Problem Set 1

1. Consider a lottery whose jackpot is $2000, and the probability of winning is .05. What is the maximum price you are willing to pay to play this lottery?

Now consider your choice between the following two lotteries: \( L \) has a jackpot of $2000, and the probability of winning is .01. \( L' \) has a jackpot of $x, and the probability of winning is .20. For what value of \( x \) would you be indifferent between buying the two lotteries?

From your answers to the questions above, determine whether or not your preferences satisfy the axioms of the von Neumann-Morgenstern expected utility theory.

2. Consider two simple lotteries \( L \) and \( L' \) in \( L \), where \( L \) is the space of all possible lotteries for a set of outcomes \( C \). Suppose that \( L \succ L' \) according to some preference relation \( \succeq \). Show that if this preference relation satisfies all the axioms of the von Neumann-Morgenstern expected utility theory, then

\[
\beta L + (1 - \beta)L' \succ \alpha L + (1 - \alpha)L'
\]

if and only if \( \beta > \alpha \).

3. Let \( \succeq \) be a preference relation on the lotteries in \( L \) which is the space of all possible lotteries for a set of outcomes \( C \). Assume that these preferences satisfy the von-Neumann-Morgenstern axioms. Let \( U : L \rightarrow \mathbb{R} \) be a vNM utility function that represents these preferences. Suppose \( \tilde{U} : L \rightarrow \mathbb{R} \) is a positive affine transformation of \( U \). Show that \( \tilde{U} \) also represents the preference relation \( \succ \). Show that \( \tilde{U} \) also has an expected utility form. Hint: use the fact that for any simple lottery \( L = (p_1, ..., p_N) \sum p_n = 1 \), and the fact that \( U \), being a von-Neumann-Morgenstern utility function, has an expected utility form (and let \( u() \) denote the corresponding function that assigns numbers to the outcomes in \( C \)).


5. An individual has a Bernoulli utility function \( u() \) which is strictly increasing and strictly concave. She is offered a bet with \( \frac{2}{3} \) probability of winning $x and probability \( \frac{1}{3} \) of losing $x dollars. Show that for \( x \) sufficiently small, the individual will take the bet.