

Are “Market Neutral” Hedge Funds Really Market Neutral?

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1 Background

- The hedge fund industry has grown from about \$50 billion in 1990 to \$1 trillion in June 2004
- The first hedge fund gained its name from its strategy of buying undervalued stocks using the proceeds of short-sales of related stocks, thereby creating “market neutral” portfolio
- Today there are a multitude of hedge fund strategies, most of which are not specifically market neutral.
- Individual hedge funds are often categorised by their *self-described* investment style.

2 Hedge fund styles and sizes (Fung & Hsieh, 1997)

- Event driven (9.6%): aka “corporate life-cycle investing”, eg spin-offs, mergers, bankruptcy re-organisations, etc.
- Global (34.3%): A catch-all category for funds that invest in any non-US/emerging market equities
- Global/macro (33.1%): Investing on the basis of macroeconomic analyses, typically via major interest rate shifts
- Market neutral (20.0%): funds that actively seek to avoid major risk factors, mainly focussing on apparent mis-pricing

3 Market neutrality - standard definition

- The standard definition uses simple linear correlations, or market “betas”.
- A fund is often said to be “market neutral” if it exhibits zero correlation with the market index, or multiple market indices:

$$\text{Corr} [r_{it}, r_{mt}] = 0, \text{ or}$$

$$\frac{\text{Cov} [r_{it}, r_{mt}]}{V [r_{mt}]} \equiv \beta_i = 0$$

- In all cases, neutrality is defined as a property of the dependence between the fund return and the market return.

4 The breadth and depth of market neutrality

- The “neutrality” of a fund can more generally be thought of as having two dimensions:
 1. *Breadth*: refers to the number of sources of “market” risk to which the fund is neutral. For example, equity market risk, bond market risk, exchange rate risk, etc. See Fung and Hsieh (2001), Mitchell and Pulvino (2001), Agarwal and Naik (2002), for example.
 2. *Depth*: refers to the “completeness” of the neutrality of the fund to these market risks. Zero correlation is one level of neutrality, but we can imagine others.
- I focus on neutrality “depth”, and consider only a single (equity) market index as a source of market risk.

5 Contribution of this paper

1. I use the preferences of a risk averse investor (with some existing exposure to market risk) to suggest various neutrality concepts:
 - “mean neutrality”, which nests the standard correlation-based definition
 - neutrality concepts based on the idea that the *risk* of a market neutral fund should not be affected by market risk:
 - “variance neutrality”
 - “Value-at-Risk neutrality”
 - “tail neutrality”
 - “complete neutrality”, which corresponds to independence of the fund to market risk

6 Contribution of this paper (cont'd)

2. I suggest statistical tests for each of the proposed neutrality concepts
 - Test a null hypothesis of neutrality against a general non-neutrality alternative hypothesis
 - Test a null hypothesis of neutrality against a non-neutrality alternative hypothesis that is disliked by risk averse investors
 - For example: risk averse investors prefer zero linear correlation to positive correlation, but prefer negative correlation to zero correlation.
 - So test null of zero correlation against alternative of positive correlation specifically, rather than against non-zero correlation more generally.

7 Contribution of this paper (cont'd)

3. I conduct a detailed study of the neutrality properties of “market neutral” hedge funds from the HFR and TASS hedge fund data bases.

- Monthly data from April 1993 to April 2003, 121 observations
- 194 live and 23 dead funds
- Robustness checks:
 - Choice of market index (S&P500, MSCI World, MSCI Europe)
 - Denomination of returns (U.S. dollars or British pounds)
 - Impact of last few observations on dead funds
 - Impact of age of fund

8 Why focus on “market neutral” hedge funds?

1. The low correlation/dependence between hedge funds and market returns is a widely-cited benefit of hedge funds
2. “Market neutral” funds actually define themselves by their relation (or lack thereof) with the market
 - Measures of neutrality may be useful for ranking “market neutral” funds.
 - If we can take the label “market neutral” at face value, then this is one less quantity that needs to be estimated from data
3. “Market neutral” hedge funds have not received a great deal of attention in the literature
 - Fung and Hsieh (2001) examine “trend following” hedge funds; Mitchell and Pulvino (2001) focus on “risk arbitrage” hedge funds; Agarwal and Naik (2002) look at a variety of categories, but not market neutral funds

9 Summary stats on the funds

	Median fund	S&P500	MSCI Europe	MSCI World
Mean*	8.0825	10.1602	8.5604	6.6308
Std dev*	7.9488	15.6600	15.9666	14.7250
Skewness	0.0261	-0.5424	-0.3655	-0.5425
Kurtosis	3.6547	3.2561	3.7360	3.2753
Min	-4.4800	-14.4431	-13.1550	-13.3503
Max	5.8800	9.7766	13.4872	9.0228
JB stat	2.5761	5.9932	4.9222	6.0362
p-value	0.2758	0.0500	0.0853	0.0489
Number of obs	42	121	121	121

The column headed “median fund” presents the median of the statistic in the row across the 217 funds with more than 6 observations. The mean and standard deviation statistics have been annualised to ease interpretation. ‘JB stat’ refers to the Jarque-Bera (1980) test of normality.

10 Summary stats on observations available

Table 2: Summary statistics on the number of observations on each fund

Min	1
0.25 percentile	19
Median	42
Mean	49.5
0.75 percentile	69
Max	121
<hr/>	
Number with ≥ 6 obs	213
Number with ≥ 18 obs	171
Number with ≥ 24 obs	150
Number with ≥ 66 obs	59

11 Asymptotic vs. bootstrap critical values

- The relatively short samples of data available make relying on asymptotic theory for critical values somewhat optimistic.
- I use a bootstrap procedure to obtain the critical values of all tests in this paper. By bootstrapping appropriately standardised test statistics a better approximation to the asymptotic distribution of the test statistic is obtained. I use the “stationary bootstrap” of Politis and Romano (1994) to allow for serial correlation, heteroskedasticity, and non-normality in returns.
- I impose the null of independence when generating the bootstrap samples by bootstrapping the fund and the market return series separately.
- A *joint test* of neutrality is obtained by looking at the bootstrap distribution of the number of tests failed (at the 5% level): if the actual number of tests failed is greater than the 95th percentile of this distribution then the fund is reported to have failed the joint test.

12 Correlation neutrality

- Looking at simple linear correlation between the each fund return and S&P500 return, over period that the fund was alive
- 171 funds with 18 or more observations:
 - Average correlation: 0.016. (the two largest were 0.934 and 0.921)
 - 29.2% of funds have significant non-zero correlation, at the 0.05 level.
 - 28.0% have significant positive correlation
- Thus over a quarter of “market neutral” funds exhibit statistically significant positive linear correlation with the market index.

13 Mean neutrality

- This is the simplest generalisation of correlation neutrality.
- It is defined as the expected return on the fund being independent of the market return:

$$\begin{aligned} E[r_{it}|r_{mt}] &= E[r_{it}] \quad \forall r_{mt}, \text{ or} \\ E[r_{it}|\mathcal{F}_{t-1}, r_{mt}] &= E[r_{it}|\mathcal{F}_{t-1}] \quad \forall t, r_{mt} \end{aligned}$$

where \mathcal{F}_{t-1} contains all information available at time $t - 1$.

- Under joint normality mean neutrality simplifies to correlation neutrality.
- Most general method for testing this would be via nonparametric regression:

$$r_{it} = \mu_i(r_{mt}) + e_{it}$$

and then testing that μ_i is flat.

14 Testing mean neutrality

- I used a simple second-order polynomial as an approximation to the conditional mean function:

$$r_{it} = \beta_0 + \beta_1 r_{mt} \delta_t + \beta_2 r_{mt}^2 \delta_t + \gamma_1 r_{mt} (1 - \delta_t) + \gamma_2 r_{mt}^2 (1 - \delta_t) + e_{it}$$
$$\delta_{it} = \begin{cases} 1 & \text{if } r_{mt} \leq 0 \\ 0 & \text{if } r_{mt} > 0 \end{cases}$$

and then tested

$$H_0 : \beta_i = \gamma_i = 0 \text{ for all } i$$

vs. $H_a : \beta_i \neq 0 \text{ or } \gamma_i \neq 0 \text{ for some } i$

- In this sample the higher-order (ie >2) polynomial terms were generally not significant.
- Applying this to funds with at least 24 observations leads to a rejection frequency of 23.3% at the 0.05 level across the 150 funds.

15 Mean neutrality on the downside

- Note that the previous test ignores the fact that risk averse investors do not dislike *all* types of dependence on the market:
 - A risk averse investor would prefer a negative relation with the market when the market return is negative, and a positive relation when the market return is positive, to zero correlation in both states.
- Thus it may not be mean neutrality that investors truly seek, but rather a restricted form of mean dependence between the fund and the market.
- Consider the following refinement of mean neutrality, which I call “mean neutrality on the downside”. This form of neutrality imposes that the expected return on the fund is neutral or negatively related to the market when the market return is negative.

$$\frac{\partial \mu_i(r_{mt})}{\partial r_{mt}} \leq 0 \text{ for } r_{mt} \leq 0$$

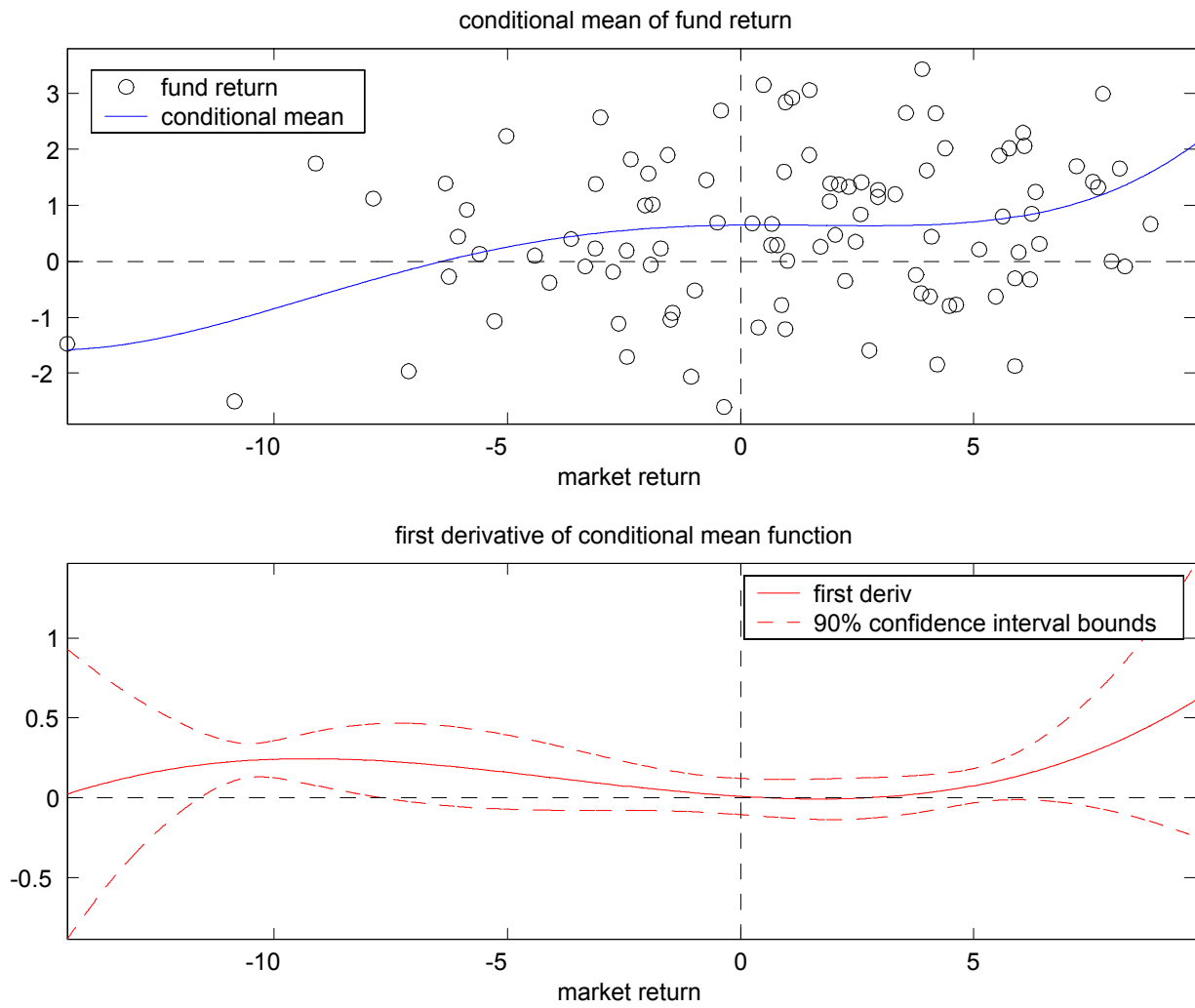


Figure 1: No evidence against mean neutrality on the downside

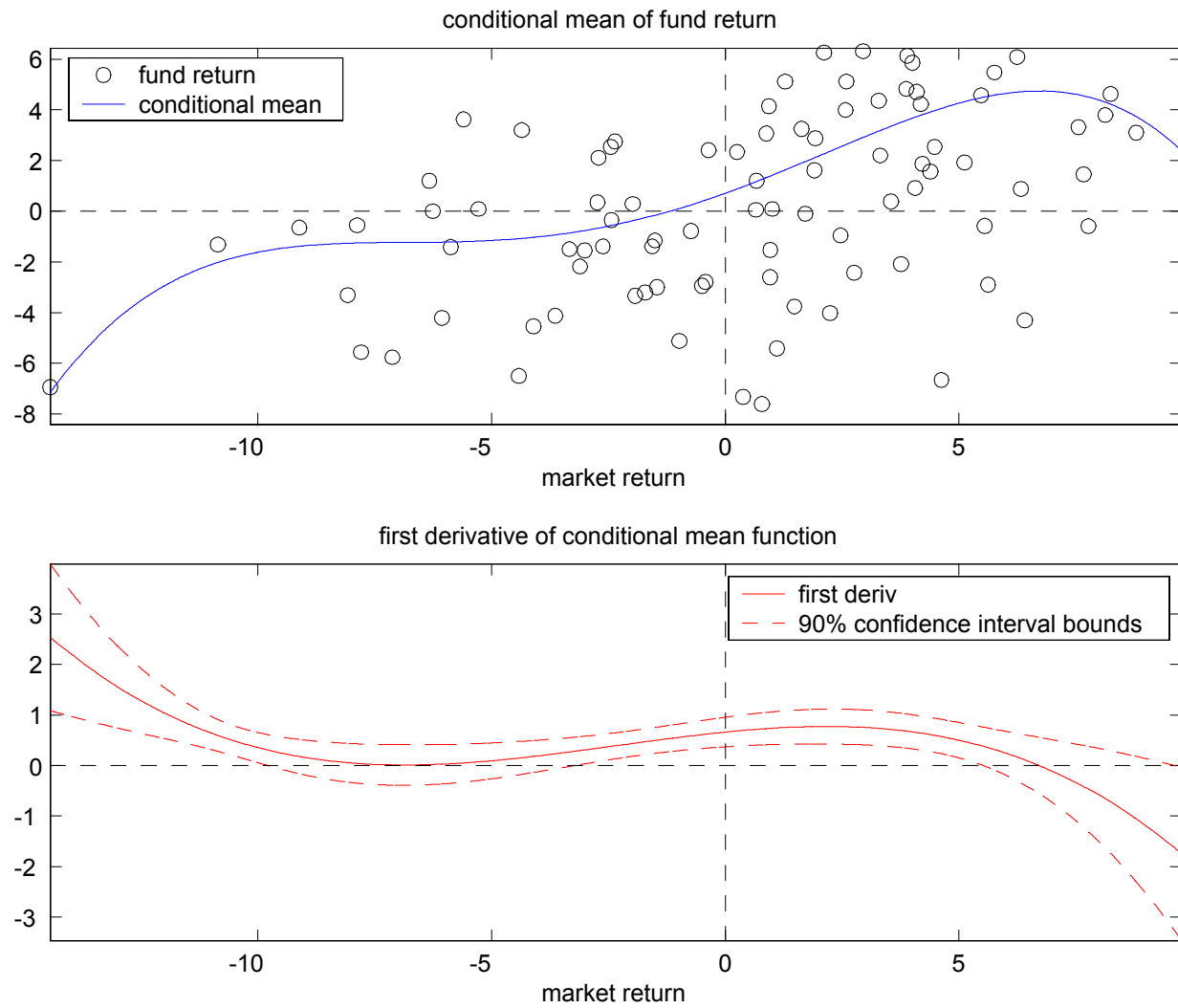


Figure 2: Weak evidence against mean neutrality on the downside

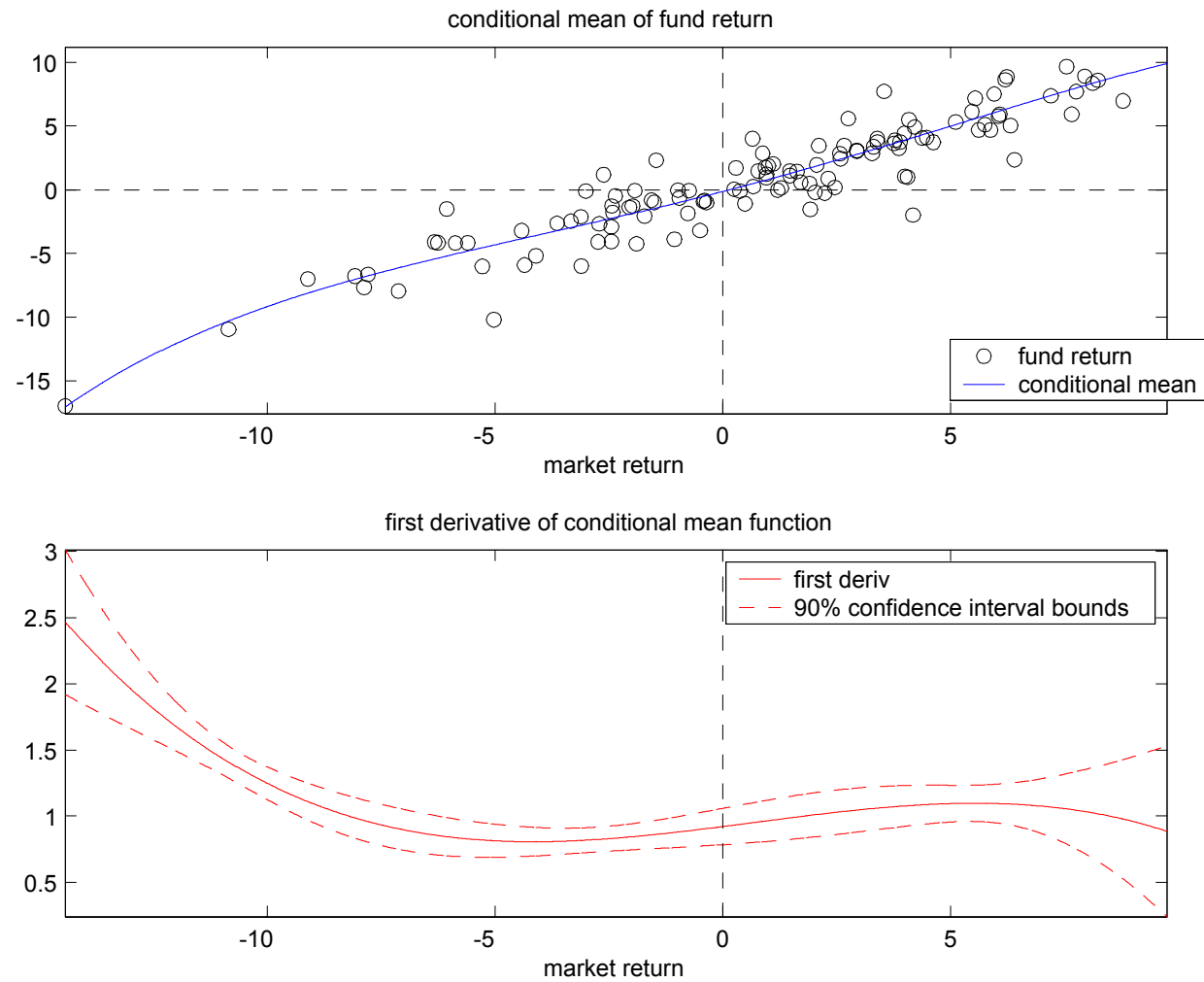


Figure 3: Strong evidence against mean neutrality on the downside

16 Testing mean neutrality on the downside

- Using the piece-wise second-order polynomial approximation we have

$$\frac{\partial \mu_i(r_{mt})}{\partial r_{mt}} = \beta_1 + 2\beta_2 r_{mt} \text{ for } r_{mt} \leq 0$$

- This first-derivative is negative for all values of $r_{mt} \leq 0$ if and only if $\beta_1 \leq 0$ and $\beta_2 \geq 0$, thus the appropriate null and alternative hypotheses are:

$$H_0 : \beta_1 \leq 0 \cap \beta_2 \geq 0$$

vs. $H_a : \beta_1 > 0 \cup \beta_2 < 0$

- I use $\max[\beta_1, -\beta_2]$ as a test statistic, and found that only 0.7% of funds fail this test of mean neutrality on the downside.

17 Neutrality of fund risk

- The natural first neutrality property to consider related to the levels of the hedge fund return: is the fund return neutral to market risk?
- A second neutrality property one might desire from a “market neutral” hedge fund relates to risk: is the *risk* of the fund neutral to market risk?
 - If risk of the fund increases at same time as risk of the market, then portfolio of fund and market will be less attractive to risk averse investors than one whose risk moves independently of the market.
- I propose three neutrality concepts that relate to risk:
 1. Variance neutrality
 2. Value-at-Risk neutrality
 3. Tail neutrality

18 Variance neutrality

- Non-increasing absolute risk aversion, suggested by Arrow (1971), leads to a preference for positive skewness, so risk averse investors prefer:

$$\text{Corr} \left[(r_{it} - \mu_i)^2, r_{mt} - \mu_m \right] \geq 0$$

- Decreasing *absolute prudence*, suggested by Kimball (1993), leads to an aversion to kurtosis, so risk averse investors prefer:

$$\text{Corr} \left[(r_{it} - \mu_i)^2, (r_{mt} - \mu_m)^2 \right] \leq 0$$

- Thus risk averse investors may care about the “variance neutrality” of a hedge fund, defined as:

$$\begin{aligned} V [r_{it} - \mu_i(r_{mt}) | r_{mt}] &= V [r_{it} - \mu_i(r_{mt})] \quad \forall r_{mt}, \text{ or} \\ V [r_{it} - \mu_i(r_{mt}) | \mathcal{F}_{t-1}, r_{mt}] &= V [r_{it} - \mu_i(r_{mt}) | \mathcal{F}_{t-1}] \quad \forall r_{mt} \end{aligned}$$

where \mathcal{F}_{t-1} contains all information available at time $t - 1$.

- As above, I approximate the conditional variance function by a polynomial:

$$\begin{aligned}
 r_{it} &= \mu_i(r_{mt}) + e_{it} \\
 e_{it} &= \sigma_i(r_{mt}) \varepsilon_{it}, \quad \varepsilon_{it} \sim (0, 1) \\
 \sigma_i^2(r_{mt}) &= \alpha_0 + \alpha_1 r_{mt} \delta_t + \alpha_2 r_{mt}^2 \delta_t \\
 &\quad + \alpha_3 r_{mt} (1 - \delta_t) + \alpha_4 r_{mt}^2 (1 - \delta_t)
 \end{aligned}$$

- Note that mean (non-) neutrality is modelled prior to testing variance neutrality.
- To test variance neutrality we examine:

$$\begin{aligned}
 H_0 &: \alpha_i = 0 \text{ for all } i \\
 \text{vs. } H_a &: \alpha_i \neq 0 \text{ for some } i
 \end{aligned}$$

- When testing for variance neutrality across the 150 funds with at least 24 observations, we were able to reject the null at the 0.05 level for 6.0% of funds when including an ARCH(1) term.

19 Variance neutrality on the downside

- Could also consider a form of neutrality such as “variance neutrality on the downside”. The preference for positive skew and aversion to kurtosis implies that risk averse investors would prefer:

$$\frac{\partial \sigma_i^2(r_{mt})}{\partial r_{mt}} \geq 0 \text{ for } r_{mt} \leq 0$$
$$\frac{\partial \sigma_i^2(r_{mt})}{\partial r_{mt}} = \alpha_1 + 2\alpha_2 r_{mt} \text{ for } r_{mt} \leq 0$$

- We can obtain a test of this condition by looking at the signs of α_1 and α_2

$$H_0 : \alpha_1 \geq 0 \cap \alpha_2 \leq 0$$

vs. $H_a : \alpha_1 < 0 \cup \alpha_2 > 0$

- I found evidence against variance neutrality on the downside for only 4.0% of funds when including an ARCH(1) term.

20 Value-at-Risk neutrality

- VaR has been proposed (and subject to some criticism) as an alternative measure of risk.

$$VaR_\alpha(r_{it}) \equiv F_i^{-1}(\alpha), \text{ or } VaR_\alpha(r_{it}) = X : \int_{-\infty}^X f_i(r) dr = \alpha$$

- I define a VaR-neutral portfolio as

$$\begin{aligned} VaR(r_{it}|r_{mt}) &= VaR(r_{it}) \quad \forall r_{mt}, \text{ or} \\ VaR(r_{it}|\mathcal{F}_{t-1}, r_{mt}) &= VaR(r_{it}|\mathcal{F}_{t-1}) \quad \forall r_{mt} \end{aligned}$$

where \mathcal{F}_{t-1} is all information available at time $t - 1$.

- More generally, this could be called “quantile neutrality”. This is a special case of complete neutrality, discussed later, which implies that all quantiles of the fund are neutral to the market.

21 Conditional VaR neutrality

- Violations of mean and variance neutrality will, generally, automatically lead to violations of VaR neutrality, so we may wish to consider “conditional VaR neutrality”:

$$VaR \left(\frac{r_{it} - \mu_i(r_{mt})}{\sigma_i(r_{mt})} \middle| r_{mt} \right) = VaR \left(\frac{r_{it} - \mu_i(r_{mt})}{\sigma_i(r_{mt})} \right) \forall r_{mt}$$

- If fund and market returns are jointly normally distributed, then VaR neutrality will follow directly from mean and variance neutrality. (This is not true for elliptical distributions more generally.)
- Further, under joint normality, conditional VaR neutrality will *always* hold, even if mean and variance neutrality do not.

22 Testing VaR neutrality

$$VaR(r_{it}|r_{mt}) = VaR(r_{it}) \forall r_{mt}$$

- With sufficient data we could use similar techniques to those employed to mean neutrality, substituting quantile regression (see Koenker and Basset, 1978) for least-squares regression.
- However we do not have a great deal of data on the funds, and the quantiles of interest are usually in the tail.
- A simple alternative approach was suggested by Christoffersen (1998):

23 Testing VaR neutrality, cont'd

- Let

$$\varepsilon_{it} \equiv \frac{r_{it} - \mu_i(r_{mt})}{\sigma_i(r_{mt})} \quad , \quad \varepsilon_{mt} \equiv \frac{r_{mt} - \mu_m}{\sigma_m}$$
$$Hit_{it} = \mathbf{1} \left\{ \varepsilon_{it} \leq \widehat{VaR}_i^\alpha \right\} \quad , \quad Hit_{mt} = \mathbf{1} \left\{ r_{mt} \leq \widehat{VaR}_m^\alpha \right\}$$

where $\alpha = 0.1, 0.05$ or 0.01 .

- Then test

$$H_0 : \Pr [Hit_{it} = 1 | Hit_{mt} = 1] = \Pr [Hit_{it} = 1 | Hit_{mt} = 0] \quad , \quad \text{vs}$$

$$H_a : \Pr [Hit_{it} = 1 | Hit_{mt} = 1] \neq \Pr [Hit_{it} = 1 | Hit_{mt} = 0]$$

via a simple likelihood-ratio test.

- We used applied Christoffersen's test to the 59 funds with more than 66 months of data, using 10% VaR, rather than the more common 5% or 1% VaR, to increase the number of informative observations.

- We found only 3.4% of funds exhibited significant VaR dependence, at the 0.05 level, controlling for mean and variance non-neutrality.

- Thus we have no evidence against VaR neutrality for these funds, when controlling for mean and variance non-neutrality.

24 VaR neutrality on the downside

- It is clear that risk averse investors would prefer the violations of the VaR to be negatively correlated, ie that:

$$\Pr [Hit_{it} = 1 | Hit_{mt} = 1] < \alpha = \Pr [Hit_{it} = 1]$$

- Thus we can again test a restricted version of VaR neutrality:

$$H_0 : \Pr [Hit_{it} = 1 | Hit_{mt} = 1] \leq \Pr [Hit_{it} = 1], \text{ vs}$$

$$H_a : \Pr [Hit_{it} = 1 | Hit_{mt} = 1] > \Pr [Hit_{it} = 1]$$

using a likelihood ratio test.

- Applying this test to the 59 funds with more than 66 months of data, again using 10% VaR, we found 3.4% of funds violated VaR neutrality on the downside, controlling for mean and variance non-neutrality.
- Thus we have no evidence against VaR neutrality on the downside for these funds, when controlling for mean and variance non-neutrality.

25 Tail neutrality

- Finally, we consider the neutrality of the hedge fund to the market during extreme events. We define tail neutrality as:

$$\tau \equiv \lim_{\varepsilon \rightarrow 0} \Pr [F_i(r_{it}) \leq \varepsilon | F_m(r_{mt}) \leq \varepsilon] = \lim_{\varepsilon \rightarrow 0} \Pr [F_i(r_{it}) \leq \varepsilon] = 0$$

- This is equivalent to VaR neutrality if we let the probability $\rightarrow 0$.
- Testing for tail neutrality can be done in a number of ways:
 1. Longin and Solnik (2001): specify a parametric copula and test restrictions on its parameter(s)
 2. Bae, Karolyi and Stulz (2003): specify a multinomial logistic regression and test restrictions on the parameters
 3. Quintos (2003): use a nonparametric approach based on EVT to derive a test statistic

26 Testing tail neutrality

- We employ the test of Quintos (2003). This test is derived to allow for GARCH in the data.
- Note that testing tail neutrality is already a “downside” type of neutrality, as the null and alternative hypotheses are:

$$H_0 : \tau = 0, \text{ vs}$$

$$H_a : \tau > 0$$

- We restricted our data set to funds with at least 100 observations. Of these 9 had sufficient data in the joint tails to conduct the test, and only 1 fund violated tail neutrality at the 0.05 level.
- We thus conclude that we have no evidence against tail neutrality for these hedge funds. This conclusion may well be the result of insufficient data.

27 Complete neutrality

- This is the strictest form of neutrality, requiring that the fund return is completely independent of market risk. That is:

$$r_{it}|r_{mt} \stackrel{d}{=} r_{it}, \text{ or}$$
$$r_{it}|\mathcal{F}_{t-1}, r_{mt} \stackrel{d}{=} r_{it}|\mathcal{F}_{t-1}$$

where \mathcal{F}_{t-1} contains all information available at time $t - 1$, and “ $\stackrel{d}{=}$ ” indicates equality in distribution.

- The joint density of the fund and market returns can be written as:

$$\begin{aligned} f(r_{it}, r_{mt}) &= f_i(r_{it}) \cdot f_m(r_{mt}) \cdot c(F_i(r_{it}), F_m(r_{mt})) \\ &= f_i(r_{it}) \cdot f_m(r_{mt}) \text{ under complete neutrality} \end{aligned}$$

where f_i and f_m are the marginal densities of the fund and market returns, and c is the copula density, or dependence function.

28 Testing for complete neutrality

- We could test for complete neutrality using a multivariate first-order stochastic dominance test.
- More simply, we can use the fact that for any *measure of concordance*, we have

$$C^- \preceq_u C_I \preceq_u C^+ \Leftrightarrow C^- \leq C_I \leq C^+ \Rightarrow \rho_S(C^-) \leq 0 \leq \rho_S(C^+)$$

and so we can obtain an approximate ordering of funds by looking at rank correlation.

- The average rank correlation across funds with at least 18 observations was 0.016. 25.7% of funds had significant rank correlation, and 24.6% of funds had significantly positive rank correlation.
- Thus about one-quarter of funds failed the test of complete neutrality.

29 Summary: are “market neutral” funds really market neutral?

- We can create a joint test of neutrality by combining the results of the 5 individual tests presented previously.
- This test is obtained by looking at the bootstrap distribution of the number of tests failed (at the 5% level): if the actual number of tests failed by a fund is greater than the 95th percentile of this distribution then the fund is reported to have failed the joint test.
- 28.1% of funds failed the joint test of general neutrality.
- 21.6% of funds failed a joint test of neutrality on the downside.
- So not all market neutral hedge funds are created equal.

30 Comparison of neutral and non-neutral funds

	Neutral fund portfolio	Non-neutral fund portfolio	Difference
Mean return [†]	9.14	13.61	-4.47*
Standard deviation [†]	3.15	6.49	-3.34*
Skewness	0.20	-0.33	0.53
Kurtosis	3.19	3.14	0.04
Correlation with market	-0.06	0.78	-0.85*
Median size	\$23m	\$15m	\$8m
Median age	46 months	88 months	-42 months*

“Non-neutral” funds are those that failed the joint test of neutrality on the downside. An asterisk denotes that the difference is significant at the 0.05 level. [†]Means and standard deviations have been annualised.

31 Are “market neutral” funds more neutral than other funds?

Proportion of funds that reject “neutral on the downside”					
	<i>Market Neutral</i>	<i>Equity Hedge</i>	<i>Equity Non-hedge</i>	<i>Event Driven</i>	<i>Funds of funds</i>
Linear correlation	0.27	0.57	0.88	0.74	0.53
Mean neutrality	0.01	0.01	0.07	0.01	0.01
Variance neutrality	0.04	0.00	0.00	0.04	0.00
VaR neutrality	0.00	0.03	0.05	0.06	0.02
Tail neutrality	0.04	0.01	0.42	0.08	0.03
Complete neutrality	0.25	0.58	0.88	0.71	0.52
Joint test	0.22	0.54	0.81	0.64	0.49

Using US dollar returns and the S&P500 as the market index. All tests are conducted at the 0.05 level.

32 Robustness checks

- Choice of market portfolio:
 - Originally considered an investor using the S&P500 as the market portfolio. No substantial differences found when using the MSCI World index or the MSCI Europe index as the market portfolio.
- Choice of currency:
 - Originally considered a US dollar returns. No substantial differences found when using British pounds or DM/Euros.
- End-game behaviour:
 - Dropping the last 6 months of observations on each fund and re-doing the analysis did not change the results substantially.

33 Conclusions and summary

- “Market neutral” hedge funds currently have about \$200 billion under management. One of the attractions of these types of funds is the low (or zero) dependence with the market.
- We proposed considering “neutrality” as a concept with breadth and depth:
 - “Breadth” refers to the number of market risks to which the fund is neutral
 - “Depth” refers to the degree of the neutrality of the fund to each risk
- We applied tests of each type of neutrality to a merged database of HFR and TASS hedge fund returns, over April 1993 to April 2003.
 - A block bootstrap was employed to deal with serial correlation, volatility clustering and non-normality in the returns data.

34 Conclusions, cont'd

- Around one-quarter of “market neutral” funds are significantly non-neutral.
- This finding suggests that the diversification benefits of “market neutral” hedge funds may vary from fund to fund, and may be better or worse than their name suggests.
- We cannot take “market neutrality” at face value; some analysis of the fund’s relationship with the market is required to determine whether the fund is offering the desired degree and type of market neutrality.