REALIZED GARCH

A look at AMZN and AMGN

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Brief Recap

- Last time, I gave a summary of Realized GARCH and HEAVY models of volatility forecasting

- I decided to focus more on REALIZED GARCH

- Ideas for moving forward as of last time:
  - Estimate Realized GARCH model using bipower variation as the realized measure
  - Bivariate model of open-to close and close-to open returns
The Model

• For Realized GARCH

\[ r_t = \sqrt{h_t} z_t, \]
\[ h_t = \omega + \beta h_{t-1} + \gamma x_{t-1}, \]
\[ x_t = \xi + \phi h_t + \tau(z_t) + u_t \]

where \( z_t \sim \text{i.i.d.}(0, 1) \) and \( u_t \sim \text{i.i.d.}(0, \sigma_u^2) \), with \( z_t \) and \( u_t \) being mutually independent.

• For GARCHX there is no measurement equation
• For GARCH, \( X=r^2 \) and there is no measurement equation
My two stocks are AMZN and AMGN
Assumptions I made in early data analysis

• For GARCH:
  • \( w=0 \)
  • \( a=.05 \)
  • \( b=.95 \)

• For GARCH-X
  • \( w=0 \)
  • \( a=0 \)
  • \( b=.5 \)
  • \( y=.b, \)

• Starting value is equal to unconditional mean
GARCH, GARCHX, Rolling Vol for AMZN
GARCH, GARCHX, Rolling Vol for AMGN
GARCH, GARCHX, RV AMZN
GARCH, GARCHX, RV for AMGN
Log-Linear Specification

• Similar to before, but do a regression of the logarithms of each process:

\[
\log h_t = \omega + \sum_{i=1}^{p} \beta_i \log h_{t-i} + \sum_{j=1}^{q} \gamma_j \log x_{t-j}
\]
\[
\log x_t = \xi + \phi \log h_t + \tau(z_t) + u_t
\]
\[
\log r_t^2 = \log h_t + \log z_t^2
\]

• From Hansen Huang and Shek (2011):
Linear vs. Log-Linear for AMZN
Linear vs. Log-Linear for AMGN
GARCH Log-Likelihoods for AMZN

\[ \ell(r, x) = -\frac{1}{2} \sum_{t=1}^{n} \left[ \log(2\pi) + \log(h_t) + \frac{r_t^2}{h_t} \right] \]

- assuming \( w=0, a+b=1 \)
- \( b \) is optimized around .98
GARCH Log-Likelihoods for AMGN

\[ \ell(r, x) = -\frac{1}{2} \sum_{t=1}^{n} \left[ \log(2\pi) + \log(h_t) + \frac{r_t^2}{h_t} \right] \]

- assuming \(w=0, \ a+b=1\)
- \(b\) is optimized around \(.97\)
Log-Likelihood For Realized GARCH: Next Step

- Assuming a Gaussian Specification:

\[
\ell(r, x; \theta) = -\frac{1}{2} \sum_{t=1}^{n} \left[ \log(h_t) + \frac{r_t^2}{h_t} + \log(\sigma_u^2) + \frac{u_t^2}{\sigma_u^2} \right]
\]

- Standard GARCH does not model a realized measure, but this can be factored:

\[
f(r_t, x_t | \mathcal{F}_{t-1}) = f(r_t | \mathcal{F}_{t-1}) f(x_t | r_t, \mathcal{F}_{t-1})
\]

\[
\ell(r, x) = -\frac{1}{2} \sum_{t=1}^{n} \left[ \log(2\pi) + \log(h_t) + \frac{r_t^2}{h_t} \right] + -\frac{1}{2} \sum_{t=1}^{n} \left[ \log(2\pi) + \log(\sigma_u^2) + \frac{u_t^2}{\sigma_u^2} \right]
\]

\[
\begin{array}{c}
\ell(r) = \ell(r) \\
\ell(x|r) = \ell(x|r)
\end{array}
\]
Questions Going forward

• Can I come up with a better starting value?
• How can I model an index of stocks rather than individual names?
• How do I do the Log-Likelihood Calculation for Realized GARCH?