Continuous Effort Case

Suppose $e \in E = [\underline{e}, \bar{e}]$.

The optimal contract $w^o(\pi \mid e)$ for implementing effort level $e$ solves the following problem $M(e)$:

$$\min_{w(\pi \mid e)} \int_{\mathbb{P}} w(\pi \mid e) \cdot f(\pi \mid e) d\pi$$

s.t.

$$\int_{\mathbb{P}} [v(w(\pi \mid e)) \cdot f(\pi \mid e)d\pi] - g(e) \geq \pi$$

(Individual Rationality (IR))

$$\int_{\mathbb{P}} [v(w(\pi \mid e)) \cdot f(\pi \mid \bar{e})d\pi] - g(\bar{e}) \geq \pi$$

$$\int_{\mathbb{P}} [v(w(\pi \mid e)) \cdot f(\pi \mid \underline{e})d\pi] - g(\underline{e}) \geq \pi \forall \bar{e} \in \{E\setminus e\}.$$  

(Incentive Compatibility (IC))

There are infinitely many IC constraints. So there may not exist any wage schedule $w(\pi \mid e)$ for which all constraints are simultaneously satisfied.
The First Order Approach

One possible method of solving this problem is to replace the IC constraints with the first order condition of the following problem:

$$\max_{e \in E} \int_{\pi} \left[ v(w(\pi | e)) \cdot f(\pi | e) d\pi \right] - g(e).$$

The FOC is:

$$\int_{\pi} \left[ v(w(\pi | e)) \cdot f_e(\pi | e) d\pi \right] - g'(e) = 0.$$

(First-Order IC Condition)
The First Order Approach (cont.)

Consider the following constrained minimization:

$$
\min_{w(\pi|e)} \int_{\pi} w(\pi | e) \cdot f(\pi | e) d\pi
$$

s.t.

$$
\int_{\pi} [v(w(\pi | e)) \cdot f(\pi | e) d\pi] - g(e) \geq \bar{u}
$$

(Individual Rationality (IR))

$$
\int_{\pi} [v(w(\pi | e)) \cdot f_e(\pi | e) d\pi] - g'(e) = 0.
$$

(First-Order IC Condition)

With Langrange multipliers $\gamma$ and $\mu$, the optimal wage schedule, $\hat{w}(\pi | e)$, must satisfy the following FOC:
\[
\frac{1}{v'(\tilde{w}(\pi \mid e))} = \gamma + \mu \left[ \frac{f_e(\pi \mid e)}{f(\pi \mid e)} \right].
\]

As before we have \( \gamma > 0 \) and \( \mu > 0 \).
First Order Approach (cont.)

The choice of $e$ that satisfies the FOIC Condition need not be unique, nor need it be a maximum.

To ensure that the effort level that satisfies the FOIC Condition is indeed a global maximum we need:

$$
\int_{\pi} \left[ v\left( w(\pi \mid e) \right) \cdot f(\pi \mid e) d\pi \right] - g(e)
$$

to be globally concave in $e$. This puts restrictions on the shapes of both $f(\pi \mid e)$ and $w(\pi \mid e)$.

The sufficient conditions for $\hat{w}(\pi \mid e) = w^o(\pi \mid e)$ are:

(1) $F(\pi \mid e)$ satisfies the MLRP.

(2) $F(\pi \mid e)$ is convex in $e$; i.e.,

$$
F\left( \pi \mid \lambda e + (1 - \lambda)e' \right) \leq \lambda F\left( \pi \mid e \right) + (1 - \lambda)F\left( \pi \mid e' \right)
$$

for all $e, e' \in E$ and $\lambda \in (0, 1)$.