Topic 3:
Moral Hazard and Principal-Agent Problems

- The Principal-Agent Problem
  - Basic Economic Environment
  - The Interaction Between P and A
  - The Profit-Maximizing Contract
  - Full Information Case
  - Hidden Action Case
    - Risk-Neutral Agent
    - Risk-Averse Agent
  - Continuous Effort: The First Order Approach

- Principal-Agent with Hidden Information
  - Basic Environment
  - Benchmark: Public Information
  - Private Information Case
The Issue

Main Issue: Actions of agents are hidden from other parties to a transaction.

Principal-Agent problem: the principal wants the agent to perform an action costly to the agent, but the action is not directly observed by the principal.

Examples:

Owners (shareholders) of a firm and firm’s manager.

Landowner and sharecropper.

Manufacturer and retailer.
The Principal-Agent Problem

Basic Economic Environment:

A risk-neutral firm owner (the principal) only cares about profits.

Let $e \in E$ be the level of effort exerted by the manager (the agent).

Profits are a random variable $\pi$, with (continuous) distribution function $F(\pi \mid e)$ on $[\underline{\pi}, \overline{\pi}]$, and associated density $f(\pi \mid e)$. 
The Principal-Agent Problem (cont.)

Basic assumptions:

(i) \( f(\pi \mid e) > 0 \) for all \( \pi \in [\underline{\pi}, \overline{\pi}] \) and \( e \in E \).

(ii) For \( e_1 > e_2 \), \( F(\pi \mid e_1) \leq F(\pi \mid e_2) \) for all \( \pi \in [\underline{\pi}, \overline{\pi}] \).

Then:

\[
\int_{\underline{\pi}}^{\overline{\pi}} \pi \cdot f(\pi \mid e_1) d\pi \geq \int_{\underline{\pi}}^{\overline{\pi}} \pi \cdot f(\pi \mid e_2) d\pi.
\]
(iii) The manager has Bernoulli utility function

\[ u(w, e) = v(w) - g(e) \]

with \( v' > 0, v'' \leq 0, \) and \( g' > 0. \)

(iv) The manager has reservation utility of \( \bar{u}. \)
The Principal-Agent Problem (cont.)

Information assumptions:

(i) $e$ is not observable to the principal (or if observable it is non-verifiable).

(ii) $\pi$ is observable and verifiable.

**Implication of (i) and (ii):** the parties can only contract on $\pi$, not on $e$. We consider wage/compensation schedules of the form $w(\pi)$.

(iii) $F(\pi \mid e)$ is common knowledge.

(iv) $u(w, e)$ and $\bar{u}$ are common knowledge.
The Interaction Between P and A

1. The owner offers the manager a compensation schedule $w(\pi)$.

2. The manager accepts or rejects this schedule.

   If the manager rejects, he earns his reservation value $\bar{u}$, the owner earns 0 profits and the interaction is over.

3. If the manager accepts, then he chooses a level of effort $e$, which generates a distribution over profits $F(\pi \mid e)$.

4. The profits of the firm are then realized and the manager is paid according to the schedule $w(\pi)$. 
Manager’s Effort Choice

If the manager accepts $w(\pi)$, her expected payoffs are:

$$\bar{\pi} \int [v(w(\pi)) \cdot f(\pi \mid e) \, d\pi] - g(e)$$

If the manager accepts she will choose $\hat{e}(w(\pi))$ where:

$$\hat{e}(w(\pi)) = \arg \max_{\hat{e}} \bar{\pi} \int [v(w(\pi)) \cdot f(\pi \mid \hat{e}) \, d\pi] - g(e)$$

Consequently, she will accept the contract iff:

$$\bar{\pi} \int [v(w(\pi)) \cdot f(\pi \mid \hat{e}) \, d\pi] - g(\hat{e}) \geq \bar{u}$$
Principal’s Design of Contract

The principal must consider the agent’s effort choice, since she will earn:

\[
\int_{\pi}^{\bar{\pi}} [\pi - w(\pi)] \cdot f(\pi \mid \tilde{e}) d\pi.
\]

The principal’s problem is to find the contract \( w(\pi) \) that maximizes her expected profits, taking into consideration:

1. whether the agent will accept the contract.

2. the agent’s choice of effort given the contract; that is \( \tilde{e}(w(\pi)) \).
Profit-Maximizing Contract for Two Effort Levels

Only two possible effort levels: $e_H > e_L$.

Assume $F(\pi \mid e_H) \leq F(\pi \mid e_L)$ and $g(e_H) > g(e_L)$.

The principal faces the following optimization problem:

$$\max_{w(\pi), e \in \{e_L, e_H\}} \int_{\pi}^{\pi} [\pi - w(\pi)] \cdot f(\pi \mid e) d\pi$$

s.t.

$$\int_{\pi}^{\pi} [\nu(w(\pi)) \cdot f(\pi \mid e) d\pi] - g(e) \geq \bar{u}.$$  

(Individual Rationality (IR))

and

$$e = \arg \max_{\bar{e}} \int_{\pi}^{\pi} [\nu(w(\pi)) \cdot f(\pi \mid \bar{e}) d\pi] - g(\bar{e})$$

(Incentive Compatibility (IC))
The Profit-Maximizing Contract (cont.)

Two-stage solution method:

First, for each effort level \( (e_H \text{ and } e_L) \), find the \( w(\pi) \) which maximizes the principal’s expected profits subject to:

(1) the agent will accept the contract (IR constraint),

(2) the agent will choose the intended effort level (IC constraint).

Second, compare the profits obtained in each case \( (e_H \text{ vs. } e_L) \), and induce the agent to choose the effort level yielding the highest expected profits.