Answer all three questions below. Your answer to the first problem will constitute 25% of your final grade, and the second two problems will constitute 37.5% each.

Use the books provided for your answers, and use one book for each problem. Write your id number, NOT your name, on all answer books.

1. (25%) Consider the following two lotteries $L$ and $L'$. $L$ gives $5, $15, and $30 with probabilities $\frac{1}{3}, \frac{5}{9},$ and $\frac{1}{9}$, respectively. $L'$ gives $10 with probability $\frac{2}{3}$ and $20 with probability $\frac{1}{3}$. What lottery would be chosen by (i) a risk neutral individual, (ii) a risk averse individual, (iii) a risk loving individual.
2. (37.5%) John Doe has Bernoulli utility \( u(y) \) and has initial wealth \( w_0 \). The payoff of a Maryland State Lottery ticket is a random variable \( x \). There is a probability \( \pi \) of winning $\Delta$ and a probability \( 1 - \pi \) of winning nothing.

In the case that Mr. Doe has yet to buy such a lottery ticket, define \( p^B \) to be the maximum price at which he is willing to buy a ticket. Alternatively, in the case that Mr. Doe already owns such a lottery ticket, define \( p^S \) to be the minimum price at which he is willing to sell it.

Show, using the result below that if \( u(y) \) exhibits “decreasing absolute risk aversion,” then \( p^B < p^S \).

Result: For an individual with utility function \( u(y) \) and wealth \( w \) who faces an uncertain return represented by a random variable \( z \), let \( c_z(w) \) denote the ‘certainty equivalent’ of \( z \). That is, \( u(w + c_z(w)) = E[u(w + z)] \). Then \( c_z(w) \) increases in \( w \) if and only if \( u(y) \) exhibits “decreasing absolute risk aversion.”
3. (37.5%) A Screening Problem: Consider two firms competing for a large group of workers. Each worker’s productivity (ability) level, $\alpha$, is private information, and $\alpha = 1$ or $\alpha = 2$. The proportion of workers that have high productivity (i.e., $\alpha = 2$) is $\lambda$. A worker can be hired at any task level $t \geq 0$. An employee’s utility depends on ability, task, and wage: 

$$U(t, w; \alpha) = w - \frac{t^2}{2\alpha},$$

where the last term is the disutility of performing task $t$. Each firm’s profits are $\pi(t, w; \alpha) = \alpha - w$ per employee hired. Notice that the firms’ profits do not depend on $t$. If a firm does not hire any workers, it earns zero profits. All workers have utility of zero if they are not hired.

The interaction between the firms and workers proceeds as follows:

1. The firms simultaneously offer contracts to workers of the form $(t, w)$ where $t$ is the task level and $w$ is the wage.

2. Each worker accepts or not a contract from one of the firms. Profits/utility are then realized.
   
   (i) Show that the workers’ utility functions satisfy the single-crossing property.
   
   (ii) If there exists a subgame perfect equilibrium in this game, what are the contracts offered in equilibrium?

   (iii) Under what conditions does a subgame perfect equilibrium exist?