Suppose that effort is observable and verifiable, so that a contract can be conditioned on effort.

The principal’s problem is:

$$\max_{w(\pi), e \in \{e_L, e_H\}} \int_{\pi} [\pi - w(\pi)] \cdot f(\pi | e) d\pi$$

s.t.

$$\int_{\pi} [v(w(\pi)) \cdot f(\pi | e) d\pi] - g(e) \geq u.\)$$

(Individual Rationality (IR))

(Only have IR constraint, since principal can impose an arbitrarily large penalty for taking the other effort level.)
Full Information Case (cont.)

Solution:

First, for $e_L$ and $e_H$, find the contract that maximizes profits s.t. the IR constraint for that effort level. For effort level $e$:

$$\max_{\pi \mid e} \int_{\pi} \left[ \pi - w(\pi \mid e) \right] \cdot f(\pi \mid e) d\pi$$

Since mean profits are independent of $w(\pi \mid e)$, this is the same as:

$$\min_{\pi \mid e} \int_{\pi} w(\pi \mid e) \cdot f(\pi \mid e) d\pi$$

s.t.

$$\int_{\pi} \left[ v(w(\pi \mid e)) \cdot f(\pi \mid e) d\pi \right] - g(e) \geq u.$$

(Individual Rationality (IR))
Proposition: Under full information, with a strictly risk averse agent, the optimal wage contract for each effort level is unique and consists of a fixed wage.

Proof. Let $\gamma$ denote the Langrangian multiplier on the IR constraint (IR will be binding). Then $w^*(\pi \mid e)$ must satisfy the FOC for this minimization problem:

$$-f(\pi \mid e) + \gamma v'(w^*(\pi \mid e)) \cdot f(\pi \mid e) = 0 \quad \text{or}$$

$$\frac{1}{v'(w^*(\pi \mid e))} = \gamma.$$ 

When $v'' < 0$, then $w^*(\pi \mid e)$ is a constant. ■
Interpretation:

We have a “fixed wage contract” \( w^*(\pi \mid e) = w_e^* \), where \( w_e^* \) is solves:

\[
v(w_e^*) - g(e) = \pi; \quad \text{ie.,} \quad w_e^* = v^{-1}[\pi + g(e)].
\]

The risk-neutral principal absorbs all of the risk of the uncertain profits, and we have the first-best level of risk sharing.

We will have \( w_e^{*H} > w_e^{*L} \), since \( g(e_H) > g(e_L) \).
Second, select the optimal effort level from the point of view of the principal, $e^*$, by solving:

$$\max_{e \in \{e_L, e_H\}} \int_{\pi} \pi \cdot f(\pi | e) d\pi - w_e^*.$$ 

Once the principal determines $e^*$, it can offer the following compensation schedule to the agent:

$$w_e^* \text{ for } e = e^*$$

$$-\infty \text{ for } e \neq e^*.$$ 

Therefore, the principal guarantees that the worker will choose the wage $w_e^*$ for $e = e^*$. 