Answer all three questions below. Each answer will constitute 1/3 of your grade.

Use the exam books provided for your answers, and use one book for each problem. Write your id number, NOT your name, on all answer books.

1. Consider a risk-averse consumer with initial wealth $w_0$. The consumer faces the risk of two kinds of automobile accidents: a minor accident with probability $p_1$ and a major accident with probability $p_2$. She cannot have both types of accidents, and $p_1 + p_2 \leq 1$. Her total loss from a minor accident is $a$ and her total loss from a major accident are $b$, where $a < b$.

The consumer has a choice between two kinds of insurance policies. One type of policy has a “deductible,” so that she pays the first $d$ dollars of a loss. (That is, in the event of a loss, if the loss is higher than $d$, the insurance company pays the consumer the amount of the loss minus $d$. If the loss is lower than $d$, she does not receive any reimbursement from the insurance company). The second type of policy has a co-insurance feature where the customer pays a fraction $0 < \alpha < 1$ of any loss. She can only buy one or the other insurance policy - no combination is available. Both policies charge the same premium $r$ to the consumer.

(i) Write down the consumer’s expected utility from purchasing each type of insurance policy.

(ii) Suppose that the expected value of the net loss to the consumer is the same under both policies. Write down the condition on $d$ and $\alpha$ such that this is true.

(iii) Suppose that the premium $r$ is equal to the expected value of a loss under each policy, and the condition in part (ii) holds so that expected losses are the same. Which type of policy will the consumer choose?
2. Consider the following principal-agent relationship. The principal's revenues are \( R = e + \epsilon \), where \( e \) is the agent's effort and \( \epsilon \) is a random variable determined after effort is chosen, where \( E(\epsilon) = 0 \) and \( \text{Var}(\epsilon) = \sigma^2 \). The principal is risk neutral; she seeks to maximize her expected profits, which are the expected revenues minus expected wage payments. Let \( \Pi = R - w \) denote the principal’s profits, where \( w \) is the wage paid to the agent. The wage may be a schedule, \( w(R) \), contingent on levels of realized revenues. The agent is risk-averse; she has an expected utility function \( U = E(w) - \lambda \text{Var}(w) - e^2/2 \), and has a reservation utility of 0.

(i) What is the level of effort that maximizes the joint expected welfare of the principal and the agent?

(ii) Suppose that effort, \( e \), and total revenues, \( R \), are both observable and verifiable. If the principal offers a wage schedule to the agent, which the agent then accepts or rejects, what will be the wage schedule and what level of effort will the agent exert?

(iii) Suppose now that only \( R \) is observable and verifiable. For simplicity, restrict attention to linear wage schedules: \( w(R) = a + bR \). Suppose again that the principal can make an offer to the agent which the agent then accepts or rejects. (a) What is the contract that the principal will offer the agent; that is determine the optimal levels of \((a,b)\) for the principal. (b) What is the slope of the incentive scheme, \( b \), when \( \lambda = 0 \)? Explain.
3. There is a continuum of individuals that differ according to their productivity \( \theta \in [\theta', \theta''] \) where \( \theta' < \theta'' \). Let \( g(\theta) \) be the density of workers of type \( \theta \), with \( g(\theta) > 0 \) for all \( \theta \in [\theta', \theta''] \).

There is a large set of firms with identical constant returns to scale technology. Firms seek to maximize profits and act as price takers. A worker with productivity \( \theta \) contributes \( \theta \) units of output to a firm; the price of output is normalized to 1.

Each individual has the choice to be self-employed, in which case he earns \( r(\theta) \). Suppose \( r(\theta) \) is a strictly decreasing function of \( \theta \).

Firms know \( g(\theta) \) and \( r(\theta) \) but cannot observe any individual’s productivity.

(i) Do workers with higher or lower productivity levels choose to work at any given wage \( w \)?

(ii) Show that if \( r(\theta) > \theta \) for all \( \theta \), then the resulting competitive equilibrium is Pareto efficient.

(iii) Suppose there exists a \( \theta \) such that \( r(\theta) < \theta \) for \( \theta > \theta \) and \( r(\theta) > \theta \) for \( \theta < \theta \). Show that any competitive equilibrium with strictly positive employment involves too much employment relative to the Pareto optimal allocation of workers.