tau}$ denote quarterly factors stemming from the four-factor model of Fama-French-Carhart (1997) and the five-factor model of Fama and French (1993, 2015), and $Z_{\tau-1}$ is a set of instruments. This specification uses 3- and 36- month lagged component betas as instruments. We report parameter estimates along with Newey-West robust t-statistics for both the FFC4 and FF5 factor models. The alpha is reported in annualized percentage terms.

		$f_{MKT,\tau}\times$			$f_{SMB,\tau}\times$			$f_{HML,\tau}\times$			$f_{MOM,\tau}\times$			R^2
	α	1	β^{LC3}	β^{LC36}	1	β^{LC3}	β^{LC36}	1	β^{LC3}	β^{LC36}	1	β^{LC3}	β^{LC36}	
β	3.5	0.3	0.2	0.0	0.1	-0.3	0.2	-0.4	0.3	0.5	0.9	1.1	0.2	58.1
	1.3	1.3	1.4	0.0	0.9	-0.5	0.3	-0.8	1.1	0.7	1.0	2.3	0.5	
Semi β	4.8	0.2	-0.3	0.1	0.0	0.1	0.3	-0.2	0.2	0.1	1.3	0.7	0.0	67.1
	2.5	1.0	-1.3	1.5	0.3	0.1	0.6	-0.9	1.3	0.1	1.9	1.8	0.0	
$\beta^{\mathcal{N}}$	6.7	0.4	-0.1	0.0	0.0	-0.3	-0.3	-0.7	0.6	0.0	2.3	1.1	-0.3	62.1
	2.5	1.8	-0.6	-0.4	0.5	-0.6	-0.4	-1.9	2.1	0.0	2.2	2.5	-0.5	
$\beta^{\mathcal{M}^-}$	1.4	0.2	-0.1	0.3	0.0	0.6	-0.1	-0.1	0.0	0.1	1.2	-0.2	0.0	40.6
	0.9	3.0	-0.4	4.1	0.0	1.5	-0.4	-0.3	-0.1	0.2	1.9	-0.3	0.1	

		$f_{MKT,\tau}\times$			$f_{SMB,\tau}\times$			$f_{HML,\tau}\times$			$f_{RMW,\tau}\times$			$f_{CMA,\tau}\times$			R^2
	α	1	β^{LC3}	β^{LC36}	1	β^{LC3}	β^{LC36}	1	β^{LC3}	β^{LC36}	1	β^{LC3}	β^{LC36}	1	β^{LC3}	β^{LC36}	
β	3.1	0.5	0.1	0.0	-0.2	0.1	0.0	-0.3	-0.6	-0.3	-0.4	1.4	1.2	2.9	1.5	2.3	55.2
	1.1	4.3	0.4	0.3	-0.8	0.4	0.1	-0.6	-1.8	-0.9	-0.9	0.8	0.7	1.7	1.1	1.7	
Semi β	5.5	0.2	0.3	0.2	-0.2	0.0	0.4	-0.4	-0.3	0.1	0.3	-0.5	2.2	-0.3	0.8	0.5	66.2
	2.8	3.5	2.5	1.5	-1.4	0.1	0.9	-1.0	-1.0	0.4	0.8	-0.4	1.8	-0.2	0.9	0.4	
$\beta^{\mathcal{N}}$	8.5	0.4	0.3	0.1	-0.4	0.0	1.0	-0.6	-0.4	-0.1	0.0	-0.4	3.4	0.0	1.4	1.4	54.7
	2.7	3.0	1.7	0.4	-2.0	0.1	1.6	-1.2	-1.0	-0.2	0.0	-0.2	1.9	0.0	1.0	0.9	
$\beta^{\mathcal{M}^-}$	2.3	0.1	0.3	0.2	-0.1	0.1	-0.3	-0.6	0.2	-0.7	0.0	2.0	2.5	1.7	2.6	2.3	45.6
	1.2	1.2	2.9	2.1	-0.6	0.7	-0.6	-1.2	0.6	-1.4	0.0	1.5	1.6	0.9	1.9	1.2	

Table S.7: **Conditional Alphas - Specification II.** The return regression is given by $r_\tau = \alpha + \sum_{k=1}^{K}(\lambda_{0,k} + \lambda'_{1,k} Z_{\tau-1}) f_{k,\tau} + \epsilon_\tau$, where $f_{k,\tau}$ denote quarterly factors stemming from the four-factor model of Fama-French-Carhart (1997) and the five-factor model of Fama and French (1993, 2015), and $Z_{\tau-1}$ is a set of instruments. This specification uses Dividend Yield and Default Spread as instruments. We report parameter estimates along with Newey-West robust t-statistics for both the FFC4 and FF5 factor models. The alpha is reported in annualized percentage terms.

		$f_{MKT,\tau}\times$			$f_{SMB,\tau}\times$			$f_{HML,\tau}\times$			$f_{MOM,\tau}\times$						R^2
	α	1	DY	DS	1	DY	DS	1	DY	DS	1	DY	DS				
β	3.7	1.0	-0.1	-1.1	0.3	-0.9	1.5	2.5	-1.0	-0.1	-0.6	0.0	0.1				54.3
	1.4	2.2	-0.1	-2.4	1.0	-0.9	1.1	2.0	-1.1	-0.6	-1.2	0.0	0.5				
Semi β	6.2	0.8	-0.4	-0.5	0.0	-1.2	1.7	1.4	-0.5	0.0	-0.1	0.0	0.1				67.6
	3.4	2.6	-1.2	-1.6	0.1	-1.8	1.9	1.6	-0.9	0.1	-0.2	0.1	0.8				
$\beta^{\mathcal{N}}$	8.7	1.4	-0.4	-1.0	0.5	-2.2	2.9	2.4	-2.4	0.0	-0.7	0.0	0.4				59.6
	3.3	3.2	-0.8	-2.2	2.1	-2.2	2.2	1.9	-2.6	-0.1	-1.6	-0.2	1.7				
$\beta^{\mathcal{M}^-}$	3.8	0.1	-0.4	-0.1	-0.5	-0.2	0.5	0.5	1.4	0.1	0.6	0.1	-0.1				49.9
	1.9	0.5	-1.4	-0.3	-2.9	-0.3	0.6	0.5	2.3	0.4	2.0	0.3	-0.8				
		$f_{MKT,\tau}\times$			$f_{SMB,\tau}\times$			$f_{HML,\tau}\times$			$f_{RMW,\tau}\times$			$f_{CMA,\tau}\times$			
	α	1	DY	DS	1	DY	DS	1	DY	DS	1	DY	DS	1	DY	DS	
β	2.8	0.9	-0.3	-2.2	0.2	2.0	-0.2	2.2	5.0	0.2	-3.7	-0.3	-0.7	0.0	-0.4	-0.2	55.6
	1.0	2.1	-0.6	-3.0	0.2	2.0	-0.2	1.5	2.8	0.1	-1.8	-1.2	-1.4	0.1	-0.7	-0.3	
Semi β	5.4	0.9	-0.6	-0.8	-0.2	1.0	-1.1	2.3	2.0	0.0	-1.8	-0.1	-0.1	0.1	0.0	-0.1	68.8
	3.2	3.3	-1.9	-1.8	-0.3	1.7	-1.7	2.5	1.8	0.0	-1.4	-0.5	-0.4	0.4	0.0	-0.3	
$\beta^{\mathcal{N}}$	7.2	0.9	-0.9	-2.0	-0.6	1.9	-0.9	3.8	5.2	0.4	-4.3	-0.1	-0.6	-0.3	0.3	0.2	57.0
	2.5	2.1	-1.6	-2.5	-0.6	1.7	-0.8	2.4	2.8	0.2	-2.0	-0.3	-1.2	-0.6	0.4	0.2	
$\beta^{\mathcal{M}^-}$	3.8	0.8	-0.3	0.3	0.3	0.1	-1.3	0.7	-1.2	-0.4	0.8	-0.1	0.4	0.4	-0.2	-0.4	45.1
	2.0	3.0	-0.8	0.6	0.5	0.2	-2.0	0.7	-1.0	-0.3	0.6	-0.4	1.1	1.7	-0.6	-0.9	

Table S.8: **Conditional Alphas - Specification III.** The return regression is given by $r_\tau = \alpha + \sum_{k=1}^{K}(\lambda_{0,k} + \lambda'_{1,k}Z_{\tau-1})f_{k,\tau} + \epsilon_\tau$, where $f_{k,\tau}$ denote quarterly factors stemming from the four-factor model of Fama-French-Carhart (1997) and the five-factor model of Fama and French (1993, 2015), and $Z_{\tau-1}$ is a set of instruments. This specification uses 3- and 36- month lagged component betas, as well as Dividend Yield and Default Spread as instruments. We report parameter estimates along with Newey-West robust t-statistics for both the FFC4 and FF5 factor models. The alpha is reported in annualized percentage terms.

		$f_{MKT,\tau}\times$					$f_{SMB,\tau}\times$					$f_{HML,\tau}\times$					$f_{MOM,\tau}\times$					
	α	1	β^3	β^{36}	DY	DS	1	β^3	β^{36}	DY	DS	1	β^3	β^{36}	DY	DS	1	β^3	β^{36}	DY	DS	R^2
β	4.1	0.5	1.0	-0.4	0.3	-0.2	0.3	-0.2	0.4	0.8	1.2	0.7	0.1	-0.3	-0.9	0.7	-1.1	-0.2	-0.6	0.1	0.2	64.3
	1.3	1.3	1.8	-0.5	1.2	-0.4	0.5	-0.3	1.4	1.1	1.4	1.1	0.2	-0.3	-0.6	0.4	-1.3	-1.0	-1.5	0.5	0.8	
Semi β	5.7	0.6	-0.9	-0.4	0.0	-0.1	0.2	-0.3	0.4	0.1	0.9	0.5	0.5	-0.7	1.8	1.1	-0.5	-0.1	0.1	0.0	0.2	76.8
	3.6	1.8	-2.7	-0.9	-0.1	-0.2	0.4	-1.3	1.7	0.1	1.6	1.0	1.1	-1.0	2.1	1.2	-0.8	-0.6	0.4	0.3	1.6	
$\beta^{\mathcal{N}}$	7.4	1.0	-0.8	-1.1	0.4	-0.3	-0.7	-0.6	0.6	0.6	2.2	0.4	-0.1	-1.1	2.8	2.6	-2.0	-0.2	-0.4	-0.1	0.5	70.0
	3.3	2.4	-1.6	-1.7	1.5	-0.9	-1.0	-1.7	2.4	0.9	2.7	0.8	-0.2	-1.2	2.2	1.8	-2.7	-0.9	-1.0	-0.3	2.4	
$\beta^{\mathcal{M}^-}$	3.2	0.1	-1.1	0.1	-0.6	0.8	0.0	-0.4	0.1	0.2	1.2	0.7	0.0	-0.3	1.1	0.2	1.5	0.1	0.7	0.0	-0.1	57.6
	1.9	0.2	-2.5	0.4	-3.3	1.9	0.1	-1.3	0.5	0.3	2.0	1.2	0.1	-0.4	1.3	0.2	2.5	0.9	2.4	0.1	-0.7	

		$f_{MKT,\tau}\times$					$f_{SMB,\tau}\times$					$f_{HML,\tau}\times$					$f_{RMW,\tau}\times$					$f_{CMA,\tau}\times$					
	α	1	β^3	β^{36}	DY	DS	1	β^3	β^{36}	DY	DS	1	β^3	β^{36}	DY	DS	1	β^3	β^{36}	DY	DS	1	β^3	β^{36}	DY	DS	R^2
β	3.1	0.9	0.4	-2.0	1.0	2.5	0.0	-0.3	-0.3	-0.2	-0.2	1.7	1.4	3.7	2.0	2.2	-0.7	0.7	4.5	-1.7	-5.0	-0.1	-0.7	-0.1	-0.3	-0.1	62.1
	1.2	2.0	0.6	-2.6	1.0	2.6	0.0	-0.7	-1.0	-0.7	-0.5	1.1	0.7	2.0	1.6	1.5	-0.7	0.4	2.5	-0.7	-2.5	-0.3	-1.5	-0.2	-0.5	-0.2	
Semi β	4.9	0.7	-0.4	-0.7	-0.1	0.9	0.6	-0.5	-0.2	0.2	0.2	-0.6	2.1	-0.5	0.6	-0.2	-0.6	2.0	2.0	0.2	-2.7	-0.1	-0.2	-0.1	-0.2	0.4	71.9
	2.9	2.4	-1.0	-1.3	-0.1	1.4	1.4	-1.3	-0.6	0.5	0.7	-0.6	2.0	-0.4	0.7	-0.2	-0.9	2.0	1.7	0.1	-2.0	-0.6	-0.7	-0.2	-0.4	0.7	
$\beta^{\mathcal{N}}$	7.3	0.9	0.1	-1.8	-0.8	2.6	0.9	-0.6	0.0	-0.1	0.1	0.4	3.9	-0.1	1.9	1.6	-1.0	1.5	5.1	0.9	-6.6	0.0	-0.5	-0.4	0.2	0.4	63.3
	2.8	1.9	0.1	-2.1	-0.8	2.6	1.8	-1.4	0.0	-0.2	0.2	0.3	2.4	0.0	1.6	1.1	-0.9	1.0	2.7	0.4	-3.2	0.0	-1.1	-1.0	0.3	0.5	
$\beta^{\mathcal{M}^-}$	3.6	0.7	0.0	0.0	0.6	0.4	-0.4	-0.5	0.3	-0.6	-0.1	2.7	1.7	2.0	1.2	2.2	-1.1	-0.5	-0.8	-1.2	0.8	-0.1	0.5	0.5	-0.2	-0.6	51.9
	2.0	2.5	0.1	0.1	0.9	0.6	-0.8	-1.0	0.8	-1.1	-0.2	2.3	1.2	1.2	0.9	1.0	-1.4	-0.4	-0.7	-0.8	0.6	-0.6	1.6	1.8	-0.4	-1.3	

S8. Alternative Semibeta Portfolios

In the main part of the paper we consider the performance of a portfolio based on averages of long-short positions in both of the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ semibetas. Alternative, long-short portfolios based on long and short positions in the opposing semibetas could also be constructed. The first such portfolio that we consider takes a value-weighted long position in the top quintile of $\beta^{\mathcal{N}}$ stocks and a short value-weighted position in the top quintile of $\beta^{\mathcal{M}^-}$ stocks. We refer to this portfolio as $\beta_H^{\mathcal{N}} - \beta_H^{\mathcal{M}^-}$. The second strategy we consider is similarly constructed by taking he opposite positions in the two bottom quintiles. We refer to this portfolio as $\beta_L^{\mathcal{M}^-} - \beta_L^{\mathcal{N}}$.

Table S.9 summarizes the performance of these two alternative semibeta portfolios. For comparison, we also include the results for the Semi β portfolio analyzed in the main part of the paper in the first panel of the table. The table shows that the long-short portfolio based on the top quintiles performs the best. That portfolio also outperforms the Semi β portfolio analyzed in the main part of the paper, with a Sharpe ratio of 1.03, and highly significant annualized FFC4 and FF5 alphas of 8.92 and 10.07%, respectively. Meanwhile, the portfolio based on the bottom quintiles result in a Sharpe ratio of 0.70, below that of the Semi β portfolio, and somewhat lower FFC4 and FF5 alphas of 4.68 and 4.89%, respectively. Nonetheless, all of the alphas are highly statistically significant with t-statistics in excess of 4.0.

Table S.9: **Betting On and Against Semibetas.** The top panel reports annualized descriptive statistics of the combined betting on and against semibeta strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced daily. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2019 sample.

	Semi β		$\beta_H^{\mathcal{N}} - \beta_H^{\mathcal{M}^-}$		$\beta_L^{\mathcal{M}^-} - \beta_L^{\mathcal{N}}$	
Avg ret	8.17		10.15		6.12	
Std dev	8.86		9.84		8.70	
Sharpe	0.92		1.03		0.70	
α	6.84	7.52	8.92	10.07	4.68	4.89
	5.92	6.49	6.50	7.50	4.08	4.12
β_{MKT}	0.28	0.25	0.29	0.23	0.28	0.26
	67.31	53.06	57.53	43.36	66.60	54.51
β_{SMB}	0.31	0.24	0.33	0.24	0.28	0.24
	38.92	28.43	35.50	24.78	35.85	27.46
β_{HML}	-0.01	0.16	-0.05	0.14	0.03	0.17
	-1.10	17.85	-5.12	14.11	3.89	18.86
β_{MOM}	-0.16		-0.17		-0.14	
	-27.83		-25.60		-25.37	
β_{RMW}		-0.25		-0.34		-0.15
		-21.46		-25.36		-13.20
β_{CMA}		-0.24		-0.29		-0.19
		-17.42		-18.28		-13.31
R^2	55.83	56.68	49.17	52.53	53.50	52.11

S9. Additional Transaction Cost Analyses

In this section we report the net of transaction cost performance for the fully-adjusted monthly-rebalanced betting on and against semibeta portfolios. We also consider alternative values for λ used in smoothing the portfolio weights, as well as an alternative procedure to help reduce transaction costs based on smoothing the semibeta estimates themselves.

Table S.10 further analyzes the sensitivity of the results for the smoothed semibeta portfolio to the choice of smoothing parameter λ used in smoothing the portfolio weights, $\omega_t^P = \lambda\omega_{t-1}^P + (1-\lambda)\omega_t^F$. We present the results for $\lambda = \{0.5, 0.7, 0.9, 0.99\}$, both with and without transaction costs.[3] In parallel to the results reported in Table 11 in the main part of the paper, the table shows that partially adjusting the weights generally improves the portfolio performance, even without considering transaction costs. However, when λ becomes too high the signal becomes too muted, and the alphas decrease. On the other hand, higher values of λ help reduce transaction costs. The value of $\lambda = 0.95$ underlying the results in main part of the paper appears a reasonable choice for judiciously balancing the signal and the cost of trading.

Instead of partially updating the weights to counter transaction costs, it is possibly to only partially update the semibeta estimates themselves:

$$\tilde{\beta}_t^j = \lambda\tilde{\beta}_{t-1}^j + (1-\lambda)\beta_t^j, \qquad j \in \{\mathcal{N}, \mathcal{M}-\}.$$

Similar exponentially weighted moving average filters are commonly used to extract the signal from noisy time series. Thus, portfolios based on the filtered semibetas may not only lead to more stable allocations, they may also entail more accurate stock selections. To facilitate comparison with the previous approach based on smoothing the weights, we use the same value of $\lambda = 0.95$. The results in the first panel of Table S.11 show that

[3]The standard errors on the estimated alphas in Table S.10 are numerically close to one in several cases, resulting in entries where the point estimate and the t-statistic are nearly identical. This is just a coincidence.

the smoothed semibetas do indeed improve on the signal, albeit only slightly, resulting in a similar Sharpe ratio and marginally higher FFC4 and FF5 alphas compared to the unsmoothed results in Table 10. Taking into account transaction costs, the performance of the smoothed semibeta portfolios are also drastically improved compared to the results for the fully adjusted portfolios in Table 11. However, partially adjusting the weights appears more effective in reducing trading costs, as manifest in much lower and mostly insignificant FFC4 and FF5 alphas in Table S.11 compared to the corresponding larger and significant alphas in Table 11 in the main part of the paper.

Going one step further, the weights defined by the smoothed semibetas could also be smoothed. Table S.12 reports the results from partially adjusting both the semibetas and the resulting weights, where we use the same $\lambda = 0.95$ for both of the filters. This partial adjustment of both the semibetas and the weights results in the best overall performance, both in terms of Sharpe ratio and FFC4 and FF5 alphas. Smoothing the betas improves the signal, while only partially adjusting the weights substantially reduces transaction costs with little to no detrimental impact on the overall portfolio performance. Even with roundtrip transaction costs of 0.5%, the Sharpe ratio is 0.51, and the FFC4 and FF5 alphas equal 3.63% and 5.87%, respectively, both of which are statistically significant.

Table S.10: **Betting On and Against Semibetas with Transaction Costs: Choice of Smoothing Parameter.** The table reports the annualized alpha coefficients and Newey-West robust t-statistics based on the Fama-French-Carhart (1997) and the five-factor model of Fama and French (1993, 2015), for various levels of the smoothing parameter λ. The portfolios are self-financing value-weighted long-short portfolios based on and an equal-weighted combination of the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ strategies, rebalanced monthly. TC is the cost of a roundtrip transaction. The sample covers 1993-2019.

Weight Adj. T-cost	Partial No		Partial Yes	
λ	FFC4	FF5	FFC4	FF5
0.50	2.33	2.97	-0.43	0.22
	2.34	2.96	-0.43	0.22
0.7	2.32	3.03	0.73	1.45
	2.46	3.21	0.77	1.53
0.9	2.24	3.32	1.70	2.79
	2.50	3.80	1.90	3.19
0.95	2.22	3.65	1.93	3.36
	2.45	4.12	2.12	3.79
0.99	1.97	3.71	1.87	3.60
	1.85	3.63	1.75	3.52

Table S.11: **Betting On and Against Semibetas with Transaction Costs: Semibeta Smoothing.** The top panel reports annualized descriptive statistics of the Betting on (Semi)Beta strategies, in a setting where semibetas are only partially adjusted each period. The bottom panel reports results from time-series regressions of the Fama-French-Carhart (1997) and the five-factor model of Fama and French (1993, 2015). The table reports the estimated regression coefficients and Newey-West robust t-statistics. Alphas are in percent, annualized. The portfolios are self-financing value-weighted long-short portfolios based on and an equal-weighted combination of the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ strategies, rebalanced monthly. TC is the cost of a roundtrip transaction. The sample covers 1993-2019.

T-cost	No		Yes	
Semibeta Adj.	Partial		Partial	
Weight Adj.	Full		Full	
Avg ret	5.40		2.26	
Std dev	12.92		12.97	
Sharpe	0.42		0.17	
α	3.64	4.76	0.49	1.62
	2.25	2.90	0.30	0.99
β_{MKT}	0.35	0.30	0.35	0.30
	59.74	45.47	59.84	45.47
β_{SMB}	0.59	0.50	0.59	0.50
	53.81	41.76	53.86	41.73
β_{HML}	-0.05	0.14	-0.05	0.14
	-4.63	10.93	-4.45	11.05
β_{MOM}	-0.18		-0.18	
	-22.33		-22.25	
β_{RMW}		-0.35		-0.35
		-21.60		-21.72
β_{CMA}		-0.27		-0.27
		-13.85		-13.81
R^2	50.43	52.21	50.17	52.00

Table S.12: **Betting On and Against Semibetas with Transaction Costs: Semibeta and Weight Smoothing.** The top panel reports annualized descriptive statistics of the Betting on (Semi)Beta strategies, in a setting where both semibetas and the corresponding portfolio weights are only partially adjusted each period. The bottom panel reports results from time-series regressions of the Fama-French-Carhart (1997) and the five-factor model of Fama and French (1993, 2015). The table reports the estimated regression coefficients and Newey-West robust t-statistics. Alphas are in percent, annualized. The portfolios are self-financing value-weighted long-short portfolios based on and an equal-weighted combination of the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ strategies, rebalanced monthly. TC is the cost of a roundtrip transaction. The sample covers 1993-2019.

T-cost	No		Yes	
Semibeta Adj.	Partial		Partial	
Weight Adj.	Partial		Partial	
Avg ret	6.20		5.93	
Std dev	11.70		11.70	
Sharpe	0.53		0.51	
α	3.90	6.14	3.63	5.87
	2.70	4.39	2.51	4.19
β_{MKT}	0.35	0.27	0.35	0.27
	66.63	47.72	66.62	47.70
β_{SMB}	0.52	0.43	0.52	0.43
	52.60	42.33	52.58	42.31
β_{HML}	-0.15	-0.03	-0.15	-0.03
	-14.52	-2.40	-14.51	-2.39
β_{MOM}	-0.03		-0.03	
	-4.31		-4.31	
β_{RMW}		-0.35		-0.35
		-25.12		-25.13
β_{CMA}		-0.29		-0.29
		-17.46		-17.46
R^2	51.47	57.55	51.46	57.55