Informal Risk Sharing with Local Information*

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Abstract

This paper considers the effect of contracting limitations in risk-sharing networks, arising for example from observability, verifiability, complexity or cultural constraints. We derive necessary and sufficient conditions for Pareto efficiency under these constraints in a general setting, and provide an explicit characterization of Pareto efficient arrangements under CARA utilities and normally distributed endowments. Contrary to other models, individuals with higher centralities become quasi-insurance providers to more peripheral individuals in our model. We show that network centrality is positively correlated with consumption volatility and we test this prediction using data on rural villages in Thailand.

Keywords: social network, risk sharing, Pareto efficiency, local information

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1 Introduction

Informal insurance arrangements in social networks have been shown to play an important role at smoothing consumption in a number of different contexts (Ellsworth 1988, Rosenzweig 1988, Deaton 1992, Paxson 1993, Udry 1994, Townsend 1994, Grimard 1997, Fafchamps and Lund 2003 and Fafchamps and Gubert 2007). A main finding in this literature is that informal insurance achieves imperfect consumption smoothing.[1] There are different theoretical explanations as to why perfect risk sharing is not possible. One leading explanation is the presence of enforcement constraints: since risk-sharing arrangements are informal, they have to satisfy incentive compatibility, implying an upper bound on the amount of transfer that individuals can credibly promise to each other. This type of explanation has been explored in a social network framework by Ambrus, Mobius, and Szeidl (2014).[2]

In this paper we explore an alternative explanation featuring imperfectness of the contracting environment. Specifically, we assume that bilateral risk sharing arrangements between a pair of agents cannot be made contingent on everyone's endowment realizations in the community (global information), but only on a pair specific subset of endowment realizations (local information). These contractibility restrictions can come from limited observability or verifiability of endowment realizations of agents located far enough on the social network, social norms and complexity costs on writing contracts, among other sources. In most of the paper we focus on the case when the local information that a pair of agents can contract on is the endowment realizations of themselves and their common neighbors, but we show how the results extend to more general contracting environments. Relative to previous models, our framework provides a number of new and testable predictions. We find that centrally located individuals become quasi insurance providers to more peripheral households.[3]

[1]Some recent papers, like Schulhofer-Wohl (2011) and Mazzocco and Saini (2012) point out that in some contexts perfect risk-sharing cannot be rejected when allowing for heterogeneous preferences. We discuss how our work is related to this literature later in this section. On the other extreme, Kazianga and Udry (2006) find a setting in which informal social insurance does not improve welfare over autarchy.

[2]See also Karlan, Mobius, Rosenblat, and Szeidl (2009), who investigate enforcement constraints in the case of a single borrowing transaction. There is also an extensive literature on the effects of limited commitment on risk-sharing possibilities for a pair of individuals instead of general networks (Coate and Ravallion (1993), Kocherlakota 1996, Ligon 1998, Fafchamps 1999, Ligon, Thomas, and Worrall 2002, Dubois, Jullien, and Magnac 2008).

[3]Throughout the paper we maintain the terminology "individuals", even though in many contexts the relevant unit of analysis is households.

Further, the current setup formalizes, and indeed generalizes, the notion of a "local sharing group" that has been invoked recently in the risk-sharing tests performed in the development literature.

Existing models of informal risk sharing in networks (Bramoullé and Kranton 2007, Ambrus, Mobius, and Szeidl 2014, Ambrus et al. 2017) assume that any bilateral arrangement between connected individuals can be conditioned on global information, meaning the community's full set of endowment realizations.[4] We find that this explanation generates qualitatively different predictions relative to models of informal insurance with enforcement constraints. Hence, our results can help future empirical work identify which type of constraint plays the key role in maintaining informal insurance arrangements away from efficiency.[5]

There is a line of theoretical literature investigating the effect of imperfect observability of incomes on informal risk sharing arrangements between two individuals in isolation: see for example Townsend (1982), Thomas and Worrall (1990), and Wang (1995). The questions investigated in this literature are fundamentally different from the ones we focus on, mainly because we are interested in questions that are inherently network related.[6]

The current framework also speaks to an ongoing debate in the development literature that emphasizes the importance of appropriately defining individuals' risk-sharing groups in empirical work. (Mazzocco and Saini 2012, Angelucci, De Giorgi, and Rasul 2017, Attanasio, Meghir, and Mommaerts 2018, Munshi and Rosenzweig 2016). A general trend in this literature considers alternative sub-groups within communities as the relevant risk-sharing units of individuals (e.g. an individual's caste or extended family). They argue that classical empirical tests of risk sharing (Townsend, 1994) must be adapted to accommodate heterogeneity in individuals' risk sharing communities. However, they only allow for a limited form of heterogeneity in which group membership is mutually exclusive and groups do not interact among

[4] Bloch, Genicot, and Ray (2008) consider various exogenously-specified transfer rules that depend on nonlocal information. See also Bourlès, Bramoullé, and Perez-Richet (2017), where individuals are motivated to send transfers to their neighbors for explicit altruistic reasons, but bilateral transfers depend on transfers among other individuals.

[5] Empirical papers trying to distinguish among different reasons of imperfectness of informal insurance contracts include Kinnan (2017) and Karaivanov and Townsend (2014). For an empirical test between full insurance versus informational constraints, see Ligon (1998).

[6] Other differences include that our analysis is static while the above papers are inherently dynamic, and in our paper individuals perfectly observe local information (but not beyond), while in the above papers incomes are not observable even between two connected individuals.

themselves.

Our paper is also related to the recent line of papers pointing out that allowing for heterogeneous preferences, in some contexts the full insurance hypothesis cannot be rejected, or at least the imperfection of the insurance can be bounded to be small: see Schulhofer-Wohl (2011), Mazzocco and Saini (2012) and Chiappori et al. (2014). In some settings this hinges on some specific type of preference heterogeneity, for example in the context of Chiappori, Samphantharak, Schulhofer-Wohl, and Townsend (2014) it requires that wealth and risk preferences are uncorrelated, which is at odds with the standard assumption of decreasing risk aversion.[7] Nevertheless, it is certainly possible that in some context informal social insurance is close to perfect. However, in other contexts empirical research found that informal social insurance is very ineffective and does not improve welfare relative to autarky (see the context in Kazianga and Udry, 2006, and for certain types of risks in the context of Goldstein, de Janvry, and Sadoulet, 2001). There are also some similarities between our work and the above literature. The latter investigates the role of heterogeneity of preferences in informal risk sharing, while our paper focuses on the role of heterogeneity in network positions. Our theory based on the latter generates novel empirical predictions, which we take to the data.

The first part of our analysis characterizes Pareto efficient risk-sharing arrangements under local information constraints for general (concave) and possibly heterogenous utility functions and endowment distributions. We show that Pareto efficiency in our context (subject to local information constraints) is equivalent to pairwise efficiency, that is the requirement that the risk-sharing agreement between any pair of neighbors is efficient, taking all other agreements between neighbors fixed. This means that any decentralized negotiation procedure that leads to an outcome in which neighbors do not leave money on the table would yield a Pareto efficient risk-sharing arrangement.

In the benchmark model with global information, the necessary and sufficient conditions for Pareto optimality, referred to as the Borch rule (Borch, 1962; Wilson, 1968) can be derived using standard techniques, and they state that the ratios of any two individuals' marginal utilities of consumptions must be equalized across states. We can generalize the Borch rule to this setting by showing that a necessary and

[7] For a recent paper finding support for preferences exhibiting decreasing risk aversion, see Paravisini, Rappoport, and Ravina (2016).

sufficient condition for Pareto optimality of a risk-sharing arrangement with local information equates the ratios of *expected* marginal utilities of consumption for each linked pair, where expectations are conditional on local states (i.e. on the realizations of the contractible endowments).

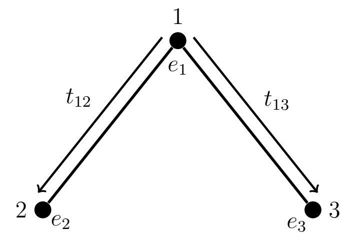
The generalized Borch rule can be used to verify the Pareto efficiency of consumption plans achieved by candidate transfer agreements in concrete specifications of our model. We provide this characterization for the case of CARA utilities and jointly normally distributed endowments in the context when local information of a pair includes the endowment realizations of the pair and common neighbors. The characterization is particularly simple for independent endowments: each individual shares her endowment realization equally among her neighbors and herself; on top of that, the arrangement can include state independent transfers, affecting the distribution of surplus but not the aggregate welfare.[8]

For the more general case of correlated endowment realizations in the CARA-normal setting, we show that efficient risk-sharing can still be achieved by transfers that are linear in endowment realizations and strictly bilateral (i.e. only contingent on the endowment realizations of the pair involved). In contrast to the local equal sharing rule that obtains in the case of independent endowments, we find that if individuals i and j are linked, increasing the exposure of i to transfers from non-common neighbors increases the share of i's endowment realization transferred to j, relative to local equal sharing, and decreases the share of j's endowment realization transferred to i. These correction terms, which are complicated functions of the network structure, take into account that more centrally located individuals are more exposed to the common shock component, and optimally correct for this discrepancy.

We show that more central individuals end up with a higher consumption variance because they serve as quasi insurance providers to more peripheral neighbors. For large random graphs we show this analytically, and for specific village networks from real world data we show it via simulations. For a fixed set of welfare weights, more central individuals are compensated for this service through higher state-independent transfers ("insurance premium"). This is contrary to the predictions from models with enforcement constraints, like (Ambrus, Mobius, and Szeidl, 2014), in which more centrally connected individuals are better insured (i.e. end up with smaller

[8]This type of transfer arrangement, which we refer to as the local equal sharing rule, was considered as an ad hoc sharing rule in Gao and Moon (2016).

Figure 1: A Simple 3-Individual Network



consumption variance).

We test this prediction using consumption and network data from the Townsend Monthly Thai Survey, which tracks villages in rural Thailand from 1998 to 2014. We find that, over different specifications and for different definitions of consumption, consumption variance positively co-moves with household's degree centrality, supporting the theoretical prediction of our model.[9]

2 Example: Three Individuals in a Line Network

2.1 Specification

Before investigating general network structures, we first consider the simplest non-trivial network, where three individuals, denoted by 1, 2 and 3, are minimally connected in a line. Despite its simplicity, this example provides some useful insights on how local information constraints affect efficient risk-sharing arrangements.

Assume that individuals have homogeneous CARA preferences of the form $u(x) = -\exp(-rx)$, and that endowments $e_1, e_2, e_3 \sim_{iid} \mathcal{N}(0, \sigma^2)$. Only linked individuals may enter into risk-sharing arrangements to mitigate endowment risks. Let t_{12} denote the net ex post transfer from individual 1 to individual 2, and t_{13} the net transfer from individual 1 to individual 3. Let x_1, x_2, x_3 denote the final consumption to individuals after the transfers are implemented, i.e., $x_1 = e_1 - t_{12} - t_{13}$, $x_2 = e_2 + t_{12}$ and $x_3 = e_3 + t_{13}$.

[9]In a separate paper, Milán et al. (2018) test the pairwise transfer scheme predicted by local information constraints against the observed food exchanges between Tsimane' households in the Bolivian Amazon. They find that bilateral transfers can be explained by network centrality, as predicted in Proposition 4 below, which provides further supporting evidence for the model.

2.2 Risk-sharing Arrangements with Global Information

First we consider the benchmark case when bilateral risk-sharing arrangements can be conditioned on global information, that is on all three individuals' endowment realizations: t_{12}, t_{13} can be arbitrary functions of the endowments e_1, e_2, e_3. Standard arguments (see Wilson, 1968) establish that Pareto efficient transfer rules t_{12}, t_{13} are the ones maximizing the social planner's problem:

$$\mathbb{E}\left[\sum_{i=1}^{3} \lambda_i u\left(x_i\right)\right] = \mathbb{E}\left[\lambda_1 u\left(e_1 - t_{12} - t_{13}\right) + \lambda_2 u\left(e_2 + t_{12}\right) + \lambda_3 u\left(e_3 + t_{13}\right)\right],$$

for some $\lambda_1, \lambda_2, \lambda_3 \in (0,1)$ s.t. $\lambda_1 + \lambda_2 + \lambda_3 = 1$. By the well-known Borch rule (Borch, 1962; Wilson, 1968), the necessary and sufficient conditions for optimality are:

$$\lambda_1 u^{'}\left(e_1 - t_{12} - t_{13}\right) = \lambda_2 u^{'}\left(e_2 + t_{12}\right) = \lambda_3 u^{'}\left(e_3 + t_{13}\right) \ \forall e_1, e_2, e_3.$$

With CARA utility, this yields $t_{12}\left(e_1, e_2, e_3\right) = \frac{1}{3}e_1 - \frac{2}{3}e_2 + \frac{1}{3}e_3 - \frac{1}{3r}\ln\left(\lambda_2^2/\lambda_1\lambda_3\right)$ and similarly for t_{13}, leading to the the final consumption plan:

$$\begin{cases} x_1 = \frac{1}{3}\left(e_1 + e_2 + e_3\right) + \frac{1}{3r}\ln\left(\lambda_2\lambda_3/\lambda_1^2\right), \\ x_2 = \frac{1}{3}\left(e_1 + e_2 + e_3\right) - \frac{1}{3r}\ln\left(\lambda_1\lambda_3/\lambda_2^2\right), \\ x_3 = \frac{1}{3}\left(e_1 + e_2 + e_3\right) - \frac{1}{3r}\ln\left(\lambda_1\lambda_2/\lambda_3^2\right). \end{cases} \tag{1}$$

That is, Pareto efficient risk-sharing arrangements in every state divide total realized endowments equally among individuals, and the equal division is then corrected by state-independent transfers that achieve the welfare weights.

2.3 Risk-sharing Arrangements with Local information

Suppose now that endowment realizations are only locally observable and verifiable, so that the transfers t_{12}, t_{13} in the risk-sharing arrangements can be contingent on local information only, that is: $t_{12} = t_{12}\left(e_1, e_2\right), t_{13} = t_{13}\left(e_1, e_3\right)$.

Achieving consumption plans on the Pareto frontier, given by (1), is impossible subject to these local information constraints. However, a necessary condition for a transfer arrangement to be socially optimal is that, for any given realization of e_1 and e_2, t_{12} should maximize $\lambda_1 u\left(e_1 - t_{12} - t_{13}\right) + \lambda_2 u\left(e_2 + t_{12}\right)$, given the distribution of

e_3 conditional on e_1 and e_2, and the implied distribution of consumption levels (net of t_{12}) induced by t_{13}.[10] In short, given t_{13}, t_{12} should maximize the planner's welfare function:

$$\max_{t_{12}} \int \left[\lambda_1 u\left(e_1 - t_{12} - t_{13}\right) + \lambda_2 u\left(e_2 + t_{12}\right)\right] f_{3|12}\left(e_3\right) de_3$$

The necessary and sufficient FOC for this maximization problem is:

$$\lambda_1 \mathbb{E}\left[u^{'}\left(e_1 - t_{12} - t_{13}\left(e_1, e_3\right)\right)\middle| e_1, e_2\right] = \lambda_2 u^{'}\left(e_2 + t_{12}\right),$$

and similarly for t_3 given t_2. Solving this system of two integral equations, we obtain the following transfer rule

$$t_{12}\left(e_1, e_2\right) = \frac{1}{3}e_1 - \frac{1}{2}e_2 - \frac{1}{24}r\sigma^2 - \frac{1}{3r}\ln\left(\lambda_1\lambda_3/\lambda_2^2\right) \tag{2}$$

and similarly for $t_{13}\left(e_1, e_3\right)$. Notice the transfers can be decomposed into three parts. The first part, $\frac{1}{3}e_1 - \frac{1}{2}e_2$, corresponds to the "local equal sharing rule", which is the local variant of the equal sharing rule. It implies that individual i's endowment e_i is equally shared by i and i's neighbors, i.e., $t_{ij} = \frac{e_i}{d_i+1} - \frac{e_j}{d_j+1}$. The second part of the equations in (2), $-\frac{1}{24}r\sigma^2$, corresponds to a state-independent transfer that can be regarded as the "insurance premium" paid by the "net insurance purchaser" to the "net insurance provider". In this case, as the final consumption are

$$\begin{cases} x_1 = \frac{1}{3}e_1 + \frac{1}{2}e_2 + \frac{1}{2}e_3 + \frac{1}{12}r\sigma^2 + \frac{1}{3r}\ln\left(\lambda_1^2/\lambda_2\lambda_3\right), \\ x_2 = \frac{1}{3}e_1 + \frac{1}{2}e_2 - \frac{1}{24}r\sigma^2 + \frac{1}{3r}\ln\left(\lambda_2^2/\lambda_1\lambda_3\right), \\ x_3 = \frac{1}{3}e_1 + \frac{1}{2}e_3 - \frac{1}{24}r\sigma^2 + \frac{1}{3r}\ln\left(\lambda_3^2/\lambda_1\lambda_2\right), \end{cases}$$

individual 1 takes extra risk exposure $\frac{1}{3}e_1 + \frac{1}{2}e_2 + \frac{1}{2}e_3$ in comparison to individuals 2 and 3, $\frac{1}{3}e_1 + \frac{1}{2}e_2$ or $\frac{1}{3}e_1 + \frac{1}{2}e_3$. Hence, individual 1 is rewarded the certainty equivalent (CE) for her intermediary role in risk sharing. The third part of the equations in (2), $-\frac{1}{3r}\ln\left(\lambda_1\lambda_2/\lambda_3^2\right)$, redistributes wealth according to the welfare weights assigned to different individuals (it is zero when $\lambda_1 = \lambda_2 = \lambda_3$).

To evaluate the welfare loss associated with risk-sharing arrangements that condition only on local information, since social welfare is a linear, strictly decreasing function of total variances under CARA utilities and normal endowments, we

[10]We show in Section 3 that this condition is actually also sufficient.

can simply compare the total variances of final consumption. With global information, the sum of consumption variances is: $TVar_G = 3 \cdot Var\left[\frac{1}{3}(e_1 + e_2 + e_3)\right] = \sigma^2$. With bilateral risk-sharing arrangements subject to the local information constraints, the sum of consumption variances increases to: $TVar_L = Var\left[\frac{1}{3}e_1 + \frac{1}{2}e_2 + \frac{1}{2}e_3\right] + Var\left[\frac{1}{3}e_1 + \frac{1}{2}e_2\right] + Var\left[\frac{1}{3}e_1 + \frac{1}{2}e_3\right] = \frac{4}{3}\sigma^2$. Hence the welfare loss arising from local information constraints is $\frac{1}{3}\sigma^2$ in the above example.

3 General Framework

3.1 Setup

Before we proceed to our main analysis, we first introduce the model setup and define some notations. Let $N = \{1, 2, ..., n\}$ be a finite set of individuals and let G be the adjacency matrix of a network structure on N. A pair of individuals i, j are linked if $G_{ij} = 1$, and by convention, $G_{ii} = 0$. Throughout the paper we assume, without loss of generality, that G represents a connected network.[11] Denote the neighborhood of i by $N_i := \{j \in N : G_{ij} = 1\}$ and the extended neighborhood of i by $\overline{N}_i := N_i \cup \{i\}$. Let $d_i := \#(N_i)$ denote individual i's degree. The state of the world is defined as the vector of endowment realizations $e \equiv (e_i)_{i \in N} \in \Omega \equiv \mathbb{R}^n$, and its distribution is characterized by a probability measure $\mathbb{P}$ on $(\Omega, \mathscr{B}(\Omega))$. We assume that the distribution of e has finite expectation.

A central assumption in our paper is that individuals can only observe the endowment realizations of their direct neighbors. (See Section 7.3 for generalization of our model beyond this assumption.) Define $N_{ij} := N_i \cap N_j$ and $\overline{N}_{ij} := \overline{N}_i \cap \overline{N}_j$. Let $I_i(e) := (e_j)_{j \in \overline{N}_i}$ be the information vector of i, and $I_{ij}(e) := (e_k)_{k \in \overline{N}_{ij}}$ be the common information vector of a linked pair ij. We may later refer to I_{ij} as the local state for ij.

We assume that only linked pairs of individuals can engage in informal risk sharing directly. An ex ante risk-sharing arrangement between linked individuals i and j is a net transfer rule $t_{ij} : \Omega \to \mathbb{R}$, which prescribes a net amount of $t_{ij}(e)$ to be transferred from i to j at each realized state e. We assume that linked pairs can only condition the net transfer on their their ex post local common information I_{ij}. Formally, we require that $t_{ij} : \Omega \to \mathbb{R}$ be $\sigma(I_{ij})$-measurable, where $\sigma(I_{ij})$ denotes the sub-σ-algebra

[11] Otherwise we may analyze each component separately.

induced by I_{ij}. By definition, $t_{ij}(e) = -t_{ji}(e)$ for every $e \in \Omega$ and linked $i, j \in N$. We refer to the profile of ex ante risk-sharing arrangements t_{ij} between all pairs of linked individuals as a transfer arrangement t.

Let $\mathcal{T}$ denote the set of all admissible transfer arrangements $t : \Omega := \mathbb{R}^n \to \mathbb{R}^{\sum_{i \in N} d_i}$ that are only contingent on the local states for all linked pairs:

$$\mathcal{T} := \left\{ t : \Omega \to \mathbb{R}^{\sum_{i \in N} d_i} \;\middle|\; \forall i, j \text{ s.t. } G_{ij} = 1, \quad \begin{array}{c} t_{ij} \text{ is } \sigma(I_{ij})\text{-measurable} \\ \text{and } t_{ij}(e) + t_{ji}(e) = 0, \; \forall e \in \Omega, \\ \text{and } \mathbb{E}[t_{ij}] \text{ is finite.} \end{array} \right\}$$

Define $\langle s, t \rangle := \mathbb{E}\left[\sum_{G_{ij}=1} s_{ij}(e)\, t_{ij}(e)\right]$ for any $s, t \in \mathcal{T}$. It follows that $\langle \cdot, \cdot \rangle$ is an inner product and $\mathcal{T}$ is a well-defined inner product space (see Lemma 1 in Appendix B.1 for a formal proof). We slightly abuse notations by treating each element in $\mathcal{T}$ as an equivalent class of transfer arrangements that are indistinguishable under the norm induced by $\langle \cdot, \cdot \rangle$. Throughout the paper, we write "$s = t$" to mean "$\langle s - t, s - t \rangle = 0$", whenever applicable.

Given a transfer arrangement $t \in \mathcal{T}$, we define the final consumption plan induced by t as $x^t : \Omega \to \mathbb{R}^n$ with $x_i^t(e) := e_i - \sum_{h \in N_i} t_{ij}(e)$. Individuals derive utilities from their own final consumption[12], and we assume that they have a strictly concave and twice differentiable utility function u, with $u' > 0$ and $u'' < 0$.

3.2 General Conditions for Pareto Efficiency

To characterize the set of Pareto efficient transfers under the local information constraint, we consider the following problem:

$$\max_{t \in \mathcal{T}} J(t) := \mathbb{E}\left[\sum_{k \in N} \lambda_k u\left(e_k - \sum_{h \in N_k} t_{kh}\right)\right] \tag{3}$$

Recall that both e and t are assumed to have finite expectation. As u is strictly concave, by Jensen's inequality, we conclude that $\mathbb{E}\left[u\left(e_k - \sum_{h \in N_k} t_{kh}\right)\right] < \infty$ for all $k \in N$, so the social welfare function $J : \mathcal{T} \to \mathbb{R} \cup \{-\infty\}$ is well defined on $\mathcal{T}$.

The following proposition provides a formal characterization of the solution to the

[12] Here we abstract away from constraints on minimum consumption levels, which clearly would further reduce the efficiency of the risk sharing contracts. However if income variances are small relative to expected income levels, then we expect the distortions to be small.

maximization problem above. Since the transfer rule t_{ij} is restricted to be measurable with respect to $\sigma(I_{ij})$, we slightly abuse notations and write it as $t_{ij} : \mathbb{R}^{d_{ij}+2} \to \mathbb{R}$ where $d_{ij} + 2 = \dim(I_{ij})$. We denote the distribution of I_{ij} on $\mathbb{R}^{d_{ij}+2}$ by $\mathbb{P}I_{ij}^{-1}$.

Proposition 1. *A profile of $t \in \mathcal{T}$ solves (3) if and only if it simultaneously solves the $\sum_{i\in N} d_i$ optimization problems in the form of (4) at $\mathbb{P}I_{ij}^{-1}$-almost all possible local states of the linked pair: $\forall i, j$ s.t. $G_{ij} = 1$, for $\mathbb{P}I_{ij}^{-1}$-almost all $\tilde{I}_{ij} \in \mathbb{R}^{d_{ij}+2}$,*

$$t_{ij}\left(\tilde{I}_{ij}\right) \in \arg\max_{\tilde{t}_{ij}\in\mathbb{R}} \mathbb{E}\left[\begin{array}{c} \lambda_i u_i\left(e_i - \tilde{t}_{ij} - \sum_{h\in N_i\setminus\{j\}} t_{ih}(e)\right) \\ +\lambda_j u_j\left(e_j + \tilde{t}_{ij} - \sum_{h\in N_j\setminus\{i\}} t_{jh}(e)\right) \end{array} \middle| I_{ij}(e) = \tilde{I}_{ij}\right] \tag{4}$$

Proposition 1 is an intuitive result. Its analogue under global information has a similar form and essentially connects the marginal utilities of consumption of two individuals in two different states (Wilson, 1968). With local information, the statement is now expressed, for every linked pair, in terms of a conditional expectation over the common information set of that pair. Therefore, equation (4) says that the set of Pareto efficient transfers call for pairwise efficient risk sharing along each link of the network, where efficiency is measured with respect to an expectation over all possible realizations of the nonlocal information.

Importantly, Proposition 1 provides a motivation for investigating Pareto efficient risk sharing subject to local information by implying that these are exactly the possible outcomes resulting from decentralized negotiation procedures satisfying the weak requirement that neighboring agents end up with agreements that are efficient at the pair level. To see this, notice that Proposition 1 establishes an equivalence between Pareto-efficient risk-sharing arrangements subject to local information constraints and stable outcomes of decentralized bilateral risk sharing arrangements between neighbors subject to the same constraints. In problem (4), at each I_{ij}, the choice of t_{ij} affects the expected utilities of only i and j, so each optimization problem in (4) can be reinterpreted as the surplus maximization problem jointly solved by the linked pair ij, given the transfer rules chosen by other linked pairs. Therefore, any bargaining procedure that leads to an agreement between any two neighboring agents that is efficient for the pair (does not leave surplus on the table) given other agreements, results in a Pareto efficient outcome at the social level.[13]

[13] A concrete example for such a negotiation procedure is split the difference negotiations, originally proposed in Stole and Zwiebel (1996) and adopted to the risk sharing context in Ambrus et al. (2017).

The next result establishes that while in general there can be multiple transfer profiles satisfying the conditions for optimality (4), they all imply the same consumption plan in all states.

Proposition 2. *All profiles of transfers $t \in \mathcal{T}$ that solve (3) lead to ($\mathbb{P}$-almost) the same consumption plan x.*

By Proposition 2, if we can find a profile of transfers so that the induced consumption plan satisfy (4), then it must correspond to a Pareto efficient risk-sharing arrangement.

For simplicity, below we will denote the conditional expectation operator $\mathbb{E}[\cdot\,|I_{ij}]$ by $\mathbb{E}_{ij}[\cdot]$. In observation of Proposition 1 and Proposition 2, we may express the necessary and sufficient condition for Pareto efficiency as a requirement on the ratio of conditional expected marginal utilities given by the next Corollary.

Corollary 1. *A profile of transfers t is Pareto efficient if and only if the ratio of the expected marginal utilities conditional on all local states is constant: for every $i, j \in N$ s.t. $G_{ij} = 1$,*

$$\frac{\mathbb{E}_{ij}\left[u_i'\left(x_i^t\right)\right]}{\mathbb{E}_{ij}\left[u_j'\left(x_j^t\right)\right]} = \frac{\lambda_j}{\lambda_i}. \tag{5}$$

This extends the Borch rule (Borch, 1962; Wilson, 1968) for Pareto efficient risk-sharing arrangements to settings with local information constraints. As opposed to the global-information case, the ratio of expected marginal utilities of consumptions among individuals do not have to be equal state by state, they only have to be equal between linked individuals in expectation, conditional on local common information.

3.3 Efficient Risk-sharing in the CARA-Normal Setting

In this section we investigate Pareto efficient risk-sharing arrangements, subject to local information constraints, under the assumption of CARA utilities and jointly normally distributed endowments with a uniform global correlation structure.

Assumption 1. *Throughout the subsequent sections we assume that individuals have homogeneous CARA utility functions $u(x) = -\exp(-rx)$, where $r > 0$ is the coefficient of absolute risk aversion. The vector of endowments $(e_i)_{i\in N}$ follows a multi-*

variate normal distribution, $e \sim \mathcal{N}(\mathbf{0}, \sigma^2 \Sigma)$ *with* $\Sigma_{ii} = 1$ *for all* i *and* $\sum_{ij} = \rho$ *for all* $i \neq j$*, for some* $\rho \in \left[-\frac{1}{n-1}, 1\right]$.[14]

3.3.1 Independent Endowments

We first analyze the case where endowments are independent, i.e., $\rho = 0$.

We use a guess and verify approach, and postulate that a linear transfer scheme, that is a scheme for which the transfer between any two connected individuals is a linear function of endowment realizations in the pair's joint information set, can achieve any Pareto efficient risk-sharing arrangement. Below we verify that the candidate linear risk-sharing arrangement is indeed optimal subject to local information constraints (in $\mathcal{T}$) using the expectational Borch rule.

Given a linear transfer scheme, the final consumptions, conditional on I_{ij}, also follow normal distribution, so $\mathbb{E}_{ij}\left[u_i'(x_i)\right] = r\exp\left[-r\left(\mathbb{E}_{ij}[x_i] - \frac{1}{2}rVar_{ij}[x_i]\right)\right]$. Define the conditional certainty equivalent $CE\left(x_i \mid I_{ij}\right) := \mathbb{E}_{ij}[x_i] - \frac{1}{2}rVar_{ij}[x_i]$. Then (5) can then be rewritten as

$$CE\left(x_i^* \mid I_{ij}\right) - \frac{1}{r}\ln\lambda_i = CE\left(x_j^* \mid I_{ij}\right) - \frac{1}{r}\ln\lambda_j. \tag{6}$$

The profile of transfer schemes t achieves Pareto efficiency if and only if (6) holds for every pair of ij such that $G_{ij} = 1$, i.e., the difference in the conditional certainty equivalents is constant at each local state for a linked pair.

We say a profile of transfer rules is *strictly bilateral* if t_{ij} is $\sigma(e_i, e_j)$-measurable. We now characterize efficient transfers subject to local information for the case of independent endowments.

Proposition 3. *Given any profile of positive welfare weights* $(\lambda_i)_{i \in N}$*, there always exists a strictly bilateral Pareto efficient profile of transfer rules in* $\mathcal{T}$ *in the form of the following "local equal sharing rules":*

$$t_{ij}^*(e_i, e_j) := \frac{e_i}{d_i + 1} - \frac{e_j}{d_j + 1} + \mu_{ij}^*,$$

for some constant $\mu_{ij}^* \in \mathbb{R}$*, for each linked pair* ij.

[14] $-\frac{1}{n-1}$ is the lower bound for a global pairwise correlation in a n-person economy; mathematically, it is the smallest ρ such that the variance-covariance matrix is positive semi-definite. For any $\rho \in \left[-\frac{1}{n-1}, 1\right]$, the variance-covariance matrix is positive semi-definite.

Proposition 3 shows that the efficient transfer $t^*_{ij}(e_i, e_j)$ subject to the local information constraint is composed of two parts: the state-contingent "sharing rule" and the state-independent "insurance premium" . Notice importantly that the state-contingent transfer scheme is linear in endowments and that, for the case of independent endowments, it boils down to the *local equal sharing rule*. Moreover, the transfers between two connected individuals only depend on endowment realizations of the two of them, not of their common neighbors – only bilateral information is required for efficient risk sharing with local information. Also, this proposition suggests that two linked individuals ij only require ex ante knowledge of the local network structure (more precisely d_i and d_j) to compute and contract on the socially optimal transfer rule t^*_{ij}.

Even though Proposition 2 guarantees Pareto efficient consumption plan is unique, for general networks there might be multiple risk-sharing arrangements that are Pareto efficient. In particular, superfluous transfers, either state-dependent or state-independent, may be freely added to a cycle of individuals in the network without changing the final consumptions. Therefore, in general the transfer scheme achieving a Pareto efficient risk-sharing arrangement is not unique. In Appendix B.7 we show that for tree networks the linear transfer scheme featured in Proposition 3 is the unique transfer arrangement that achieves a given Pareto efficient risk-sharing arrangement.

3.3.2 Correlated Endowments

We now turn to the case of correlated endowments with $\rho \neq 0$. To maintain analytical tractability, in Assumption 1 we assume a symmetric correlation structure, where any two individuals' endowments have the same correlation coefficient $\rho \in \left[-\frac{1}{n-1}, 1\right]$. Equivalently, we are assuming that each individual's endowment can be decomposed additively into two independent components: a common shock and an idiosyncratic shock, i.e., $e_i = \sqrt{\rho}\tilde{e}_0 + \sqrt{1-\rho}\tilde{e}_i$, with $(\tilde{e}_k)_{k=0}^n \sim_{iid} \mathcal{N}(0, \sigma^2)$.

We first characterize the precise conditions under which a *linear* transfer arrangement can achieve Pareto efficiency in $\mathcal{T}$, the space of *all* admissble transfers rules, linear or nonlinear, that satisfy the local information constraints.

Here we illustrate the main ideas in the case of *minimally-connected networks*, and defer the formal treatment of general network structures to Appendix B.3. Notice that, under minimal connectedness, $I_{ij} = (e_i, e_j)$, so transfer t_{ij} must be strictly bilateral. Then, the local FOC for optimality can be written as

$$
\begin{aligned}
t_{ij} = \frac{1}{2}e_i - \frac{1}{2}e_j - \frac{1}{2r}\ln\mathbb{E}\left[\exp\left(r\sum_{k\in N_i\setminus\{j\}} t_{ik}\left(e_i,e_k\right)\right)\middle| e_i,e_j\right] \\
+\frac{1}{2r}\ln\mathbb{E}\left[\exp\left(r\sum_{k\in N_j\setminus\{i\}} t_{jk}\left(e_j,e_k\right)\right)\middle| e_i,e_j\right] + \frac{1}{2r}\ln\frac{\lambda_j}{\lambda_i}
\end{aligned} \tag{7}
$$

Postulating a linear transfer scheme of the form, $t_{ij}\left(e_i,e_j\right) = \alpha_{ij}e_i - \alpha_{ji}e_j + \mu_{ij}\ \forall G_{ij} = 1$, we can substitute the postulated linear forms of t_{ik} into (7) and obtain expressions for the above conditional expectations in terms of linear combinations of endowments based on the conditional distribution $e_k|_{e_i,e_j} \sim \mathcal{N}\left(\frac{\rho}{1+\rho}\left(e_i+e_j\right),\ \frac{1+\rho-2\rho^2}{1+\rho}\cdot\sigma^2\right)$. We can therefore explicitly derive the conditional expectation terms in the above formula and, after collecting terms and reconciling with the postulated formula for t_{ij}, arrive at the following system of equations:[15]

$$
\alpha_{ij} = \frac{1}{2}\left[1 - \sum_{k\in N_i\setminus\{j\}}\alpha_{ik} + \frac{\rho}{1+\rho}\left(\sum_{k\in N_i\setminus\{j\}}\alpha_{ki} - \sum_{k\in N_j\setminus\{i\}}\alpha_{kj}\right)\right] \quad \forall ij \text{ s.t. } G_{ij}=1. \tag{8}
$$

In equation (8), the net transferred share α_{ij} of e_i from i to j is given by the half of the "remaining share" after deducting the transfers to i's other neighbors $N_i\setminus\{j\}$, corrected by an adjustment for inflows of non-local endowments. The $\frac{1}{2}$ multiplier is analogous to the equal sharing rule in the independent endowments case, but last term in the square brackets is new.[16] We refer to it as an informational effect, for the following reason: $\sum_{k\in N_i\setminus\{j\}}\alpha_{ki}$ is the sum of i's shares of i's other neighbors' endowments $\left(e_k\right)_{k\in N_i\setminus\{j\}}$, and the conditional expectation of each k's endowment changes linearly with the realization of e_i by a factor of $\frac{\rho}{1+\rho}$. Similarly, $\sum_{k\in N_j\setminus\{i\}}\alpha_{kj}$ is the sum of j's shares of j's other neighbors' endowments $\left(e_k\right)_{k\in N_j\setminus\{i\}}$, and the conditional expectation of each k's endowment also changes linearly with the realization of e_i

[15] Rigorously there should be another set of equations that verify the guess for the state-independent constant transfers μ, which in general involve both α and μ. However, Lemma 6 in Appendix A.4 implies that, given any admissible α, there exist some μ such that (α,μ) satisfies the set of verification equations for the constant transfers. Hence, system (8) (which involves only α) constitutes the essential condition for Pareto efficiency. We therefore omit the conditions on μ and delay our discussion about state-independent transfers to Section 7.2.

[16] This term disappears when $\rho = 0$.

by a factor of $\frac{\rho}{1+\rho}$. Due to the symmetric correlation structure, the realization of e_i provides the same amount of local information about all non-local endowment realizations e_k for $k \notin \overline{N}_{ij}$, and thus its informational effect can be calculated as a simple net summation of endowment shares. As a higher realized e_i predicts that both i and j are more likely to obtain higher amounts of inflows from uncommon neighbors, this commonly recognized information can be used by the pair ij to (imperfectly) share the non-local risk exposures.[17] After pooling the conditional expectations of non-local inflows, i and j again share the *remaining* shares of e_i and e_j equally. It is worth pointing out that i carries out this kind of "equal sharing" with all her neighbors, and the inflow/outflow shares (α_{ij}) must make all the sharing simultaneously equal (in expectation).

We now turn to the case of *general network structures*. In Appendix A.5, we show in Proposition 7 that a linear, strictly bilateral and Pareto efficient profile of transfer rules $t_{ij}(e_i, e_j) := \alpha_{ij}e_i - \alpha_{ji}e_j + \mu_{ij}$ correspond to the solution of a complicated system of linear equations, which is difficult to solve. Instead, we show in Proposition 8 that we may *equivalently* solve an alternative optimization problem that minimizes total consumption variances among all linear transfer rules, as defined below.

Specifically, let α be a linear profile of transfer rules in $\mathcal{T}$, and consider the following optimization problem that minimizes the sum of each individual's consumption variance under the risk-sharing arrangements defined by α:

$$\min_{\alpha} \sum_{i \in N} Var\left[\left(1 - \sum_{j \in N_i} \alpha_{ij}\right) e_i + \sum_{j \in N_i} \alpha_{ji} e_j\right]. \tag{9}$$

Let $\tilde{\Lambda}_i$ be the Lagrange multiplier associated with i's outflow constraint $\sum_{j \in \overline{N}_i} \alpha_{ij} = 1$ and denote $\Lambda_i := \frac{\tilde{\Lambda}_i}{2(1-\rho)}$. Then, taking the FOC for the Lagrangian, we have

$$\begin{cases} \alpha_{ji} = \Lambda_j - \frac{\rho}{1-\rho}\left(\alpha_{ii} + \sum_{k \in N_i} \alpha_{ki}\right) \quad \forall j \in \overline{N}_i, \forall i \in N & (10.1) \\ \sum_{j \in \overline{N}_i} \alpha_{ij} = 1 \quad \forall i \in N & (10.2) \end{cases} \tag{10}$$

This is a system of $\left(\sum_i d_i + 2n\right)$ equations in $\left(\sum_i d_i + 2n\right)$ variables (α, Λ). We now

[17]To be precise, by "inflow" we mean the undertaking of a share of someone else's income endowment, which may be positive or negative; by "outflow" we mean the distribution of a share of one's own endowment to someone else, which may also be positive or negative. In particular, a negative inflow is not the same as an outflow. Instead, i's inflow from j is the same as j's outflow to i.

characterize the set of Pareto efficient linear and bilateral transfer rules in our setting as the solution to this problem. This yields the following result:

Proposition 4. [18] *For any $\rho \in \left(-\frac{1}{n-1}, 1\right)$ and any network structure G, or for $\rho = -\frac{1}{n-1}$ and any G such that $\max_{i \in N} d_i < n-1$, there exists a unique solution to system (10) given by the following: $\forall i \in N,\ \forall j \in \overline{N}_i$,*

$$\alpha_{ji} = \Lambda_j - \frac{\rho}{1+\rho d_i} \sum_{k \in \overline{N}_i} \Lambda_k \tag{11}$$

where Λ_i is given by:

- *(Fixed point representation):*

$$\Lambda_i = \frac{1}{d_i+1}\left(1 + \sum_{j \in \overline{N}_i} \sum_{k \in \overline{N}_j} \frac{\rho}{1+\rho d_j} \Lambda_k\right) \tag{12}$$

- *(Closed-form representation): writing $\Lambda = (\Lambda_i)_{i=1}^n$*

$$\Lambda = \left(\overline{D} - \overline{G}\Psi\overline{G}\right)^{-1} \mathbf{1}$$

  where $\overline{D}$ is a diagonal matrix with its i-th diagonal entry being $d_i + 1$, Ψ is a diagonal matrix with its i-th diagonal entry being $\frac{\rho}{1+\rho d_i}$, and $\overline{G} := G + \mathbf{I}_n$.

- *(Explicit representation): For $\rho \in [0, 1)$,*

$$\Lambda_i = \frac{1}{d_i+1} + \sum_{q \in \mathbb{N}} \sum_{j \in N} \sum_{\pi_{ij} \in \Pi_{ij}^{2q}} W(\pi_{ij}) \tag{13}$$

  where $W(\pi_{ij})$, the weight of each path $\pi_{ij} = (i_0, i_1, i_2, \dots i_q)$ of length q from i to j (i.e. $i_0 = i$ and $i_q = j$), is given by,

$$W(\pi_{ij}) := \frac{1}{d_{i_0}+1} \cdot \frac{\rho}{1+\rho d_{i_1}} \cdot \frac{1}{d_{i_2}+1} \cdot \frac{\rho}{1+\rho d_{i_3}} \cdot \dots \cdot \frac{1}{d_{i_q}+1} \tag{14}$$

In particular the proof shows that the system of equations (12) has a unique

[18] See Appendix B.8 for Pareto efficient risk-sharing arrangements in the boundary cases of $\rho \in \left\{-\frac{1}{n-1}, 1\right\}$.

solution, characterized by the explicit representation. The representation reveals that the form in which the network determines the Pareto efficient linear transfer arrangements has to do with interaction at distance two (i.e. neighbors of neighbors).

Intuitively, the network interaction terms in (10.1) define substitutability across the shares going to j. This implies that individuals *at most two links apart* (i.e. with a common neighbor, i) affect each others' transfer shares directly. But indirect effects play a crucial role here as well. To see this, notice that these two households not only interact through their transfer to i, but might also exchange resources with other partners, and these other relations affect what i receives from them, given their constraints in (10.2). This is the main message behind equation (11), where these inter-dependencies along the network have been worked out, and we can express the share from j to i as a function of some constants Λ's, that accumulate all these indirect effects. These values can be shown to depend across individuals according to the recursive formulation in (12), which is reminiscent of Katz-Bonacich, Page Rank, and other global network measures, albeit with two crucial differences: 1) The centrality of i depends on the centralities not of direct neighbors, but of neighbors of neighbors (i.e. at length two), and 2) the weights are not a simple geometric series (as in the Bonacich measure), but instead depend explicitly on the degree of the direct neighbors that are linking i with all of her length-2 neighbors.

The recursive formulation in (12) admits a unique solution, for any network. This provides an alternative characterization of the centrality as the accumulation of *weighted even paths*, which exhibit the two crucial differences described above (i.e. length two and path-specific weights). However, notice that (12) sums over individuals in $\bar{N}_i$ and $\bar{N}_j$. In other words, self-loops are allowed. This implies that we are not in a situation where an individual that is, say, at distance 3 from i will not matter for i's centrality measure. On the contrary, she will in fact matter because self-loops will allow us to reach any individual that is weakly connected to i. However, the weighting scheme crucially depends on the even-length paths that we can compute, starting from i. In other words, while this measure ultimately relates individuals at all distances in the network, the specific weights between each pair of individuals require counting only the even-length paths that connect them.

This complicated weighting scheme unfortunately makes comparative statics on the network difficult to analyze. To see this notice that when a link is removed from the network a number of even-length paths disappear, lowering the total elements summing in (13). However, this also lowers the degree (or connectivity) of the two individuals involved in that link. This increases the weights associated to all even-length paths that go through each of these two individuals, as shown in equation (14). As such, it is difficult in general to know which way the centrality measure moves as links are deleted. In any case, it is often helpful to think of the general result in Proposition 4 as capturing the extent to which the sender's indirect interactions in the network cannot be accessed by any other of the receiver's partners. In Section A.8 of the Supplementary Appendix we go over a simple example with a small network of five individuals, and we show how to weight paths in order to construct the relevant network centrality measure, and how to obtain the predicted transfers α'_{ji} of Proposition 4. We also provide simulations of transfers for different values of ρ and discuss the implications of increasing the uniform correlation within the network.

4 Network Centrality and Consumption Volatility

In this section, we present one of the model's major theoretical implications, which is about the relationship between network centrality and consumption volatility in risk-sharing communities. This will lead to an empirical prediction of the model that contrasts previous models of informal insurance in networks.

We first derive in Subsection 4.1 analytical results for the correlation between network centrality and consumption volatility according to our theoretical model. For star networks, we derive closed-form formula for individual consumption variances under any endowment correlation parameter, and show that the center's final consumption is always more volatile than the peripherals'. Alternatively, assuming that endowments are independent and that networks are sampled according to an Erdos-Renyi random graph generating process, we derive exact formula for the asymptotic covariance between degree centrality and consumption variance, which turns out unambiguously positive.

Next, in Subsection 4.2 we run simulations of endowment shocks in *real-world village network* structures, constructed from two data sets provided, respectively, by Field and Pande, and by Banerjee, Chandrasekhar, Duflo and Jackson. We compute the consumption variances of individuals implied by our theoretical model with local information constraints, as well as consumption variances implied by the AMS (Ambrus, Mobius, and Szeidl, 2014) model with "link capacity constraints". We find statistically significant positive correlations between consumption variances and a few measures of network centrality, which contrasts sharply with the negative correlations we find under the AMS setting.

In Section 5, we also provide empirical evidence in support of this prediction using real consumption and network data from the Townsend Thai Project.

4.1 Analytical Results

4.1.1 Star Networks

We first provide analytical results for the positive relationship between network centrality and consumption volatility in star networks.

Let c denote the center individual, who is connected to $n-1$ peripheral individuals, and none of the peripheral individuals are connected to each other. We use p to refer to a generic peripheral individual.

It is straightforward to show that a linear risk-sharing arrangement achieving Pareto efficiency subject to local information constraints specifies the following endowment shares to be transferred:

$$\alpha_{cp} = \frac{2+2\left(n-1\right)\rho}{n\left(2+n\rho\right)}, \quad \alpha_{pc} = \frac{1+\rho}{2+n\rho}, \quad \gamma_{cp} = \frac{\left(n-2\right)\rho}{2+n\rho}.$$

It can be shown that the difference in consumption variances in efficient contracts satisfies

$$Var\left(x_c\right) - Var\left(x_p\right) = \frac{\left(n-2\right)\left(1+\left(n-1\right)\rho\right)\left(1-\rho^2\right)}{\left(2+n\rho\right)^2} \geq 0$$

with equality only at $\rho \in \left\{-\frac{1}{n-1}, 1\right\}$. In particular, $Var\left(x_c\right) - Var\left(x_p\right) \to \frac{1-\rho^2}{\rho}$ as $n \to \infty$, and hence the consumption variance of the center can be much higher than the consumption variance of a periphery individual when ρ is low and n is high.

4.1.2 Erdos-Renyi Random Graphs

We now proceed to characterize the (large-network) asymptotic relationship between network centrality and consumption volatility under the Erdos-Renyi random graph setting, which lends great tractability to the analysis.

To formalize our results, write $\mathbb{P}^{ER}, \mathbb{E}^{ER}$ as the probability measure and expectation operator with respect to the Erdos-Renyi random graph generating process

$\mathcal{G}^{ER}(n,p)$: for each $n \geq 2$ and $p \in (0,1)$, let

$$G_{ij} \equiv G_{ji} \sim_{i.i.d.} \ Bernoulli\left(p\right), \ \forall i,j \in \{1,...,n\}.$$

Fixing a sequence of $\{p_n\} \subseteq (0,1)$, we write $\mathbb{P}_n^{ER}, \mathbb{E}_n^{ER}$ for the Erdos-Renyi random graph generating process $\mathcal{G}^{ER}(n,p_n)$.

For each network structure G_n drawn from $\mathbb{P}_n^{ER}$, we write $d_i(G_n)$ as individual i's degree in G_n. We write e to denote a generic realization of the endowment vector, and take the distributions of e and G_n to be statistically independent from each other. Furthermore, we focus on the simple case with independent endowment shocks, i.e., the global correlation parameter $\rho = 0$. By previous results, we know that any Pareto efficient risk-sharing arrangements take the form of the local equal sharing rule, so that the final consumption allocation is given by

$$x_i(G_n)[e] := \sum_{j \in \overline{N}_i(G_n)} \frac{1}{d_j(G_n)+1} e_j,$$

and the individual consumption variance is given by

$$Var(x_i(G_n)) = \sum_{j \in \overline{N}_i(G_n)} \frac{1}{(d_j(G_n)+1)^2}$$

where $Var(\cdot)$ denotes the variance operator with respect to the endowment shocks e conditional on realized network structure being G_n.

Proposition 5. *Let $\{G_n\}$ be a sequence of Erdos-Renyi random graphs generated by $\mathcal{G}^{ER}(n,p_n)$. Suppose $p_n = p$ for all n. Then:*

$$\lim_{n\to\infty} nCov_n^{ER}\left[Var(x_i(G_n)), \ d_i(G_n)\right] \to \frac{1-p}{p} > 0.$$

We also derive similar asymptotic results in Proposition 11 in Appendix B.12 for the case of sparse network, where $np_n \to \lambda$ for some constant $\lambda \in (0,\infty)$.

To see the intuition behind the proof, consider the following two opposite effects of a larger degree d_i on individual i's consumption variance $Var(x_i)$. On the one hand, a larger d_i implies a smaller consumption variance coming from one's own endowment shock $\frac{1}{(d_i+1)^2}$, but on the other hand it implies exposure to endowment shocks from a greater number of neighbors, as measured by $\sum_{j\in N_i} \frac{1}{(d_j+1)^2}$. The intuition for

Proposition 5 lies in the greater importance of more degree-central individuals to the society for risk reduction under local information constraints. As risk can only be shared locally, more central individuals are exposed to risk from more *sources*, which are pooled for more effective risk reduction. Given our focus on Pareto efficiency, intuitively more central individuals should undertake a larger amount of *total risk*, effectively playing informally the role of an "quasi-insurance provider".

Below, we illustrate our analytical asymptotic results under the Erdos-Renyi random graph setting with finite-sample simulation results using real-world network structures. In the following section we investigate the empirical relationship between network centrality and consumption volatility with survey data from Thailand on consumption and social interactions in village economies.

4.2 Simulation Results

Our theoretical prediction of the positive correlation between consumption variance and degree centrality, as formalized in Proposition 5, generates a sharp comparison with the predictions of Ambrus, Mobius, and Szeidl (2014), who investigate how "link capacity constraints" affect the efficiency of risk-sharing arrangements. Under their framework, a more central individual, say, with a larger degree, tend to have "weaker" *total* capacity constraints for any given realization of endowment realization, so she is more likely to have *lower* consumption volatility. In contrast, our current model focuses on local information constraints, in which case more risk is pooled at individuals with larger degrees, giving them *higher* consumption volatility on average (asymptotically).

We illustrate the contrast between our theoretical prediction and that of AMS numerically in this subsection. Specifically, we estimate the correlations predicted by our model and by the model in AMS, between an individual's centrality and consumption variance, via simulated endowment realizations in two real-world village networks from India from two different databases, each randomly selected and provided to us by the researchers who collected the data.[19] The simulation results also

[19] The first network was provided to us by Erica Field and Rohini Pande, who collected it from villages in the districts of Thanjavur, Thiruvarur and Pudukkotai (Tamil Nadu) in India. In a subset of the villages, complete within-village network data was collected by surveying all households. The second network is from data collected by Abhijit Banerjee, Arun Chandrasekhar, Esther Duflo and Matthew Jackson in Karnataka, India (they collected complete within-village network data in 75 villages), used for example in the Banerjee, Chandrasekhar, Duflo, and Jackson (2018). From both

Table 1: Correlation between Centralities and Consumption Variances

	Capacity	(A) Field & Pande Degree	(A) Field & Pande Eigenvector	(B) BCDJ Degree	(B) BCDJ Eigenvector
AMS	0.5	-0.8943^{***}	-0.4585^{***}	-0.3552^{***}	-0.3155^{***}
	1.0	-0.6885^{***}	-0.2313^{***}	-0.2913^{***}	-0.2573^{***}
	1.5	-0.5430^{***}	-0.0898^{***}	-0.2843^{***}	-0.2511^{***}
Our Model		0.1994^{***}	0.1659^{***}	0.2084^{***}	0.0858

*** denotes statistical significance at 1%-level.

serve to illustrate whether our analytic results in Proposition 5 for *large Erdos-Renyi random graphs* hold numerically in *finite real-world* network structures.

In both simulations, we randomly drew the endowment $e_i^{(t)}$ of each household according to the standard normal distribution for $T = 5000$ times: $\left\{e_i^{(t)}\right\}_{i,t} \sim_{iid} \mathcal{N}(0,1)$. We assumed that all households have CARA utility functions with $\lambda = 1$. We then computed the final consumptions of each household under the equally-weighted Utilitarian optimal risk-sharing arrangement subject to local information constraints, using the results from subsection 4.1, and the sample variance of final consumptions for each household (note that the variance does not depend on the planner's weights). Following this we computed the sample correlation between degree/eigenvector centrality and consumption variance. Similarly, we computed the constrained efficient consumptions implied by the model in AMS, and the sample correlation between the centrality measures and consumption variance under three levels of capacity constraints (the maximum amount that can be transferred through any link, at any state): 0.5, 1 and 1.5. The results are summarized in Table 1. All results are highly statistically significant with p-values very close to zero, except for the correlation between consumption variance and eigenvalue centrality for one of the two datasets).[20]

Under the AMS model, we observe a negative correlation between centrality and consumption variance. In AMS, transfers along links are subject to capacity con-

datasets we received the network of financial connection for one randomly selected village with complete network data. From the original network we created the undirected "AND" network, that is, we defined a link between two households whenever both households indicated each other as a borrowing relationship. We excluded households that became isolated in the "AND" network.

[20]The p-values, calculated from standard t-tests against the null hypotheses of zero correlations, are at orders of magnitudes below 10^{-10}, except the case noted.

straints. As a result, centrally located households tend to have a lower consumption variance, because capacity constraints are less likely to be binding for them locally, and for typical endowment realizations they end up pooling risk with a larger set of other households.[21] This holds for all capacity values we used in the simulations, but the relationship is more highlighted for relatively stricter capacity constraints.[22]

Under our current model, we observe the opposite sign: sample correlation between both degree and eigenvector centrality on the one hand, and consumption variance on the other hand is positive (as noted above, not significantly for eigenvalue centrality when using the BCDJ data). This is consistent with the theoretical results we obtain in Proposition 5 for large Erdos-Renyi random graphs.

The sharp contrast in predictions between our model and the AMS model suggests a potential way to empirically differentiate local information constraints and link capacity constraints, or to empirically assess their relative relevance in real-world risk-sharing communities. We proceed to provide empirical evidences in favor of our current model in the next subsection.

5 Empirical Evidence: Townsend Thai Monthly Survey

In this section, we provide empirical evidences for the relevance of our theoretical model and its predictions. Specifically, we compute (eight versions of) sample standard deviations of detrended real monthly consumption per capita for the surveyed households as measures of their consumption volatility, and obtain (five versions of) network degrees of the surveyed households.[23]. We then regress consumption volatility on network degrees along with a portfolio of economically relevant control variables and village fixed effects. We find statistically significant positive correlation between consumption volatility and network degrees, which demonstrates that the analytical

[21]Using terminology from AMS, more centrally-located households typically end up on larger "risk-sharing islands."

[22]As capacities increase, centrality in the AMS model matters less, since capacity constraints are less likely to bind.

[23]The publicly released network-related data in the Townsend Thai Project are encoded, for the concern of privacy policies, in such a way that we are not able to construct the full network structure among the surveyed households. Yet the publicly released data nevertheless allow us to construct precise measures of the *network degrees* of the surveyed households, which we use for our main empirical analysis.

and simulation results obtained (in Section 4) under our theoretical model of local information constraints are robustly consistent with the empirical patterns of informal risk sharing in real-world village communities.

5.1 Data Description and Variable Construction

The Townsend Thai Monthly Survey (Townsend, 2016), initiated in 1998 as part of the Townsend Thai Project, is an ongoing survey that provides monthly household-level data on a wide range of aspects, including in particular household consumption and social interactions, for approximately 720 households from 16 villages in Thailand. The data used in this section are based on the 196 months from August 1998 (labeled as month 1) to December 2014 (labeled as month 196), which constitutes the entire sample publicly available at the writing of this draft. Below we provide a brief description about a subset of the data from Townsend Thai that are most relevant to our empirical analysis. A more comprehensive and detailed description of the survey can be found in Samphantharak and Townsend (2010), Townsend and Suwanik (2016), Pawasutipaisit et al. (2016) and also online at the official website of the Townsend Thai Project.

We focus mainly on two categories of variables in the monthly surveys: household consumption and social interactions. For each household, we use the former to construct measures of consumption volatility, and the latter to construct measures of degree centrality. We provide below a brief description of our variable construction procedures, and defer more details to Appendix A.9.

Consumption Variance Using data on household sizes and expenditures from Townsend Thai Survey and data on monthly *Consumer Price Index* from the Thailand Ministry of Commercewe, we first compute *real monthly household consumption per capita*. We then keep only those households whose consumption are recorded for at least 100 (not necessarily consecutive) out of 196 months, and compute the *detrended*[24] real consumption per capita using linear detrending of logged real consumption per capital. Finally, for each household we compute the *sample standard*

[24]Given that real consumption per capita is rising over a time frame of 196 months, detrending is required for the construction of a sensible measure of consumption volatility. This is because, without detrending, a deterministic path of real consumption per capita at a fixed positive growth rate will generate positive variance in consumption without any uncertainty.

deviation of detrended real monthly household consumption per capita across all months as a measure of this household's consumption volatility.

Using the same procedure but different variants of definitions, we generate eight measures of consumption volatility, named respectively, "*sd_x*", "*sd_x_nm*", "*sd_x_re*", "*sd_x_nm_re*", "*sd_lx*", "*sd_lx_nm*", "*sd_lx_re*" and "*sd_lx_nm_re*". The postscripts "*_x*" or "*_lx*" indicate whether the standard deviations are computed on monetary scales or log scales, respectively. The presence of the postscript "*_nm*", short for "no maintenance", indicates that the definition of consumption excludes certain extraordinary maintenance expenditures. The postscript "*_re*", short for "resident", indicates that the definition of household sizes (for the calculation of per capital consumption) only counts household members that resided in the household for at least fifteen days in a certain month.

Network Degrees Using data on social interactions from the Townsend Thai Survey, we construct five different versions of network degrees, using five definitions of network links. The first measure of network degree is constructed based on records of within-village gift (and remittance), borrowing and lending transactions from the Townsend Thai Monthly Survey. Among the many types of social and economic interactions among households recorded by the survey, "gift and remittance" as well as "borrowing" and "lending" are arguably most relevant to the purpose of informal risk sharing. The second and the third measures of network degrees we use are based on records of reported kinship or neighbor relationships. Such records are obtained from numerous records of within-village interactions from five modules of the survey: household assets, agricultural assets, gifts & remittance, borrowing, and lending. Our fourth degree measures is constructed by taking a union of the kinship and neighborhood links. The fifth degree measure is constructed by taking a union of the transaction links (gifts, remittance, borrowing or lending) and the relationship links (kinship or neighborhood).

Control Variables Clearly, given the complexity of real-world household heterogeneities, a multitude of other factors may be related to the risk exposure and the consumption volatility of the surveyed households. To account for such heterogeneities subject to data availability, we also construct control variables that are reasonable related to risk preference, risk behavior or risk sources pertaining to each household using data from the Townsend Thai Monthly Survey. Specifically, we construct a series of binary and real-valued variables based on monthly information on the households' real saving balances, use of institutional finance (borrowing from commercial banks, BAAG, PCG, Rice Bank, Agricultural Co-operation or other institutions), use of personal finance (borrowing from money lenders, store owners, input suppliers, relatives or friends), health insurance payment, life insurance payment and ROSCA payment. We also compute sample deviations of linearly detrended log real *income* per capita based on income statement section of the Townsend Thai Monthly Survey Household Financial Accounting dataset Townsend (2017) from month 1-172. All these constructed variables, along with village-level fixed effects, are included in the regression analysis as control variables.

Summary Statistics With the three types of variables, i.e., the consumption variables, the network variables and the control variables, constructed above, we obtain a merged data set and focus on the households for whom all three types of variables are not missing.This effectively leaves us with a sample size of around 600 households. We now present some summary statistics for this subset of households in Table 2.

Table 2: Summary Statistics

variable	*obs.*	*mean*	*s.d.*	*min*	*max*
(a) Monthly real consumption per capita					
x	132,997	1625	4214	0	773,411
x_re	132,997	2497	7059	0	1,160,116
x_nm	132,997	1444	2408	0	181,988
x_nm_re	132,997	2226	4606	0	444,195
(b) Household consumption volatility					
sd_x	607	2425	3520	98	55287
sd_x_re	607	4142	5922	173	84839
sd_x_nm	607	1661	1618	98	12854
sd_x_nm_re	607	3031	3504	171	32259
sd_lx	607	0.483	0.124	0.206	0.911
sd_lx_re	607	0.471	0.118	0.198	0.945
sd_lx_nm	607	0.431	0.114	0.178	0.898
sd_lx_nm_re	607	0.420	0.105	0.177	0.813
# months	607	189.115	18.420	100	196
(c) Household network degrees					
GRBL	599	3.770	2.956	1	24
K	607	2.129	1.870	0	10
N	607	3.684	2.980	0	22
KN	607	3.807	3.029	0	22
GRBLKN	607	4.008	3.088	1	24

5.2 Main Empirical Results

We now present our main empirical results on the positive correlation between degree centrality and consumption variance.

Table 3 reports pairwise sample correlation coefficients between (i) household degrees according to five versions of network definitions and (ii) standard deviations of six versions of detrended real monthly household consumption per capita or its logged forms. Clearly, all sample correlation coefficients are positive and statistically significant.

Table 4 reports results from fives sets of regressions of *sample standard deviation of detrended real consumption per capita* on *own network degree, average counterparty degree,* along with a list of control variables for risk heterogeneity and village-level fixed effects. All five sets of regressions use the same four representative dependent variables, "*sd_x*", "*sd_x_nm_re*", "*sd_lx*", "*sd_lx_nm_re*". Clearly, most estimated coefficients on *network degrees* are positive and statistically significant, after controlling for variations in average counterparty degree, village fixed effects and a list of household-level control variables.

Table 5 reports results from the same five sets of regressions as those run in Table 4, except that regressions in Table 5 include standard deviations of detrended real income per capita as an additional control variable. Specifically, all regressions with *regular-scale* consumption volatility measures ("*sd_x*", "*sd_x_nm_re*") as dependent variables add "*sd_y*" as an additional right-hand-side variable, while those with *log-scale* consumption volatility measures ("*sd_lx*", "*sd_lx_nm_re*") add "*sd_ly*" instead. Again, most estimated coefficients on *network degrees* remain positive and statistically significant.

It is noticeable that the coefficients in regressions of the dependent variable "*sd_x*", though remain positive, become statistically insignificant. This, however, is not particularly surprising, as the construction of variable "*sd_x*" involves very occasional extraordinary (*maintenance*) expenditures, which we find, if not expressed in log scale, blows up the variance, skewness and kurtosis of the whole sample remarkably as discussed earlier.

In summary, Table 3, Table 4 and Table 5 demonstrate remarkably robust consistency between the theoretical results obtained previously under our model and the empirical patterns of informal risk sharing in real-world village communities.

Table 3: Pairwise Correlation Coefficients

	s.d. of detrended real consumption per capita							
	sd_x	*sd_x_re*	*sd_x_nm*	*sd_x_nm_re*	*sd_lx*	*sd_lx_re*	*sd_lx_nm*	*sd_lx_nm_re*
Network degrees								
(a) GRBL	0.1816	0.1793	0.3170	0.2743	0.2496	0.2563	0.2359	0.2518
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
(b) K	0.1018	0.1114	0.1287	0.1540	0.1449	0.1318	0.1431	0.1377
	(0.0121)	(0.0060)	(0.0015)	(0.0001)	(0.0003)	(0.0011)	(0.0004)	(0.0007)
(c) N	0.1740	0.1655	0.2873	0.2395	0.2308	0.2339	0.2235	0.2351
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
(d) KN	0.1707	0.1655	0.2886	0.2428	0.2327	0.2352	0.2270	0.2383
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
(e) GRBLKN	0.1742	0.1668	0.2919	0.2461	0.2386	0.2452	0.2259	0.2414
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Note: P-values are reported in parentheses under corresponding correlation coefficients. The numbers of observations (households) are 599 for row (a), and 607 for rows (b)-(e).

Table 4: Regression with control variables and village fixed effects

	coefficient	*robust s.e.*	*p-value*	*obs.*	R^2
(a) GRBL network					
sd_x	108.229	58.667	0.066	582	0.1993
sd_x_nm_re	288.345	55.886	0.000	582	0.1589
sd_lx	0.0083	0.0018	0.000	582	0.2091
sd_lx_nm_re	0.0070	0.0013	0.000	582	0.1998
(b) K network					
sd_x	58.106	84.309	0.491	590	0.1897
sd_x_nm_re	229.256	98.458	0.020	590	0.1221
sd_lx	0.0066	0.0031	0.035	590	0.1875
sd_lx_nm_re	0.0044	0.0024	0.068	590	0.1715
(c) N network					
sd_x	98.247	56.015	0.080	590	0.1965
sd_x_nm_re	246.268	53.198	0.000	590	0.1475
sd_lx	0.0070	0.0019	0.000	590	0.2081
sd_lx_nm_re	0.0058	0.0014	0.000	590	0.1960
(d) KN network					
sd_x	98.594	54.336	0.070	590	0.1969
sd_x_nm_re	247.487	53.603	0.000	590	0.1493
sd_lx	0.0070	0.0018	0.000	590	0.2091
sd_lx_nm_re	0.0058	0.0014	0.000	590	0.1972
(e) GRBLKN network					
sd_x	106.736	51.762	0.040	590	0.1988
sd_x_nm_re	246.632	50.432	0.000	590	0.1510
sd_lx	0.0073	0.0018	0.000	590	0.2124
sd_lx_nm_re	0.0060	0.0013	0.000	590	0.1996

Note: Some households are dropped in the regressions due to missing values in control variables, reducing the sample sizes to 582 for (a) and 590 for (b)-(e), respectively.

Table 5: Regression with income volatility, control variables and village fixed effects

	coefficient	*robust s.e.*	*p-value*	*obs.*	R^2
(a) GRBL network					
sd_x	84.261	56.930	0.139	582	0.2388
sd_x_nm_re	234.494	54.500	0.000	582	0.2315
sd_lx	0.0080	0.0019	0.000	582	0.2241
sd_lx_nm_re	0.0069	0.0014	0.000	582	0.2017
(b) K network					
sd_x	49.292	84.740	0.561	590	0.2326
sd_x_nm_re	214.264	99.047	0.031	590	0.2046
sd_lx	0.0061	0.0032	0.058	590	0.2070
sd_lx_nm_re	0.0043	0.0025	0.084	590	0.1751
(c) N network					
sd_x	73.989	54.177	0.173	590	0.2364
sd_x_nm_re	206.068	50.615	0.000	590	0.2210
sd_lx	0.0066	0.0019	0.001	590	0.2238
sd_lx_nm_re	0.0057	0.0014	0.000	590	0.1979
(d) KN network					
sd_x	76.515	52.534	0.146	590	0.2369
sd_x_nm_re	210.719	51.505	0.000	590	0.2232
sd_lx	0.0067	0.0018	0.000	590	0.2247
sd_lx_nm_re	0.0057	0.0014	0.000	590	0.1990
(e) GRBLKN network					
sd_x	84.885	50.136	0.091	590	0.2383
sd_x_nm_re	210.083	48.735	0.000	590	0.2245
sd_lx	0.0070	0.0018	0.000	590	0.2280
sd_lx_nm_re	0.0059	0.0013	0.000	590	0.2015

Note: Some households are dropped in the regressions due to missing values in control variables, reducing the sample sizes to 582 for (a) and 590 for (b)-(e), respectively.

6 Implications of the Theory for Empirical Tests of Risk Sharing

The performance of risk-sharing communities has been repeatedly tested in data since the work of Cochrane (1991), Mace (1991) and Townsend (1994). Their original approach developed empirical tests of full insurance that related household consumption and income. Indeed, the well known Borch rule – equating the ratio of marginal utilities across households – imposes that, under full insurance, household consumption should not respond to idiosyncratic movements in income after controlling for aggregate shocks. This implication can be tested in the following popular regression:

$$log(c_{it}) = \alpha_i + \beta_1 log(y_{it}) + \beta_2 log(\bar{y}_t) + \epsilon_{it} \tag{15}$$

where c_{it} and y_{it} correspond to household $i's$ consumption and income at time t, and where $\bar{y}_t = \sum_i y_{it}$ represents aggregate village income at time t.[25] Full insurance implies that $\beta_1 = 0$ and $\beta_2 = 1$. An overwhelming proportion of studies have rejected the full-insurance hypothesis in a wide number of settings. As a result, a great deal of work has followed, that seeks to explain this stylized fact.

On the theory side, we have argued that this paper complements an ongoing effort to model the relevant contracting frictions in informal risk sharing environments.[26] In this section we argue that our framework also responds to a recent strand of the literature that suggests modifying the classical Townsend test in order to accommodate various forms of heterogeneity. Some of this work argues that the standard consumption regression in (15) is misspecified if, for instance, households hold heterogeneous risk preferences.[27] More relevant to the current discussion, several other studies have also suggested that households within a village indeed access different risk sharing groups, and that controlling for aggregate-level shocks, as in (15), would incorrectly estimate income coefficients: $\bar{y}$ should be group-specific. In a couple well-known examples, Mazzocco and Saini (2012) argue that the relevant sharing group in India is the caste (rather than the village), while Attanasio, Meghir, and Mommaerts (2018) test for efficient insurance within extended families in the U.S.[28]

This paper refines and generalizes the modified tests that evaluate the performance of insurance mechanisms on local sharing groups. Rather than taking groups as separate, perfectly insured communities, the current framework allows for a fully

[25]Village-time fixed effects are traditionally used to capture aggregate shocks at the village level.

[26]For example Thomas and Worrall (1990), Kocherlakota (1996), Ambrus, Mobius, and Szeidl (2014), and Kinnan (2017).

[27]See for instance Mazzocco and Saini (2012) and Schulhofer-Wohl (2011).

[28]In similar procedures Hayashi, Altonji, and Kotlikoff (1996) consider whether extended families can be viewed as collective units sharing risk efficiently. Munshi and Rosenzweig (2016) also find that the caste is the relevant group to explain migration patterns in rural India. Most relevant here, Fafchamps and Lund (2003) address the failure of efficient insurance in the data suggesting that households receive transfers not at the village level, but from a network of family and friends

general social structure with interconnected sharing groups that are specific to each household, and which may overlap in complicated ways along any given network. We show how, under the local information constraints of our model, not defining the relevant local sharing group biases the estimates of risk-sharing tests. More importantly, we show that controlling for this bias will not eliminate the correlation between household consumption and income: the structure of the network, coupled with the information constraints, induces imperfect risk-sharing and generates heterogeneity in sharing behavior. The current framework therefore allows us to decompose the standard Townsend coefficient β_1 into an underlying distribution of household-specific coefficients that capture the varying risk-sharing possibilities induced by the network structure, and which can be interpreted economically in terms of consumption volatility (as shown in the previous section).

To fix ideas, consider the simple network with three individuals and independent endowments in section 2 and set $\lambda_i = 1$; all arguments below can be extended to general networks, correlated endowments, and any profile of Pareto weights. If we write down final consumption for each household in the form of the classical risk-sharing specification of equation (15), we have that,

$$\begin{cases} c_{1t} = \alpha_1 + \left(\frac{1}{3} - \frac{1}{2}\right) y_{1t} + \frac{1}{2}\bar{y}_t + \epsilon_{1t}, \\ c_{2t} = \alpha_2 + \left(\frac{1}{2} - \frac{1}{3}\right) y_{2t} + \frac{1}{3}\bar{y}_t + \left(\epsilon_{2t} - \frac{1}{3}y_{3t}\right), \\ c_{3t} = \alpha_3 + \left(\frac{1}{2} - \frac{1}{3}\right) y_{3t} + \frac{1}{3}\bar{y}_t + \left(\epsilon_{3t} - \frac{1}{3}y_{2t}\right), \end{cases}$$

where $\alpha_1 = \frac{1}{12}r\sigma^2$ and $\alpha_2 = \alpha_3 = \frac{1}{24}r\sigma^2$ correspond to state-independent transfers and are represented as household-specific intercepts. These equations reflect three important themes of this paper as they relate to empirical tests of risk-sharing: 1) coefficients on own income are generically different from zero for all households. i.e. $\alpha_{ii} \neq \alpha_{ij}$, 2) these coefficients vary according to households' network position, and 3) imposing the common sharing group on all households generates biased estimates: notice the last two equations contain weighted incomes in the error term. The classical risk sharing test in (15) pools these equations and obtains a unique estimate for β_1; given the previous discussion we expect this estimate to be biased, different from zero, and positive.

In order to obtain unbiased estimates for β_1, consider estimating (15) with the relevant local sharing group instead. In this case, we show coefficients are properly

Table 6: Simulated Risk-Sharing Test under the Model for Two Simple Economies

	Dependent variable: Consumption			
	Star Network		*Circle Network*	
	Common Group	Local Group	Common Group	Local Group
	(1)	(2)	(3)	(4)
Income	0.201	0.027	0.121	0.001
	(0.002)	(0.002)	(0.001)	(0.001)
Agg. Income	0.780	0.977	0.845	0.998
	(0.001)	(0.002)	(0.002)	(0.001)
Observations	300,000	300,000	400,000	400,000
R^2	0.529	0.692	0.477	0.654

Note: Income data simulated from log-normal distribution with $\sigma^2 = 4$ and $t = 100,000$.
Model estimated on logged data.
Values in parentheses are standard errors.

estimated, but we still obtain heterogeneous estimates, β_i, for the coefficients on own income. As a result, the risk sharing test still delivers positive estimates – not surprisingly, since risk sharing is not efficient under information constraints. To see this, rewrite again our consumption equations in the form of (15), but now allow for household-specific aggregates, $\bar{y}_{it} = \sum_{j \in N_i} y_{jt}$, that sum over the incomes of $i's$ sharing partners. In this case we have,

$$\begin{cases} c_{1t} = \alpha_1 + \left(\frac{1}{3} - \frac{1}{2}\right) y_{1t} + \frac{1}{2}\bar{y}_{1t} + \epsilon_{1t}, \\ c_{2t} = \alpha_2 + \left(\frac{1}{2} - \frac{1}{3}\right) y_{2t} + \frac{1}{3}\bar{y}_{2t} + \epsilon_{2t}, \\ c_{3t} = \alpha_3 + \left(\frac{1}{2} - \frac{1}{3}\right) y_{3t} + \frac{1}{3}\bar{y}_{3t} + \epsilon_{3t}, \end{cases}$$

Because aggregate income terms are now household-specific (i.e. $\bar{y}_i$), the additional terms in the error disappear and we obtain unbiased estimators. Notice, however, that coefficients to own income are different from zero so long as $\alpha_{ii} \neq \alpha_{ij}$. This implies that the pooled regression will again deliver positive coefficient for β_1, even with the appropriate local aggregates. In this context, the pooled estimate in fact represents the average of the underlying heterogeneity in risk-sharing possibilities across households, which respond to network effects and relate to consumption volatility as specified by the theoretical results above.

Finally, notice that under sufficiently symmetric structures, we cannot reject this localized version of the Townsend test, because in "regular" networks $\alpha_{ii} - \alpha_{ij} = 0$.

This means we are able to generalize the discussion on appropriate local aggregates in Townsend regressions – the theory is sufficiently rich to accommodate previous models of local sharing groups, as well as many other local structures. In fact, a well-defined local version of the Townsend test may fail to reject full insurance not only if castes or extended families are perfectly connected partitions (as stressed in the previous literature), but also if the social structure is sufficiently symmetric. As an extreme example, consider the circle network in which all individuals are identically positioned. Although all local sharing groups overlap and none of them are perfectly connected, this network structure would nonetheless generate sufficient regularity to "pass" an appropriately defined version of the risk-sharing test.

The previous discussion can be observed compactly in Table 6, where the risk-sharing test is performed on simulated income data for the three individual "star" network discussed above, and the four individual "circle" network that exhibits perfect symmetry. The test is performed both with a common aggregate income term (columns 1 and 3) and with appropriately defined local sharing groups (columns 2 and 4). Notice that coefficients on own income are biased upwards by a whole order of magnitude when imposing a common aggregate income term but remain positive and significant in the star network, where the lack of symmetry keeps the pooled coefficient estimate away from zero. However, as discussed above, the circle network "passes" the Townsend test (coefficient to income is not significant) under appropriately specified local aggregate income terms.

7 Extensions

7.1 Sptial Correlation Structure

In the previous section we considered a symmetric correlation structure, in which the correlation between the endowments of two individuals did not depend on their positions on the network. An alternative specification, however, is to incorporate the possibility of spatially correlated endowments, that is correlation that decays with social distance.[29] As we illustrate below (and in more details in Appendix B.9), this type of correlation structure can be detrimental to the efficiency of informal risk

[29]There are many reasons why this correlation structure is more realistic for certain types of endowment shocks: for example, as shown in Fafchamps and Gubert (2007) and in Conley and Udry (2010), social distance tends to be highly correlated with geographic proximity.

sharing with local information constraints.

For concreteness, take the same environment as in Section 3.3 (identical CARA utilities and jointly normally distributed endowments), but assume that the correlation between e_i and e_j geometrically decays with the social distance between i and j : $Corr\left(e_i, e_j\right) = \varrho^{dist(i,j)}$, where the social distance $dist\left(i, j\right)$ is formally defined as the length (i.e., the number of links) of the shortest path connecting i and j in network G. Also, for analytical simplicity we focus on circle networks with $n = 2m + 1$ individuals. In order to make comparable the risk-sharing efficiencies under geometrically decaying spatial correlation structure with that under the uniform global correlation structure analyzed in Section 3.3, we control the "shareable risk" to be the same across the two specifications by setting $\rho = \rho_m\left(\varrho\right) := \frac{\varrho(1-\varrho^m)}{m(1-\varrho)}$, where ρ is the uniform global pairwise correlation, while ϱ is the rate of decay in the geometrically decaying correlation structure. Then informal risk sharing subject to the local information constraint achieves drastically different levels of asymptotic efficiency under the two correlation structures.

Proposition 6. *Let $x_i^{unif}\left(\rho\right), x_i^{geo}\left(\varrho\right)$ denote the Pareto efficient consumption plan subject to the local information constraint under the uniform and the geometrically decaying correlation structures, parametrized by ρ and ϱ respectively, and let $Var_{unif,\rho}$, $Var_{geo,\rho}$ correspond to the variance operators under the two probability distributions induced by the two correlation structures. Then:*

$$\lim_{\varrho\to 1}\lim_{m\to\infty} Var_{unif,\rho_m(\varrho)}\left(x_i^{unif}\left(\rho_m\left(\varrho\right)\right)\right) = \frac{1}{3},$$

$$\lim_{\varrho\to 1}\lim_{m\to\infty} Var_{geo,\rho}\left(x_i^{geo}\left(\varrho\right)\right) = 1.$$

Hence, for ϱ close to 1 and sufficiently large m, uniform correlation leads to significant risk sharing (yielding payoffs close to that under independent endowments), while geometrically decaying correlation yields payoffs very close to the autarky payoffs, even though the two correlation structures lead to the same payoffs if global information can be used for risk sharing.

This difference in risk-sharing efficiency, driven by the difference in underlying correlation structures, is a peculiar feature of the local information constraint considered in this paper. With global information, a geometrically decaying correlation structure does not in itself imply risk-sharing inefficiency relative to the uniform correlation structure. For example, in a large ring network considered above, shocks that are spatially far away from each other are almost independent, and each given individual is spatially far away from most of the individuals in the network. Hence, under global information mostly shocks with low correlations are pooled together, thus yielding significant risk reduction. However, with local information, only spatially close shocks are pooled, rendering risk sharing virtually ineffective due to the high local correlation.

This might help explain why it is the case that while in most settings empirical research found that informal insurance works well, Kazianga and Udry (2006) found a setting in which informal insurance does not seem to help, and Goldstein, de Janvry, and Sadoulet (2001) found that certain types of endowment shocks are not well insured through informal risk sharing. In particular, this may be due to high correlation between endowments of neighboring households in the above settings, for the types of endowment shocks investigated.

7.2 Endogenous Network Formation

So far our analysis focused on characterizing Pareto efficient risk-sharing arrangements subject to local information constraints on an exogenously given network, implicitly assuming that the network structure is mainly shaped by predetermined factors such as kinship. Here we briefly discuss some implications of allowing for endogenous link formation in the context of informal risk sharing with local information constraints, in the CARA-normal environment of Section 3.3. The approach we take is similar as in Ambrus et al. 2017, who consider network formation in a risk-sharing framework with global information contracts, and propose a two-stage game in which in the first stage individuals can simultaneously indicate other individuals they want to link with. If two individuals each indicated each other, the link is formed, and the two connecting individuals each incur a cost of $c \geq 0$.[30] The solution concept we use is pairwise stability. In the second stage, whatever network is formed in the first stage, it is assumed that individuals agree on a Pareto efficient risk-sharing arrangement subject to local information constraints.

In our analysis of the CARA-normal framework so far, state independent transfers played a very limited role. However, when we allow for endogenous network formation,

[30] This simple game of network formation was originally considered in Myerson (1991). See also Jackson and Wolinsky (1996).

it becomes crucial how the network structure influences state independent transfers, and hence the distribution of surplus created by risk sharing, as it directly affects incentives to form links. Therefore, it is important to specify exactly which Pareto efficient risk-sharing arrangement prevails for each possible network that can form. Different ways of specifying state-independent transfers can lead to very different conclusions regarding network formation, as we demonstrate below.

A benchmark case is when all state-independent transfers are set to 0, which case is extensively investigated by Gao and Moon (2016) who assume local equal sharing with no state-independent transfers as an ad hoc sharing rule. They show that, even with zero cost of linking, an individual i's benefit for establishing an extra link with j falls very fast with the existing number of links the individual i has, as with more existing neighbors (larger d_i) the marginal reduction in self-endowment exposure $\left(\frac{1}{d_i+1} - \frac{1}{d_i+2}\right)$ is small relative to the additional exposure to j's endowment $\frac{1}{d_j+2}$. Typically this implies severe underinvestment into social links.

An alternative approach is pursued by Ambrus et al. (2017), in the context of risk-sharing arrangements with global information: they assume that the profile of state-independent transfers is determined according to the Myerson value. The Myerson value, proposed in Myerson (1980), is a network-specific version of the Shapley value that allocates surplus according to average incremental contribution of individuals to total social surplus.[31] In particular, Ambrus et al. (2017) show that with state-independent transfers specified as above (for whatever network is formed), if individuals are ex ante symmetric then there is never underinvestment, that is given any stable network, there is no potential link that is not established, even though its net social value would be strictly positive. Below we show that the same conclusion holds in our setting with local information constraints, in the case of CARA utilities and independently an jointly normally distributed endowments. The detailed specification and the proof are available in Appendix B.11.

Proposition 7. *Suppose that, for any given network structure, the Pareto efficient consumption plan subject to the local information constraint is implemented, and the state-independent transfers are induced by the Myerson values. Consider the first-stage network formation game in which each individual pays a private cost of c for*

[31] Ambrus et al. (2017) also provide micro-foundations, in the form of a decentralized bargaining procedure between neighboring individuals that leads to state independent transfers achieving the Myerson value allocation.

each of her established links. Then, there is no underinvestment in social links in any pairwise stable network.

We leave a more detailed investigation of network formation in the context of risk sharing with local information constraints to future research.

7.3 General Contractibility Constraints

So far we have focused on a particular specification of contractibility constraints as formalized in Subsection 3.1. In particular, we have taken the *informational network* to coincide exactly with the *physical (transfer) network*, and moreover interpret the *informational network* as encoding a collection of contractibility constraints induced by ex post common observability of endowment realizations.

However, our methods and results carry over to environments with more general forms of contractibility constraints. From another perspective, our model, particularly the local information constraints we define, admits alternative interpretations as "reduced-form" representations of more general forms of contractibility constraints.

As before, let G denote a generic undirected and unweighted network structure defined on N. We now interpret G as *physical (transfer) network*: two individuals i and j can enter into a risk-sharing transfer contract if and only if they are linked in G, or $G_{ij} = 1$. As before, N_i and N_{ij} denote i's neighborhood and ij's common neighborhood under G, respectively.

We now specify a more general form of contractibility constraints. Suppose now that, for each linked pair of individuals ij in G, their bilateral transfer contract t_{ij} can be (effectively) contingent on the ex post realizations of the income shocks of individuals in some predetermined set $Q_{ij} \subseteq N \setminus \{i, j\}$, in addition to their own income shocks e_i and e_j. In other words, t_{ij} can be contingent on the ex post realizations of e_k for all $k \in \overline{Q}_{ij} := \{i, j\} \cup Q_{ij}$. We write Q (and equivalently $\overline{Q}$) to denote the joint requirements of pairwise contractibility constraints Q_{ij} for all linked individuals in G.[32]

Clearly, by taking $Q_{ij} = N_{ij}$ for all linked ij, we reduce the model back to the special case of local information constraints as formalized in Subsection 3.1. By taking

[32]Alternatively, we could specify Q without reference to G. However, as we take both G and Q as primitives, this expositional difference is inconsequential.

$Q_{ij} = N\backslash\{i,j\}$ for all ij, we reduce the model back to the simple "global-information" benchmark.

It should be emphasized that Q should be interpreted as the "reduced-form" representations of all effective contractibility constraints, i.e., Q_{ij} completely encode all relevant and effective constraints on ex post information that individuals i and j ex ante anticipate their bilateral net transfer t_{ij} to be contingent on. For example, primitive features of the environment that may be relevant to contractibility considerations include the extent of observability, the technology of communication, the opportunities for ex post strategic interactions that may effectively implement truthful information transmission in equilibrium. Moreover, the net bilateral transfer rule t_{ij} may not take the form of a complete ex ante contract, but may instead incorporate the mutual understanding of all relevant ex post strategic interactions and their expected equilibrium payoffs from those games.

We do not seek to provide a complete construction and analysis of the vast possibilities of ex post interactions, which are largely separable from the central problem considered in this paper. For illustration purpose, in Appendix B.13 we present a more rigorous formulation and analysis of one potential specification of ex post interactions that facilitate (ex post) on-equilibrium information transmission beyond a primitive individually observable information structure. In this subsection, however, we take Q as the primitive, and discuss how our methods and results can be adapted to accommodate the contractibility constraints encoded by Q.

We first discuss the results that generalize for an arbitrary Q. Clearly, under general contracting constraints encoded by Q, the social planner's problem 3 remains a convex optimization problem: the objective function remains concave, while the choice space (space of admissible transfer arrangements under G and Q) remains a convex set.

Corollary 2. *Propositions 1 and 2 carry over with proper notational adaptions.*

Consequently Corollary 1 (the localized Borch rule) remains valid, too.

We then again specialize to the CARA-normal setting as considered in Subsection 3.3. We first provide a sufficient condition under which Propositions 3 and 4 generalize almost exactly.

Proposition 8. *Suppose that G is connected as before and that the following hold simultaneously:*

(a) There exists an undirected and unweighted supergraph of G, denoted $G^{'}$, such that the contractibility constraints Q satisfies that $Q_{ij} = N_i^{'} \cap N_j^{'}$ for all linked ij in the original network G, where $N_i^{'}$ denotes i's neighborhood in the supergraph network $G^{'}$.

(b) For every pair ij linked in $G^{'}$, there exists a path in G from i to j such that, for any individual k that lies on this path, we have that jk are also linked in $G^{'}$.

Then the constrained Pareto efficient consumption plan under (G, Q) is given by the consumption plan induced by the linear transfer rules $t^* \left(G^{'}\right)$, or equivalently the transfer shares $\alpha^* \left(G^{'}\right)$, as defined in Propositions 3 and 4.

Condition (a) essentially requires that all contractibility constraints are induced by common neighborhoods under an "informational network" $G^{'}$ that is a supergraph of the physical transfer network G. Condition (b) essentially requires that the physical transfer network G is rich enough to channel, potentially via a path of individuals in G, any net bilateral transfer scheme between two informationally linked individuals in $G^{'}$. Simple examples of $G^{'}$ that satisfies condition (b) includes a supergraph of G which add (informational) links between some distance-2 pairs of individuals in the physical network G, and a supergraph of G which add links between all individuals within a distance of k from each other in the physical network G.

Under conditions $(a)(b)$, only the "informational network" $G^{'}$ is relevant in determining the constrained Pareto efficient consumption plan, or equivalently the risk sharing transfer arrangements up to superfluous cyclical transfers, which can be computed by exactly the same formulas given by Propositions 3 and 4 with the informational network $G^{'}$ as the relevant network structure.

In Appendix A.11, we now provide some illustrative examples for the constrained Pareto efficient transfers under failures of condition (a) or (b). In particular, we show that the "localized Borch rule" (Corollary 1) remains useful (by Corollary 2), and the *local equal sharing rule*, with a potential adaption to accommodate informational directedness, still captures the essential feature of Pareto efficiency.

8 Conclusion

This paper analyzes informal risk sharing arrangements under local information constraints, when bilateral transfers can only depend on endowment realizations of a subset of individuals. We characterize the Pareto efficient consumption allocations in this setting, and provide closed-form descriptions of the bilateral transfer arrangements that lead to them in a widely studied context of CARA utilities and jointly normally distributed endowments. We show that more central individuals have more volatile consumption and we test this implication using data from rural villages in Thailand. This model generalizes the notion of a local sharing group that has been invoked recently in the risk-sharing tests performed in the development literature.

The model provides numerous further implications for empirical work. In a first approach, Milán et al. (2018) show that the current framework fits the observed sharing behavior of indigenous communities in the Bolivian Amazon. However, further empirical work is needed to distinguish local information constraints from other similar contractual frictions, such as the hidden income model identified by Kinnan (2017) as the relevant friction in Thai data. Indeed, in future work we plan to derive a dynamic version of the model that provides testable predictions between current consumption and past information, which can be compared to those of other proposed risk-sharing frictions. Another empirical project to follow from this work takes the model's predictions on bilateral exchanges in order to develop a complete model of spillover effects across individuals that can be used to structurally estimate the underlying network structure following techniques in Manresa (2016) and most recently in De Paula, Rasul, and Souza (2018) .

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Supplement to “Informal Risk Sharing with Local Information”

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A Main Proofs and Supporting Materials

The proofs for all the lemmas stated in this section are available in Appendix B.

Define $J(t) := \mathbb{E}\left[\sum_{k\in N} \lambda_k u_k \left(e_k - \sum_{h\in N_k} t_{kh}\right)\right]$, the objective function in equation (3).

Lemma 1. *$\mathcal{T}$ with $\langle\cdot,\cdot\rangle$ forms an inner product space.*

Lemma 2. *J is concave on $\mathcal{T}$.*

Lemma 3. *J is Gâteaux-differentiable.*

Lemma 4. *For any $t \in \mathcal{T}$ that solves (4), we have $J'(t) = \mathbf{0}$.*

Lemma 5. *The set of consumption plan induced by the profiles of transfer rules t in $\mathcal{T}$ is convex.*

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A.1 Proof of Proposition 1

Proof. We first prove the "only if" part. Note that, given any $t \in \mathcal{T}^*$, $\forall i, j$,

$$\mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k-\sum_{h\in N_k}t_{kh}\right)\right]=\mathbb{E}\left[\mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k-\sum_{h\in N_k}t_{kh}\right)\middle| I_{ij}\right]\right]$$
$$\leq \mathbb{E}\left[\max_{t_{ij}\in\mathbb{R}}\mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k-\sum_{h\in N_k}t_{kh}\right)\middle| I_{ij}\right]\right]$$

This is because, conditional on I_{ij}, t_{ij} must be constant across all possible states, and thus the maximization of the conditional expectation is to solve for the optimal real number t_{ij}. For t to be a solution for problem (3), suppose there exists linked ij such that t_{ij} does not solve the problem (4). Then, by the inequality above, there exists another t_{ij}, specified for each different realization of I_{ij} and hence each possible state of nature, that leads to higher value of $\mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k-\sum_{h\in N_k}t_{kh}\right)\right]$, contradicting the optimality of t for problem (3). Note that the "$\mathbb{P}$-almost-all" quantifier applies here.

For the "if" part, notice that by Lemma 4, t solves all (4) simultaneously implies that $J^{'}(t) = \mathbf{0}$. As $J : \mathcal{T} \to \mathbb{R}$ is concave by Lemma 2 and Gâteaux-differentiable by Lemma 3, we can apply a mathematical result on convex optimization in normed space, specifically Theorem 3.24 and Proposition 3.20 in Peypouquet (2015), to conclude that asserting that if $J^{'}(t) = \mathbf{0}$, then $J(t)$ is the unique global maximum. ∎

A.2 Proof of Proposition 2

Proof. Following the proof of Lemma 2, we can easily show, by the strict concavity of $u_i(\cdot)$, that the objective function in (3) is strictly concave in the consumption plan x. Lemma 5 shows that the set of admissible consumption plan induced by the set of transfer rules in $\mathcal{T}$ is convex. Hence, there is at most of one consumption plan that solves (3) . ∎

A.3 Proof of Corollary 1

Proof. By the concavity (shown in Lemma 2) of the objective function in (4), the FOC is both sufficient and necessary for maximization. The FOC w.r.t t_{ij}, is

$$\mathbb{E}\left[\lambda_i u_i^{'}\left(e_i - \sum_{h\in N_i} t_{ih}\left(e\right)\right) + \lambda_j u_j^{'}\left(e_j - \sum_{h\in N_j} t_{jh}\left(e\right)\right)\cdot(-1)\middle| I_{ij}\right] = 0$$

Rearranging the above we have

$$\frac{\mathbb{E}_{ij}\left[u_i^{'}\left(x_i^t\right)\right]}{\mathbb{E}_{ij}\left[u_j^{'}\left(x_j^t\right)\right]} = \frac{\mathbb{E}\left[u_i^{'}\left(e_i - \sum_{h\in N_i} t_{ih}\left(e\right)\right)\middle| I_{ij}\right]}{\mathbb{E}\left[u_j^{'}\left(e_j - \sum_{h\in N_j} t_{jh}\left(e\right)\right)\middle| I_{ij}\right]} = \frac{\lambda_j}{\lambda_i}.$$

∎

A.4 Proof of Proposition 3

Proof. Let x_i^* be the consumption plan induced by the transfer t^* described above. Then

$$\begin{aligned}
CE\left(x_i^*|I_{ij}\right) &= \mathbb{E}_{ij}\left[e_i - \sum_{k\in N_i} t_{ik}^*\right] - \frac{1}{2}rVar_{ij}\left[e_i - \sum_{k\in N_i} t_{ik}^*\right] \\
&= e_i - \frac{e_i}{d_i+1} + \frac{e_j}{d_j+1} - \mu_{ij}^* - \sum_{k\in N_{ij}}\left(\frac{e_i}{d_i+1} - \frac{e_k}{d_k+1} + \mu_{ik}^*\right) \\
&\quad - \sum_{k\in N_i\backslash\overline{N}_j}\left(\frac{e_i}{d_i+1} - \frac{\mathbb{E}_{ij}\left[e_k\right]}{d_k+1} + \mu_{ik}^*\right) - \frac{1}{2}rVar\left[\sum_{k\in N_i\backslash\overline{N}_j}\frac{e_k}{d_k+1}\right] \\
&= \frac{e_i}{d_i+1} + \frac{e_j}{d_j+1} + \sum_{k\in N_{ij}}\frac{e_k}{d_k+1} - \sum_{k\in N_i}\mu_{ik}^* - \frac{1}{2}r\sigma^2\cdot\sum_{k\in N_i\backslash\overline{N}_j}\frac{1}{(d_k+1)^2}.
\end{aligned}$$

The necessary and sufficient condition for t^* to be Pareto efficient is given by (6). Plugging the above into (6) and canceling out the terms dependent on local information $(e_k)_{k\in\overline{N}_{ij}}$, we arrive at the following condition for Pareto efficiency:

$$\sum_{k\in N_i}\mu_{ik}^* + \frac{1}{2}r\sigma^2\cdot\sum_{k\in N_i\backslash\overline{N}_j}\frac{1}{(d_k+1)^2} + \frac{1}{r}\ln\lambda_i = \sum_{k\in N_j}\mu_{jk}^* + \frac{1}{2}r\sigma^2\cdot\sum_{k\in N_j\backslash\overline{N}_i}\frac{1}{(d_k+1)^2} + \frac{1}{r}\ln\lambda_j \tag{16}$$

Any profile of state-independent transfers μ^* that solves the above system (16) makes t^* efficient under weightings λ.

Notice that, if $CE\left(x_i^* \middle| I_{ij}\right) - \frac{1}{r}\ln\lambda_i = CE\left(x_j^* \middle| I_{ij}\right) - \frac{1}{r}\ln\lambda_j$ holds for any e,

$$\begin{aligned} CE\left(x_i^*\right) - \frac{1}{r}\ln\lambda_i &= \mathbb{E}\left[CE\left(x_i^* \middle| I_{ij}\right) - \frac{1}{r}\ln\lambda_i\right] - \frac{1}{2}rVar\left[CE\left(x_i^* \middle| I_{ij}\right) - \frac{1}{r}\ln\lambda_i\right] \\ &= CE\left(x_j^*\right) - \frac{1}{r}\ln\lambda_j \end{aligned}$$

Hence, with G assumed WLOG to be connected, we have

$$\begin{aligned} CE\left(x_i^*\right) - \frac{1}{r}\ln\lambda_i &= \frac{1}{n}\sum_{k\in N}\left(CE\left(x_k^*\right) - \frac{1}{r}\ln\lambda_k\right) \\ &= -\frac{r\sigma^2}{2n}\sum_{k\in N}\frac{1}{d_k+1} - \frac{1}{nr}\sum_{k\in N}\ln\lambda_k \end{aligned} \tag{17}$$

On the other hand, as $x_i^* = \frac{e_i}{d_i+1} + \sum_{k\in N_i}\left(\frac{e_k}{d_k+1} - \mu_{ik}^*\right)$,

$$CE\left(x_i^*\right) = -\sum_{k\in N_i}\mu_{ik}^* - \frac{1}{2}r\sigma^2\sum_{k\in \overline{N}_i}\frac{1}{\left(d_k+1\right)^2} \tag{18}$$

Equating the expressions for $CE\left(x_i^*\right)$ in (17) and (18), we obtain

$$\sum_{k\in N_i}\mu_{ik}^* = \frac{1}{2}r\sigma^2\left(\frac{1}{n}\sum_{k\in N}\frac{1}{d_k+1} - \sum_{k\in\overline{N}_i}\frac{1}{\left(d_k+1\right)^2}\right) + \frac{1}{r}\left(\frac{1}{n}\sum_{k\in N}\ln\lambda_k - \ln\lambda_i\right). \tag{19}$$

Lemma 6. *Given any real vector $c \in \mathbb{R}^n$ such that $\sum_{i\in N} c_i = 0$, there exists a real vector $\mu \in \mathbb{R}^{\sum_i d_i}$ such that $\mu_{ik} + \mu_{ki} = 0$ for every linked pair ik and*

$$\sum_{k\in N_i}\mu_{ik} = c_i.$$

The solution is unique if and only if the network is minimally connected.

Lemma 6 has established that there indeed exists a solution μ^* to (19). Given any solution μ^* to (19), as $\overline{N}_i \backslash \left(N_i \backslash \overline{N}_j\right) = \overline{N}_{ij}, \quad$ we have

$$
\begin{aligned}
&\sum_{k\in N_i} \mu^*_{ik} + \frac{1}{2} r\sigma^2 \sum_{k\in N_i\backslash \overline{N}_j} \frac{1}{(d_k+1)^2} + \frac{1}{r}\ln\lambda_i \\
=&\frac{1}{2} r\sigma^2 \left(\frac{1}{n}\sum_{k\in N} \frac{1}{d_k+1} - \sum_{k\in \overline{N}_{ij}} \frac{1}{(d_k+1)^2} \right) + \frac{1}{nr}\sum_{k\in N} \ln\lambda_k \\
=&\sum_{k\in N_j} \mu^*_{jk} + \frac{1}{2} r\sigma^2 \sum_{k\in N_j\backslash \overline{N}_i} \frac{1}{(d_k+1)^2} + \frac{1}{r}\ln\lambda_j
\end{aligned}
$$

implying that μ^* also solves the system of equations (16). Hence, t^* is Pareto efficient. ∎

A.5 Preparatory Derivations for Proposition 4

As previewed in Section 3.3.2, we now explain in more details two preparatory steps for our main result, Proposition 4, which characterizes the Pareto efficient transfer shares under CARA-Normal setting with correlation parameter ρ.

First, we show that the Pareto efficient profile of linear and strictly bilateral transfer rules: $t_{ij}(e_i, e_j) := \alpha_{ij}e_i - \alpha_{ji}e_j + \mu_{ij}$ correspond to the solution of a complicated system of linear equations.

Lemma 7. *If there exist a vector γ such that (α, γ) jointly solve the following system of linear equations,*

$$
\begin{cases}
\alpha_{ij} = \frac{1}{2}\left(1 - \sum_{k\in N_i\backslash\{j\}} \alpha_{ik} + \gamma_{ij}\right) & (20.1) \\
0 = \alpha_{ki} - \alpha_{kj} + \gamma_{ij} \quad \forall k \in N_{ij} & (20.2) \\
\gamma_{ij} = \frac{\rho}{1+(d_{ij}+1)\rho}\left(\sum_{k\in N_i\backslash\overline{N}_j} \alpha_{ki} - \sum_{k\in N_j\backslash\overline{N}_i} \alpha_{kj}\right) & (20.3)
\end{cases}
\forall i,j \ s.t.\ G_{ij} = 1 \qquad (20)
$$

then, given any constant vector c with $c_{ij} = c_{ji}$ for all $ij \in G$, the profile of linear and strictly bilateral transfer rules defined by

$$
t_{ij}(e_i, e_j) := \alpha_{ij}e_i - \alpha_{ji}e_j + c_{ij}, \quad \forall ij \in G
$$

for all $ij \in G$ are Pareto efficient in $\mathcal{T}$.

We next show that instead of solving the set of linear equations (20) that imply

Pareto efficiency in $\mathcal{T}$, we may solve an alternative optimization problem (9) that minimizes total consumption variances among all linear transfer rules.

Lemma 8. $\forall \rho \in \left(-\frac{1}{n-1}, 1\right)$, *if system* (10) *admits a unique solution, then the solution also solves system* (20)*: i.e., a profile of linear and strictly bilateral transfer rules is Pareto efficient in* $\mathcal{T}$ *if it uniquely minimizes the sum of consumption variances among all profiles of linear and strictly bilateral transfer rules in* $\mathcal{T}^*$.

Finally, we show in Proposition 4 in the main text that, for any given network, system (10) indeed admits a unique solution that can be expressed in closed form. The solution depends on the pairwise correlation ρ and on the positions of individuals in the network, and can be represented as a linear function of accumulated paths along the network.

A.6 Proof of Proposition 4

Proof. Let $\overline{G} := G + \mathbf{I}_n$ so that $\overline{G}_{ii} = 1\ \forall i \in N$. The optimality conditions given in equation (10.1) and (10.2) can be rewritten as

$$\alpha_{ji} = \overline{G}_{ij} \left(\Lambda_j - \frac{\rho}{1-\rho} \sum_{k \in N} \overline{G}_{ik} \alpha_{ki} \right) \tag{21}$$

Let $\overline{\alpha}_i := (\alpha_{1i}, \alpha_{2i}, \ldots, \alpha_{ni})'$ denote the vector of i's inflow shares, $\mathbf{\Lambda} = (\Lambda_1, \Lambda_2, \ldots, \Lambda_n)'$ the vector of rescaled constraint multipliers, and g_i represent the i-th column of $\overline{G}$. Then (21) can be rewritten in vector form as

$$\left(\mathbf{I} + \frac{\rho}{1-\rho} g_i g_i^{'} \right) \alpha_i = diag\left(g_i\right) \mathbf{\Lambda}$$

where $diag\left(g_i\right)$ is a diagonal matrix with g_i's entries on the diagonal. Left-multiplying both sides by $\left(\mathbf{I} - \frac{\rho}{1+\rho d_i} g_i g_i^{'} \right)$, which is well-defined for any $\rho > -\frac{1}{n-1}$ and any G, we have

$$\alpha_i = \left(\mathbf{I} - \frac{\rho}{1+\rho d_i} g_i g_i^{'} \right) diag\left(g_i\right) \mathbf{\Lambda}$$

As $g_i g_i^{'} \cdot diag\left(g_i\right) = g_i g_i^{'}$, the above becomes

$$\alpha_i = \left(diag\left(g_i\right) - \frac{\rho}{1+\rho d_i} g_i g_i^{'} \right) \mathbf{\Lambda} \tag{22}$$

Now, notice that (10.3) implies

$$1 = \sum_{j \in N} \alpha_{ij} = (d_i + 1)\, \Lambda_i - \sum_{j \in N} \overline{G}_{ij} \left(\frac{\rho}{1 + \rho d_j} \sum_{k} \overline{G}_{jk} \Lambda_k \right) \tag{23}$$

and thus we have

$$\Lambda_i = \frac{1}{d_i + 1} \left(1 + \sum_{j \in \overline{N}_i} \sum_{k \in \overline{N}_j} \frac{\rho}{1 + \rho d_j} \Lambda_k \right).$$

This establishes the recursive representation of the solution.

To obtain the closed-form solution, rewrite equation (23) as

$$\mathbf{1} = \sum_{i \in N} \left(diag\,(g_i) - \frac{\rho}{1 + \rho d_i} g_i g_i^{'} \right) \mathbf{\Lambda} = \left(\overline{D} - \overline{G} \Psi \overline{G} \right) \Lambda$$

where $\overline{D}$ is a diagonal matrix with its i-th diagonal entry being $d_i + 1$, and Ψ is a diagonal matrix with its i-th diagonal entry being $\frac{\rho}{1+\rho d_i}$. Notice that $\forall \xi \in \mathbb{R}^n \backslash \{\mathbf{0}\}$,

$$\begin{aligned}
\xi^{'} \left(\overline{D} - \overline{G} \Psi \overline{G} \right) \xi &= \sum_{i \in N} (d_i + 1)\, \xi_i^2 - \sum_{i \in N} \frac{\rho}{1 + \rho d_i} \left(\sum_{j \in \overline{N}_i} \xi_j \right)^2 \\
&\geq \sum_{i \in N} (d_i + 1)\, \xi_i^2 - \sum_{i \in N} \frac{1}{1 + d_i} \left(\sum_{j \in \overline{N}_i} \xi_j \right)^2 \\
&\geq \sum_{i \in N} (d_i + 1)\, \xi_i^2 - \sum_{i \in N} \frac{1}{1 + d_i} \cdot (1 + d_i) \sum_{j \in \overline{N}_i} \xi_j^2 \\
&= \sum_{i \in N} (d_i + 1)\, \xi_i^2 - \sum_{i \in N} (d_i + 1)\, \xi_i^2 \\
&= 0
\end{aligned}$$

where the equality holds if and only if $\rho = 1$ and $\xi = c \cdot \mathbf{1}$ for some $c > 0$. Hence, $\forall \rho \in \left(-\frac{1}{n-1}, 1 \right)$, $\left(\overline{D} - \overline{G} \Psi \overline{G} \right)$ is positive definite and thus invertible. Hence,

$$\begin{aligned}
\Lambda &= \left(\overline{D} - \overline{G} \Psi \overline{G} \right)^{-1} \mathbf{1}, \\
\alpha_i &= \left(diag\,(g_i) - \frac{\rho}{1 + \rho d_i} g_i g_i^{'} \right) \left(\overline{D} - \overline{G} \Psi \overline{G} \right)^{-1} \mathbf{1}.
\end{aligned}$$

Finally, we solve for the inverse matrix above as a series of powers of $\overline{G}$. Notice that

$$\left(\overline{D}-\overline{G}\Psi\overline{G}\right)^{-1}=\left(\overline{D}^{\frac{1}{2}}\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)\overline{D}^{\frac{1}{2}}\right)^{-1}=\overline{D}^{-\frac{1}{2}}\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)^{-1}\overline{D}^{-\frac{1}{2}}$$

where the middle term $\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)$ is also invertible and positive definite for $\rho\in\left(-\frac{1}{n-1},1\right)$ due to the positive definiteness of $\overline{D}-\overline{G}\boldsymbol{\Psi}\overline{G}$ and the invertibility of $\overline{D}$. For $\rho\in(0,1)$, notice that $\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}$ is also positive definite, so its eigenvalues must be positive. Also, its largest eigenvalue φ_{max} must be smaller than 1. Otherwise, there exists a nonzero vector ξ such that

$$\xi'\left(I-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{\frac{1}{2}}\right)\xi=(1-\varphi_{max})\,\xi'\xi<0$$

contradicting the positive definiteness of $\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)$. Then, we may write

$$\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)^{-1}=\mathbf{I}+\sum_{k=1}^{\infty}\left(\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)^{k}$$

and thus

$$\begin{aligned}\left(\overline{D}-\overline{G}\Psi\overline{G}\right)^{-1}&=\overline{D}^{-1}+\overline{D}^{-\frac{1}{2}}\sum_{k=1}^{\infty}\left(\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)^{k}\overline{D}^{-\frac{1}{2}}\\&=\overline{D}^{-1}+\sum_{k=1}^{\infty}\left(\overline{D}^{-1}\overline{Q}\right)^{k}\overline{D}^{-1}\end{aligned}$$

where $\overline{Q}:=\overline{G}\Psi\overline{G}$ can be interpreted as the weighted square of the extended adjacency matrix. Consider the set of all paths of length q between i and j under G as

$$\Pi_{ij}^{q}(G)=\left\{(i_0,i_1,i_2,\ldots i_q)\mid i_0=i,\ i_q=j\ and\,\overline{G}_{i_n i_{n+1}}=1\ for\ n=0,1,\ldots q-1\right\}$$

For every $\pi_{ij}\in\Pi_{ij}^{q}(G)$, let $W(\pi_{ij})$ denote the weight associated to this path. It is not difficult to see that,

$$W(\pi_{ij})=\frac{1}{d_i+1}\frac{\rho}{1+\rho d_{i_1}}\frac{1}{d_{i_2}+1}\frac{\rho}{1+\rho d_{i_3}}\cdots\frac{1}{d_j+1}$$

Then

$$\begin{aligned}
\Lambda_i &= \left[\left(\overline{D}-\overline{G}\Psi\overline{G}\right)^{-1}\mathbf{1}\right]_i \\
&= \left(\overline{D}^{-1}\mathbf{1}\right)_i + \left(\sum_{k=1}^{\infty}\left(\overline{D}^{-1}\overline{Q}\right)^k\overline{D}^{-1}\mathbf{1}\right)_i \\
&= \frac{1}{d_i+1} + \left[\sum_{k=1}^{\infty}\left(\overline{D}^{-1}\overline{G}\Psi\overline{G}\right)^k\right]_i\overline{D}^{-1}\mathbf{1} \\
&= \frac{1}{d_i+1} + \sum_{j\in N}\left[\sum_{k=1}^{\infty}\left(\overline{D}^{-1}\overline{G}\Psi\overline{G}\right)^k\right]_{ij}\cdot\frac{1}{d_j+1} \\
&= \frac{1}{d_i+1} + \sum_{j\in N}\sum_{k=1}^{\infty}\left(\overline{D}_{[i]}^{-1}\overline{G}\Psi\overline{G}\overline{D}^{-1}...\overline{D}^{-1}\overline{G}\Psi\overline{G}_{(j)}\right)\frac{1}{d_j+1} \\
&= \frac{1}{d_i+1} + \sum_{j\in N}\sum_{q=1}^{\infty}\sum_{\pi_{ij}\in\Pi_{ij}^{2q}}\left(\frac{1}{d_i+1}\cdot\frac{\rho}{1+\rho d_{i_1}}\cdot\frac{1}{d_{i_2}+1}\cdot...\right)\frac{1}{d_j+1} \\
&= \frac{1}{d_i+1} + \sum_{q=1}^{\infty}\sum_{j\in N}\sum_{\pi_{ij}\in\Pi_{ij}^{2q}}W\left(\pi_{ij}\right)
\end{aligned}$$

This concludes the proof. ∎

A.7 Proof of Proposition 5

Proof. To start with, notice that as $d_i \sim B\left(n-1,p\right)$, we have

$$\frac{1}{n}d_i = \frac{n-1}{n}\cdot\frac{1}{n-1}d_i \xrightarrow{a.s.} p$$

and

$$\frac{d_i-np}{\sqrt{np\left(1-p\right)}} = \sqrt{\frac{n-1}{n}}\cdot\frac{d_i-\left(n-1\right)p}{\sqrt{\left(n-1\right)p\left(1-p\right)}} - \frac{p}{\sqrt{np\left(1-p\right)}} \xrightarrow{d} \mathcal{N}\left(0,1\right).$$

For each n, set $\overline{d}_n$ by

$$\overline{d}_n := \left(\mathbb{E}_n^{ER}\left[\left.\frac{1}{\left(d_j\left(G_n\right)+1\right)^2}\right| ij\in G_n\right]\right)^{-\frac{1}{2}} - 1$$

$$= \left(\mathbb{E}_n^{ER} \left[\frac{1}{\left(2 + \tilde{d}_j\right)^2} \right] \right)^{-\frac{1}{2}} - 1 \quad \text{where } \tilde{d}_j \sim B\left(n-2, p\right)$$

so that

$$\mathbb{E}_n^{ER} \left[\left. \frac{1}{\left(\frac{1}{n} d_j\left(G_n\right) + \frac{1}{n}\right)^2} \right| ij \in G_n \right] - \frac{1}{\left(\frac{1}{n}\overline{d}_n + \frac{1}{n}\right)^2} = 0.$$

Notice that

$$\frac{1}{n}\overline{d}_n = \left(\mathbb{E}_n^{ER} \left[\frac{1}{\left(\frac{1}{n}2 + \frac{n-2}{n} \cdot \frac{1}{n-2}\tilde{d}_j\right)^2} \right] \right)^{-\frac{1}{2}} - \frac{1}{n} \to \left(\frac{1}{\left(0+p\right)^2} \right)^{-\frac{1}{2}} = p.$$

Now, consider

$$\begin{aligned}
& Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_i\left(G_n\right)\right] \\
=& Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right) - \frac{p}{\left(\frac{1}{n}\overline{d}_n + \frac{1}{n}\right)^2},\ d_i\left(G_n\right) - \left(n-1\right)p \right] \\
=& \mathbb{E}_n^{ER}\left[\left(Var\left(x_i\left(G_n\right)\right) - \frac{p}{\left(\frac{1}{n}\overline{d}_n + \frac{1}{n}\right)^2} \right) \cdot \left(d_i\left(G_n\right) - \left(n-1\right)p\right) \right] \\
& - \mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right) - \frac{p}{\left(\frac{1}{n}\overline{d}_n + \frac{1}{n}\right)^2} \right] \mathbb{E}_n^{ER}\left[d_i\left(G_n\right) - \left(n-1\right)p\right] \\
=& \mathbb{E}_n^{ER}\left[\left(Var\left(x_i\left(G_n\right)\right) - \frac{p}{\left(\frac{1}{n}\overline{d}_n + \frac{1}{n}\right)^2} \right) \cdot \left(d_i\left(G_n\right) - \left(n-1\right)p\right) \right] \\
=& \mathbb{E}_n^{ER}\left[\mathbb{E}_n^{ER}\left[\left. Var\left(x_i\left(G_n\right)\right) - \frac{p}{\left(\frac{1}{n}\overline{d}_n + \frac{1}{n}\right)^2} \right| d_i\left(G_n\right) \right] \cdot \left(d_i\left(G_n\right) - \left(n-1\right)p\right) \right]
\end{aligned}$$

where the second last equality follows from the fact that

$$\mathbb{E}_n^{ER}\left[d_i\left(G_n\right) - \left(n-1\right)p\right] = 0.$$

Then,

$$\mathbb{E}_n^{ER}\left[\left. nVar\left(x_i\left(G_n\right)\right) \right| d_i\left(G_n\right) \right]$$

$$
\begin{aligned}
=&\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}d_i+\frac{1}{n}\sum_{j\in N_i}\left[\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right]\middle| d_i\left(G_n\right)\right]\\
=&\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}d_i+\frac{1}{n}\sum_{j\in N_i}\left\{\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}\middle| d_i\left(G_n\right)\right]-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right\}\\
=&\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}d_i\\
\xrightarrow{a.s.}&\frac{1}{\left(p+0\right)^2}\cdot 0+\frac{1}{\left(p+0\right)^2}\cdot p=\frac{1}{p}
\end{aligned}
$$

where the last equality follows from the definition of $\overline{d}_n$. By appropriate centering, we now have

$$
\begin{aligned}
&\frac{1}{\sqrt{n}}\left(d_i-\left(n-1\right)p\right)\cdot\sqrt{n}\left(n\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)\middle| d_i\left(G_n\right)\right]-\frac{np}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right)\\
=&\frac{1}{\sqrt{n}}\left(d_i-\left(n-1\right)p\right)\cdot\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{\sqrt{n}}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{\sqrt{n}}\left(d_i-np\right)\right]\\
=&\frac{1}{\sqrt{n}}p\cdot\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{\sqrt{n}}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{\sqrt{n}}\left(d_i-np\right)\right]\\
&\frac{\sqrt{n}}{n}\cdot\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{\sqrt{n}}\left(d_i-np\right)+\frac{p\left(1-p\right)}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\left(\frac{d_i-np}{\sqrt{np\left(1-p\right)}}\right)^2\\
\xrightarrow{d}&0+0+\frac{p\left(1-p\right)}{p^2}\cdot\chi_1^2
\end{aligned}
$$

so that

$$
\begin{aligned}
&\mathbb{E}_n^{ER}\left[\frac{1}{\sqrt{n}}\left(d_i-np\right)\cdot\sqrt{n}\left(\mathbb{E}_n^{ER}\left[nVar\left(x_i\left(G_n\right)\right)\middle| d_i\left(G_n\right)\right]-\frac{p}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right)\right]\\
\to&\mathbb{E}\left[\frac{p\left(1-p\right)}{p^2}\cdot\chi_1^2\right]=\frac{1-p}{p}.
\end{aligned}
$$

In summary, we have

$$
\begin{aligned}
&nCov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_i\left(G_n\right)\right]\\
=&\mathbb{E}_n^{ER}\left[\sqrt{n}\mathbb{E}_n^{ER}\left[nVar\left(x_i\left(G_n\right)\right)-\frac{np}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\middle| d_i\left(G_n\right)\right]\cdot\frac{1}{\sqrt{n}}\left(d_i\left(G_n\right)-\left(n-1\right)p\right)\right]
\end{aligned}
$$

$$\rightarrow \frac{1-p}{p} > 0.$$

∎

A.8 An Example on Constructing Transfers with a Simple Network

In this section we provide additional intuition on how to construct bilateral transfers from a given fixed network. We show how to construct the Lagrangean centrality measure by accumulating weighted paths, and how this leads to a precise prediction on gross transfers as a function of the underlying correlation parameter ρ.

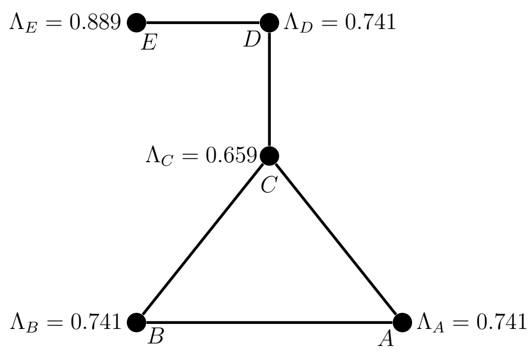


Figure 2: Centrality Measures in a Five-Player Network with $\rho = 0.4$

We consider a simple network where A, B and C form a triangle, and C is linked to D who is linked to E. This network is shown in Figure 2 above, where the Lagrangean Centrality measures appear next to each node. These measures are calculated by the accumulation of weighted paths, as indicated in Proposition 4. These weights are path-specific and depend on the degree of each node involved in each even-length path. To fix ideas, consider as an example all paths of length 4 starting from E. Since we are considering self-links (or loops) there are many such paths – for instance one such path is $\{E-E-E-E-E\}$, while another such path is $\{E-D-C-B-A\}$ and even another one is $\{E-D-C-B-B\}$, or $\{E-D-C-C-C\}$, and so on. As you can see, there are many such paths. For each one of these paths, the weight assigned to it depends on the degrees of those involved in the path, as described in the last equation of Proposition 4. For instance, for the path $\{E-E-E-E-E\}$ the

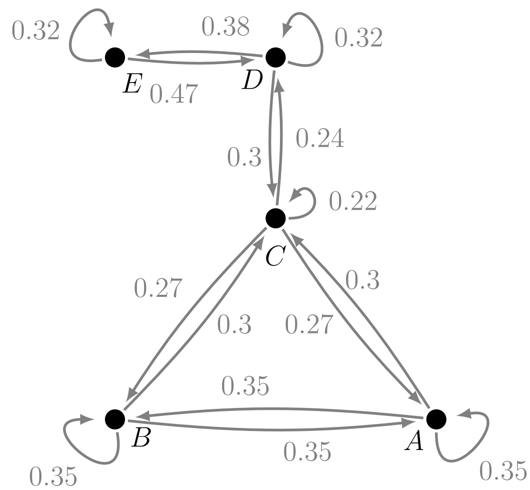


Figure 3: Gross Transfers in a Five-Player Network $\rho = 0.4$

entire path only involves individual E, which has $\bar{d}_E = 2$, so we therefore compute the following weight:

$$W(\{E - E - E - E - E\}) = \frac{1}{2+1} \cdot \frac{\rho}{1+2\rho} \cdot \frac{1}{2+1} \cdot \frac{\rho}{1+2\rho} \cdot \frac{1}{2+1}$$

Another example involves the path $\{E - D - C - B - A\}$, which involves all nodes in the network. In this case, we have that

$$W(\{E - D - C - B - A\}) = \frac{1}{2+1} \cdot \frac{\rho}{1+3\rho} \cdot \frac{1}{4+1} \cdot \frac{\rho}{1+3\rho} \cdot \frac{1}{3+1}$$

We could continue this way and compute the weights of all such paths of length 4 and 6 and 8 and so on. The centrality measures, Λ_i, shown in Proposition 4 can be interpreted as the sum of the weights of all even-length paths starting from i in this way. In Figure 2 above we compute these measures for the case in which the uniform correlation parameter ρ is equal to 0.4. Once we have the measures Λ_i for all individuals i we can compute the flows α_{ij} from i to j as a linear function of these centrality measures, as shown in the first equation of Proposition 4. Figure 3 shows the resulting *gross* flows across every pair of connected individuals for the case with $\rho = 0.4$

Notice from Figure 3 that individual C transfers a larger share of its endowment to individuals A and B than to individual D, since B and A can share this efficiently, but D cannot share these proceeds with E, given the informational constraints that make it impossible for t_{DE} to depend on e_C. Moreover, we can see that individual C takes on a larger share from others' endowments than the share it sends out. As explained earlier this generates a larger volatility of consumption for individual C, which acts as a net provider of insurance to the rest of the community. Finally, notice that the transfer from E to D is larger than the transfer from D to E: because E is more peripheral, it is optimal that it unloads a greater share of risk unto D than viceversa, since D is insured in turn by C.

Of course, for different values of ρ the Lagrangean centralities vary widely, and therefore the resulting transfers are also very different. In fact, notice that when $\rho = 0$ it is clear from the second equation in Proposition 4 that $\Lambda_i = \frac{1}{d_i+1}$ which is the reciprocal of the number of connections of individual i (including itself). In that case, it is clear from the first equation in Proposition 4 that

$$\alpha_{ji} = \Lambda_j = \frac{1}{d_j + 1}$$

which corresponds to the local equal-sharing rule, as described in section 3.3.1 on independent endowments. Figure 4 shows simulation results for different values of ρ. Notice that as we increase the value of $\rho \in \left(-\frac{1}{n-1}, 1\right)$ gross transfers between any pair of individuals converge, and therefore all net transfers tend to zero.

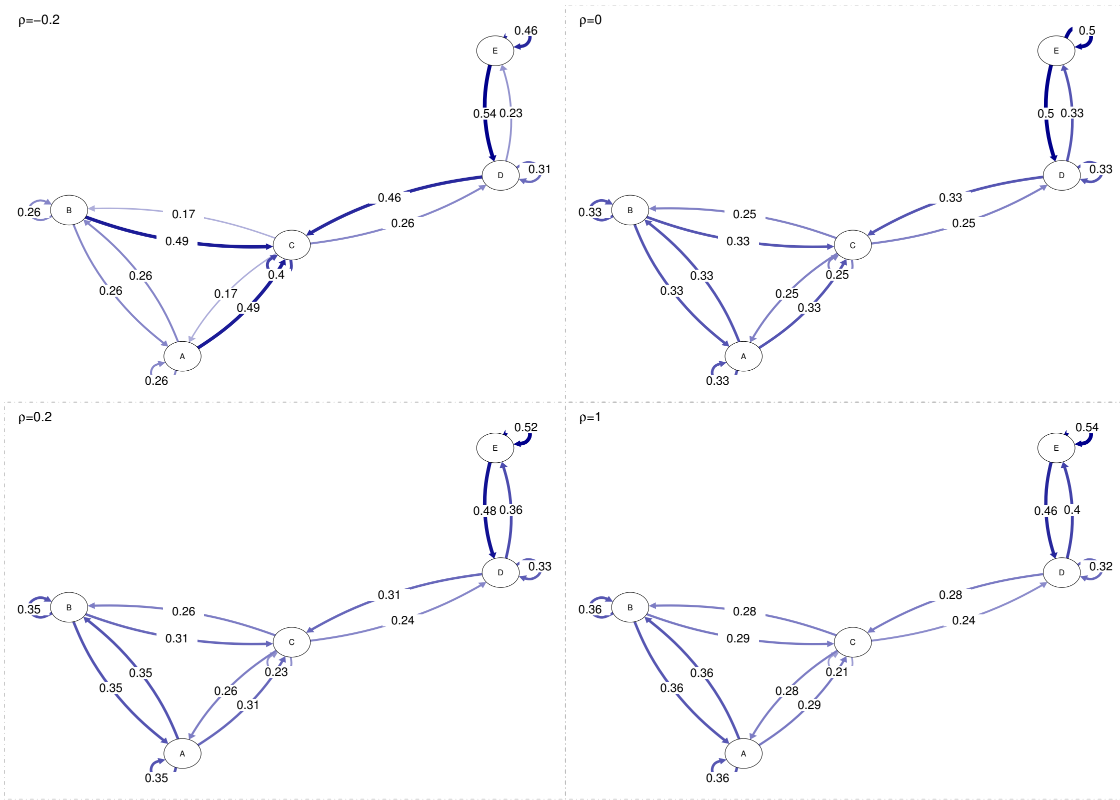


Figure 4: Predicted Bilateral Transfers for Different Uniform Correlation Values ρ

A.9 Details on Variable Constructions

More Information on the Townsend Thai Monthly Survey An initial village-wide census enumerated all households and structures in the surveyed villages, allowing unique identification of a given household or a given structure in a given village across all 196 months. The variable "*newid*" in the publicly released data set, constructed by the data provider using an unknown scrambling algorithm in conformation with public data dissemination protocols, serves as the unique identifier for each household, but may also be used to uniquely identify the village in which the household resides. For each given village, the variable "*census structure number*" uniquely identifies a certain structure in the given village as recorded in the initial census.

The subsequent monthly surveys cover a randomly sampled subset of households. From each village, 45 households were randomly selected, and the subsequent monthly survey recorded updates from the same households over time in all villages. Some of surveyed households moved out of a village, and some new households were added to the monthly survey during the sampled periods.

Consumption Variance We first describe how we construct measures of consumption variance for each household.

First, we construct *real monthly household consumption per capita* using data on household expenditures For a typical household (*newid*) in a given month (*month*), the Townsend Thai Monthly Survey include a quite comprehensive range of finely categorized monthly household expenditures. Among them some "shorter-term" categories, such as expenses on food, gasoline and daily commutes, are recorded at biweekly frequency[36] while other "longer-term" categories, such as utilities, rents and clothing, are recorded at monthly frequency. All expenditures are recorded in nominal Thailand *bahts*.

We construct our monthly consumption variables in two ways for robustness. The first version is constructed by aggregating over all categories of expenditures except for the category called "*other expenses*". The second version is constructed by aggregating over all categories of expenditures, excluding not only "*other expenses*" but also the categories labeled as "*maintenance of houses and private vehicles*". The reason for excluding these maintenance expenditures is that we occasionally observe a very few number (about 8 among 133,188 household-month records) of extremely large expenditures in these categories in certain months, causing the sample variances (across 133,188 records) to almost quadruple as well as the sample skewness and kurtosis to blow up. As such occasional extraordinary expenditures are hard to justify as fluctuations in *monthly* consumption, the second version of consumption variables are constructed without these *maintenance* categories, and will be indicated by an postscript "*_nm*", short for "*no maintenance*", whenever applicable.

Second, we compute *real* monthly household consumption *per capita*, using data on household sizes from the Townsend Thai Monthly Survey, as well as data on monthly *Consumer Price Index* from the Thailand Ministry of Commerce. We construct two measures of households sizes for robustness. The first version is constructed based on the number of household members that are registered on the household composition roster and are interviewed by the survey for each household in each month. The second version is constructed based on the number of household members who

[36] To be precise, these categories are recorded at weekly frequency for months 1-24, and at biweekly frequency from month 25 onwards.

are not only interviewed but also indicate that they have resided in the household for at least fifteen days of the month. We indicate all variables constructed from the second version of sizes with the postscript "*_re*", short for "*resident*", whenever applicable.

Third, we keep only households whose consumption are recorded for at least 100 (not necessarily consecutive) out of 196 months. This selects a subsample of 689 households from a total of 780 distinct households, excluding in particular households that moved out early or were added to the monthly survey late during the 196 months. As we are interested in measuring consumption volatility across time, we impose this subsample selection to ensure that we have a reasonable number of time periods for each household and that the sample variance to be computed later are reasonably accurate. Note that the selected households contribute an overwhelming proportion (129,944 among 133,188, or 97.56 percent) of household-month observations on real consumption per capita.

Fourth, we compute *detrended* real consumption per capita with linear detrending of logged real consumption per capital. Given that real consumption per capita should be rising over a time frame of 196 months, detrending is required for the construction of a sensible measure of consumption volatility. This is because, without detrending, a deterministic path of real consumption per capita at a fixed positive growth rate will generate positive variance in consumption without any uncertainty. We model growth in real consumption per capita with a standard time trend, which is consistent with the theory of balanced growth paths. Specifically, *separately* for each household, we regress logged real consumption per capita on a time trend and obtain the predicted values and residuals from the regressions.

We define *detrended real consumption per capita* as the difference between the *observed* real consumption per capita and the *predicted* real consumption per capita, computed as the exponent of predicted values from the trend regression of *logged* real consumption per capita. We also define *detrended log real consumption per capita,* computed as the residuals from the trend regression of *log* real consumption per capita.

Finally, we compute the *sample standard deviations* of detrended real monthly household consumption per capita, for each household across all months in which this given household is surveyed. We compute four measures of consumption volatility, named respectively, "*sd_x*", "*sd_x_nm*", "*sd_x_re*" and "*sd_x_nm_re*", depending on the definitions of consumption variables and household sizes as discussed above. We also compute another set of volatility measures, using sample standard deviations of detrended *log* real monthly household consumption per capita, which are named "*sd_lx*", "*sd_lx_nm*", "*sd_lx_re*" and "*sd_lx_nm_re*", respectively.

Network Degrees We now describe how we construct measures of network degrees for each household in each village. For robustness, we consider five definitions of network links, leading to five different versions of network degrees.

The first measure of network degree is constructed based on records of within-village gift and remittance, borrowing or lending transactions from the Townsend Thai Monthly Survey. Among the many types of social and economic interactions among households recorded by the survey, "gift and remittance" as well as "borrowing" and "lending" are arguably most relevant to the purpose of informal risk sharing.

Each month, each surveyed household, identified by a "*newid*", reports whether this household has given or received any gift or remittance during the month. The data set records each *gift and remittance giving, gift and remittance receiving, borrowing* and *lending* transactions separately, and includes some information on the counterparties of each transaction. In particular, the data identify whether the a surveyed household has a gift or remittance transaction with some counterparty household that resides in the *same village* as the surveyed household, and also identify the "*census structure number*" of the counterparty household whenever available.

We define the *"degree"* of a certain household, identified by "*newid",* as the number of unique *same-village* counterparties who the household has ever interacted with over the 196 months, each of whom resides at some structure identified by a certain "*census structure number".*

The second, third and fourth measures of network degrees we use are based on records of reported kinship or neighbor relationships from the Townsend Thai Monthly Survey. Such records are obtained from numerous records of within-village interactions from five modules of the survey: household assets, agricultural assets, gifts & remittance, borrowing, and lending. Again, each record identifies a household who reports a certain interaction, the "*census structure number*" of the counterparty, whether the counterparty has kinship relationships with the given household, and whether the counterparty relationship is a neighbor of the given household in the village. Our second and third degree measures count the unique number of *same-village* counterparties who are reported, respectively, as kins and neighbors. Our fourth degree measures is constructed by taking a union of the kinship and neighborhood links.

The fifth, and the last, degree measure is constructed by taking a union of the transaction links (gifts, remittance, borrowing or lending) and the relationship links (kinship or neighborhood).

Control Variables We extract monthly information on the households' real saving balances, use of institutional finance (borrowing from commercial banks, BAAG, PCG, Rice Bank, Agricultural Co-operation or other institutions), use of personal finance (borrowing from money lenders, store owners, input suppliers, relatives or friends), health insurance payment, life insurance payment and ROSCA payment. We construct a series of binary and real-valued variables based on these information, and then compute both inter-temporal averages and inter-temporal sums of these variables across all 196 months for each household. We also calculate the inter-temporal sample standard deviation of real saving balances to capture household-specific variation in savings. See below for more details.

Saving Saving may be used for consumption smoothing. We aggregate the current balances of all accounts that each household as savings at each month, and adjust them to real terms with the CPI data. We then compute inter-temporal averages as well as standard deviations over the whole sampled periods as two control variables.

Institutional finance Institutional finance, recorded by a binary variable, represents whether a household has borrowed from commercial banks, BAAG, PCG, Rice Bank, Agricultural Co-operation or other institutions at each month (1 is yes and 0 is no), which may be regarded as proxy for access to formal financial institutions that may help with consumption smoothing. We compute inter-temporal average and sum of this variable for each household.

Personal finance Personal finance, recorded by a binary variable, represents whether the household have borrowed from money lenders, store owners, input suppliers or individuals such as relatives, friends or other individuals at each month (1 is yes and 0 is no). We compute inter-temporal average and sum of this variable for each household.

Health insurance Health insurance, recorded by a binary variable, represents whether the household has ever paid for health insurance fee at each month (1 is yes and 0 is no). For those households that have paid health insurance fees, the amount of health insurance premia paid is also recorded. We compute inter-temporal averages and sums of these variables for each household.

Life insurance Life insurance, recorded by a binary variable, represents whether the household has any life insurance policies or funeral fund memberships at each month(1 is yes and 0 is no). We also construct a binary variable that records whether the household paid any money into the insurance policy/funeral fund since the last interview, and a double variable that represents the amount paid to policy. We compute inter-temporal averages and sums of these variables for each household.

ROSCA ROSCA, recorded by a binary variable, represents whether the household is participating in any ROSCAs that have not finished in each month (1 is yes and 0 is no). We also construct a binary variable that records whether the household have paid any money into ROSCA since the last interview, and a double variable that represents the amount paid to ROSCAs. We compute inter-temporal averages and sums of these variables for each household.

Lastly, we obtain monthly *net operating income* for each household from the income statement section of the Townsend Thai Monthly Survey Household Financial Accounting dataset Townsend (2017), which covers months 1-172. We compute *linearly detrended log real income per capita* from the dataset, and compute its sample standard deviations ("*sd_ly*") as well as the sample standard deviations of the differences between observed real income per capita and detrended real income per capita ("*sd_y*").

All these constructed variables, along with village-level fixed effects, are included in the regression analysis as control variables.

A.10 Alternative Model Specifications

The main results in Section 3.3 are developed under the CARA-normal setting (Assumption 1) with a global correlation structure. We now consider the extendability of those results under some alternative model specifications.

Quadratic Utility Function

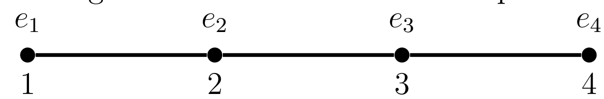
As to the specification of utility functions, we could alternatively work with quadratic utility functions, $u_i(x_i) = x_i - \frac{1}{2}rx_i^2$ for $i \in N$, which also admits a mean-variance expected utility representation. Noting that $u_i'(x_i) = 1 - rx_i$, the conditional Borch rule in Proposition 1 takes the form of $\lambda_i (1 - r\mathbb{E}_{ij}[x_i]) = \lambda_j (1 - r\mathbb{E}_{ij}[x_j])$. With equal Pareto weightings ($\lambda = \mathbf{1}$) and normal endowments, it can be shown that this leads to exactly the same system of linear equations as in (20).[37] Hence, the linear transfer shares given in Proposition 4 also characterize a Pareto efficient risk-sharing arrangement under the quadratic-normal setting. However, the Pareto efficient frontier traced out by all admissible Pareto weightings will correspond to a collection of different state-dependent transfer shares α.

Relaxing the Normality of the Endowments Distribution

The family of normal distributions have two properties that are technically essential to the proof of Pareto efficiency via the conditional Borch rule. First, a linear combination of a jointly normal vector remains normal, which allows us to explicitly characterize the distribution of final consumption $x_i = e_i - \sum_{j \in N_i} \alpha_{ji} e_j$ when transfer rules are linear. Second, normal distributions admit linear conditional expectations in the form of (25), which allows us to transform the conditions for Pareto efficiency into a system of linear equations on transfer shares. The assumption of normality can be relaxed slightly: if the endowment vector has a joint elliptical distribution,[38] then both properties carry over,[39] and thus the transfer shares given by Proposition 4 continue to characterize the Pareto efficient risk-sharing arrangements. Without the joint normality (or ellipticity) assumption, linear risk-sharing arrangements are in generally not Pareto efficient. For example consider again the 3-individual line

[37] The proof is available in Appendix B.10.
[38] Normal distribution is a special case of elliptical distribution.
[39] See, for example, Fang, Kotz, and Ng (1990), Theorem 2.16 & 2.18.

Figure 5: Four-Individual Example



network with the random endowment vector $e = (Y, Z, -Z^3)$ where Y, Z are independent standard normal random variables. As there are effectively no uncertainty, the unique Pareto efficient profile of transfer rules is given by $t_{12}(e_1, e_2) = \frac{1}{3}e_1 - \frac{2}{3}e_2 - \frac{1}{3}e_2^3$, $t_{13}(e_1, e_3) = \frac{1}{3}e_1 - \frac{2}{3}e_3 - \frac{1}{3}e_3^{1/3}$ which are clearly nonlinear.

Heterogeneity in Expected Endowments

Throughout Section 4 we maintained the specification that endowment distributions have zero mean. However, we argue that, as risk sharing is the sole concern of this paper, the specification of zero mean is a warranted normalization. For concreteness, let $\overline{y}_i$ be the expected level of endowment for individual i, and $y_i = \overline{y}_i + e_i$ be the random realization of endowment, where e_i is assumed to have zero mean. Clearly y and e induce the same local information structures $\sigma\left(y_k : k \in \overline{N}_{ij}\right) \equiv \sigma\left(e_k : k \in \overline{N}_{ij}\right)$, so it makes no differences whether the risk-sharing arrangements are specified to be contingent on y or e. Hence our results remain valid regardless of whether "endowments" or "endowment shocks" are shared. Moreover, neither does it make any difference whether the linear "guess" is taken to be $t_{ij} = \alpha_{ij}e_i - \alpha_{ji}e_j + \sum_{k \in N_{ij}} \beta_{ijk}e_k + \mu_{ij}$ or $t_{ij} = \alpha_{ij}y_i - \alpha_{ji}y_j + \sum_{k \in N_{ij}} \beta_{ijk}y_k + \tilde{\mu}_{ij}$: both will lead to the same system of linear equations in (20), so the Pareto efficient state-dependent transfer shares are given by exactly the same formulas in Proposition 4, irrespective of the value of mean-income vector $\overline{y}$. Any difference induced by $\overline{y}$ is completely absorbed by the state-independent transfers μ, which are irrelevant to Pareto efficiency in our framework.

A.11 General Contractibility Constraints

We now provide an example in which either condition (a) or (b) fails, and illustrate how our key methods and results remain adaptable and relevant.

Example 1. Consider a physical transfer network G in the form of a 4-individual line as depicted in Figure 5. For illustration simplicity suppose that endowments are independent, i.e., $\rho = 0$. We suppress constant transfers (with respect to endowment realizations) for notational simplicity.

First, consider the contractibility constraints Q given by $Q_{12} = \{4\}$ and $Q_{23} = Q_{34} = \emptyset$. Condition (a) holds because Q can be induced by common neighborhoods under $G' = \{12, 23, 34, 14, 24\}$, a supergraph of G. However, condition (b) fails, because individual 3, who lies on the unique path connecting individuals 4 and 1, is not informationally connected with 1 in G'. Nevertheless, the localized Borch rule remains valid, and it is thus straightforward to compute the constrained Pareto efficient transfer rule, which is still given by the local equal sharing rule:

$$t^{**}_{i,i+1}(e) = \begin{cases} \frac{1}{2}e_1 - \frac{1}{3}e_2, & i = 1, \\ \frac{1}{3}e_2 - \frac{1}{3}e_3, & i = 2, \\ \frac{1}{3}e_3 - \frac{1}{2}e_4, & i = 3. \end{cases}$$

In this case, the additional contractible information e_4 for individuals 12 is effectively useless.

Alternatively, consider the contractibility constraints Q given by $Q_{12} = \{3\}$ and $Q_{23} = Q_{34} = \emptyset$. Now condition (a) fails as the joint requirements of $Q_{12} = \{3\}$ and $Q_{23} = \emptyset$ imply that Q cannot be induced by any undirected supergraph of G: instead, Q can be seen as induced by a *directed* supergraph of G, given by $G' = \{12, 13, 21, 23, 32, 34, 43\}$. Condition (b), if adapted properly to accommodate the form of directedness, can be regarded as satisfied. However, taking Q as the primitive, the constrained Pareto efficient transfer rule can be still computed according to the localized Borch rule given Proposition 8.

$$t^{**}_{i,i+1}(e) = \begin{cases} \frac{1}{2}e_1 - \frac{1}{3}e_2 - \frac{1}{4}e_3, & i = 1, \\ \frac{1}{3}e_2 - \frac{2}{4}e_3, & i = 2, \\ \frac{1}{4}e_3 - \frac{1}{2}e_4, & i = 3, \end{cases}$$

which gives the consumption plan:

$$x^{**}_{i}(e) = \begin{cases} \frac{1}{2}e_1 + \frac{1}{3}e_2 + \frac{1}{4}e_3, & i = 1, \\ \frac{1}{2}e_1 + \frac{1}{3}e_2 + \frac{1}{4}e_3, & i = 2, \\ \frac{1}{3}e_2 + \frac{1}{4}e_3 + \frac{1}{2}e_4, & i = 3, \\ \frac{1}{4}e_3 + \frac{1}{2}e_4, & i = 4. \end{cases}$$

Despite that t^{**} under (G, Q) no longer exactly takes the form of a local equal sharing rule as defined in Proposition 3, it clearly still carries some essential similarity to "local equal sharing". Specifically, we might as well interpret t^{**} as a generalized version of local equal sharing rule that now accommodates some informational or contractual *directedness*: individuals 12 can now make their transfer rule t_{12} effectively contingent on the endowment realization of e_3, for whatever reasons we do not specify here, while individuals 23 cannot do the same. If we define the "in-neighborhood" $N_i^{'}$ under the directed informational network $G^{'}$ as $N_i^{'} := \left\{ j \in N : ji \in G^{'} \right\}$, then x^{**} can be characterized exactly by *directed local equal sharing*.

The example above illustrates how the intuition of local equal sharing remains highly relevant under more general contractibility constraints, as well as how the localized Borch rule remains valid and useful in characterizing the constrained Pareto efficient risk sharing arrangements. However, we do not further pursue the adaption of Propositions 3 and 4 in the absence of conditions $(a)(b)$, though probably empirically relevant and technically nontrivial, in this paper.

B Additional Materials

B.1 Proofs of Lemmas 1-5

Lemma. 1: *$\mathcal{T}^*$ with $\langle \cdot, \cdot \rangle$ forms an inner product space.*

Proof. We first show that $\langle \cdot, \cdot \rangle$ is a well-defined inner product. Symmetry immediately follows from the definition. Linearity in the first argument follows from the linearity of the expectation operator:

$$\begin{aligned} \langle \alpha s + \beta t,\ r \rangle &= \mathbb{E}\left[\sum_{G_{ij}=1} \left(\alpha s_{ij} + \beta t_{ij}\right) r_{ij}\right] = \alpha \mathbb{E}\left[\sum_{G_{ij}=1} s_{ij} r_{ij}\right] + \beta \mathbb{E}\left[\sum_{G_{ij}=1} t_{ij} r_{ij}\right] \\ &= \alpha \langle s,\ r \rangle + \beta \langle t,\ r \rangle . \end{aligned}$$

Positive definiteness is also obvious: $\langle t, t \rangle = \mathbb{E}\left[\sum_{G_{ij}=1} t_{ij}^2 (e)\right] \geq 0$ and $\langle t, t \rangle = 0$ if and only if $t = \mathbf{0}$, i.e., $t_{ij}(\omega) = 0$ for all linked ij and $\mathbb{P}$-almost all $e \in \Omega$.

We then show that $\mathcal{T}$ is a linear space. $\forall s, t \in \mathcal{T}$, $\forall \alpha, \beta \in \mathbb{R}$, $\alpha s (I_{ij}) + \beta t (I_{ij})$ is also $\sigma (I_{ij})$-measurable, and

$$\alpha s_{ij}(e) + \beta t_{ij}(e) = -\left(\alpha s_{ji}(e) + \beta t_{ji}(e)\right).$$

Finiteness of expectation is obvious. Hence, $\alpha s + \beta t \in \mathcal{T}$. ∎

Lemma. 2: *The objective function in (3)*

$$J(t) := \mathbb{E}\left[\sum_{k \in N} \lambda_k u_k \left(e_k - \sum_{h \in N_k} t_{kh}(e)\right)\right]$$

is concave on $\mathcal{T}$.

Proof. $\forall s, t \in \mathcal{T}$, $\forall \alpha \in [0,1]$,

$$
\begin{aligned}
& J\left(\alpha s+(1-\alpha) t\right) \\
&= \mathbb{E}\left[\sum_i \lambda_i u_i\left(e_i-\sum_{j \in N_i}\left(\alpha s_{ij}(e)+(1-\alpha) t_{ij}(e)\right)\right)\right] \\
&= \sum_i \lambda_i \mathbb{E}\left[u_i\left(\alpha\left(e_i-\sum_{j \in N_i} s_{ij}(e)\right)+(1-\alpha)\left(e_i-\sum_{j \in N_i} t_{ij}(e)\right)\right)\right] \\
&\geq \sum_i \lambda_i \mathbb{E}\left[\alpha u_i\left(e_i-\sum_{j \in N_i} s_{ij}(e)\right)+(1-\alpha) u_i\left(e_i-\sum_{j \in N_i} t_{ij}(e)\right)\right] \\
&= \alpha \mathbb{E}\left[\sum_i \lambda_i u_i\left(e_i-\sum_{j \in N_i} s_{ij}(e)\right)\right]+(1-\alpha) \mathbb{E}\left[\sum_i \lambda_i u_i\left(e_i-\sum_{j \in N_i} t_{ij}(e)\right)\right] \\
&= \alpha J(s)+(1-\alpha) J(t).
\end{aligned}
$$

∎

Lemma. 3: *J is Gâteaux-differentiable.*

Proof. $\forall s, t \in \mathcal{T}$, for $\alpha > 0$,

$$\begin{aligned}
&\frac{J(t+\alpha s) - J(t)}{\alpha} \\
&= \mathbb{E}\left[\sum_i \lambda_i \left[\frac{u_i\left(e_i - \sum_{j\in N_i} t_{ij}(e) - \alpha \sum_{j\in N_i} s_{ij}(e)\right) - u_i\left(e_i - \sum_{j\in N_i} t_{ij}(e)\right)}{\alpha}\right]\right] \\
&= \mathbb{E}\left[\sum_i \lambda_i \left[-\frac{u_i^{'}\left(e_i - \sum_{j\in N_i} t_{ij}(e) - \alpha \tilde{s}(e)\right) \cdot \alpha \sum_{j\in N_i} s_{ij}(e)}{\alpha}\right]\right] \\
&\qquad\qquad\qquad\qquad \text{for some } \tilde{s}_{ij}(e) \text{ between } 0 \text{ and } \sum_{j\in N_i} s_{ij}(e) \\
&= -\mathbb{E}\left[\sum_i \lambda_i \left[u_i^{'}\left(e_i - \sum_{j\in N_i} t_{ij}(e) - \alpha\tilde{s}\right) \cdot \sum_{j\in N_i} s_{ij}(e)\right]\right] \\
&\to -\mathbb{E}\left[\sum_i \lambda_i \left[u_i^{'}\left(e_i - \sum_{j\in N_i} t_{ij}(e)\right) \cdot \sum_{j\in N_i} s_{ij}(e)\right]\right] \text{ as } \alpha \to 0 \\
&= -\sum_i \lambda_i \mathbb{E}\left[u_i^{'}\left(e_i - \sum_{j\in N_i} t_{ij}(e)\right) \mathbf{1}_{i\times N_i} \cdot s(e)\right] \\
&= \sum_i \lambda_i < f_i, s >
\end{aligned}$$

where

$$f_i(e) := -u_i^{'}\left(e_i - \sum_{j\in N_i} t_{ij}(e)\right) \mathbf{1}_{i\times N_i}$$

and $\mathbf{1}_{i\times N_i}$ is vector of 0 and 1s that equals 1 for the (directed) link ij for any $j \in N_i$ so that $\mathbf{1}_{i\times N_i} \cdot s(e) = \sum_{j\in N_i} s_{ij}(e)$. Define $J^{'}(t) : \mathcal{T} \to \mathbb{R}$ by

$$J^{'}(t)\, s = \sum_i \lambda_i < f_i, s > .$$

Clearly $J^{'}(t)$ is a linear operator on $\mathcal{T}$, and is thus the Gâteaux-derivative of J. ∎

Lemma. 4: *For any $t \in \mathcal{T}$ that solves (4), we have*

$$J^{'}(t) = \mathbf{0}.$$

Proof. To solve (4)

$$\max_{\tilde{t}_{ij}\in\mathbb{R}} J^{(ij,I_{ij})}\left(\tilde{t}_{ij}\right) := \mathbb{E}\left[\lambda_i u_i\left(e_i - \tilde{t}_{ij} - \sum_{h\in N_i} t_{ih}\right) + \lambda_j u_j\left(e_j + \tilde{t}_{ij} - \sum_{h\in j} t_{jh}\right)\middle| I_{ij}\right]$$

we first notice the objective function $J^{(ij,I_{ij})}\left(\tilde{t}_{ij}\right)$ is strictly concave in $\tilde{t}_{ij}$ on $\mathbb{R}$. Hence, the sufficient and necessary condition for optimality is given by the FOC:

$$\mathbb{E}\left[\lambda_i u_i^{'}\left(e_i - \sum_{h\in N_i} t_{ih}(e)\right)\middle| I_{ij}\right] = \mathbb{E}\left[\lambda_j u_j^{'}\left(e_j - \sum_{h\in N_j} t_{jh}(e)\right)\middle| I_{ij}\right]$$

Then, $\forall s \in \mathcal{T}$,

$$\begin{aligned}
J^{'}(t)\, s &= -\mathbb{E}\left[\sum_i \lambda_i \left[u_i^{'}\left(e_i - \sum_{j\in N_i} t_{ij}(e)\right)\cdot \sum_{j\in N_i} s_{ij}(e)\right]\right] \\
&= -\frac{1}{2}\sum_{G_{ij}=1}\mathbb{E}\left[\left(\lambda_i u_i^{'}\left(e_i - \sum_{h\in N_i} t_{ih}(e)\right) - \lambda_j u_j^{'}\left(e_j - \sum_{h\in N_j} t_{jh}(e)\right)\right)\cdot s_{ij}(e)\right] \\
&= -\frac{1}{2}\sum_i\sum_{j\in N_i}\mathbb{E}\left[s_{ij}(I_{ij})\cdot \mathbb{E}\left[\lambda_i u_i^{'}\left(e_i - \sum_{h\in N_i} t_{ih}(e)\right) - \lambda_j u_j^{'}\left(e_j - \sum_{h\in N_j} t_{jh}(e)\right)\middle| I_{ij}\right]\right] \\
&= -\frac{1}{2}\sum_i\sum_{j\in N_i}\mathbb{E}\left[s_{ij}(I_{ij})\cdot 0\right] \\
&= 0.
\end{aligned}$$

Hence $J^{'}(t) = \mathbf{0}$. ∎

Lemma. 5:*The set of consumption plan induced by the profiles of transfer rules t in $\mathcal{T}$ is convex.*

Proof. Let $x, x^{'}$ be two profiles of consumption plans induced by $t, t^{'}$ respectively. Then $\forall\lambda \in [0,1]$,

$$\begin{aligned}
\lambda x_i(e) + (1-\lambda)\, x_i^{'}(e) &= \lambda\left[e_i - \sum_{j\in N_i} t_{ij}(e)\right] + (1-\lambda)\left[e_i - \sum_{j\in N_i} t_{ij}^{'}(e)\right] \\
&= e_i - \sum_{j\in N_u}\left[\lambda t_{ij}(e) + (1-\lambda)\, t_{ij}^{'}(e)\right]
\end{aligned}$$

Thus $\left(\lambda x + (1-\lambda) x'\right)$ can be induced by $\left(\lambda t + (1-\lambda) t'\right)$. $\mathcal{T}$, as an inner product space, is convex, so the set of consumption plans induced by the profiles of transfer rules in $\mathcal{T}$ must also be convex. ∎

B.2 Proof of Lemma 6

Lemma. 6: *Given any real vector $c \in \mathbb{R}^n$ such that $\sum_{i\in N} c_i = 0$, there exists a real vector $\mu \in \mathbb{R}^{\sum_i d_i}$ such that $\mu_{ik} + \mu_{ki} = 0$ for every linked pair ik and*

$$\sum_{k\in N_i} \mu_{ik} = c_i. \tag{24}$$

The solution is unique if and only if the network is minimally connected.

Proof. With the restrictions that $\mu_{ik} = -\mu_{ki}$ for all linked pair ik, (24) constitutes a system of n linear equations with $\frac{1}{2}\sum_{i\in N} d_i$ variables μ_{ik}. Summing up all the n equations, we have

$$0 = \sum_{i<k,\, G_{ik}=1} (\mu_{ik} + \mu_{ki}) = \sum_{i\in N} c_i = 0.$$

Hence, the n linear equations impose at most $(n-1)$ linearly independent conditions.

Viewing (24) in vector form,

$$C\mu = c$$

where C is a $n \times \frac{1}{2}\sum_{i\in N} d_i$ matrix. Note that in each column of C, denoted C_{ij} for $i<j$, there are either no nonzero entries (when $G_{ij}=0$), or just two nonzero entries: 1 on the i-th row and -1 on the j-th row when $G_{ij}=1$. Suppose $G_{ij}=1$. Then, given any subset of individuals S that include i and j, if the rows of C corresponding to S are linearly dependent, these rows must sum to 0: this can be true only if all entries ik with $i \in S$ and $k \notin S$ are zero, implying that S form a component under G, and thus G is not connected if $\#(S) < n$. This is in contradiction with the supposition that G is connected when $\#(S) < n$. Hence, C must have exactly $(n-1)$ linearly independent rows.

Let $\tilde{C}$ and $\tilde{c}$ be the first $(n-1)$ rows of C and c. Then, as $\tilde{C}$ has full row rank, there always exists a solution to $\tilde{C}\mu = \tilde{c}$, and any of the solutions μ must also solve the equation $C\mu = c$. The solution is unique if and only if the component is minimally connected, when there are precisely $(n-1)$ links and thus $\tilde{C}$ is an invertible square matrix.

We can obtain one particular solution using the following algorithm. First, we can arbitrarily select a subset of links that minimally connect the nodes, i.e., the graph restricted to this subset of links is minimally connected. Then, there must exist at least one peripheral node, and we can first easily obtain μ_{ij} for all such peripheral nodes $i \in P_1 := \{k \in N : \ d_k = 1\}$. Then, we can look for new peripheral nodes ignoring the links involving nodes in P_1, and obtain μ_{ij} for all $i \in P_2 := \{k \in N : \ k \notin P_1 \wedge G_{kj} = 1 \text{ for some } j \in P_1\}$ with all previously calculated μ's taken as given. We iterate this process until we exhaust all nodes. Then we are left with a profile of μ that solves (24). ∎

B.3 Proof of Lemma 7

Proof. For general network structures, the analysis is very similar to the above, but there are several complications. As $I_{ij} = \left(e_i, e_j, e_{N_{ij}}\right)$, the transfer rule t_{ij} can be contingent on $e_{N_{ij}} := (e_k)_{k \in N_{ij}}$ in addition to e_i, e_j. Furthermore, as the knowledge of the ex post realization of $e_{N_{ij}}$ brings in extra information about the distribution of non-local endowment realizations, Pareto efficiency requires that t_{ij} be contingent on $e_{N_{ij}}$. Specifically,

$$e_k|_{e_i, e_j, e_{N_{ij}}} \sim \mathcal{N}\left(\frac{\rho}{1 + (d_{ij} + 1)\,\rho}\left(e_i + e_j + \sum_{k \in N_{ij}} e_k\right),\ V_{d_{ij}+2}\right) \tag{25}$$

where $d_{ij} := \#\left(N_{ij}\right)$ and $V_{d_{ij}+2}$ denotes the variance of e_k conditional on observing $(d_{ij} + 2)$ endowment realizations.[40]

We again postulate a linear transfer rule: $t_{ij} = \alpha_{ij} e_i - \alpha_{ji} e_j + \sum_{k \in N_{ij}} \beta_{ijk} e_k + \mu_{ij}$, and plug in the postulated form to obtain a system of verification equations. Again, we ignore the verification equations for the state-independent transfers μ, and defer the discussion of μ to Section 6.3. After some tedious algebraic transformations, we again arrive at a rather complicated system of linear equations in (α, β) that defines the condition for Pareto efficiency, namely system (26) as shown below.

Lemma 9. *A linear profile of transfer rules $t = (\alpha, \beta, \mu)$ is Pareto efficient if $\forall ij$ s.t.*

[40] See, for example, Eaton (2007), p116-117.

$G_{ij} = 1$,

$$\begin{cases} \alpha_{ij} & = \frac{1}{2}\left(1 - \sum_{k \in N_i \backslash \{j\}} \alpha_{ik} + \sum_{k \in N_{ij}} \beta_{jki} + \gamma_{ij}\right) \\ \beta_{ijk} & = \frac{1}{2}\Big[\alpha_{ki} - \alpha_{kj} + \sum_{h \in N_{ijk}} \left(\beta_{ihk} - \beta_{jhk}\right) \\ & \quad - \sum_{h \in N_{ik} \backslash \overline{N}_j} \beta_{ihk} + \sum_{h \in N_{jk} \backslash \overline{N}_i} \beta_{jkh} + \gamma_{ij}\Big] \quad \forall k \in N_{ij} \\ \gamma_{ij} & = \frac{\rho}{1+(d_{ij}+1)\rho}\Big[\sum_{k \in N_i \backslash \overline{N}_j} \left(\alpha_{ki} - \sum_{h \in N_{ik} \backslash \overline{N}_j} \beta_{ikh}\right) \\ & \quad - \sum_{k \in N_j \backslash \overline{N}_i} \left(\alpha_{kj} - \sum_{h \in N_{jk} \backslash \overline{N}_i} \beta_{jkh}\right) \\ & \quad - \sum_{k \in N_{ij}} \left(\sum_{h \in N_{ik} \backslash \overline{N}_j} \beta_{ikh} - \sum_{h \in N_{jk} \backslash \overline{N}_i} \beta_{jkh}\right)\Big] \end{cases} \tag{26}$$

Instead of solving for this complicated system directly, we first present an innocuous simplification of it. Due to the possible existence of cycles and superfluous transfers along cycles, this system may in general admit multiple solutions. For example, given a complete triad ijk, we can make a superfluous transfer of a ϵ share of e_i from i to j, j to k and k to i by adding ϵ to α_{ij}, β_{jki}, and subtracting ϵ from α_{ik}. It can then be checked that this operation is indeed superfluous, in the sense that $\left(\alpha_{ij} + \epsilon, \beta_{jki} + \epsilon, \beta_{kji} - \epsilon, \alpha_{ik} - \epsilon\right)$, keeping everything else fixed, still solves the system of equations for Pareto efficiency with the induced final consumption plan left unchanged. Since any amount of superfluous cycles are redundant, we can set $\beta_{ijk} = 0$ for all triads ijk without loss of Pareto efficiency. Hence, in the following, we establish that there exists some vector of strictly bilateral transfer shares $(\alpha^*, \beta^* \equiv \mathbf{0})$ that solves (26) and thus achieves Pareto efficiency. In other words, the strictly bilateral linear transfer rules that we characterize below are the "simplest" Pareto efficient rules in terms of minimizing the sum of state-contingent transfers.

By setting $\beta = \mathbf{0}$, we achieve a significant simplification of (26) and obtain the system (20), which is repeated here for easier reference:

$$\begin{cases} \alpha_{ij} = \frac{1}{2}\left(1 - \sum_{k \in N_i \backslash \{j\}} \alpha_{ik} + \gamma_{ij}\right) & \text{(??.1)} \\ 0 = \alpha_{ki} - \alpha_{kj} + \gamma_{ij} \quad \forall k \in N_{ij} & \text{(??.2)} \\ \gamma_{ij} = \frac{\rho}{1+(d_{ij}+1)\rho}\left(\sum_{k \in N_i \backslash \overline{N}_j} \alpha_{ki} - \sum_{k \in N_j \backslash \overline{N}_i} \alpha_{kj}\right) & \text{(??.3)} \end{cases} \forall i, j \text{ s.t. } G_{ij} = 1$$

The first equation (20.1) states that the share of e_i transferred from i to j is half of the remaining share after i's transfers to i's other neighbors plus the informational adjustment term between ij. With $\gamma \equiv 0$, which is implied by $\rho = 0$, α will be simply reduced to the local equal sharing rule. The second equation (20.2) requires that the difference in the shares of e_k undertaken by i and j is equal to the informational effect between ij, so that it is indeed optimal for ij to set $\beta_{ijk} = 0$. This confirms again that strict bilaterality ($\beta = \mathbf{0}$) is not an assumption, as (20.2) also incorporates the efficiency requirements for $\beta = \mathbf{0}$. The third equation (20.3) defines the auxiliary variable γ_{ij}. We interpret γ_{ij} as the net informational effect because it is the rate at which locally observed endowment realizations affect the pair ij's joint expectation of non-local endowments. Notice that γ_{ij} is the same across $k \in \overline{N}_{ij}$ because each element of $(e_k)_{k \in \overline{N}_{ij}}$ provides exactly the same amount of information to the linked pair ij for their joint inference on non-local endowments. Given α, $\left|\gamma_{ij}\right|$ is decreasing in d_{ij}, indicating that the magnitude of the informational effect (for any single endowment realization) is decreasing in the amount of local information. Below we proceed to show the existence and provide a closed-form characterization of a solution to (20).

We first prove that (20.2) are implied by (20.1) and (20.3). By differencing (20.1) for ki and for kj we get: $\alpha_{ki} - \alpha_{kj} = \gamma_{ki} - \gamma_{kj}$. Hence, in the presence of (20.1) equation (20.2) is equivalent to, for all triads ijk, $\gamma_{ij} + \gamma_{jk} + \gamma_{ki} = 0$. This is reminiscent of the Kirchhoff Voltage Law for electric resistor networks, which states that the sum of voltage differences across any closed cycle must sum to zero. It turns out that the Kirchhoff Voltage Law indeed holds in our setting for any cycle in a general network.

Lemma 10. ***"Kirchhoff Voltage Law"****: $\forall \rho \in \left(-\frac{1}{n-1}, 1\right)$, if (20.1) and (20.3) admit a unique solution (α, γ), this solution also satisfy (20.2); furthermore, given any cycle $i_1 i_2 ... i_m i_1$, γ satisfies the "Kirchhoff Voltage Law" $\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1} = 0$.*

Intuitively, Pareto optimality requires that ij share equally the net difference in the conditional expectations of *nonlocal* inflow exposures (captured by γ_{ij}) by creating an opposite net difference in their *local* inflow exposures, as specified in equation (20.2). This adjustment guarantees the expectational Borch rule in equation (5), and therefore Pareto efficiency. To see this, notice that conditional expectation and variance of consumption will differ only by a constant across different local states (I_{ij}). Together, this implies that conditional CE's differ only by a constant, as required.

Given the redundancy of (20.2) in the presence of (20.1) and (20.3), we may now conclude that any solution to the system consisting of (20.1) and (20.3) defines a linear and Pareto efficient profile of transfer rules in $\mathcal{T}$. ∎

B.4 Proof of Lemma 8

Proof. Write system (20) in the following form:

$$\begin{cases} 2\alpha_{ij} + \sum_{k\in N_i\backslash\{j\}} \alpha_{ik} - \gamma_{ij} = 1, \; (1)_{ij} \quad \forall G_{ij} = 1 \\ \gamma_{ij} = \frac{\rho}{1+(d_{ij}+1)\rho}\left(\sum_{k\in N_i\backslash \overline{N}_j} \alpha_{ki} - \sum_{k\in N_j\backslash \overline{N}_i} \alpha_{kj}\right), \; (3)_{ij} \quad \forall G_{ij} = 1. \end{cases}$$

This is a system of $2\sum_i d_i$ equations in $2\sum_i d_i$ variables (α, γ). Notice that this system can have at most one solution by Proposition 2, as each distinct solution to the above system will define a distinct consumption plan.

Write system (10) in the following form: $\forall ij$ s.t. $G_{ij} = 1$, and $\forall i \in N$

$$\begin{cases} \alpha_{ji} = \Lambda_j - \frac{\rho}{1-\rho}\left(\sum_{k\in N_i} \alpha_{ki} + \alpha_{ii}\right), \; (12)_{ji} \quad \forall G_{ij} = 1 \\ \alpha_{ii} = \Lambda_i - \frac{\rho}{1-\rho}\left(\sum_{k\in N_i} \alpha_{ki} + \alpha_{ii}\right), \; (12)_{ii} \quad \forall i \in N \\ \alpha_{ii} + \sum_{k\in N_i} \alpha_{ik} = 1, \; (13)_i \quad \forall i \in N \end{cases}$$

This is a system of $(\sum_i d_i + 2n)$ equations in $(\sum_i d_i + 2n)$ variables (α, Λ). Suppose that this system has a unique solution. [41]

We now show that there exist $\sum_i d_i$ linearly independent sequences of row operations that produce the tautology "0 = 0". Given that the system (12)(13) has a unique solution $\left((\alpha_{ij})_{G_{ij}=1}, \alpha\right)$, this will imply that the $(\alpha_{ij})_{G_{ij}=1}$, along with γ defined by (3), will also solve system (1)(3).

Notice that, by the proof of Proposition 10 in Appendix B.6, (1) and (3) imply that

$$(1+\rho)\,\gamma_{ij} = \rho\left(\sum_{h\in N_i} \alpha_{hi} - \sum_{h\in N_j} \alpha_{hj} + \alpha_{ij} - \alpha_{ji}\right) \quad (5)_{ij}.$$

In other words, $(5)_{ij}$ can be obtained by a sequence of row operations on (1) and (3).

Consider a fixed linked pair ij with $i < j$.

By $(1-\rho) \times \left((12)_{ji} - (12)_{ij} + (12)_{ii} - (12)_{jj}\right)$, we have

$$(1-\rho)\left(\alpha_{ji} - \alpha_{ij} + \alpha_{ii} - \alpha_{jj}\right) + 2\rho\left(\sum_{k\in N_i} \alpha_{ki} + \alpha_{ii} - \sum_{k\in N_j} \alpha_{kj} - \alpha_{jj}\right) = 0,$$

[41] It indeed has a unique solution given by Proposition 4.

which is equivalent to

$$(1+\rho)(\alpha_{ii}-\alpha_{jj}+\alpha_{ij}-\alpha_{ji})+2\rho\left(\sum_{k\in N_i}\alpha_{ki}-\sum_{k\in N_j}\alpha_{kj}+\alpha_{ij}-\alpha_{ji}\right)=0.$$

Plugging $⑤_{ij}$ into the second term above, we have

$$(1+\rho)(\alpha_{ii}-\alpha_{jj})+(1-\rho)(\alpha_{ji}-\alpha_{ij})+2(1+\rho)\gamma_{ij}=0,$$

which, divided by $(1+\rho)$ on both sides, is equivalent to

$$\alpha_{ii}-\alpha_{jj}+\alpha_{ji}-\alpha_{ij}+2\gamma_{ij}=0. (14)_{ij}$$

By $(13)_i - (1)_{ij}$, we have

$$\alpha_{ii}-\alpha_{ij}+\gamma_{ij}=0. (15)_{ij}$$

By $(13)_j - (1)_{ji}$, we have $\alpha_{jj}-\alpha_{ji}+\gamma_{ji}=0$. As $(3)_{ij}+(3)_{ji}$ implies $\gamma_{ij}+\gamma_{ji}=0$, we have

$$\alpha_{jj}-\alpha_{ji}-\gamma_{ij}=0. (16)_{ij}$$

By $(14)_{ij}-(15)_{ij}+(16)_{ij}$, we reach the tautology "$0=0$".

Now, consider $(12)_{ji}+(12)_{ij}-(12)_{ii}-(12)_{jj}$, which leads to

$$\alpha_{ji}+\alpha_{ij}-\alpha_{ii}-\alpha_{jj}=0. (17)_{ij}$$

Then $(17)_{ij}+(15)_{ij}+(16)_{ij}$ leads to the tautology "$0=0$".

In summary of the above, for each fixed linked pair ij with $i<j$, we have established that

$$\begin{cases}\xi_{ij}: & \frac{1-\rho}{1+\rho}\left((12)_{ji}-(12)_{ij}+(12)_{ii}-(12)_{jj}\right)-(13)_i+(13)_j+\zeta_{ij}'(1)+\eta_{ij}'(3)=\mathbf{0}'.\\ \tilde{\xi}_{ij}: & (12)_{ji}+(12)_{ij}-(12)_{ii}-(12)_{jj}+(13)_i+(13)_j+\tilde{\zeta}_{ij}'(1)+\tilde{\eta}_{ij}'(3)=\mathbf{0}'.\end{cases}$$

for some conformable vector $\zeta_{ij},\tilde{\zeta}_{ij},\eta_{ij},\tilde{\eta}_{ij}$. Clearly, the two tautology-generating row operations above are linear independent: any linear combination of the two operations that cancels out $(12)_{ji}$ cannot cancel out $(12)_{ij}$.

Moreover, $(12)_{ij}, (12)_{ji}$ do not show up in any tautology-generating row operation within $\left\{\xi_{hk}, \tilde{\xi}_{hk} : (i,j) \neq \{h,k\}\right\}$, so $\left\{\xi_{ij}, \tilde{\xi}_{ij}\right\}$ must be linearly independent from $\left\{\xi_{hk}, \tilde{\xi}_{hk} : (i,j) \neq \{h,k\}\right\}$.

Hence, we have constructed a set of $\sum_i d_i$ linearly independent tautology-generating row operations $\left\{\xi_{ij}, \tilde{\xi}_{ij} : G_{ij} = 1, \; i < j\right\}$. ∎

B.5 Proof of Lemma 9

Lemma. 9: *A linear profile of transfer rules $t = (\alpha, \beta, \mu)$ is Pareto efficient if $\forall ij$ s.t. $G_{ij} = 1$,*

$$\begin{cases} \alpha_{ij} & = \frac{1}{2}\left(1 - \sum_{k \in N_i \setminus \{j\}} \alpha_{ik} + \sum_{k \in N_{ij}} \beta_{jki} + \gamma_{ij}\right) \\ \beta_{ijk} & = \frac{1}{2}\Big[\alpha_{ki} - \alpha_{kj} + \sum_{h \in N_{ijk}} \left(\beta_{ihk} - \beta_{jhk}\right) \\ & \quad - \sum_{h \in N_{ik} \setminus \overline{N}_j} \beta_{ihk} + \sum_{h \in N_{jk} \setminus \overline{N}_i} \beta_{jkh} + \gamma_{ij}\Big] \quad \forall k \in N_{ij} \\ \gamma_{ij} & = \frac{\rho}{1 + (d_{ij}+1)\rho}\Big[\sum_{k \in N_i \setminus \overline{N}_j} \left(\alpha_{ki} - \sum_{h \in N_{ik} \setminus \overline{N}_j} \beta_{ikh}\right) \\ & \quad - \sum_{k \in N_j \setminus \overline{N}_i} \left(\alpha_{kj} - \sum_{h \in N_{jk} \setminus \overline{N}_i} \beta_{jkh}\right) \\ & \quad - \sum_{k \in N_{ij}} \left(\sum_{h \in N_{ik} \setminus \overline{N}_j} \beta_{ikh} - \sum_{h \in N_{jk} \setminus \overline{N}_i} \beta_{jkh}\right)\Big] \end{cases} \tag{26}$$

Proof. For each $k \in N_i \setminus \{j\}$, we then have

$$\begin{aligned} \sum_{k \in N_i \setminus \{j\}} t_{ik} = e_i \sum_{k \in N_i \setminus \{j\}} \alpha_{ik} - \sum_{k \in N_i \setminus \{j\}} \alpha_{ki} e_k + \sum_{k \in N_i \setminus \{j\}} \sum_{h \in N_{ik}} \beta_{ikh} e_h + c_{ij} \\ = e_i \sum_{k \in N_i \setminus \{j\}} \alpha_{ik} - \sum_{k \in N_{ij}} \alpha_{ki} e_k + \sum_{k \in N_{ij}} \left(\beta_{ikj} e_j + \sum_{h \in N_{ijk}} \beta_{ikh} e_h\right) + \sum_{k \in N_i \setminus \overline{N}_j} \sum_{h \in N_{ijk}} \beta_{ikh} e_h \\ - \sum_{k \in N_i \setminus \overline{N}_j} \alpha_{ki} e_k + \sum_{k \in N_{ij}} \sum_{h \in N_{ik} \setminus \overline{N}_j} \beta_{ikh} e_h + \sum_{k \in N_i \setminus \overline{N}_j} \sum_{h \in N_{ik} \setminus \overline{N}_j} \beta_{ikh} e_h + c_{ij} \end{aligned}$$

so that

$$t_{ij} = \frac{1}{2} e_i - \frac{1}{2} e_j - \frac{1}{2} \sum_{k \in N_i \setminus \{j\}} \alpha_{ik} e_i + \frac{1}{2} \sum_{k \in N_j \setminus \{i\}} \alpha_{jk} e_j - \frac{1}{2} \sum_{k \in N_{ij}} \left(\beta_{ikj} e_j - \beta_{jki} e_i\right)$$

$$
\begin{aligned}
&+\frac{1}{2}\sum_{k\in N_{ij}}\left[\left(\alpha_{ki}-\alpha_{kj}\right)e_k-\sum_{h\in N_{ijk}}\left(\beta_{ikh}-\beta_{jkh}\right)e_h\right]\\
&-\frac{1}{2}\sum_{k\in N_i\setminus\overline{N}_j}\sum_{h\in N_{ijk}}\beta_{ikh}e_h+\frac{1}{2}\sum_{k\in N_j\setminus\overline{N}_i}\sum_{h\in N_{ijk}}\beta_{jkh}e_h\\
&-\frac{1}{2}\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\left(e_i+e_j+\sum_{k\in N_{ij}}e_k\right)\sum_{k\in N_{ij}}\left(\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\\
&+\frac{1}{2}\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\left(e_i+e_j+\sum_{k\in N_{ij}}e_k\right)\sum_{k\in N_i\setminus\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)\\
&-\frac{1}{2}\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\left(e_i+e_j+\sum_{k\in N_{ij}}e_k\right)\sum_{k\in N_j\setminus\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\\
=&\frac{1}{2}\left\{1-\sum_{k\in N_i\setminus\{j\}}\alpha_{ik}+\sum_{k\in N_{ij}}\beta_{jki}+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\left[\sum_{k\in N_i\setminus\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)\right.\right.\\
&\left.\left.-\sum_{k\in N_j\setminus\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)-\sum_{k\in N_{ij}}\left(\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\right]\right\}\cdot e_i\\
&-\frac{1}{2}\left\{1-\sum_{k\in N_j\setminus\{i\}}\alpha_{jk}+\sum_{k\in N_{ij}}\beta_{ikj}+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\left[\sum_{k\in N_j\setminus\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\right.\right.\\
&\left.\left.-\sum_{k\in N_i\setminus\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)-\sum_{k\in N_{ij}}\left(\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)\right]\right\}\cdot e_j\\
&+\frac{1}{2}\sum_{k\in N_{ij}}\left\{\alpha_{ki}-\alpha_{kj}+\sum_{h\in N_{ijk}}\left(\beta_{ihk}-\beta_{jhk}\right)-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ihk}+\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right.\\
&+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\left[\sum_{k\in N_i\setminus\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)-\sum_{k\in N_j\setminus\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\right.\\
&\left.\left.-\sum_{k\in N_{ij}}\left(\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\right]\right\}\cdot e_k+C_{ij}
\end{aligned}
$$

The last equality is obtained by collecting terms and switching summand indexes. ∎

B.6 Proof of Lemma 10

Lemma. 10: ***"Kirchhoff Voltage Law"***: *$\forall \rho \in \left(-\frac{1}{n-1}, 1\right)$, if (20.1) and (20.3) admit a unique solution (α, γ), this solution also satisfy (20.2); furthermore, given any cycle $i_1 i_2 \ldots i_m i_1$, γ satisfies the "Kirchhoff Voltage Law" $\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + \ldots + \gamma_{i_m i_1} = 0$.*

Proof. We begin by proving the first part, which only involves triads. We rewrite (20) in the following way:

$$\begin{cases} 2\alpha_{ij} + \sum_{k \in N_i \setminus \{j\}} \alpha_{ik} - \gamma_{ij} = 1, \quad \forall G_{ij} = 1 & ① \\ \alpha_{ki} - \alpha_{kj} + \gamma_{ij} = 0 \quad \forall k \in N_{ij}, \quad \forall G_{ij} = 1; & ② \\ \gamma_{ij} = \frac{\rho}{1+(d_{ij}+1)\rho} \left(\sum_{k \in N_i \setminus \overline{N}_j} \alpha_{ki} - \sum_{k \in N_j \setminus \overline{N}_i} \alpha_{kj} \right), \quad \forall G_{ij} = 1; & ③ \end{cases}$$

In matrix form we write

$$\begin{bmatrix} A \\ M \end{bmatrix} \begin{pmatrix} \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} b \\ \mathbf{0} \end{pmatrix} \qquad \begin{matrix} ① \wedge ③ \\ ② \end{matrix}$$

where α, γ are both $\sum_i d_i$-dimensional vectors, A is a $(2\sum_i d_i) \times (2\sum_i d_i)$ square matrix, $b := \begin{pmatrix} \mathbf{1}_{\sum_i d_i} \\ \mathbf{0}_{\sum_i d_i} \end{pmatrix}$ is a $(2\sum_i d_i)$-dimensional vector, M is a $\left(\sum_{G_{ij}=1} d_{ij}\right) \times (2\sum_i d_i)$ rectangular matrix, and $\mathbf{0}$ is a $\left(\sum_{G_{ij}=1} d_{ij}\right)$-dimensional vector. The upper block $A \begin{pmatrix} \alpha \\ \gamma \end{pmatrix} = b$ corresponds to equations in ① and ③, while the lower block $M \begin{pmatrix} \alpha \\ \gamma \end{pmatrix} = \mathbf{0}$ corresponds to equations in ②.

Note that $③_{ij}$ in the definition above is not written in its canonical form, i.e., it is not written in such way that the LHS of the equality sign is a linear combination of unknown variables (α, γ) while the right-hand side (RHS) is a constant scalar. In the following, we interpret any written linear equation to be representative of the underlying canonical form obtained by moving all linear combinations of (α, γ) on the RHS of "=" to the LHS (left-hand side) while moving all constants on the LHS to the RHS. For example, we interpret $③_{ij}$ to represent a canonical form such that the coefficient before the unknown variable γ_{ij} is 1 and the coefficient before α_{ki} for some $k \in N_i \backslash \overline{N}_j$ to be $-\frac{\rho}{1+(d_{ij}+1)\rho}$.[42]

Given that the system consisting of (1) and (3) admit a unique solution, its coefficient matrix A and its augmented matrix $\tilde{A} = [A|\, b]$ must have full rank $2\sum_i d_i$. To prove that the unique solution of (1) and (3) also satisfies (2), it suffices to show that the augmented matrix for the system of (1), (3) and (2)

$$\left[\begin{array}{c|c} A & b \\ M & \mathbf{0} \end{array}\right]$$

still have rank $2\sum_i d_i$. We show this by demonstrating the existence of $\sum_{G_{ij}=1} d_{ij}$ nonzero and linearly independent vector $\xi \in \mathbb{R}^{2\sum_i d_i + \sum_{G_{ij}=1} d_{ij}}$ such that

$$\xi' \begin{bmatrix} A & b \\ M & \mathbf{0} \end{bmatrix} = (0, 0, ..., 0)_{2\sum_i d_i + 1}\,.$$

We first fix any linked triad ijk.

Multiplying $(3)_{ij}$ (the ij-th equation in (3)) with $(1 + (d_{ij} + 1)\,\rho)$, we obtain $[1 + (d_{ij} + 1)\,\rho]\,\gamma_{ij} = \rho \left(\sum_{h \in N_i \backslash \overline{N}_j} \alpha_{hi} - \sum_{h \in N_j \backslash \overline{N}_i} \alpha_{hj}\right)$, which is equivalent to

$$[1 + (d_{ij} + 1)\,\rho]\,\gamma_{ij} = \rho \left[\sum_{h \in N_i} \alpha_{hi} - \sum_{h \in N_j} \alpha_{hj} - \sum_{h \in N_{ij}} (\alpha_{hi} - \alpha_{hj}) - \alpha_{ji} + \alpha_{ij}\right] \quad (4)_{ij}.$$

Adding $(2)_{ijh}$ for all $h \in N_{ij} \backslash \{k\}$ to $(4)_{ij}$, we get

$$[1 + (d_{ij} + 1)\,\rho]\,\gamma_{ij} = \rho \left[\sum_{h \in N_i} \alpha_{hi} - \sum_{h \in N_j} \alpha_{hj} + (d_{ij} - 1)\,\gamma_{ij} - \alpha_{ki} + \alpha_{kj} - \alpha_{ji} + \alpha_{ij}\right]$$

which is equivalent to

$$(1 + 2\rho)\,\gamma_{ij} = \rho \left(\sum_{h \in N_i} \alpha_{hi} - \sum_{h \in N_j} \alpha_{hj} + \alpha_{kj} - \alpha_{ki} + \alpha_{ij} - \alpha_{ji}\right) \quad (5)_{ij}.$$

[42]This convention should resolve any ambiguity about the signs of coefficients before (α, γ) in all the equations written out thereafter.

Summing up $⑤_{ij}, ⑤_{jk}, ⑤_{ki}$, we have

$$\begin{aligned}(1+2\rho)\left(\gamma_{ij}+\gamma_{jk}+\gamma_{ki}\right) &= \rho\left[(\alpha_{kj}-\alpha_{ki}+\alpha_{ij}-\alpha_{ji})+(\alpha_{ik}-\alpha_{ij}+\alpha_{jk}-\alpha_{kj})\right. \\ &\qquad \left.+(\alpha_{ji}-\alpha_{jk}+\alpha_{ki}-\alpha_{ik})\right] \\ &= 0\end{aligned}$$

For $n=3$ and $\rho > -\frac{1}{2}$, or for $n \geq 4$, we have $1+2\rho > 0$ and thus

$$\gamma_{ij}+\gamma_{jk}+\gamma_{ki}=0. \quad ⑥_{ijk}$$

Alternatively, taking $①_{ki} - ①_{kj} + ②_{ijk}$, we obtain $\gamma_{ij}-\gamma_{kj}+\gamma_{ki}=0$. By $③_{jk} + ③_{kj}$, we have $\gamma_{jk}+\gamma_{kj}=0$ and thus

$$\gamma_{ij}+\gamma_{jk}+\gamma_{ki}=0. \quad ⑦_{ijk}$$

Then $⑥_{ijk} - ⑦_{ijk}$ leads to the tautology "$0=0$". Let $\xi^{ijk} \in \mathbb{R}^{2\sum_i d_i + \sum_{G_{ij}=1} d_{ij}}$ be a vector that characterizes all the row operations conducted above. Clearly

$$\left(\xi^{ijk}\right)'\begin{bmatrix} A & b \\ M & \mathbf{0}\end{bmatrix} = \mathbf{0}'.$$

Notice that we can obtain one ξ^{ijk} for each ordered triad (i,j,k). Clearly each ξ^{ijk} is nonzero: in particular, the entries of ξ that correspond to equations $①_{ki}$ and $①_{kj}$ must be nonzero, $\xi_{①_{ki}} \neq 0$, $\xi_{①_{kj}} \neq 0$, because $①_{ki}, ①_{kj}$ are used to obtain $⑦_{ijk}$ and nowhere else.

Fixing k, for a row operation in question $\xi^{i_1 i_2 i_3}$, coefficients corresponding to $①_{kh}$ for $h \in N_k$ may be nonzero only if $i_3 = k$. Hence, $\left\{\xi^{i_1 i_2 k} : i_1, i_2 \in N_k, G_{i_1 i_2}=1\right\}$ must be linearly independent from $\left\{\xi^{i_1 i_2 i_3} : i_1, i_2 \in N_k, G_{i_1 i_2}=1, i_3 \neq k\right\}$. We now consider $\left\{\xi^{i_1 i_2 k} : i_1, i_2 \in N_k, G_{i_1 i_2}=1\right\}$. Notice that $①_{ki}, ①_{kj}$ show up in the form of "$①_{ki} - ①_{kj}$" during the process. Hence, summing up along general cycles[43] is the only possible type of row operations that can cancel out all coefficients before $①_{ki}$ for all $i \in N_k$. However, this operation does not lead to the tautology $\left(\mathbf{0}', 0\right)$, because the coefficients before $②_{i_1 i_2 k}, ..., ②_{i_m i_1 k}$ are all kept nonzero. (Notice that these only show up in $\xi^{i_1 i_2 k}$ in the step leading to $⑦_{ijk}$ and nowhere else).

[43] By general cycles we mean cycles that may involve "self cycles" of the form "$i_1 i_2 i_1$".

Hence, no nontrivial linear combination of $\left\{\xi^{i_1 i_2 k} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1\right\}$ is zero, so $\left\{\xi^{i_1 i_2 k} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1\right\}$ is linearly independent. In summary, we conclude that $\left\{\xi^{i_1 i_2 i_3} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1\right\}$ are linearly independent, so we have established the existence of $\sum_{G_{ij}=1} d_{ij}$ nonzero and linearly independent vector $\xi \in \mathbb{R}^{2\sum_i d_i + \sum_{G_{ij}=1} d_{ij}}$.

We now prove the second part, the statement for cycles of any size. Note that we still have $(5)_{ij}: \; (1+\rho)\gamma_{ij} = \rho\left(\sum_{h\in N_i}\alpha_{hi} - \sum_{h\in N_j}\alpha_{hj} + \alpha_{ij} - \alpha_{ji}\right)$. Given any cycle $i_1 i_2 ... i_m i_1$, summing up $(5)_{i_1 i_2}, (5)_{i_2 i_3}, ..., (5)_{i_m i_1}$, we have

$$(1+\rho)\left(\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1}\right) = \rho\left(\alpha_{i_1 i_2} + ... + \alpha_{i_m i_1} - \alpha_{i_2 i_1} - ... - \alpha_{i_1 i_m}\right) \qquad (10)$$

By $(1)_{i_1 i_2} - (1)_{i_2 i_1}$ and $\gamma_{ij} + \gamma_{ji} = 0$, we have $\alpha_{i_1 i_2} - \alpha_{i_2 i_1} = \sum_{h\in N_{i_2}}\alpha_{i_2 h} - \sum_{h\in N_{i_1}}\alpha_{i_1 h} + 2\gamma_{i_1 i_2}$. Summing over $i_1 i_2, ..., i_m i_1$,

$$\alpha_{i_1 i_2} + ... + \alpha_{i_m i_1} - \alpha_{i_2 i_1} - ... - \alpha_{i_1 i_m} = 2\left(\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1}\right) \qquad (11)$$

Then $(10) + \rho \times (11)$ gives $(1-\rho)\left(\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1}\right) = 0$. For $\rho < 1$, we have $\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1} = 0$. ∎

B.7 Uniqueness in Minimally Connected Networks

Proposition 9. *Under the independent CARA-Normal setting, if the network is minimally connected, then there is a unique profile of transfer rules in $\mathcal{T}^*$ that is Pareto efficient, and it takes the form of the local equal sharing rule.*

Proof. Consider minimally connected network G. For Pareto efficiency, we need for all linked pair ij

$$\frac{\mathbb{E}_{ij}\left[u_i'(x_i)\right]}{\mathbb{E}_{ij}\left[u_j'(x_j)\right]} = c_{ij}.$$

As the network is minimally connected, we have $N_{ij} = \emptyset$. Notice that

$$\mathbb{E}\left[re^{-r\left(e_i - t_{ij} - \sum_{k\in N_i\setminus\{j\}} t_{ik}(e_i,e_k)\right)}\middle| e_i, e_j\right] = c_{ij}\mathbb{E}\left[re^{-r\left(e_j + t_{ij} - \sum_{h\in N_j\setminus\{i\}} t_{jh}(e_j,e_h)\right)}\middle| e_i, e_j\right].$$

[44]By independence,

$$
\begin{aligned}
&\mathbb{E}\left[re^{-r\left(e_i-t_{ij}-\sum_{k\in N_i\backslash\{j\}}t_{ik}(e_i,e_k)\right)}\middle| e_i\right]=c_{ij}\mathbb{E}\left[re^{-r\left(e_j+t_{ij}-\sum_{h\in N_j\backslash\{i\}}t_{jh}(e_j,e_h)\right)}\middle| e_j\right]\\
\Leftrightarrow &e^{-r(e_i-t_{ij})}\cdot\prod_{k\in N_i\backslash\{j\}}\mathbb{E}\left[e^{rt_{ik}(e_i,e_k)}\middle| e_i\right]=c_{ij}e^{-r(e_j+t_{ij})}\cdot\prod_{h\in N_j\backslash\{i\}}\mathbb{E}\left[e^{rt_{jh}(e_j,e_h)}\middle| e_j\right]\\
\Leftrightarrow &e_i-t_{ij}-\frac{1}{r}\sum_{k\in N_i\backslash\{j\}}\ln\mathbb{E}\left[e^{rt_{ik}(e_i,e_k)}\middle| e_i\right]=e_j+t_{ij}-\frac{1}{r}\sum_{h\in N_j\backslash\{i\}}\ln\mathbb{E}\left[e^{rt_{jh}(e_j,e_h)}\middle| e_j\right]-\frac{1}{r}\ln c_{ij}\\
\Leftrightarrow &t_{ij}=\frac{1}{2}e_i-\frac{1}{2}e_j-\frac{1}{2r}\sum_{k\in N_i\backslash\{j\}}\ln\mathbb{E}\left[e^{rt_{ik}(e_i,e_k)}\middle| e_i\right]+\frac{1}{2r}\sum_{h\in N_j\backslash\{i\}}\ln\mathbb{E}\left[e^{rt_{jh}(e_j,e_h)}\middle| e_j\right]+\frac{1}{2r}\ln c_{ij} \qquad (27)\\
&=\frac{1}{2}e_i-\frac{1}{2}e_j-\frac{1}{2r}\sum_{k\in N_i\backslash\{j\}}\ln T_{ik}+\frac{1}{2r}\sum_{h\in N_j\backslash\{i\}}\ln T_{jh}+\frac{1}{2r}\ln c_{ij}
\end{aligned}
$$

where

$$T_{ik}:=\mathbb{E}\left[e^{rt_{ik}(e_i,e_k)}\middle| e_i\right]$$

Then, taking conditional expectations of (27), we have

$$
\begin{aligned}
T_{ij}&=e^{r\left(\frac{1}{2}e_i+\frac{1}{2}r\sigma^2-\frac{1}{2r}\sum_{k\in N_i\backslash\{j\}}\ln T_{ik}+\frac{1}{2r}\ln\alpha_{ij}\right)}\cdot\mathbb{E}\left[e^{r\frac{1}{2r}\sum_{h\in N_j\backslash\{i\}}\ln T_{jh}}\middle| e_i\right]\\
&=e^{r\left(\frac{1}{2}e_i+\frac{1}{2}r\sigma^2-\frac{1}{2r}\sum_{k\in N_i\backslash\{j\}}\ln T_{ik}+\frac{1}{2r}\ln\alpha_{ij}\right)}\cdot\prod_{h\in N_j\backslash\{i\}}\mathbb{E}\left[T_{jh}^{\frac{1}{2}}\right]
\end{aligned}
$$

and

$$\frac{1}{r}\ln T_{ij}=\frac{1}{2}e_i+\frac{1}{2}r\sigma^2-\frac{1}{2}\sum_{k\in N_i\backslash\{j\}}\frac{1}{r}\ln T_{ik}+\frac{1}{2r}\ln\alpha_{ij}+\sum_{h\in N_j\backslash\{i\}}\ln\mathbb{E}\left[T_{jh}^{\frac{1}{2}}\right].$$

Introducing notation

$$\tilde{T}_{ij}=\frac{1}{r}\ln T_{ij},$$

we have

$$\tilde{T}_{ij}=\frac{1}{2}e_i-\frac{1}{2}\sum_{k\in N_i\backslash\{j\}}\tilde{T}_{ik}+c_{ij}$$

[44]We hope the unfortunate notational coincidence of the endowment vector e and the natural exponential power $e^{\cdot}$ will not result in any confusion.

$\Rightarrow$

$$\sum_{j\in N_i}\tilde{T}_{ij} = \frac{d_i}{2}e_i - \frac{1}{2}\cdot(d_i-1)\sum_{j\in N_i}\tilde{T}_{ik} + \sum_{j\in N_i}c_{ij}$$

$\Rightarrow$

$$\sum_{j\in N_i}\tilde{T}_{ij} = \frac{d_i}{d_i+1}e_i + \frac{2}{d_i+1}\sum_{j\in N_i}c_{ij}$$

$\Rightarrow$

$$\begin{aligned}\tilde{T}_{ij} &= \frac{1}{2}e_i - \frac{1}{2}\sum_{k\in N_i\setminus\{j\}}\tilde{T}_{ik} + c_{ij}\\ &= \frac{1}{2}e_i - \frac{1}{2}\sum_{k\in N_i}\tilde{T}_{ik} + \frac{1}{2}\tilde{T}_{ij} + c_{ij}\end{aligned}$$

$\Rightarrow$

$$\frac{1}{2}\tilde{T}_{ij} = \frac{1}{2}\left(e_i - \frac{d_i}{d_i+1}e_i - \frac{2}{d_i+1}\sum_{k\in N_i}c_{ik}\right) + c_{ij}$$

$\Rightarrow$

$$\tilde{T}_{ij} = \frac{1}{d_i+1}e_i - \frac{1}{d_i+1}\sum_{k\in N_i}c_{ik} + c_{ij}$$

Hence, by (27), we have

$$\begin{aligned}t_{ij} &= \frac{1}{2}e_i - \frac{1}{2}e_j - \frac{1}{2}\sum_{k\in N_i\setminus\{j\}}\left(\frac{1}{d_i+1}e_i - \frac{1}{d_i+1}\sum_{k'\in N_i}c_{ik'} + c_{ik}\right)\\ &\quad + \frac{1}{2}\sum_{h\in N_j\setminus\{i\}}\left(\frac{1}{d_j+1}e_j - \frac{1}{d_j+1}\sum_{h'\in N_j}c_{jh'} + c_{jh'}\right) + \frac{1}{2r}\ln\alpha_{ij}\\ &= \frac{1}{2}\left(1 - \frac{d_i-1}{d_i+1}\right)e_i - \frac{1}{2}\left(1 - \frac{d_i-1}{d_i+1}\right)e_j + C_{ij}\\ &= \frac{1}{d_i+1}e_i - \frac{1}{d_i+1}e_j + C_{ij}.\end{aligned}$$

∎

B.8 Linear Pareto Efficient Transfer Shares for Boundary Correlation Parameters

Proposition 10.

- *For $\rho = -\frac{1}{n-1}$ and any network structure G such that $\max_{i \in N} d_i = n-1$, let i^* be any individual with $d_{i^*} = n-1$. Then a Pareto efficient profile of transfer rules is given by*

$$\alpha_{ji^*} = 1, \quad \alpha_{i^*j} = \alpha_{jk} = 0, \quad \forall j, k \in N \backslash \{i^*\}.$$

- *For $\rho = 1$ and any network structure G, any profile of transfer rules that satisfies the Kirchhoff Circuit Law as defined below is Pareto efficient:*

$$\sum_{j \in \overline{N}_i} \alpha_{ij} = \sum_{j \in \overline{N}_i} \alpha_{ji} \quad \forall i \in N.$$

Proof. For $\rho = -\frac{1}{n-1}$ and G s.t. $\max_{i \in N} d_i = n-1$, the profile of transfer rules given above attains zero variance in consumption for each individual, and is thus Pareto efficient. For $\rho = 1$, any profile of transfer rules that satisfies the Kirchhoff Circuit Law achieves the same profile of consumption plan as the null transfer (autarky), which is clearly Pareto efficient. ∎

B.9 Detailed Specification and Proof for Proposition 6

Specifically, we assume that the correlation between e_i and e_j geometrically decays with social distance between i and j :

$$Corr(e_i, e_j) = \varrho^{dist(i,j)},$$

where the social distance $dist(i, j)$ is formally defined as the length (i.e., the number of links) of the shortest path connecting i and j in network G. For notational simplicity we set $\sigma^2 = 1$.

For tractability, we restrict attention to circle networks with $n = 2m+1$ individuals. A n-circle consists of n individuals and n links: $G_{i,i+1} = 1$ for $i = 1, ..., n$.[45] For any linked pair $i, i+1$ along a n-circle (with $n \geq 4$), the conditional distribution of e_{i-1} (and similarly for e_{i+2}) is

$$e_{i-1}|_{e_i, e_{i+1}} \sim \mathcal{N}(\varrho e_i, 1-\varrho).$$

[45] We, for notational simplicity, define individual $n+1$ to be individual 1, and individual 0 to be individual n.

Following a similar argument as in Section 4.2, we obtain the following condition for Pareto efficiency subject to local information constraints:

$$\begin{cases} \alpha_{i,i+1} & = \frac{1}{2}\left(1-\alpha_{i,i-1}+\varrho\alpha_{i-1,i}\right) \\ \alpha_{i+1,i} & = \frac{1}{2}\left(1-\alpha_{i+1,i+2}+\varrho\alpha_{i+2,i+1}\right) \end{cases}$$

for all $i \in N$. Then, the unique and symmetric solution for the above system is given by

$$\alpha_{ij}^{*} \equiv \alpha^{geo}\left(\varrho\right) = \frac{1}{3-\varrho} \quad \forall G_{ij} = 1.$$

Under α^{*}, the final consumption for each individual is

$$x_i^{geo}\left(\varrho\right) = \frac{1}{3-\varrho}e_{i-1} + \frac{1-\varrho}{3-\varrho}e_i + \frac{1}{3-\varrho}e_{i+1}$$

with a variance of

$$Var_{geo,\varrho}\left(x_i^{geo}\left(\varrho\right)\right) = \frac{1+\varrho}{3-\varrho}.$$

In comparison, under the symmetric correlation structure in Section 4.2, the condition for Pareto efficiency on a n-circle is

$$\alpha_{i,i+1} = \frac{1}{2}\left[1-\alpha_{i,i-1}+\frac{\rho}{1+\rho}\left(\alpha_{i-1,i}-\alpha_{i+2,i+1}\right)\right]$$

with its unique and symmetric solution being

$$\alpha_{ij} \equiv \alpha^{unif}\left(\rho\right) = \frac{1}{3} \quad \forall G_{ij} = 1,$$

which is exactly the local equal sharing rule. This implies a final consumption of

$$x_i^{unif}\left(\rho\right) = \frac{1}{3}e_{i-1} + \frac{1}{3}e_i + \frac{1}{3}e_{i+1}$$

with a variance of

$$Var_{unif,\rho}\left(x_i^{unif}\left(\rho\right)\right) = \frac{1+2\rho}{3}.$$

We compare the correlation structures by setting ρ and ϱ to be such that each individual's consumption variance is equalized across the two correlation structures under the global equal sharing rule (which achieves first best risk sharing):

$$x_i^{FB} = \frac{1}{n} \sum_{k \in N} e_k.$$

The consumption variances that this sharing rule implies for the two correlation structures are:

$$\begin{aligned} Var_{unif,\rho}\left(x_i^{FB}\right) &= \frac{1 + 2m\rho}{2m+1} \\ Var_{geo,\varrho}\left(x_i^{FB}\right) &= \frac{1 + 2\sum_{k=1}^{m} \varrho^k}{2m+1} = \frac{2\frac{1-\varrho^{m+1}}{1-\varrho} - 1}{2m+1}, \end{aligned}$$

The first-best total variances under the two correlations structures are equal if and only if

$$\begin{aligned} Var_{unif,\rho}\left(x_i^{FB}\right) = Var_{geo,\varrho}\left(x_i^{FB}\right) &\Leftrightarrow \frac{1 + 2m\rho}{2m+1} = \frac{2\frac{1-\varrho^{m+1}}{1-\varrho} - 1}{2m+1} \\ &\Leftrightarrow \rho = \rho_m(\varrho) := \frac{\varrho\left(1 - \varrho^m\right)}{m\left(1 - \varrho\right)}. \end{aligned}$$

Noticing that the total variances without risk sharing at all are both equal to $(2m + 1)$ under either correlation structure, setting $\rho = \rho_m(\varrho)$ implies that the total amount of sharable risk is equalized between the two correlation structures. Next we compare the consumption variances given Pareto efficient risk-sharing arrangements subject to local information constraints.

Notice that

$$Var_{unif,\rho}\left(x_i^{unif}(\rho)\right) \leq Var_{geo,\varrho}\left(x_i^{geo}(\varrho)\right) \quad \Leftrightarrow \quad \rho \leq \overline{\rho}(\varrho) := \frac{2\varrho}{3-\varrho}.$$

Hence, whenever

$$m > \frac{(3-\varrho)\left(1-\varrho^m\right)}{2\left(1-\varrho\right)}$$

we will have $\rho(\varrho) < \overline{\rho}(\varrho)$ and thus $Var_{uni}^{\rho}\left(x_i^{unif}(\rho)\right) < Var_{geo}^{\varrho}\left(x_i^{geo}(\varrho)\right)$. In other words, fixing ϱ, efficient risk sharing subject to the local information constraint performs strictly better under the uniform correlation setting than under the geometrically decaying setting.

Moreover, the difference can be very stark. As $m \to \infty$,

$$\rho = \rho(\varrho) = \frac{\varrho(1-\varrho^m)}{m(1-\varrho)} \to 0,$$

and thus

$$Var_{unif,\rho}\left(x_i^{unif}(\rho)\right) = \frac{1+2\rho}{3} \to \frac{1}{3}, \quad \text{as } m \to \infty$$

while

$$Var_{geo,\rho}\left(x_i^{geo}(\varrho)\right) = \frac{1+\varrho}{3-\varrho} \quad \forall m.$$

When also taking $\varrho \to 1$ (after taking $m \to \infty$), we get

$$\begin{aligned}
&\lim_{\varrho\to 1}\lim_{m\to\infty} Var_{unif,\rho(\varrho)}\left(x_i^{unif}(\rho(\varrho))\right) = \frac{1}{3},\\
&\lim_{\varrho\to 1}\lim_{m\to\infty} Var_{geo,\rho}\left(x_i^{geo}(\varrho)\right) = 1.
\end{aligned}$$

B.10 Quadratic Utility Functions

With quadratic utility functions $u_i(x_i) = x_i - \frac{1}{2}rx_i^2$, the localized Borch rule requires that

$$\frac{\lambda_j}{\lambda_i} = \frac{\mathbb{E}_{ij}\left[u_i'(x_i)\right]}{\mathbb{E}_{ij}\left[u_i'(x_j)\right]} = \frac{\mathbb{E}_{ij}\left[1-rx_i\right]}{\mathbb{E}_{ij}\left[1-rx_j\right]}$$

$$\begin{aligned}
\Leftrightarrow\quad & \lambda_i - \lambda_i r\left(\mu_i + e_i - t_{ij} - \sum_{h\in N_i} E_{ij}\left[t_{ih}\right]\right) = \lambda_j - \lambda_j\left(\mu_j + e_j + t_{ij} - \sum_{h\in N_j}\mathbb{E}_{ij}\left[t_{jh}\right]\right)\\
\Leftrightarrow\quad & r(\lambda_i+\lambda_j)\,t_{ij} = -(\lambda_i-\lambda_j) + \lambda_i r\left(\mu_i + e_i - \sum_{h\in N_i\setminus j}\mathbb{E}_{ij}\left[t_{ih}(I_{ih})\right]\right)\\
&\qquad\qquad - \lambda_j r\left(e_j - \sum_{h\in N_j\setminus i}\mathbb{E}_{ij}\left[t_{jh}(I_{jh})\right]\right)\\
\Leftrightarrow\quad & t_{ij} = \frac{\lambda_i}{\lambda_i+\lambda_j}\left(\mu_i + e_i - \sum_{h\in N_i\setminus j}\mathbb{E}_{ij}\left[t_{ih}(I_{ih})\right]\right)\\
&\qquad - \frac{\lambda_j}{\lambda_i+\lambda_j}\left(\mu_j + e_j - \sum_{h\in N_j\setminus i}\mathbb{E}_{ij}\left[t_{jh}(I_{jh})\right]\right) - \frac{\lambda_i-\lambda_j}{r(\lambda_i+\lambda_j)}
\end{aligned}$$

Postulating a bilateral linear rule:

$$t_{ij}\left(I_{ij}\right)=\alpha_{ij}e_i-\alpha_{ji}e_j+c_{ij}$$

Notice that this is equivalent to specifying $t_{ij}\left(I_{ij}\right)=\alpha_{ij}y_i-\alpha_{ji}y_j+c_{ij}$ as we allow μ_{ij} are simultaneously determined along with:

$$\begin{aligned}
x_i&=\left(1-\sum_{j\in N_i}\alpha_{ij}\right)e_i+\sum_{j\in N_i}\alpha_{ji}e_j+\mu_i-\sum_{j\in N_i}c_{ij}\\
&\equiv\left(1-\sum_{j\in N_i}\alpha_{ij}\right)y_i+\sum_{j\in N_i}\alpha_{ji}y_j+\left(\sum_{j\in N_i}\alpha_{ji}\mu_i-\sum_{j\in N_i}\alpha_{ij}\mu_j\right)-\sum_{j\in N_i}c_{ij}
\end{aligned}$$

Plugging in the postulation,

$$\begin{aligned}
t_{ij}=&\frac{\lambda_i}{\lambda_i+\lambda_j}\left(\left(1-\sum_{h\in N_i\backslash j}\alpha_{ih}\right)e_i+\sum_{h\in N_{ij}}\alpha_{hi}e_h+\sum_{h\in N_i\backslash\overline{N}_j}\alpha_{hi}\mathbb{E}_{ij}\left[e_h\right]\right)\\
&-\frac{\lambda_j}{\lambda_i+\lambda_j}\left(\left(1-\sum_{h\in N_j\backslash i}\alpha_{jh}\right)e_j+\sum_{h\in N_{ij}}\alpha_{hj}e_h+\sum_{h\in N_j\backslash\overline{N}_i}\alpha_{hi}\mathbb{E}_{ij}\left[e_h\right]\right)\\
&+\frac{\lambda_i\left(\mu_i-\sum_{h\in N_i\backslash j}c_{ih}\right)-\lambda_j\left(\mu_j-\sum_{h\in N_j\backslash i}c_{jh}\right)}{\lambda_i+\lambda_j}-\frac{\lambda_i-\lambda_j}{r\left(\lambda_i+\lambda_j\right)}\\
=&\frac{\lambda_i}{\lambda_i+\lambda_j}\left(\left(1-\sum_{h\in N_i\backslash j}\alpha_{ih}\right)e_i+\sum_{h\in N_{ij}}\alpha_{hi}e_h+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\sum_{h\in N_i\backslash\overline{N}_j}\alpha_{hi}\cdot\sum_{k\in\overline{N}_{ij}}e_k\right)\\
&-\frac{\lambda_j}{\lambda_i+\lambda_j}\left(\left(1-\sum_{h\in N_j\backslash i}\alpha_{jh}\right)e_j+\sum_{h\in N_{ij}}\alpha_{hj}e_h+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\sum_{h\in N_j\backslash\overline{N}_i}\alpha_{hi}\cdot\sum_{k\in\overline{N}_{ij}}e_k\right)\\
&+\frac{\lambda_i\left(\mu_i-\sum_{h\in N_i\backslash j}c_{ih}\right)-\lambda_j\left(\mu_j-\sum_{h\in N_j\backslash i}c_{jh}\right)}{\lambda_i+\lambda_j}-\frac{\lambda_i-\lambda_j}{r\left(\lambda_i+\lambda_j\right)}
\end{aligned}$$

In the special case of equal weighting: $\lambda_i=\lambda_j$, we have

$$\alpha_{ij}=\frac{1}{2}\left(1-\sum_{h\in N_i}\alpha_{ih}+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\left(\sum_{h\in N_i\backslash\overline{N}_j}\alpha_{hi}-\sum_{h\in N_j\backslash\overline{N}_i}\alpha_{hi}\right)\right)$$

$$c_{ij} = \frac{1}{2}\left(\mu_i - \mu_j\right) - \frac{1}{2}\left(\sum_{h\in N_i\setminus j} c_{ih} - \sum_{h\in N_j\setminus i} c_{jh}\right)$$

Note that system of linear equations in α is exactly the same one as in Section 4.

B.11 Detailed Specification and Proof for Proposition 7

In our setting, for a given network G, individual i's Myerson value is defined by

$$MV_i\left(G\right) := \sum_{S\subseteq N} \frac{\left(\#\left(S\right)-1\right)\left(n-\#\left(S\right)\right)}{n!}\cdot\frac{1}{2}r\sigma^2\left[TVar\left(G|_{\left(S\setminus\{i\}\right)}\right)+\sigma^2 - TVar\left(G|_S\right)\right]$$

where $\#\left(S\right)$ denotes the number of individuals in a subset S of N, and $G|_S$ denotes the subgraph of G restricted to the subset S of individuals. Given the CARA-normal specification, $TVar\left(G|_{\left(S\setminus\{i\}\right)}\right)+\sigma^2 - TVar\left(G|_S\right)$ is the surplus from risk reduction through i's links in S.

Notice that, given any $S\subseteq N$,

$$TVar\left(G|_{S\setminus\{i\}}\right) - TVar\left(G|_S\right) = 1 - \frac{1}{d_i\left(G|_S\right)+1} + \sum_{k\in N_i\left(G|_S\right)} \frac{1}{d_k\left(G|_S\right)\left[d_k\left(G|_S\right)+1\right]},$$

which is strictly increasing in $d_i\left(G|_S\right)$ but strictly decreasing in $d_k\left(G|_S\right)$ for each $j\in N_k\left(G|_S\right)$. Moreover, for any $k\in N$, $d_k\left(G|_S\right)$ is weakly increasing in S, i.e., $S\subseteq S' \Rightarrow d_k\left(G|_S\right)\leq d_k\left(G|_{S'}\right)$.

Consider any pairwise stable network G under the Myerson-value transfers. Then, if i,j are linked, it must be that

$$MV_i\left(G\right) - MV_i\left(G-ij\right) \geq c.$$

Fixing ij, for each $S\subseteq N$, we have

$$\begin{aligned} & TVar\left(G-ij|_{S\setminus\{i\}}\right) - TVar\left(G-ij|_S\right) \\ = & \begin{cases} TVar\left(G|_{S\setminus\{i\}}\right) - TVar\left(G|_S\right), & \text{if } j\notin S \\ 1-\frac{1}{d_i\left(G|_S\right)} + \sum_{k\in N_i\left(G|_S\right)\setminus\{j\}} \frac{1}{d_k\left(G|_S\right)\left[d_k\left(G|_S\right)+1\right]}, & \text{if } j\in S \end{cases}\end{aligned}$$

so

$$
\begin{aligned}
&\left[TVar\left(G|_{S\setminus\{i\}}\right) - TVar\left(G|_S\right)\right] - \left[TVar\left(G-ij|_{S\setminus\{i\}}\right) - TVar\left(G-ij|_S\right)\right] \\
=&\mathbb{1}\{j\in S\}\cdot\left[\frac{1}{d_i\left(G|_S\right)\left[d_i\left(G|_S\right)+1\right]} + \frac{1}{d_j\left(G|_S\right)\left[d_j\left(G|_S\right)+1\right]}\right] \\
\geq&\mathbb{1}\{j\in S\}\cdot\left[\frac{1}{d_i\left(G\right)\left[d_i\left(G\right)+1\right]} + \frac{1}{d_j\left(G\right)\left[d_j\left(G\right)+1\right]}\right]
\end{aligned}
$$

Averaging over all possible $S \subseteq N$, we get

$$
MV_i(G) - MV_i(G-ij) \geq \frac{1}{2}\cdot\left[\frac{1}{d_i\left(G\right)\left[d_i\left(G\right)+1\right]} + \frac{1}{d_j\left(G\right)\left[d_j\left(G\right)+1\right]}\right]
$$

as

$$
\sum_{S\subseteq N}\frac{\left(\#(S)-1\right)\left(n-\#(S)\right)}{n!}\mathbb{1}\{j\in S\} = Pr\{i \text{ arrives later than } j\} = \frac{1}{2}.
$$

From the perspective of social efficiency, the link ij in G is (strictly) socially efficient if

$$
\frac{1}{d_i\left(G\right)\left[d_i\left(G\right)+1\right]} + \frac{1}{d_j\left(G\right)\left[d_j\left(G\right)+1\right]} > 2c.
$$

Thus we can conclude that, given any pairwise stable network G under the Myerson-value transfers, whenever a link ij is (strictly) socially efficient, it will be present in G, because the increments in both i's and j's private benefits strictly exceed the cost of linking c:

$$
\begin{aligned}
MV_i(G) - MV_i(G-ij) &> \frac{1}{2}\cdot 2c = c \\
MV_j(G) - MV_j(G-ij) &> \frac{1}{2}\cdot 2c = c.
\end{aligned}
$$

B.12 Network Centrality and Consumption Variance: More Analytical Results

We first investigate the limit covariance between degree centrality and consumption variance under the *sparse* asymptotics.

Proposition 11. *Suppose $np_n \to \lambda > 1$. Then:*

$$\lim_{n\to\infty} Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_i\left(G_n\right)\right] = \kappa\left(\lambda\right) := \mathbb{E}\left[\frac{\xi^3+4\xi^2+2\left(2-\lambda\right)\xi-3\lambda}{\left(\xi+1\right)^2\left(\xi+2\right)^2}\right],$$

where $\xi \sim Poisson\left(\lambda\right)$.

Numerical computation of $\kappa\left(\lambda\right)$ in Mathematica shows that $\kappa\left(\lambda\right)$ is positive whenever λ exceeds a threshold $\overline{\lambda} \approx 3.8803$. See below for proof of this proposition and numerical plots of $\kappa\left(\lambda\right)$.

In the sparse case, the sign of the limit covariance is now characterized by the asymptotic average degree parameter λ. If the asymptotic average degree λ is sufficiently large, or exceeds $\overline{\lambda} \approx 3.88$ more precisely, the correlation between consumption variance and degree centrality is positive asymptotically. Given that average degrees in many real-world networks are probably larger than $\overline{\lambda}$, our theory predicts a positive asymptotic correlation between degree centrality and consumption volatility even under the sparse asymptotics.

Proof. Suppose $np_n \to \lambda > 1$. In this case it is well known that

$$d_i\left(G_n\right) \xrightarrow{d} Poission\left(\lambda\right),$$

i.e.,

$$\mathbb{P}_n^{ER}\left[d_i\left(G_n\right)=k\right] \to e^{-\lambda}\frac{\lambda^k}{k!}, \quad \forall k \in \mathbb{N}.$$

Now we set $\overline{d}_n$ as, for each n,

$$\begin{aligned}\overline{d}_n &:= \left(\mathbb{E}_n^{ER}\left[\left.\frac{1}{\left(d_j\left(G_n\right)+1\right)^2}\right| ij \in G_n\right]\right)^{-\frac{1}{2}} - 1 \\ &= \left(\mathbb{E}_n^{ER}\left[\frac{1}{\left(2+\tilde{d}_j\right)^2}\right]\right)^{-\frac{1}{2}} - 1 \quad \text{where } \tilde{d}_j \sim B\left(n-2, p_n\right). \\ &\to d_\infty := \left(\mathbb{E}\left[\frac{1}{\left(2+Poisson\left(\lambda\right)\right)^2}\right]\right)^{-\frac{1}{2}} - 1\end{aligned}$$

Again,

$$\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)\middle|\, d_i\left(G_n\right)\right] = \frac{1}{\left(d_i+1\right)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i$$

and thus

$$\begin{aligned}
&Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_i\left(G_n\right)\right]\\
=&\mathbb{E}_n^{ER}\left[\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)\middle|\, d_i\left(G_n\right)\right]\cdot d_i\left(G_n\right)\right] - \mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)\right]\cdot\mathbb{E}_n^{ER}\left[d_i\left(G_n\right)\right]\\
=&\mathbb{E}_n^{ER}\left[\frac{d_i}{\left(d_i+1\right)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i^2\right] - \mathbb{E}_n^{ER}\left[\frac{1}{\left(d_i+1\right)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i\right]\cdot\mathbb{E}_n^{ER}\left[d_i\right]\\
=&\mathbb{E}_n^{ER}\left[\frac{d_i}{\left(d_i+1\right)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i^2\right] - \mathbb{E}_n^{ER}\left[\frac{1}{\left(d_i+1\right)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i\right]\cdot\mathbb{E}_n^{ER}\left[d_i\right]\\
=&\mathbb{E}_n^{ER}\left[\frac{d_i}{\left(d_i+1\right)^2}\right] - \mathbb{E}_n^{ER}\left[\frac{1}{\left(d_i+1\right)^2}\right]\cdot\mathbb{E}_n^{ER}\left[d_i\right] + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot Var_n^{ER}\left[d_i\right]\\
\rightarrow&\kappa\left(\lambda\right) := \mathbb{E}\left[\frac{\xi-\lambda}{\left(\xi+1\right)^2}\right] + \frac{\lambda}{\left(d_\infty+1\right)^2},\quad \text{where } \xi\sim Poisson\left(\lambda\right)\\
=&\mathbb{E}\left[\frac{\xi}{\left(\xi+1\right)^2}\right] - \lambda\mathbb{E}\left[\frac{1}{\left(\xi+1\right)^2} - \frac{1}{\left(\xi+2\right)^2}\right],\quad \text{where } \xi\sim Poisson\left(\lambda\right)\\
=&\mathbb{E}\left[\frac{\xi^3+4\xi^2+2\left(2-\lambda\right)\xi-3\lambda}{\left(\xi+1\right)^2\left(\xi+2\right)^2}\right],\quad \text{where } \xi\sim Poisson\left(\lambda\right)
\end{aligned}$$

which may be positive or negative depending on λ. ∎

Numerical computation of $\kappa\left(\lambda\right)$ in in Mathematica shows that $\kappa\left(\lambda\right)$ is positive for large enough λ with a cutoff $\overline{\lambda}\approx 3.8803$. See Figure 6 for numerical plots of $\kappa\left(\lambda\right)$.

Proposition 12. *Consumption variance and neighbor degree:*

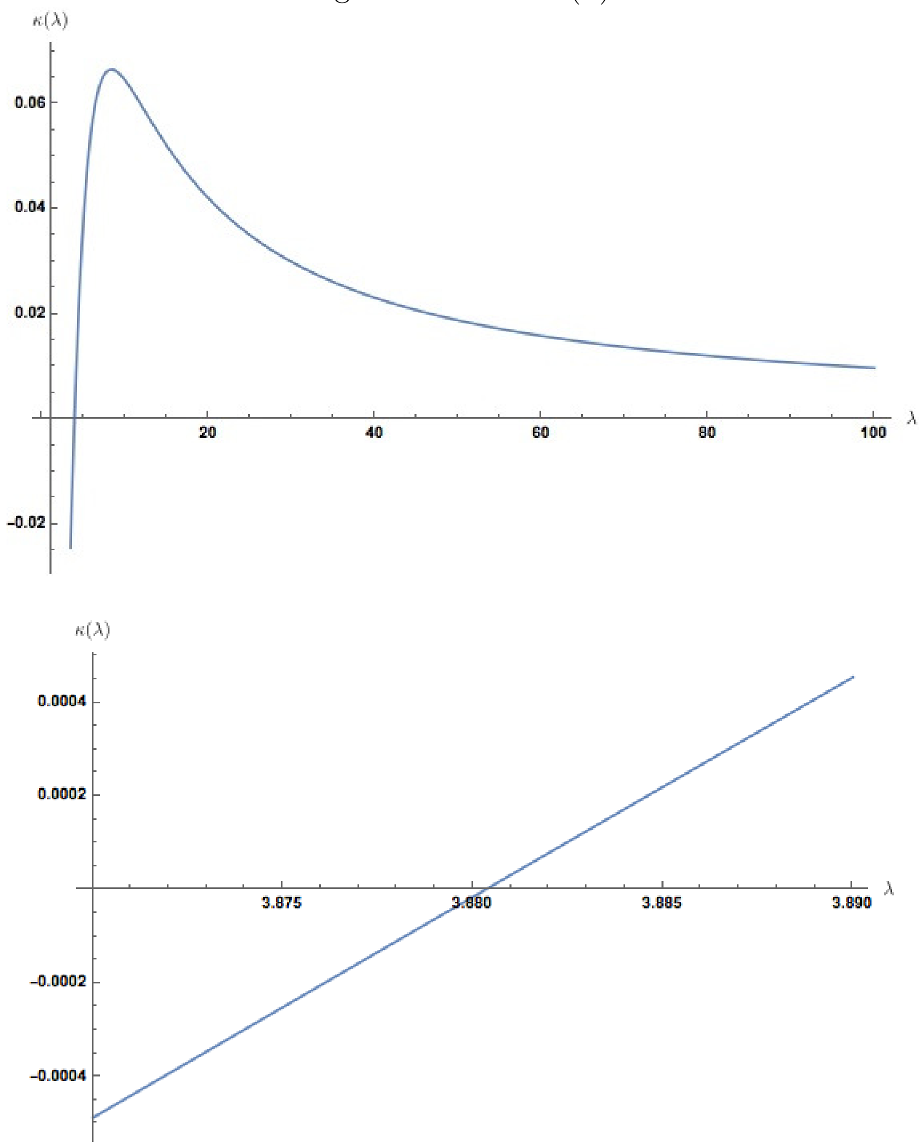
(a). Finite-sample:

$$Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_j\left(G_n\right)\middle|\, ij\in G_n\right] < 0,\ \forall n\in\mathbb{N}.$$

(b). Asymptotic's in the Dense Case: Suppose $p_n = p$ for all n.

$$\lim_{n\to\infty} n^{\frac{3}{2}} Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_j\left(G_n\right)\middle|\, ij\in G_n\right] = 0.$$

Figure 6: Plot of $\kappa(\lambda)$



(c). Asymptotic's in the Sparse Case: Suppose $np_n \to \lambda > 1$.

$$\lim_{n\to\infty} Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_j\left(G_n\right)\right| ij \in G_n] < 0.$$

The intuition underlying the negative correlation between consumption variance $Var(x_i)$ and neighbor degree centrality d_j is very simple: with all other things fixed, a "more popular" friend j of i with a larger degree d_j means that i undertakes a smaller share of e_j under the local equal sharing rule, thus leading to a smaller consumption variance for i. Mathematically, it is clear that $Var(x_i)$ is decreasing in d_j:

$$\frac{\partial}{\partial d_j} Var\left(x_i\right) = -\frac{1}{\left(\overline{d}+1\right)^2} = -\frac{2\left(d_j+1\right)}{\left(\overline{d}+1\right)^2} < 0.$$

Proof. Part (a): Now, consider

$$\begin{aligned}
&\mathbb{E}_n^{ER}\left[nVar\left(x_i\left(G_n\right)\right)\right| d_j\left(G_n\right),\ ij \in G_n] \\
=&\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}d_i+\frac{1}{n}\sum_{k\in N_i}\left[\frac{1}{\left(\frac{1}{n}d_k+\frac{1}{n}\right)^2}-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right]\right| d_j\left(G_n\right),\ ij \in G_n\right] \\
=&\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\right| d_j\left(G_n\right),\ ij \in G_n\right]\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\mathbb{E}_n^{ER}\left[\frac{1}{n}d_i\left(G_n\right)\right| d_j\left(G_n\right),\ ij \in G_n\right] \\
&+\frac{1}{n}\left[\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right]+\frac{1}{n}\sum_{k\in N_i\setminus\{j\}}\left\{\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}\right| d_j\left(G_n\right),\ ij \in G_n\right]-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right\}
\end{aligned}$$

We then set

$$\begin{aligned}
\overline{d}_n :=& \left(\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_k+\frac{1}{n}\right)^2}\right| d_k\left(G_n\right),\ \{ij, ik\} \subseteq G_n\right]\right)^{-\frac{1}{2}} - 1 \\
=& \left(\mathbb{E}_n^{ER}\left[\frac{1}{\left(2+\tilde{d}_k\right)^2}\right]\right)^{-\frac{1}{2}} - 1 \quad \text{where } \tilde{d}_k \sim B\left(n-2, p_n\right)
\end{aligned}$$

which implies that

$$\mathbb{E}_n^{ER}\left[nVar\left(x_i\left(G_n\right)\right)\right| d_j\left(G_n\right),\ ij \in G_n]$$

$$
\begin{aligned}
=&\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}d_i+\frac{1}{n}\sum_{k\in N_i}\left[\frac{1}{\left(\frac{1}{n}d_k+\frac{1}{n}\right)^2}-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right]\right| d_j\left(G_n\right),\ ij\in G_n\Bigg]\\
=&\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}\left(1+\left(n-2\right)p_n\right)+\frac{1}{n}\left[\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right]\\
=&\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}\left(1+\left(n-2\right)p_n\right)+\frac{1}{n}\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}
\end{aligned}
$$

Thus we have

$$
\mathbb{E}_n^{ER}\left[n^2Var\left(x_i\left(G_n\right)\right)-\frac{1+\left(n-2\right)p_n}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right| d_j\left(G_n\right),\ ij\in G_n\Bigg]=\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}.
$$

Then

$$
\begin{aligned}
&n^{\frac{3}{2}}Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_j\left(G_n\right)\right| ij\in G_n]\\
=&Cov_n^{ER}\left[n^2Var\left(x_i\left(G_n\right)\right)-\frac{1+\left(n-2\right)p_n}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2},\ \frac{1}{\sqrt{n}}\left(d_j\left(G_n\right)-1-\left(n-2\right)p_n\right)\right| ij\in G_n\Bigg]\\
=&\mathbb{E}_n^{ER}\left[\left(n^2Var\left(x_i\left(G_n\right)\right)-\frac{1+\left(n-2\right)p_n}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right)\frac{1}{\sqrt{n}}\left(d_j-1-\left(n-2\right)p_n\right)\right| ij\in G_n\Bigg]\\
&-\mathbb{E}_n^{ER}\left[n^2Var\left(x_i\left(G_n\right)\right)-\frac{1+\left(n-2\right)p_n}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right]\frac{1}{\sqrt{n}}\mathbb{E}_n^{ER}\left[d_j-1-\left(n-2\right)p_n\right]\\
=&\mathbb{E}_n^{ER}\left[\left(n^2Var\left(x_i\left(G_n\right)\right)-\frac{1+\left(n-2\right)p_n}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right)\frac{1}{\sqrt{n}}\left(d_j-1-\left(n-2\right)p_n\right)\right| ij\in G_n\Bigg]\\
=&\mathbb{E}_n^{ER}\left[\mathbb{E}\left[n^2Var\left(x_i\left(G_n\right)\right)-\frac{1+\left(n-2\right)p_n}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right| d_j\left(G_n\right),\ ij\in G_n\Bigg]\frac{1}{\sqrt{n}}\left(d_j-1-\left(n-2\right)p_n\right)\right| ij\in G_n\Bigg]\\
=&\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}\frac{1}{\sqrt{n}}\left(d_j-1-\left(n-2\right)p_n\right)\right| ij\in G_n\Bigg]\\
<&0
\end{aligned}
$$

as $d_j|_{ij\in G_n}\sim 1+B\left(n-2,p_n\right)$ and $\mathbb{E}_n^{ER}\left[d_j-1-\left(n-2\right)p_n\right| ij\in G_n]=0$.

Part (b): Suppose $p_n = p$ for all $n \in \mathbb{N}$. As

$$\frac{1}{n} d_j \xrightarrow{a.s.} p,$$

$$\frac{1}{\sqrt{n}} \left(d_j - 1 - (n-2) p \right) \Big|_{ij \in G_n} \xrightarrow{d} \mathcal{N} \left(0, p (1-p) \right),$$

we have

$$n^{\frac{3}{2}} Cov_n^{ER} \left[Var \left(x_i \left(G_n \right) \right), \ d_j \left(G_n \right) \middle| ij \in G_n \right] \to \mathbb{E} \left[\frac{1}{p^2} \cdot \mathcal{N} \left(0, p (1-p) \right) \right] = 0.$$

Note that the result suggests the possibility of obtaining stronger asymptotic results with alternative centering constants and convergence rates. However, we refrain from further pursuing this path of analysis, as the finite-sample result above already demonstrates well our main point: there is a negative correlation between consumption variance and neighbor degree.

Part (c): Suppose $np_n \to \lambda > 1$. As

$$d_j|_{ij \in G_n} \xrightarrow{a.s.} 1 + Poisson \left(\lambda \right),$$

we have

$$\begin{aligned} Cov_n^{ER} \left[Var \left(x_i \left(G_n \right) \right), \ d_j \left(G_n \right) \middle| ij \in G_n \right] &= \mathbb{E}_n^{ER} \left[\frac{1}{\left(d_j + 1 \right)^2} \left(d_j - 1 - (n-2) p_n \right) \middle| ij \in G_n \right] \\ &\to \mathbb{E} \left[\frac{1}{\left(\xi + 2 \right)^2} \left(\xi - \lambda \right) \right] \quad \text{where } \xi \sim Poisson \left(\lambda \right) \\ &< 0. \end{aligned}$$

■

B.13 Risk Sharing with Ex Post Communication

In our specification of the local information constraints in Subsection 3.1, we required that the risk-sharing contract between linked ij must be contingent on their ex post common knowledge $I_{ij} := \left(e_k \right)_{k \in \overline{N}_i \cap \overline{N}_j}$ only.

However, one may argue that this requirement might be too stringent. If individuals are allowed to interact ex post in strategic ways in the presence of, for example, repeated risk sharing, ex post communication mechanisms, or ex post side contracts, then ex post information might be effectively transmitted to individuals who do not observe it directly.

In Subsection 7.3, we abstract from the detailed specification of such ex post interactions, but instead use the contractibility constraints Q as a reduced-form summary of all effects of ex post interactions on the effective contractibility of risk sharing arrangements.

Here, we present a more detailed specification and analysis of ex post interactions. It should be emphasized that there are clearly many other reasonable specifications that may produce different results than the one to be presented below. However, the specification adopted here will produce a contractibility structure Q as specified in Subsection 7.3 that satisfy both conditions (a) and (b) in Proposition 8.

Fix any connected network G. Consider now that, after endowment realizations but before transfer payments, a single-round of simultaneous communication is allowed: simultaneously, each individual i may send a message $m_{ij} \in \mathscr{M}_{ij}$ to each individual $j \in N \setminus \{i\}$, where $\mathscr{M}_{ij}$ denotes an arbitrary message space. The (local) observability of *messages* is determined by a communication protocol, which we take to be a primitive of the environment. For example, a few simplest communication protocols that lead to different levels of observability of messages are:

(a) *Global communication*: m_{ij} is publicly observable by all individuals Equivalently, we might as well take $m_{ij} \equiv m_i$ and $\mathscr{M}_{ij} \equiv \mathscr{M}_i$, i.e., each individual can only send a public message that then becomes global common knowledge. For example, a global message be thought of as a Tweet, which everyone can observe (if he wants to).

(b) *Local announcement*: m_{ij} is locally observable by the sender i and i's neighbors. Again, we might as well take $m_{ij} \equiv m_i$ and $\mathscr{M}_{ij} \equiv \mathscr{M}_i$. For example, a local announcement can be thought of as a message i posts on his *own* Facebook timeline.

(c) *Local comment*: m_{ij} is locally observable by the receiver j and j's neighbors. For example, a local comment can be thought of as a message i leaves on j's Facebook timeline.

(d) *Private communication*: m_{ij} is only privately observable by the sender i and receiver j. A variety of communication technologies such as personal meeting, phone calls, online chats fit into this category.

Given a communication protocol, for each linked pair ij, their ex post local common knowledge before transfers are carried out not only include the endowment realizations they can commonly observe, denoted $I_{ij} = (e_k)_{k \in \overline{N}_{ij}}$, but also include the communication messages they can commonly observe, denoted M_{ij}, which will differ across the four communication protocols above.

Again, we require that the bilateral transfer contract t_{ij} be contingent only on ex post local common knowledge, i.e., t_{ij} be $\sigma(I_{ij}, M_{ij})$-measurable. As before, we abstract from ex post enforcement issues of the contract t_{ij} per se, but focus on the strategic aspects of ex post messages. We proceed to analyze the four communication protocols above separately.

(a) Global Communication

Under the global communication protocol, the *first-best* consumption plan induced by global equal sharing can be achieved.

Specifically, let each individual i submits ex post a public report m_i of her own endowment realization e_i. As the whole vector of reports m is globally observable, for any linked pair ij, they make make their transfer contract t_{ij} effectively contingent on $I_{ij} = (e_k)_{k \in \overline{N}_{ij}}$ and m.

Consider the following specification of t_{ij}:

$$t_{ij}(I_{ij}, m) = \tilde{t}_{ij}\left(I_{ij}, m_{N \setminus \overline{N}_{ij}}\right) + |e_i - m_i| - |e_j - m_j|,$$

where $m_{N \setminus \overline{N}_{ij}}$ denotes the reports from individuals outside $\overline{N}_{ij}$. Clearly, t_{ij} respects the measurability constraints.

Let $m_i^s : \mathbb{R}^{\#(\overline{N}_i)} \to \mathbb{R}$ denote individual i's reporting strategy. Given the risk sharing arrangement t specified above, it is easy to see that, after endowment realizations, it is a Nash equilibrium for each individual to report his own e_i truthfully, i.e., setting $m^s(e) \equiv e$. This is because, given the endowment realizations e and the induced local state I_{ij} for each $j \in N_i$, strategically individual i should choose m_i to maximize his final consumption under t:

$$x_i^t(e,m) = e_i - \sum_{j\in N_i} t_{ij}(e,m)$$
$$= \left[e_i - \sum_{j\in N_i} \tilde{t}_{ij}\left(I_{ij}, m_{N\setminus\overline{N}_{ij}}\right) + \sum_{j\in N_i} |e_j - m_j|\right] - d_i|e_i - m_i|,$$

which depends on m_i only via the last term, $-d_i|e_i - m_i|$. It is then a dominant strategy for individual i to set $m_i = e_i$.

Anticipating this global truthful revelation of endowment realizations, it is obvious that $\tilde{t}$ should be configured to implement the global equal sharing rule[46], with the understanding that $m_{N\setminus\overline{N}_{ij}} = e_{N\setminus\overline{N}_{ij}}$ in equilibrium ex post.

In summary, global communication as specified above completely solves all information problems, and effectively (in the sense of ex post dominant strategy implementability) produces an informational network G' that is given by the complete network. Then, the local Borch rule, applied to the complete network G' (or the corresponding contractibility structure Q as defined in 7.3), immediately implies that the first best global equal sharing can be achieved.

(b) Local Announcements

With local announcement, the effective (ex post dominant strategy implementable) informational network G' is effectively given by connecting all 2nd-order neighbors in the physical network G.

Let x'^* denote the constrained Pareto efficient consumption plan computed according to Proposition 4 with the informational network G'. As G is connected, there exists a profile of bilateral transfer rules $\tilde{t}$ defined on physical transfer links in G that satisfies the following two conditions:

i). $\tilde{t}$ induces the consumption plan x'^*; and

ii). for every $i \in N$, individual i's exposure to e_k for any individual k of distance 2 to individual i in G is implemented by $\tilde{t}$ completely through a shortest path between i and k in G.

Notice that the existence of such a transfer arrangement $\tilde{t}$ is guaranteed by the complete arbitrariness in the configuration of superfluous cyclical transfers.

[46]This is clearly feasible under connectedness of G.

The key implication of condition *ii*) is that, for any linked pair ij in G, whenever there exists some individual k of distance 2 to *both* i *and* j, then the net share β_{ijk} of e_k transferred from i to j must be exactly 0. This is because, either i's or j's exposure of e_k should be completely channeled via their shortest paths (of length 2) to individual k, which necessarily does not include link ij; moreover, any other individual's shortest path to k does not include link ij, either. Similarly, another implication of condition *ii*) is that, for any lined pair ij in G and any $k \in \overline{N}_{ij}$, between ij there is zero share of e_k being transferred, i.e., $\beta_{ijk} = 0$.

With the two implications of condition *ii*) in mind, we deduce that, as $\tilde{t}_{ij}$ can be contingent on endowment realizations of individuals in

$$\overline{N}_{ij}^{'} = \overline{N}_{ij} \cup \left(N_i \backslash \overline{N}_j\right) \cup \left(N_j \backslash \overline{N}_i\right) \cup \{k : k \text{ is of distance 2 to both } i \text{ and } j\},$$

i.e., the common neighborhood of ij under the supergraph $G^{'}$ that treats distance-2 individuals in G as linked (in $G^{'}$), the transfer arrangement $\tilde{t}$ admits the following linear representation:

$$\tilde{t}_{ij}(e) = \tilde{\alpha}_{ij} e_i - \tilde{\alpha}_{ji} e_j + \sum_{k \in N_i \backslash \overline{N}_j} \tilde{\beta}_{ijk} e_k - \sum_{k \in N_j \backslash \overline{N}_i} \tilde{\beta}_{ijk} e_k.$$

We now proceed to construct a risk sharing arrangement t using ex post messages based on $\tilde{t}$.

This can be achieved by letting each individual i submit a report m_i of all endowment realizations i observes, i.e., $(e_k)_{k \in \overline{N}_i}$. As m_i is observed by i's neighbors, the bilateral transfer contract t_{ij} between i and i's neighbor j can be contingent on I_{ij} as well as (m_i, m_j).

Consider the following specification of t_{ij}:

$$\begin{aligned} t_{ij}(I_{ij}, m_i, m_j) &= \tilde{t}_{ij}\left(I_{ij}, (m_{ik})_{k \in N_i \backslash \overline{N}_j}, (m_{jk})_{k \in N_j \backslash \overline{N}_i}\right) + C\left(|e_j - m_{ij}| - |e_i - m_{ji}|\right) \\ &= \tilde{\alpha}_{ij} e_i - \tilde{\alpha}_{ji} e_j + \sum_{k \in N_i \backslash \overline{N}_j} \tilde{\beta}_{ijk} m_{ik} - \sum_{k \in N_j \backslash \overline{N}_i} \tilde{\beta}_{jik} m_{jk} \\ &\quad + C\left(|e_j - m_{ij}| - |e_i - m_{ji}|\right) \end{aligned}$$

where m_{ik} denotes individual i's report of e_k and C is constant given by

$$C := \max_{ij \in G} \sum_{k \in N_i \setminus \overline{N}_j} \left| \tilde{\beta}_{ikj} \right|. \tag{28}$$

Again, i's final consumption under t is given by

$$\begin{aligned} x_i^t(e,m) &= \left(1 - \sum_{j \in N_i} \tilde{\alpha}_{ij}\right) e_i + \sum_{j \in N_i} \left(\tilde{\alpha}_{ji} e_j - C\left|e_j - m_{ij}\right| + C\left|e_i - m_{ji}\right|\right) \\ &\quad - \sum_{j \in N_i} \left(\sum_{k \in N_i \setminus \overline{N}_j} \tilde{\beta}_{ijk} m_{ik} - \sum_{k \in N_j \setminus \overline{N}_i} \tilde{\beta}_{jik} m_{ji} \right) \\ &= \left[\left(1 - \sum_{j \in N_i} \tilde{\alpha}_{ij}\right) e_i + \sum_{j \in N_i} \left(\tilde{\alpha}_{ji} e_j + C\left|e_i - m_{ji}\right| + \sum_{k \in N_j \setminus \overline{N}_i} \tilde{\beta}_{jik} m_{ji} \right) \right] \\ &\quad - \sum_{j \in N_i} \left(C\left|e_j - m_{ij}\right| + \sum_{k \in N_i \setminus \overline{N}_j} \tilde{\beta}_{ikj} m_{ij} \right), \end{aligned}$$

which by (28) strictly increases in m_{ij} whenever $m_{ij} < e_j$, and strictly decreases in m_{ij} whenever $m_{ij} > e_j$. Hence, ex post individual i's dominant strategy is to set $m_{ij} = e_j$.

Lastly, notice that no information about third-order neighbors can be possibly transmitted under the protocol of local announcements.

In summary, all individuals will truthfully report $(e_k)_{k \in \overline{N}_i}$ in an ex post dominant strategy equilibrium, together achieving the constrained Pareto efficient consumption plan with respect to the augmented informational network G', a supergraph of the physical transfer network G that links all distance-2 neighbors in G.

(c) Local Comments

With local comments, the effective (ex post dominant strategy implementable) informational network G' is effectively given by connecting all neighbors within a distance of 3 in the physical network G.

Now, each individual i may submit to each neighbor $j \in N_i$ a report m_{ij} of the endowment realizations i observe, i.e., $I_i \equiv (e_k)_{k \in \overline{N}_i}$. Specifically, $m_{ij} \in \mathbb{R}^{|\overline{N}_i|}$ and we write m_{ijk} to denote i's report of e_k to individual j.

Now that for any linked pair ij, they can commonly observe endowment realizations I_{ij}, all comments received by i and all comments received by j (say, displayed on i's and j's Facebook pages). This facilitates transmission of distance-3 information: for any path $i - j - k - h$ in G, individual i can now observe on j's comment book a comment left by k about h's endowment realization e_h. We now simply need to properly construct the bilateral contracts to incentivize truthful reporting ex post.

A formal procedure can be constructed in a similar way to the procedure for (b) "local announcements" above. For avoid repetition, we now just provide a description of the key idea.

If i lies about own endowment realization e_i to $j \in N_i$, this is immediately detectable by j. Then by properly specifying a punishment transfer from i to j based on $|e_i - m_{iji}|$, we can incentivize i to truthfully reports his own endowment realizations to his neighbors. This implies that, each individual i can now observe a truthful report of his 2nd-order neighbors, based on their truthful comments sent to i's first-order neighbors.

Now consider a linked pair ij. If i lies to j about e_k for some $k \in N_i \backslash \overline{N}_j$, this is detectable by j because j also observes k's report to i, namely m_{ki}, which includes a truthful report m_{kik} of e_k. Contract t_{ij} may then specify a sufficient punishment transfer from i to j, which ensures the truthfulness of m_{ijk} about e_k in a Nash equilibrium. Hence, each individual i can now observe a truthful report of his 3rd-order neighbor's endowment e_k, which is included in a report from one of i's 2nd-order neighbor to one of i's (1st-order) neighbor.

Now, suppose that both i and j "effectively know" e_k for some $k \notin \overline{N}_{ij}$. If $k \in N_i \cup N_j$, then k submits a truthful report to either i or j, which is commonly observable by i and j, so t_{ij} can be optimally contingent on m_{kik} or m_{kjk}. If $k \notin N_i \cup N_j$, there are two possibilities.

First, if k is a 2nd-order neighbor of i (or j with similar arguments), then there must exist some $h \in N_i$ that submits a report m_{hi} to i, which is commonly observed by ij and includes a truthful report of e_k, so t_{ij} can be optimally contingent on m_{hik}.

Second, if k is a 3rd-order neighbor of both i and j, there are three sub-cases. In sub-case 1, ij are both linked to h, of whom k is a 2nd-order neighbor. Then ij commonly observe a report received by h, which includes a truthful report of e_k. In sub-case 2, there exists a path $k \to i$ and a path $k \to j$ that both pass through some $h \in N_k$, but the condition for sub-case 1 does not hold. Then there is no report of e_k

that is commonly observed by ij, but h is a diagonal node for link ij in a pentagon subgraph. This does not affect risk-sharing efficiency due to the redundancy of link ij: efficient exposure to e_k can be channeled completely through the two paths from h to i and j respectively, and it has been shown in the above arguments that e_k or a truthful report of e_k is commonly observable by the two contracting parties in each link along the two paths. In sub-case 3, any two paths $k \to i$ and $k \to j$ must be disjoint (except at k), in which case k is the diagonal node to link ij in a heptagon subgraph. Again this does not affect risk-sharing efficiency due to the redundancy of link ij: efficient exposures to e_k can still be channeled completely through the two paths. In particular consider the path $k - i_2 - i_1 - i$, and notice that a truthful report of e_k from i_2 to i_1 is commonly observable by both i_1 and i, thus being contractible.

This completes the proof that "local comment" leads to an implementable informational network with effective local observability of 2nd-order and 3rd-order neighbors in the physical network G. Lastly, notice that no information about 4th-order neighbors is possible with local comment, so no other contracts can do better.

(d) Private Communication

With private communication, the informational network remains unchanged. This is because when messages are completely private there is no information spillover to any other party. In the meanwhile, the ex post messaging game is a zero-sum game (as the messages are mapped into net transfers). Hence, given each local state I_{ij}, both i and j are guaranteed the value of the ex post game in any Nash equilibrium, so the equilibrium payoffs do not depend on nonlocal endowment realizations. Thus the implementable informational network remains unchanged.