1. An Akerlof “Market for Lemons:” There are a continuum of potential buyers of used cars, and a continuum of potential sellers, where each set is normalized to be of measure one. Each seller has one car to sell, and each buyer desires to purchase one car. The quality, \( x \), of each seller’s car is private information. Buyers know the distribution of quality, which is uniform on \([1, 2]\). A buyer’s monetary-valued utility from a car of quality \( x \) is \( \alpha x \); a seller’s monetary-valued utility of retaining his car of quality \( x \) is simply \( x \). Consequently, if a seller sells a car of quality \( x \) at a price \( p \), her payoff is \( p - x \) and the buyer’s payoff is \( \alpha x - p \).

   (i) Determine the competitive equilibrium in this used car market for every value of \( \alpha \in [1, 2] \).

   (ii) For what values of \( \alpha \) does the competitive equilibrium involve the Pareto optimal number of trades?

2. MWG 13.C.5 (Advertising as a Signal of Product Quality)

3. A Screening Problem: Consider an employer and a potential employee. The employee’s productivity (ability) level, \( \alpha \), is private information, and \( \alpha = 1 \) or \( \alpha = 2 \). The employee can be hired at any task level \( t \), and there is a continuum of possible task levels \( t \in [0, 5] \). The employer offers contracts to the employee of the form \((t, w)\) where \( t \) is the task level and \( w \) is the wage. The employee’s utility depends on ability, task, and wage: \( U(t, w; \alpha) = w - \frac{t^2}{2\alpha} \), where the last term is the disutility of performing task \( t \) which is increasing in \( t \) and decreasing in ability \( \alpha \). The employer’s profits are \( V(t, w; \alpha) = \alpha t - w \).

   If the employee is not hired, the employer earns zero profits. The employee’s reservation payoffs depend on her ability, where \( u^0(\alpha) \) is weakly increasing in \( \alpha \) and \( u^0(1) = 0 \).

   (i) Derive the optimal contracts for the two types of employee, \((t_1^*, w_1^*)\) and \((t_2^*, w_2^*)\), in the case where the employer can observe the employee’s ability.

   (ii) Now consider the case where ability is not observable to the employer. Suppose the employer believes that the employee has ability 1 and 2 with equal probability. Set up the optimization problem to determine the contracts that maximize her expected profits.
(iii) Solve for the optimal contracts in two cases: (a) \( u^0(2) = u^0(1) = 0 \), and (b) \( u^0(2) \in \left( \frac{1}{4}, 4 \right) \). Explain the presence or absence of “task distortions” and “information rents” in the contracts.

4. Adverse Selection/Hidden Information in Insurance Markets

Consider a strictly risk averse individual with a utility function \( u(\cdot) \) that depends only on money and who has initial wealth \( w \). She has some chance of falling sick and her treatment would cost \( c \). She either a high risk with probability \( \pi_H \) of falling sick, or low risk with probability \( \pi_L \), where \( 1 > \pi_H > \pi_L > 0 \). The individual knows whether she is high or low risk, but any insurer only knows that she is high risk with probability \( \lambda \) (and low risk with probability \( 1 - \lambda \)).

Suppose there is a single risk-neutral insurer who acts as a monopolist. The insurer offers insurance policies of the form \((p, r)\) where \( p \) is the premium paid by the insuree and \( r \) is the amount of money the insurance companies pays to reimburse the insuree for medical bills.

(i) Under any insurance contract \((p, r)\), let \( x \) denote the final wealth of the individual when she has no illness (i.e., \( x = w - p \)) and let \( y \) denote the final wealth of the individual when she does fall ill (i.e., \( y = w - p - c + r \)). Draw indifference curves of the two risk types in \((x, y)\) space passing through the individual’s initial wealth level. Establish that the indifference curves of the two risk types satisfy a form of the “single crossing property.” Interpret this property in terms of the marginal rate of substitution of income in the case of illness and case of no illness.

(ii) Characterize in the diagram the optimal insurance contracts the monopolist would offer if it could observe and verify the individual’s risk type. Let \((p^*_L, r^*_L)\) and \((p^*_H, r^*_H)\) denote these “first-best” contracts. Explain the relationship between \( p^*_L \) and \( p^*_H \) and \( r^*_L \) and \( r^*_H \).

(iii) Now consider the case where the individual’s risk type is not observable. What would happen if the monopolist insurer offered the two contracts \((p^*_L, r^*_L)\) and \((p^*_H, r^*_H)\)? That is, which risk type would choose which contract? Write down the insurer’s optimization problem in this case of hidden information. Characterize the optimal contract in a diagram. For which risk type is the contract distorted away from the first-best contract? Which risk-type earns rents?