Signaling: Spence

Main idea: get out of adverse selection by allowing workers with high $\theta$ to “signal” their type by taking some action that workers with low $\theta$ would not want to imitate.
Situation 1: Focus on Costs of Signaling

Two types: $\theta_H > \theta_L > 0$

Let $\lambda = \text{prob}\{\theta = \theta_H\} \in (0, 1)$.

Let $e$ denote the level of education.

Let $c(e, \theta)$ be the cost of education for a worker of type $\theta$, where:

$$c(0, \theta) = 0;$$

$$c_e(e, \theta) > 0; \ c_{ee}(e, \theta) > 0;$$

$$c_\theta(e, \theta) < 0;$$

$$c_{e\theta}(e, \theta) < 0$$

Assumption $c_{e\theta}(e, \theta) < 0$ is the Spence-Mirrlees condition or “single crossing property”.
Situation 1: Focus on Costs of Signaling

Assume that education does not affect a worker’s $\theta$.

The worker’s utility is $u(w, e, \theta) = w - c(e, \theta)$.

Assume that $r(\theta_H) = r(\theta_L) = 0$.

The equilibrium wage is now $w^* = E(\theta)$ and all workers work, which is the P.O. outcome.
Signaling (cont.)

Two firms with CRTS technology.

Timing of interaction between the firms and a worker:

0. Nature selects worker’s type, which is private information.

1. Worker chooses an education level $e$.

2. Firms observe $e$ and simultaneously make wage offers to the worker.
Look for a PBE of this game: specify both the strategies and beliefs of the agents.

Firms’ beliefs have to be consistent with workers equilibrium strategies (satisfy Bayes’ rule whenever possible).

Specify “off-the-equilibrium path” beliefs (what firms believe after observing an unexpected education level)

Let $\mu(e)$ be the firms’ common belief over the worker’s type, conditional on observed $e$.

At stage 2, the firms believe with probability $\mu(e)$ that $\theta = \theta_H$. 
A set of strategies and the belief function $\mu(e)$ are a PBE if:

(i) The worker’s strategy is optimal given the firms’ strategies.

(ii) The belief function $\mu(e)$ is derived from the worker’s strategy using Bayes’ rule whenever possible.

(iii) The firms’ wage offers constitute a Nash equilibrium in the simultaneous wage-setting game in which the probability that the worker is high productivity is $\mu(e)$. 
In stage 2, Bertrand competition between firms guarantees that:

$$w^*(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L.$$  

The worker will accept this wage (no outside option).

In stage 1: the worker knows that the wage she will be offered depends on her choice of $e$.

The worker’s utility is $u(w, e) = w - c(e, \theta)$.  

Separating Equilibria

Let $e^*(\theta)$ be the worker’s equilibrium choice of education.

In a separating equilibrium $e^*(\theta_H) \neq e^*(\theta_L)$.

Characterize a S.E.:

(1) In any SE $w^*(e^*(\theta_H)) = \theta_H$ and $w^*(e^*(\theta_L)) = \theta_L$.

(2) In any SE $e^*(\theta_L) = 0$.

Suppose $e^*(\theta_L) > 0$. In a SE $w^*(e^*(\theta_L)) = \theta_L$. Worker then receives in equilibrium

$$w^*(e^*(\theta_L)) - c(e^*(\theta_L), \theta_L) = \theta_L - c(e^*(\theta_L), \theta_L)$$

If she deviates to $e = 0$ the lowest wage she can earn is $\theta_L$ (as in equilibrium) and saves education costs.

Therefore, $e^*(\theta_L) > 0$ cannot be part of a SE.
Separating Equilibria (cont.)

Conditions (1) and (2) give the following elements of a Separating Equilibrium:

\[ w^*(e^*(\theta_L)) = \theta_L. \quad \text{and} \quad e^*(\theta_L) = 0 \]
\[ w^*(e^*(\theta_H)) = \theta_H \]
Separating Equilibria (cont.)

Consider now $e^*(\theta_H)$:

Type $\theta_L$ earns in equilibrium: $U^*_L = \theta_L - c(0, \theta_L)$.

We also know that $w^*(e^*(\theta_H)) = \theta_H$.

Let $\epsilon$ be the minimum $e$ consistent with a SE:

$$U^*_L = \theta_L - c(0, \theta_L) = \theta_H - c(\epsilon, \theta_L)$$

Therefore the low type would not want to deviate to $\epsilon$. 
Separating Equilibria (cont.)

The high type would also not want to deviate to $e = 0$:

With $e^*(\theta_H) = e$, she earns $U_H^* = \theta_H - c(e, \theta_H)$.

She would not deviate to $e = 0$ when:

$$\theta_H - c(e, \theta_H) \geq \theta_L - c(0, \theta_H)$$

that is,

$$\theta_H - \theta_L \geq c(e, \theta_H) - c(0, \theta_H)$$

From our derivation of $e$ we have,

$$\theta_L - c(0, \theta_L) = \theta_H - c(e, \theta_L)$$

which implies

$$\theta_H - \theta_L = c(e, \theta_L) - c(0, \theta_L)$$

Substituting for $\theta_H - \theta_L$ into the above inequality:

$$c(e, \theta_L) - c(0, \theta_L) \geq c(e, \theta_H) - c(0, \theta_H)$$

Which is true by single-crossing property.
Separating Equilibria (cont.)

Then:

\[ e^*(\theta_L) = 0, \quad e^*(\theta_H) = \varepsilon, \]
\[ w^*(e^*(\theta_L)) = \theta_L, \quad w^*(e^*(\theta_H)) = \theta_H \]

and no worker would want to deviate.

Off-the-equilibrium path beliefs:

\[ \mu(e) = 0 \quad \text{for} \quad e < \varepsilon = e^*(\theta_H) \]
\[ \mu(e) = 1 \quad \text{for} \quad e \geq \varepsilon = e^*(\theta_H) \]

These beliefs give us the wage profile:

\[ w^*(e) = \theta_L \quad \text{for} \quad e < \varepsilon = e^*(\theta_H) \]
\[ w^*(e) = \theta_H \quad \text{for} \quad e \geq \varepsilon = e^*(\theta_H) \]
Separating Equilibria (cont.)

Neither type has an incentive to deviate:

- A low type would not deviate to any $0 < e < \epsilon$ (it is already earning $\theta_L$ and $\epsilon$ implies higher costs) and would not deviate to any $e \geq \epsilon$ (by construction of $\epsilon$).

- A high type would not deviate to any $0 \leq e < \epsilon$ (it would choose 0 and earn $\theta_L$) and would not deviate to a higher $e$. 
Separating Equilibria (cont.)

There are possibly many SE.

Let $\overline{e}$ be the highest $e$ consistent with a SE. That is, $\overline{e}$ satisfies:

$$U_H^* = \theta_H - c(\overline{e}, \theta_H) = \theta_L - c(0, \theta_H)$$

These SE. can be Pareto ranked.
Compare workers’ welfare in the SE vs. in the no-signaling case.

With no signaling both types earn:

\[ E(\theta) = \lambda \theta_H + (1 - \lambda) \theta_L \]

In the SE high productivity workers earn:

\[ w^*(e^*(\theta_H)) - c(e^*(\theta_H), \theta_H) = \theta_H - c(e^*(\theta_H), \theta_H). \]

For \( \lambda \) sufficiently high, they are doing worse off than in the no-signaling case.

Low productivity workers are always worse off. In the SE:

\[ w^*(e^*(\theta_L)) - c(e^*(\theta_L), \theta_L) = \theta_L - 0. \]

The level of \( \lambda \) does not affect the existence of a SE.
Pooling Equilibria

In a pooling equilibrium:

\[ e^*(\theta_H) = e^*(\theta_L) = e^* \quad \text{and} \quad w^*(e^*) = E(\theta) \]

Pooling equilibria can be pareto ranked, and again the Pareto inferior equilibria are sustained by “pessimistic” off the equil. path beliefs on the part of firms.
Multiple Equilibria & the Intuitive Criterion

**Objective:** restrict firms’ off-the-equilibrium path beliefs.

Consider the SE examined earlier:

For a type $\theta_L$ worker, the deviation to $e' > e > \bar{e}$ is dominated by her equilibrium payoffs for any possible off-the-equilibrium path beliefs.
IC: eliminate a PBE if it is sustained by beliefs that place positive probability on a type for whom a deviation would be equilibrium dominated.

Having observed $e' > e > \bar{e}$, the firms cannot believe with any positive probability that this worker is a low productivity worker.

Therefore, they must have $\mu(e) = 1$, and offer the worker a wage $\theta_H$.

The high-productivity worker would then deviate to this lower level of education.
Situation 2: Costs and Benefits of Signaling

Suppose:

\[ r = r(\theta_L) = r(\theta_H) \]

\[ \theta_L < r < \theta_H \text{ and} \]

\[ E(\theta) < r. \]

In this case, no workers are employed in a competitive equilibrium.

The Pareto efficient outcome involves the high productivity workers being employed, since \( r < \theta_H \).

There exist separating equilibria in which high productivity workers are employed.

In any SE:

\[ e^*(\theta_L) = 0 \text{ and } w^*(e^*(\theta_H)) = \theta_H \text{ and } w^*(e^*(\theta_L)) = \theta_L. \]

Low productivity workers, will not choose to be employed, since \( r > \theta_L \).
These separating equilibria involve a *Pareto improvement* over the competitive case:

For $e^*(\theta_H) < e''$, high productivity workers are better off.

They are best off for $e^*(\theta_H) = \bar{e}$. (in the competitive case they were not employed and earned $r$).

Low productivity workers have the same level of utility (they earn $r$ in both cases).

Firms are also equally well off (in the competitive case they employed no workers; in the SE. they employ high productivity workers, but pay them exactly their value)

Hence, we have a Pareto improvement: high productivity workers are strictly better off, and neither of the other two parties are worse off.