Intra-Day and Inter-Market Volatility in Foreign Exchange Rates

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Four foreign exchange spot rate series, recorded on an hourly basis for a six-month period in 1986 are examined. A seasonal GARCH model is developed to describe the time-dependent volatility apparent in the percentage nominal return of each currency. Hourly patterns in volatility are found to be remarkably similar across currencies and appear to be related to the opening and closing of the world's major markets. Robust LM tests designed to deal with the extreme leptokurtosis in the data fails to uncover any evidence of misspecification or the presence of volatility spillover effects between the currencies or across markets.

1. INTRODUCTION

A substantial body of evidence has now accumulated on the behaviour of different types of asset-pricing data, including stock returns, commodity prices and exchange rates. In particular, daily or weekly exchange rates have been found by Meese and Singleton (1982), Corbae and Ouliaris (1986) and Baillie and Bollerslev (1989a) and others to be well-approximated by martingales. While the first difference of asset-pricing data over short horizons may be uncorrelated, the price changes tend not to be independent but to exhibit "volatility clustering". This is where periods of large absolute changes tend to cluster together followed by periods of relatively small absolute changes. This phenomenon was originally noted by Mandelbrot (1963) and Fama (1965), and has led to the application of Engle's (1982) Autoregressive Conditional Heteroscedasticity (ARCH) approach in describing the volatility for several financial time series. For instance, Domowitz and Hakkio (1985), Engle and Bollerslev (1986), McCurdy and Morgan (1987, 1988, 1989), Hsieh (1988, 1989), Baillie and Bollerslev (1989b), Diebold and Nerlove (1989) and Hodrick (1989) have all used the ARCH methodology in modelling foreign exchange rate data.

This occurrence of time-dependent conditional heteroscedasticity could be due to an increased volume of trading and/or variability of prices following the acquisition of new information by the market. A market can be perfectly efficient but still exhibit ARCH
effects in its price changes if "news" or information arrives at uneven intervals of time. This is similar to Stock's (1987) notion of time-deformation, where regular intervals of calendar time may be less appropriate to record economic series compared to recording a price after the completion of a certain number of transactions; see also Lamoureux and Lastrapes (1990).

While a number of studies have now examined the characteristics and changes in volatility on individual markets trading foreign currency, stocks, bonds and commodities; comparatively little attention has been given to the relationships between asset prices and volatility on different financial assets or markets. However, with the apparent increasing integration of international financial markets it is of interest to see if the prices in any one market are informationally efficient with respect to the prices determined in other markets, and also whether or not there are volatility spillovers between different financial markets. Similarly, it is of interest to know whether news is fully and immediately incorporated in all international foreign exchange rate markets, and the relative importance of news associated with particular currencies and market locations.

Some previous studies have examined the relationships between the means of prices in different markets; e.g. Hogan and Sharpe (1984) and Ito and Roley (1987) have both looked at the transmission of news in foreign exchange markets; while Dwyer and Hafer (1988) and King and Wadhwani (1990) consider equity markets.

The possible transmission of volatility between markets has been addressed in two recent papers by Engle, Ito and Lin (1990) and Hamao, Masulis and Ng (1990). The former paper uses four observations per day on the Japanese yen/U.S. dollar exchange rate and reports evidence in favour of a spillover effect in volatility between the different market locations; whereas Hamao, Masulis and Ng (1990) find a causal effect in the variance from the U.S. to the Japanese stock market only, and not conversely.

The contribution of this paper is to consider the detailed relationship between the return and volatility of four major floating foreign exchange rates vis-a-vis the U.S. dollar on an hourly basis as they are quoted on the different currency markets around the world. Section 2 of the paper describes the data and reports some preliminary tests for unit roots, autocorrelation, and time-dependent conditional heteroskedasticity. Anticipating the results, all four exchange rates appear to be well-approximated by martingales with very small first-order moving-average effects, but substantial volatility clustering. Section 3 then develops a seasonal GARCH model to characterize the interesting and distinctly unusual volatility processes. A series of misspecification tests indicate that the conditional mean and variance functions of the model provide a reasonably good representation of the data. The use of hourly data allows both currency specific and market specific factors to be clearly identified. Most of the volatility is found to be relatively short-lived and little persistence is found between different days. The distinct pattern in the volatility for all the currencies across the trading day is discussed in Section 4. This pattern is consistent with the model developed by Admati and Pfleiderer (1988), where traders with diverse information find it optimal to exploit their private information at times when other traders are also active, generating even greater levels of volume of trade and price volatility. Interestingly, very similar patterns in volatility across the day also hold true for non-U.S. dollar exchange rates. Section 5 of the paper then considers several tests for causality in the mean and the variance for each of the different currencies and market locations. Little evidence in favour of any simple or systematic spillover effects in the returns or volatility is forthcoming. Relative to the normal distribution, all the estimated models exhibit severe excess kurtosis, which invalidates standard asymptotically-based inference procedures. To circumvent this problem, we rely throughout the paper on the robust inference
procedures recently developed by Wooldridge (1987, 1990) and Bollerslev and Wooldridge (1990). Since this provides one of the first empirical implementations of these ideas, a detailed discussion of how these techniques apply in the present context is given in the appendix. Finally, Section 6 provides a brief conclusion.

2. DATA AND PRELIMINARY TESTING

When examining the possible relationships between different currencies across different markets it would seem desirable to use data measured at fairly fine intervals of time. Unlike the different stock exchanges and securities markets around the world, the foreign exchange market is virtually continuously active with the same basic assets being traded in several different locations. Money Market Services (MMS) have recorded the value of the spot exchange rate for the British pound (BP), the West German deutschmark (DM), the Swiss franc (SF) and the Japanese yen (YEN) vis-a-vis the U.S. dollar on an hourly basis from 0:00 a.m. January 2, 1986 through 11:00 a.m. July 15, 1986; where local time is London, or Greenwich mean time. The data set constitutes a total of 3191 trading hours and is taken from the average of the last five bid rates recorded at the end of each hour by the fifty largest banks in the foreign exchange market;¹ see also Goodhart and Giugale (1988) and Whistler (1988) where the same data set have been analysed from a different perspective. The foreign exchange market is an electronic market active 24 hours a day with no particular geographical location. However, it is natural to think of the trading as proceeding according to time zones. Indeed, the vast majority of trading is concentrated in the three major markets of London, New York and Tokyo. Of these three markets, the New York and the London markets are by far the largest in terms of total volume traded. The U.S. dollar, the BP, the DM, the SF and the YEN are the most actively traded currencies on both markets. The Tokyo market remains relatively restricted. More than 90% of the Tokyo market deals in the YEN/U.S. dollar, but there is also some activity in the other currencies included in this study; see Tygier (1988). For most hours in our sample the last five bid rates occur in the last one or two minutes of the hour, but between the hours 21 and 23, after the closing of the New York market and before the opening of the Australasian markets, the foreign exchange market is relatively inactive. In this period the last five bid rates may be spread over as long an interval as five minutes; see Goodhart (1990).

The exchange rate for currency $i$ at day $t$ and hour $\tau$ is denoted by $s(i)_{t,\tau}$, where $i = BP, DM, SF, YEN$; $t = 1, 2, \ldots, 132$; and $\tau = 1, 2, \ldots, 24$. All rates are quoted as the amount of currency $i$ per U.S. dollar, with the exception of the BP which is quoted as the number of U.S. dollars per BP. For the subsequent analysis we have also divided the world into three time-zones; the Asian (A) market which is defined as being open between hour 23 and hour 7, the European (E) market which is open from hour 8 through 16, and the United States (U) which is open between hours 14 and 22.²

As a preliminary data analysis we applied the unit root testing methodology of Phillips (1987) and Phillips and Perron (1988) and failed to reject the null hypothesis of a unit root in the logarithm of any of the four exchange rate series.³ These results are consistent with those of Goodhart and Giugale (1988), and also in accord with the findings for daily and weekly exchange rates in Corbae and Ouliaris (1986) and Baillie and

¹. We are very grateful to Charles Goodhart for providing the data used in this study.
². Europe goes on daylight saving time from the last weekend of March, whereas the U.S. until 1988 was on daylight saving time from the last weekend of April. This causes a small misalignment in the definition of the time zones around these dates.
³. Full details of the unit root tests are available from the authors on request.
Bollerslev (1989a). In light of this preliminary analysis we shall subsequently only consider the first differences for each of the four exchange rates,

\[ y(i)_{t, \tau} = 100 \cdot \left[ \log (s(i)_{t, \tau}) - \log (s(i)_{t, \tau-1}) \right], \]

(1)
corresponding to the approximate percentage nominal return on currency \( i \) obtained from time \( t, \tau - 1 \) to \( t, \tau \).

It is well-known, e.g. Working (1960), that even if the unobservable return series corresponding to every one-hour interval is serially uncorrelated, the effect of averaging the last five bid rates will generate negative first-order serial correlation in the returns. This negative autocorrelation may be further enhanced by the non-synchronicity of the last five bid rates of the hour; see e.g. Lo and MacKinlay (1990). In order to accommodate both of these effects an MA (1) process is estimated for the conditional mean returns,

\[ y(i)_{t, \tau} = \mu(i) + \theta(i) \epsilon(i)_{t, \tau-1} + \epsilon(i)_{t, \tau}. \]

(2)

Table II reports the Quasi Maximum Likelihood Estimates (QMLE) obtained for this simple MA (1) under the auxiliary assumptions of homoskedasticity and conditional normality,

\[ \epsilon(i)_{t, \tau} | \Psi_{t, \tau-1} \sim N(0, \omega_0(i)) \]

(3)

where \( \Psi_{t, \tau} \) is the set of all relevant and available information at time \( t, \tau \).

Given the severe kurtosis and heteroskedasticity present in the data, as documented further below, conventional standard errors must be regarded of questionable value. Accordingly, the standard errors reported in Table I, and for all the subsequent model estimates, are based on the robust procedures in Weiss (1986) and Bollerslev and Wooldridge (1990); details of which are given in the appendix. In the present model with homoskedastic errors, these robust standard errors are identical to the standard errors originally developed by White (1982). Compared to the more traditional standard errors computed from the outer product of the gradient, the robust standard errors for \( \hat{\theta}(i) \) in Table I are about twice as high. Nonetheless, with the exception of the DM, the most frequently traded of the four currencies, the small negative MA (1) coefficients are significant at the 5\% level. Similar results are reported in Whistler (1988). As discussed above, we shall interpret this small, albeit significant, MA (1) coefficient, as being spuriously induced by the averaging and the non-synchronicity of the last five bid rates. An alternative explanation, with implications for market efficiency, is that different investors either observe different news or interpret the same news differently. This could create a negative serial correlation, as a result of a process of price adjustments where the price bounces forth and back between centres with different information. However,

4. We also applied Johansen's (1988) test and failed to find any evidence for the existence of a cointegrating vector in a vector autoregression for the four rates. This is contrary to the analysis in Baillie and Bollerslev (1989a), who on using 1245 daily observations over a longer six-year period found significant evidence for the presence of one cointegrating factor in a seven-dimensional system of U.S. dollar exchange rates. However, as illustrated by Shiller and Perron (1985) for univariate tests of unit roots, the power is much more closely related to the length of the sampling interval as opposed to the number of observations. Thus, with only 133 daily observations, this difference might be due to lack of power.

5. If the market was closed at hour \( t, \tau - 1 \) the first observation preceding hour \( t, \tau \) for which the market was open was taken as observation \( t, \tau - 1 \).

6. The estimates here and throughout the paper were obtained by the Berndt, Hall, Hall and Hausman (1972) (BHHH) algorithm using numerical first-order derivatives.

7. For the sake of brevity, throughout the paper we report only robust standard errors and test statistics. All the non-robust counterparts may be obtained from the authors on request.

8. Note, since all returns are calculated from the bid prices, there is no negative serial correlation induced from fluctuations across the bid/ask spread.
TABLE I

Quasi-maximum likelihood estimates

\[ y(i)_{t,r} = \mu_0(i) + \theta_1(i)e_{i,r-1} + \epsilon(i)_{i,r} \]

\[ \epsilon(i)_{i,r} | \Psi_{i,r-1} \sim N(0, \omega_0(i)) \]

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th>DM</th>
<th>SF</th>
<th>YEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0(i) )</td>
<td>0.0014</td>
<td>-0.0037</td>
<td>-0.0043</td>
<td>-0.0074</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0041)</td>
<td>(0.0040)</td>
<td>(0.0030)</td>
</tr>
<tr>
<td>( \theta_1(i) )</td>
<td>-0.0626</td>
<td>-0.0210</td>
<td>-0.0519</td>
<td>-0.0605</td>
</tr>
<tr>
<td></td>
<td>(0.0246)</td>
<td>(0.0231)</td>
<td>(0.0243)</td>
<td>(0.0266)</td>
</tr>
<tr>
<td>( \omega_0(i) )</td>
<td>0.0352</td>
<td>0.0347</td>
<td>0.0406</td>
<td>0.0315</td>
</tr>
<tr>
<td></td>
<td>(0.0017)</td>
<td>(0.0019)</td>
<td>(0.0020)</td>
<td>(0.0018)</td>
</tr>
</tbody>
</table>

|       |       |       |       |       |
| \( m_3(i) \) | -0.1573 | 0.1664 | 0.2001 | 0.0671 |
| \( m_4(i) \) | 8.7404 | 10.0346 | 8.3844 | 11.8276 |
| \( Q_{20}(i) \) | 31.7233 | 16.8221 | 14.7653 | 27.5153 |
| \( Q_{20}^2(i) \) | 113.1777 | 71.1417 | 97.5161 | 104.2166 |

Key. Asymptotic robust standard errors are in parentheses. The statistics \( m_3(i) \) and \( m_4(i) \) denote the residual skewness and kurtosis. \( Q_{20}(i) \) and \( Q_{20}^2(i) \) give the Ljung-Box portmanteau test for up to the 20th-order serial correlation in the residuals and the squared residuals, respectively.

Goodhart and Figliouli (1988) on using three days of minute-by-minute data from the foreign exchange market, find little support for this alternative explanation. A further possibility is that the significant serial correlation, could be due to the presence of a time-dependent risk premium. McCurdy and Morgan (1989) in their analysis of spot and futures exchange rates argue for the significance of a short-lived time-varying risk premium in daily futures data.

Returning to Table I, the summary statistics \( m_3(i) \) and \( m_4(i) \) denote the usual sample estimates of skewness and kurtosis in the residuals \( \hat{\epsilon}(i)_{i,r} \). Under the null hypothesis of normality, \( \sqrt{6/T} m_3(i) \) and \( \sqrt{24/T} m_4(i) \) both have asymptotic standard normal distributions. As already noted, severe excess kurtosis is present in all the residual series; see also Wasserfallen and Zimmerman (1985) and Wasserfallen (1989).

Table I also presents the standard Ljung and Box (1978) portmanteau test statistics \( Q_{20}(i) \) and \( Q_{20}^2(i) \) for up to 20th-order serial correlation in \( \hat{\epsilon}(i)_{i,r} \) and \( \hat{\epsilon}(i)_{i,r}^2 \), respectively. As noted by Diebold (1987) and Cumby and Huizinga (1988), with heteroskedastic and/or leptokurtic errors the standard chi-square critical values for the Ljung-Box tests based on the estimated residuals are generally inappropriate, leading to a rejection of the null hypothesis too often. Nevertheless, the Ljung-Box tests are indicative of misspecification, and the very high values for \( Q_{20}^2(i) \) strongly suggest the presence of conditional heteroskedasticity.

As previously noted, given the leptokurtosis in the data, conventional \( T \cdot R_U^2 \), where \( R_U^2 \) is the unccentred multiple correlation coefficient, type LM tests based on the outer product of the gradient (OPG) are not asymptotically justified in the present context. Consequently, Table II presents the results from calculating a series of robust LM tests, due to Wooldridge (1987, 1990), for the models estimated in Table I. The robust LM tests are computationally very simple, requiring just two auxiliary linear regressions; the details of which are outlined in the appendix. Given fairly general regularity conditions and under the null hypothesis, \( T \cdot R_U^2 \) from the last of these two auxiliary regressions is distributed asymptotically as a chi-square with \( q \) degrees of freedom, where \( q \) is the number of restrictions being tested.
Turning to the results, the four test statistics for ARMA (1, 1) errors in Table II, i.e. $\phi_1(i) \neq 0$, indicate that any spuriously induced serial correlation is well-approximated by the MA (1) process. Interestingly, the robust test statistics are all less than the conventional $T \cdot R_{1j}^2$ OPG LM tests which are not reported here. This is in accordance with the Monte-Carlo results in Bollerslev and Wooldridge (1990), and holds true for all the test statistics reported in Table II. Similarly, none of the four robust tests for MA (4) errors suggest misspecified dynamics in the conditional mean for $y(i)_{t, r}$.

While there is little evidence of any additional serial correlation, hardly surprisingly the robust LM tests for ARCH (1) and ARCH (4) effects are all highly significant at virtually any level.\textsuperscript{9} While volatility clustering and consequent ARCH effects are likely to be present in neighbouring hourly observations, it is also possible that seasonal effects on a daily basis may be present. Accordingly, the statistic labelled $\alpha_{24}(i) \neq 0$ refers to a test for the significance of $\epsilon(i)_{t-1, r}$ in explaining the conditional variance at time $t$, $\tau$; i.e. whether high volatility yesterday at hour $\tau$ implies a high volatility today at hour $\tau$. For all the currencies except the YEN this test is also highly significant. Of course, this may simply reflect an unconditional pattern in volatility across the day not captured by the simple model in (2) and (3). That such a pattern exists is evident from the tests for the inclusion of hourly seasonal dummies in the variance equation; i.e. $\omega_1(i) \neq 0, \ldots, \omega_{23}(i) \neq 0$. All four test statistics are very extreme in the asymptotic chi-square distribution with 23 degrees of freedom.

Finally, Table II also reports a test for the significance of a vacation dummy in the variance equation, $\delta(i)$. The vacation dummy takes the value unity each time the world foreign exchange market reopens following a closure due to a vacation, including weekends. In our sample such closures occur 39 times, 28 of which are weekends or extended weekend vacations.

3. A MODEL FOR HOURLY EXCHANGE RATE MOVEMENTS

On the basis of the preliminary data analysis in the preceding section, the following model was estimated for each of the hourly first differences of the logarithmic exchange rates,\textsuperscript{10}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
& BP & DM & SF & YEN \\
\hline
$\phi_1(i) \neq 0$ & 0.0015 & 0.8266 & 0.3263 & 3.2829 \\
$\theta_2(i) \neq 0, \ldots, \theta_6(i) \neq 0$ & 2.5657 & 3.4042 & 0.4410 & 7.2799 \\
$\alpha_1(i) \neq 0$ & 18.2580 & 11.9898 & 13.4391 & 16.3312 \\
$\alpha_2(i) \neq 0, \ldots, \alpha_6(i) \neq 0$ & 32.2130 & 24.3272 & 15.0565 & 27.6275 \\
$\omega_1(i) \neq 0, \ldots, \omega_{23}(i) \neq 0$ & 7.0125 & 11.0095 & 12.6432 & 3.5139 \\
$\delta(i) \neq 0$ & 172.1541 & 157.4834 & 162.9810 & 111.3201 \\
\hline
\end{tabular}
\end{table}

\textbf{Key.} For the notation, see the model in equations (4) through (6).

\textsuperscript{9} In the present situation, where all the test statistics are relatively clear cut, the OPG and the robust LM tests all lead to the same conclusions using a 5% test. In other situations, however, the inference from the two tests may differ markedly, as illustrated below.

\textsuperscript{10} If the markets were closed at hour $t$, $r - 1$, the corresponding variables were taken at the first hour preceding $t$, $r$ for which the markets were open. Similarly, if the markets were closed at hour $t - 1$, $r$, the corresponding variables were taken at the first hour on day $t - 1$ preceding hour $r$ for which the markets were open.
\[ y(i)_{t+1} = \mu_0 + \theta_1(i)\epsilon(i)_{t,\tau-1} + \epsilon(i)_{t,\tau} \]  

\[ \epsilon(i)_{t,\tau} | \Psi_{t,\tau-1} \sim N(0, \sigma(i)_{t,\tau}^2) \]  

\[ \sigma(i)_{t,\tau}^2 = \omega(i) - [\alpha_1(i) + \beta_1(i)]\omega_{t-1} + \delta(i)V_{t,\tau} \]  

\[ + \alpha_1(i)\epsilon(i)_{t,\tau-1}^2 + \beta_1(i)\sigma(i)_{t,\tau-1}^2 + \alpha_{24}(i)\epsilon(i)_{t-1,\tau-1}^2. \]  

Here \( V_{t,\tau} \) denotes the vacation dummy equal to one for the hour immediately following the market being closed and zero otherwise. The conditional variance in equation (6) has a GARCH \((1, 1)\) formulation in its fourth and fifth terms, as in Bollerslev (1986), and a seasonal ARCH variable in its sixth term. The initial two terms use a multiplicative parameterization in \( \alpha_1(i) \) and \( \beta_1(i) \) so that, apart from any deterministic vacation effects, the unconditional variance for hour \( \tau \) is equal to \( \omega(i) \). The parameterization also implies that the effect of the vacation dummy in the conditional variance is geometrically declining.\(^\text{11}\)

Table III presents the details of the estimated models. Remarkable similarities are seen to exist between currencies. Virtually all the estimated parameters, except the unconditional expected returns \( \bar{\mu}_0(i) \), are significantly different from zero at the usual 5% level. Although not reported here, the conventional standard errors are reasonably close to the robust standard errors for the conditional mean parameter estimates, but were generally much less than the robust standard errors for the conditional variance parameter estimates. In fact for many of the variance parameters, the robust and the conventional standard errors differ by a factor of two.\(^\text{12}\)

That the standardized residuals for the models in Table III possess a large degree of leptokurtosis is evident from the \( m_4(i) \) statistics reported at the end of the table. Although the degree of leptokurtosis is somewhat less than for the homoskedastic models in Table II, it is nonetheless very significant.\(^\text{13}\) This corresponds to previous findings in the literature for daily exchange rates in Baillie and Bollerslev (1989b), Gallant, Hsieh and Tauchen (1989), Hsieh (1988, 1989), and McCurdy and Morgan (1987, 1988), and for the intra day rates in Engle, Ito and Lin (1990), and highlights the importance of the robust inference procedures.\(^\text{14}\)

Returning to the actual estimates in Table III, the hourly dummy variables \( \bar{\omega}_1(i), \ldots, \bar{\omega}_{24}(i) \) are nearly all significant, and exhibit a very characteristic pattern across the four currencies. We shall return to a more detailed discussion of this pattern in the next section.

The significant estimates for \( \alpha_1(i) \) and \( \beta_1(i) \) indicate the apparent autocorrelation of news, the YEN being the most persistent, although for none of the currencies are there

\(^{11}\) As in the analysis for daily exchange rates in Baillie and Bollerslev (1989b), it would be possible to allow for a one-period impulse effect of the vacation dummy using a multiplicative parameterization; i.e. \( \delta(i)[V_{t,\tau} - \alpha_1(i) + \beta_1(i)]V_{t-1,\tau-1} - \alpha_{24}(i)V_{t-1,\tau-1} \). Through some experimentation, this parameterization was found to be less appropriate in the present context. It takes more than one hour for the markets to adjust to the extra information flows that accumulate following market closures.

\(^{12}\) For instance, the non-robust standard errors for \( \bar{\omega}_1(i) \) based on the outer product of the gradient equals 0.0120, 0.0173, 0.0153, and 0.0148 respectively, compared to the robust standard errors reported in table III equal to 0.0280, 0.0253, 0.0328, and 0.0280.

\(^{13}\) If the models are correctly specified, by Jensen's inequality the coefficients of kurtosis for the standardized residuals should be less than the kurtosis for the raw residuals. Thus, an increase in \( m_4(i) \) from Table I to Table III would be indicative of model misspecification; see Hsieh (1989).

### REVIEW OF ECONOMIC STUDIES

#### TABLE III

Quasi-maximum likelihood estimates

\[
y(i)_{x,r} = \mu_0(i) + \theta_1(i)e_{i,r-1} + \varepsilon(i)_{x,r}
\]

\[
e(i)_{x,r} | \Psi_{i,r-1} \sim N(0, \sigma(i)_{x,r}^2)
\]

\[
\sigma(i)_{x,r}^2 = \omega_x(i) - [\alpha_x(i) + \beta_x(i)\omega_{r-1}(i)] + \delta(i)V_{x,r} + \alpha_x(i)e(i)_x^2 + \beta_x(i)\sigma(i)_{x,r-1}^2 + \alpha_{24}(i)e(i)_{x-2,r}^2
\]

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th>DM</th>
<th>SF</th>
<th>YEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_0(i))</td>
<td>0.0025</td>
<td>-0.0026</td>
<td>-0.0015</td>
<td>-0.0020</td>
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<td>(\theta_1(i))</td>
<td>-0.0379</td>
<td>-0.0243</td>
<td>-0.0421</td>
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</tr>
<tr>
<td>(\omega_x(i))</td>
<td>0.0144</td>
<td>0.0273</td>
<td>0.0391</td>
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<tr>
<td>(\omega_{24}(i))</td>
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<td>0.0160</td>
<td>0.0201</td>
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<td>(\omega_{30}(i))</td>
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<td>(\omega_{50}(i))</td>
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<td>0.0131</td>
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<td>(\omega_{60}(i))</td>
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<td>0.0133</td>
<td>0.0142</td>
<td>0.0118</td>
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<tr>
<td>(\omega_{70}(i))</td>
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<td>(\omega_{90}(i))</td>
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<td>(\omega_{130}(i))</td>
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<td>0.0280</td>
<td>0.0370</td>
<td>0.0174</td>
</tr>
<tr>
<td>(\omega_{140}(i))</td>
<td>0.0258</td>
<td>0.0149</td>
<td>0.0153</td>
<td>0.0083</td>
</tr>
<tr>
<td>(\omega_{150}(i))</td>
<td>0.0354</td>
<td>0.0386</td>
<td>0.0402</td>
<td>0.0274</td>
</tr>
<tr>
<td>(\omega_{160}(i))</td>
<td>0.0385</td>
<td>0.0482</td>
<td>0.0539</td>
<td>0.0264</td>
</tr>
<tr>
<td>(\omega_{170}(i))</td>
<td>0.0560</td>
<td>0.0676</td>
<td>0.0716</td>
<td>0.0409</td>
</tr>
<tr>
<td>(\omega_{180}(i))</td>
<td>0.0871</td>
<td>0.0706</td>
<td>0.0826</td>
<td>0.0598</td>
</tr>
<tr>
<td>(\omega_{190}(i))</td>
<td>0.0646</td>
<td>0.0759</td>
<td>0.0881</td>
<td>0.0369</td>
</tr>
<tr>
<td>(\omega_{200}(i))</td>
<td>0.0608</td>
<td>0.0636</td>
<td>0.0833</td>
<td>0.0367</td>
</tr>
<tr>
<td>(\omega_{210}(i))</td>
<td>0.0512</td>
<td>0.0304</td>
<td>0.0550</td>
<td>0.0307</td>
</tr>
<tr>
<td>(\omega_{220}(i))</td>
<td>0.0434</td>
<td>0.0374</td>
<td>0.0288</td>
<td>0.0191</td>
</tr>
<tr>
<td>(\omega_{230}(i))</td>
<td>0.0205</td>
<td>0.0197</td>
<td>0.0164</td>
<td>0.0148</td>
</tr>
<tr>
<td>(\omega_{240}(i))</td>
<td>0.0195</td>
<td>0.0247</td>
<td>0.0098</td>
<td>0.0267</td>
</tr>
<tr>
<td>(\delta(i))</td>
<td>0.0081</td>
<td>0.0148</td>
<td>0.0147</td>
<td>0.0149</td>
</tr>
<tr>
<td>(\omega_{250}(i))</td>
<td>0.0098</td>
<td>0.0184</td>
<td>0.0234</td>
<td>0.0317</td>
</tr>
<tr>
<td>(\omega_{260}(i))</td>
<td>0.0200</td>
<td>0.0039</td>
<td>0.0047</td>
<td>0.0063</td>
</tr>
<tr>
<td>(\delta(i))</td>
<td>0.0684</td>
<td>0.0738</td>
<td>0.1266</td>
<td>0.0447</td>
</tr>
<tr>
<td>(\omega_{270}(i))</td>
<td>0.0248</td>
<td>0.0360</td>
<td>0.0432</td>
<td>0.0172</td>
</tr>
</tbody>
</table>
any evidence of Integrated GARCH, or IGARCH, characteristics; see Engle and Bollerslev (1986). This autocorrelation in the conditional second moments for each of the four currencies is consistent with both the so-called "meteor shower" and "heat wave" hypothesis in the terminology of Engle, Ito and Lin (1990). According to the meteor shower hypothesis, news on the same currency is transmitted through time and different market locations as the globe turns on its axis, whereas in the heat wave hypothesis news is country- or market-specific.

It is also interesting to note, that even after controlling for the intra-day pattern in the unconditional variance through \( \hat{\omega}_1(i), \ldots, \hat{\omega}_{24}(i) \), the estimates for \( \alpha_{24}(i) \), although numerically small, are significant at the 5% level for the BP, the SF and the YEN. Thus, with the exception of the DM, the estimates are supportive of a heat-wave type phenomenon in the arrival of news to the market. A high (low) volatility today at hour \( \tau \) is likely to increase (decrease) the volatility at hour \( \tau \) the following day also. This finding is contrary to the study by Engle, Ito and Lin (1990), who on using less frequently sampled data, found little evidence for the heat wave hypothesis in the YEN/U.S. dollar market.

Finally, it is worth noting the highly significant vacation dummy estimates for all the currencies. If the amount of new information that accumulates is proportional to the length of time that the market is closed, then volatility should be approximately 38 times greater following vacations when compared to the average volatility across the day; 38 being the average hourly length of a vacation in the sample. The effects as measured by the estimates for \( \delta(i) \) are all much smaller, which is consistent with the notion of information as essentially a private phenomenon, with market participants acting on their own acquisition of new information. This also confirms previous empirical findings for the U.S. stock market in French and Roll (1986) and Baillie and DeGennaro (1990), and the analysis in Baillie and Bollerslev (1989b) for daily spot exchange rates and Harvey and Huang (1990) for foreign currency futures.

To assess the general descriptive validity of the models, a series of misspecification tests were performed. The results of these robust LM tests to check the model specification conditional on the own past history of each particular currency are given in Table IV.

The first set of tests for up to a fourth-order moving-average term in the conditional mean equation, i.e. \( \theta_3(i) \neq 0, \ldots, \theta_4(i) \neq 0 \), are all insignificant at the 5% level.\textsuperscript{15} Another

\textsuperscript{15} Of course, the results for the YEN are borderline. In fact, the conventional OPG LM test takes the value 9.015, which exceeds the 95% critical value of 7.815. However, when actually estimating the model with an MA (4) error structure, all but the first MA coefficient are individually insignificant at the 5% level using the robust standard errors.
FIGURES 1-4
Estimated hourly variance dummies. Foreign currency per U.S. dollar

FIGURE 1

TABLE IV
Robust LM diagnostic tests for the models in Table III

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th>DM</th>
<th>SF</th>
<th>YEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_z(i) \neq 0, \ldots, \theta_4(i) \neq 0$</td>
<td>1.6358</td>
<td>4.7727</td>
<td>4.9589</td>
<td>7.7903</td>
</tr>
<tr>
<td>$1_A \mu_0(i) \neq 0$</td>
<td>1.0211</td>
<td>1.6085</td>
<td>0.1728</td>
<td>1.0966</td>
</tr>
<tr>
<td>$1_M \mu_0(i) \neq 0$</td>
<td>0.6646</td>
<td>1.9378</td>
<td>0.4055</td>
<td>0.4977</td>
</tr>
<tr>
<td>$1_U \mu_0(i) \neq 0$</td>
<td>2.0838</td>
<td>0.9765</td>
<td>0.3879</td>
<td>0.7017</td>
</tr>
<tr>
<td>$1_A \mu_0(i) \neq 0, \ldots, 1_7 \mu_0(i) \neq 0$</td>
<td>2.7989</td>
<td>5.8418</td>
<td>2.8445</td>
<td>2.4453</td>
</tr>
<tr>
<td>$\alpha_2(i) \neq 0$</td>
<td>0.0483</td>
<td>2.8538</td>
<td>1.3753</td>
<td>0.0121</td>
</tr>
<tr>
<td>$\beta_2(i) \neq 0$</td>
<td>0.0157</td>
<td>0.0758</td>
<td>3.0737</td>
<td>0.2208</td>
</tr>
<tr>
<td>$1_A \omega_0(i) \neq 0, \ldots, 1_7 \omega_0(i) \neq 0$</td>
<td>2.2639</td>
<td>1.9695</td>
<td>3.3984</td>
<td>6.4320</td>
</tr>
<tr>
<td>$\gamma(i) \neq 0$</td>
<td>0.0015</td>
<td>0.3740</td>
<td>0.0110</td>
<td>0.6052</td>
</tr>
</tbody>
</table>

Key: $1_A, 1_E$ and $1_U$ represent indicator variables for the Asian, European and U.S. markets respectively. $1_M, \ldots, 1_U$ denote indicator variables for different days of the week. $\gamma(i)$ is a GARCH in the mean term.

The possibility not captured by the model is that systematic mean return may exist across the three major market locations in the world. For instance, Ito and Roley (1987) observed
that in the first half of the 1980s the U.S. dollar tended to appreciate against the YEN in the New York market, and depreciate in the European market. The indicator variables $1_A$, $1_E$, and $1_U$ denote when the Asian, European and U.S. markets respectively are open; i.e. $1_A$ is equal to one for hours 23 through 7 and zero otherwise, $1_E$ is one for hours 8 through 16 only, and $1_U$ is one for the hours 14 through 22. The hypothesis that the mean returns were different for the three market locations during the depreciation of the U.S. dollar in the first half of 1986, i.e. $1_A\mu_0(i) \neq 0$, $1_E\mu_0(i) \neq 0$ and $1_U\mu_0(i) \neq 0$, find little support from the data. The statistic $1_M\mu_0(i) \neq 0, \ldots, 1_{19}\mu_0(i) \neq 0$ denotes a similar test for difference in the mean return across the five days of the week. Under the null hypothesis of no difference, the test statistic should be the realization of an asymptotic chi-square distribution with 4 degrees of freedom. Hardly surprising, no simple day of the week pattern is evident.

The statistic labelled $\alpha_2(i) \neq 0$ refers to a test for a second-order ARCH term, $\varepsilon(i)^2_{t-i-2}$, in the conditional variance equation of the model, while $\beta_{24}(i) \neq 0$ is a test for the presence of $\sigma(i)^2_{t-1-i}$ in the conditional variance at time $t-i$. No support is found for either of these two hypotheses for any of the four currencies. In their analysis of daily exchange rate data Baillie and Bollerslev (1989b) find some weak support for a day of the week pattern in the conditional variance, as do Harvey and Huang (1990) for foreign currency futures. However, after taking account of vacation effects through $\delta(i)$, the $1_M\omega_0(i) \neq 0, \ldots, 1_{19}\omega_0(i) \neq 0$ statistics do not suggest the presence of any significant pattern in the
hourly data. The hourly dynamics of the own currency volatility appear to be well specified by the relatively simple variance function in (6), and it seems that a seasonal ARCH term, $\varepsilon(i)^2_{t-1,7}$, is more appropriate for modelling seasonal volatility or heat wave effects than the corresponding seasonal conditional variance.

Finally, the row labelled $\gamma(i)\neq 0$ gives a test for GARCH in the mean effects; i.e. whether or not the conditional mean return is linearly related to its own conditional variance. As argued in McCurdy and Morgan (1987, 1988), the own conditional variance could serve as a proxy for a time-varying risk premium. No evidence is forthcoming for this idea.

In summary, the robust test statistics in Table IV all indicate a reasonably good fit for the four univariate time-series models conditional on the own past history of each of the exchange rates.

4. UNCONDITIONAL INTRA-DAY VOLATILITY

As already mentioned, the estimated hourly dummy variables in the conditional variance functions possess a very characteristic and remarkably similar pattern across all the four currencies.

Table V provides a simple summary of the average estimates of the unconditional variances for each of the currencies across the three different market locations. The U.S.
market is on average the most volatile with the European market second, whereas the Asian market generally shows much less volatility. This phenomenon is true even for the YEN, albeit not as pronounced as for the European currencies, reaffirming earlier evidence for the YEN/U.S. dollar rate in Ito and Roley (1987).

To indicate the different nature of the volatility within each market location, the estimated hourly dummies are graphed in Figures 1–4. The increase in volatility that occurs around and immediately preceding the times of the openings of the London and New York markets is particularly striking in all the currencies. There appears to be a
similar but much smaller increase in volatility associated with the opening of the Tokyo market at hour 1. Perhaps not surprisingly, the YEN appears the most volatile currency at the time of the opening of the Asian market. Increases in volatility are also apparent between the hours of 15 and 17, when trading takes place simultaneously in the European and U.S. markets (see also Wasserfallen (1989)). This phenomenon may well be due to a U-shaped pattern of trading, where heavy trading occurs at the beginning and near the end of the trading day, suggesting that the different markets may not be fully integrated. For instance, under a scenario where intra-day trading is dominated by market makers, many of whom hold open positions during the day but few overnight in their particular local market, trading and volatility might be concentrated in time at the opening and closing of each major market location. Similar characteristics on other domestic securities exchanges have been observed by Wood, McInish and Ord (1985), Jain and Joh (1986), and Linn and Lockwood (1989), who found that the variability of U.S. stock returns within a day tend to have a distinctive U-shaped pattern. It is also interesting to note the drop in volatility around the lunch hour in Tokyo and London, i.e. hours 4 and 12, whereas no “lunch hour effect” is apparent for the New York market.

A possible explanation for these type of results is to be found in the work of Admati and Pfleiderer (1988), building on the earlier work of Kyle (1985) who developed a model
with informed and liquidity-based traders operating in a market where private information is only useful for one period. Admati and Pfleiderer (1988) use a dynamic model to capture the interaction between strategic informed traders and strategic liquidity traders, and find the model will generate patterns of concentrated trading volume and price variability. In a related context, Foster and Viswanathan (1988) present a theoretical adverse selection model with uninformed and informed traders, with the latter group acting strategically to take advantage of their superior set of private information; see also the recent study by Pagano (1989).

Of course, the increase in volatility around the openings of the major markets might also be related to the systematic release of news at that time. For instance, U.S. economic data are typically announced at 8:30 a.m. New York time; i.e. between hours 12 and 13.

Similarly, all the exchange rates analysed so far have been vis-a-vis the U.S. dollar. It is certainly possible that this could create a similar pattern in volatility across currencies if most of the market relevant news occurring are related to the U.S. dollar. Very interestingly however, the same phenomenon also holds true for other bilateral exchange rates. For instance, the hourly dummy variables obtained when estimating the model in equations (4) through (6) for the DM, the SF, and the YEN vis-a-vis the BP are depicted

16. If the number and precision of informed traders is constant over time, then the information content and variability of equilibrium prices will also be time-invariant.
in Figures 5–7. Whereas these three pictures are not quite as pronounced as the same pictures for the U.S. dollar rates, very much the same type of pattern is seen to exist. This suggests that it is not merely U.S. dollar news that accounts for the volatility across the day, but that market depth may also play a very important role.

5. CAUSALITY TESTS IN MEAN AND VARIANCE

The misspecification tests discussed in Section 3, were concerned with testing the model specification based on only the past history of each individual currency. Tables VI and VII however, present robust LM tests for misspecification based on information contained in other currencies.

The presence of other currencies lagged innovations in the conditional mean equation is tested by a sequence of robust LM tests in Table VI. The possibility that other currencies innovations are only important when certain markets are open is also tested, and in all but one case, the YEN to the SF during the Asian market, the hypothesis of no effect cannot be rejected.\(^{17}\) Hence, hardly surprising with an hourly sampling frequency, there appears no systematic evidence that any one rate Granger-causes any of the other rates.

\(^{17}\) With 60 test statistics, although not independent, using a size of 5%, some tests are likely to come out significant just by chance. Thus, considerable caution should be exercised in judging against type I errors.
### TABLE VI

Robust LM tests for causality in the mean

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th>DM</th>
<th>SF</th>
<th>YEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1{(\hat{e}(BP)_{t-1})}</td>
<td>1.8778</td>
<td>0.0659</td>
<td>0.0253</td>
<td>2.0516</td>
</tr>
<tr>
<td>1{(\hat{e}(BP)_{t-1})}</td>
<td>0.6110</td>
<td>0.6258</td>
<td>1.1223</td>
<td>1.2575</td>
</tr>
<tr>
<td>1{(\hat{e}(BP)_{t-1})}</td>
<td>0.0111</td>
<td>2.3170</td>
<td>0.0225</td>
<td>2.0592</td>
</tr>
<tr>
<td>1{(\hat{e}(DM)_{t-1})}</td>
<td>0.5132</td>
<td>—</td>
<td>1.6227</td>
<td>0.1675</td>
</tr>
<tr>
<td>1{(\hat{e}(DM)_{t-1})}</td>
<td>0.0014</td>
<td>0.1286</td>
<td>0.2275</td>
<td>2.2181</td>
</tr>
<tr>
<td>1{(\hat{e}(DM)_{t-1})}</td>
<td>0.1060</td>
<td>2.3422</td>
<td>0.4787</td>
<td>0.1040</td>
</tr>
<tr>
<td>1{(\hat{e}(DM)_{t-1})}</td>
<td>0.3169</td>
<td>0.4033</td>
<td>1.5068</td>
<td>2.3445</td>
</tr>
<tr>
<td>1{(\hat{e}(SF)_{t-1})}</td>
<td>2.0255</td>
<td>0.3109</td>
<td>—</td>
<td>0.5408</td>
</tr>
<tr>
<td>1{(\hat{e}(SF)_{t-1})}</td>
<td>0.0445</td>
<td>2.4509</td>
<td>0.5782</td>
<td>1.4745</td>
</tr>
<tr>
<td>1{(\hat{e}(SF)_{t-1})}</td>
<td>0.6456</td>
<td>0.7999</td>
<td>0.3503</td>
<td>0.0677</td>
</tr>
<tr>
<td>1{(\hat{e}(SF)_{t-1})}</td>
<td>2.1271</td>
<td>3.2193</td>
<td>3.3499</td>
<td>0.9855</td>
</tr>
<tr>
<td>1{(\hat{e}(YEN)_{t-1})}</td>
<td>0.1365</td>
<td>1.3912</td>
<td>0.4960</td>
<td>—</td>
</tr>
<tr>
<td>1{(\hat{e}(YEN)_{t-1})}</td>
<td>0.4284</td>
<td>0.9530</td>
<td>5.3004</td>
<td>1.1132</td>
</tr>
<tr>
<td>1{(\hat{e}(YEN)_{t-1})}</td>
<td>0.5773</td>
<td>2.8400</td>
<td>0.4826</td>
<td>2.0685</td>
</tr>
<tr>
<td>1{(\hat{e}(YEN)_{t-1})}</td>
<td>1.8355</td>
<td>0.0865</td>
<td>0.9768</td>
<td>0.0085</td>
</tr>
</tbody>
</table>

**Key.** See Table IV.

### TABLE VII

Robust LM tests for causality in the variance

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th>DM</th>
<th>SF</th>
<th>YEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1{(\hat{e}(BP)^2_{t-1})}</td>
<td>0.0163</td>
<td>0.0895</td>
<td>0.4380</td>
<td>9.9862</td>
</tr>
<tr>
<td>1{(\hat{e}(BP)^2_{t-1})}</td>
<td>0.0200</td>
<td>0.1033</td>
<td>0.5809</td>
<td>0.8236</td>
</tr>
<tr>
<td>1{(\hat{e}(BP)^2_{t-1})}</td>
<td>0.5662</td>
<td>0.7963</td>
<td>0.8622</td>
<td>0.8079</td>
</tr>
<tr>
<td>1{(\hat{e}(DM)^2_{t-1})}</td>
<td>1.8032</td>
<td>—</td>
<td>5.4553</td>
<td>0.4817</td>
</tr>
<tr>
<td>1{(\hat{e}(DM)^2_{t-1})}</td>
<td>1.2696</td>
<td>0.1089</td>
<td>4.8831</td>
<td>2.6534</td>
</tr>
<tr>
<td>1{(\hat{e}(DM)^2_{t-1})}</td>
<td>0.0127</td>
<td>0.3448</td>
<td>0.9706</td>
<td>0.8447</td>
</tr>
<tr>
<td>1{(\hat{e}(DM)^2_{t-1})}</td>
<td>4.8988</td>
<td>0.0332</td>
<td>0.7331</td>
<td>0.3760</td>
</tr>
<tr>
<td>1{(\hat{e}(SF)^2_{t-1})}</td>
<td>2.4287</td>
<td>0.3806</td>
<td>—</td>
<td>0.1059</td>
</tr>
<tr>
<td>1{(\hat{e}(SF)^2_{t-1})}</td>
<td>0.8099</td>
<td>0.0141</td>
<td>1.9689</td>
<td>3.8664</td>
</tr>
<tr>
<td>1{(\hat{e}(SF)^2_{t-1})}</td>
<td>0.0915</td>
<td>0.1253</td>
<td>1.9340</td>
<td>3.2760</td>
</tr>
<tr>
<td>1{(\hat{e}(SF)^2_{t-1})}</td>
<td>4.5580</td>
<td>1.3089</td>
<td>1.2698</td>
<td>3.0284</td>
</tr>
<tr>
<td>1{(\hat{e}(YEN)^2_{t-1})}</td>
<td>1.2243</td>
<td>0.2024</td>
<td>2.1566</td>
<td>—</td>
</tr>
<tr>
<td>1{(\hat{e}(YEN)^2_{t-1})}</td>
<td>1.7413</td>
<td>0.4055</td>
<td>2.1424</td>
<td>0.1500</td>
</tr>
<tr>
<td>1{(\hat{e}(YEN)^2_{t-1})}</td>
<td>1.6720</td>
<td>0.0038</td>
<td>0.7742</td>
<td>0.3779</td>
</tr>
<tr>
<td>1{(\hat{e}(YEN)^2_{t-1})}</td>
<td>0.0029</td>
<td>0.4587</td>
<td>1.2488</td>
<td>1.5321</td>
</tr>
</tbody>
</table>

**Key.** See Table IV.

Alternatively, the tests in Table VI can be seen as indicators for the degree of serial correlation induced by the time averaging and non-synchronicity of the bid rates. Following Lo and MacKinlay (1990), if these effects differ in a systematic way between the currencies, the matrix of cross-autocovariances will be asymmetric; i.e. \( \text{Corr}(y(i)_{t+1}, y(j)_{t+1}) \neq \text{Corr}(y(i)_{t}, y(j)_{t+1}) \) for \( i, j = \text{BP, DM, SF, YEN} \). This is not borne out by the test statistics reported in Table VI. The tests for the different lagged own-currency effects for each of the market locations, also suggests that the degree of serial correlation does not differ in a statistically significant way across the three markets.
Turning to Table VII, a series of analogous robust LM tests to investigate the possibility of any causal relationships in the variance are presented. The idea of causality in variance was first introduced in Granger, Robins and Engle (1984). The tests reported here are performed by testing for the significance in the conditional variance of lagged squared innovations from other currencies, both continuously and during certain market locations. Five out of the 60 robust LM tests are significant at the 5% level. The strongest relationship between volatility is found for the BP affecting the YEN during the Asian market. There is some weaker evidence for volatility spillovers from the DM to the BP during the U.S. market, the DM to SF overall and during the Asian market, the SF to the BP during the U.S. market, and the SF to the YEN during the Asian market. Interestingly, none of the test statistics for any of the other rates causing a change in volatility for the DM are significant. Otherwise, little support is forthcoming for the idea of a simple systematic causal relationship in volatility between the hourly exchange rates.

The test statistics involving the own lagged squared innovations also support the constancy of the ARCH (1) coefficients across the different market locations for all four currencies.

6. CONCLUSION

The hourly data on four major exchange rates recorded continuously on different world markets appears largely consistent with the efficient markets hypothesis, so that the logarithmic first differences are well approximated by a first-order moving-average process, with a small but significantly negative MA (1) coefficient, possibly induced by the time-averaging and non-synchronicity of the bid rates. The first differences in the logarithms of the exchange rates also have remarkably similar patterns of volatility over the different hours of the day. For each exchange rate, the volatility appears to be highly serially correlated, and well represented by a seasonal GARCH model with hourly dummy variables. This is in accord with the so called meteor shower hypothesis, whereby news is transmitted through time and different market locations. The presence of a seasonal ARCH term also suggests some heat wave, or market-specific news characteristics. The distinctive pattern of intra-day volatility, which seems to hold true for other bilateral exchange rates, merits further investigation, and may well be supportive of some of the recent market micro-structure theories building on the strategic interaction of information- and liquidity-based traders. Interestingly enough, the implementation of a battery of robust LM tests failed to uncover much evidence of any systematic volatility spillovers between different currencies or market locations through time.

APPENDIX

Let,

\[ \mu_r(\theta) = E_r(y_r) \]
\[ \sigma^2_r(\theta) = \text{Var}_r(y_r) \]

denote the conditional mean and variance for \( y_r \), as functions of the \( p \times 1 \) vector of unknown parameters \( \theta \). Also define

\[ e_r(\theta) = y_r - \mu_r(\theta). \]

18. Of the 60 corresponding non-robust OPG LM tests 13 are significant at the 5% level. Again, this is in accordance with the Monte Carlo findings in Bollerslev and Wooldridge (1990).
Following Bollerslev and Wooldridge (1990), if the model for \( y_t \) correctly parameterizes \( \mu_t(\theta) \) and \( \sigma_t^2(\theta) \), the Quasi Maximum Likelihood Estimator (QMLE) for \( \theta \), say \( \hat{\theta}_T \), obtained under the auxiliary assumption of conditional normality, will under fairly general regularity conditions be \( \sqrt{T} \) consistent for the true parameters, \( \theta_0 \), and asymptotically normally distributed. Furthermore, a consistent estimate for the asymptotic covariance matrix for \( \hat{\theta}_T \) is readily available, as

\[
\sqrt{T}(\hat{\theta}_T - \theta_0) \overset{D}{\rightarrow} N(0, I)
\]

(A1)

where,

\[
\hat{\theta}_T = T^{-1} \sum_{t=1}^{T} [\nabla_\theta \mu_t(\hat{\theta}_T) \nabla_\theta \mu_t(\hat{\theta}_T) \sigma_t^{-2}(\hat{\theta}_T) + 0.5 \nabla_\theta \sigma_t^2(\hat{\theta}_T) \nabla_\theta \sigma_t^2(\hat{\theta}_T) \sigma_t^{-4}(\hat{\theta}_T)]
\]

(A2)

\[
\hat{\sigma}_T = T^{-1} \sum_{t=1}^{T} [\nabla_\theta \mu_t(\hat{\theta}_T) \nabla_\theta \sigma_t^{-2}(\hat{\theta}_T) \sigma_t^{-4}(\hat{\theta}_T) + 0.5 \nabla_\theta \sigma_t^2(\hat{\theta}_T) \sigma_t^{-4}(\hat{\theta}_T) \sigma_t^{-4}(\hat{\theta}_T)]
\]

(A3)

Note, the expressions in (A2) and (A3) involve first derivatives of the conditional mean and variance functions only. This is particularly appealing when numerical derivatives are being used. Also, when the assumption of conditional normality is satisfied, the usual equalities hold true; i.e.

\[
E(\hat{\theta}_T - \hat{\theta}_T)^2 = E(\hat{\theta}_T) = E(\hat{\theta}_T).
\]

(A4)

Just as the standard covariance matrix estimator obtained under the assumption of conditional normality is invalid with non-normal errors, the usual regression based Lagrange Multiplier (LM) tests are no longer applicable. However, from Wooldridge (1987, 1990) and Bollerslev and Wooldridge (1990) a robust LM test may be computed from a simple set of two auxiliary regressions. Assume that the hypothesis of interest about \( \theta_0 \) can be expressed as,

\[
\theta_0 = r(\phi_0)
\]

(A5)

where \( \phi_0 \) is \( m \times 1 \), and the function \( r(\phi) \) is continuously differentiable on the interior of the parameter space. Also, let "-" denote the estimates under the null hypothesis given by (A4). Then instead of relying directly on the score for the normal quasi-log likelihood, \( \hat{\eta}_t \),\( \tilde{\eta}_t \), where,

\[
\hat{\eta}_t = \begin{bmatrix} \epsilon_t(\theta) \\ \sigma_t^2(\theta) - \sigma_t^2(\theta) \end{bmatrix}
\]

(A6)

the idea is to first purge from \( \hat{\phi}_t = \tilde{\eta}_t \), its linear projection onto \( \hat{\theta}_t = \tilde{\eta}_t^\top \Psi_t \), where \( \Psi_t \) denotes the \( 2 \times m \) matrix of derivatives for the mean and variance functions under the null hypothesis, i.e.

\[
\Psi_t = \begin{bmatrix} \nabla_\theta \mu_t(r(\phi)) \\ \nabla_\theta \sigma_t^2(r(\phi)) \end{bmatrix}
\]

(A7)

That is accomplished by running the matrix regression of \( \hat{\phi}_t \) on \( \hat{\theta}_t \), and saving the \( 2 \times p \) matrix residuals,

\[
\tilde{\eta}_t = \hat{\phi}_t - \hat{\theta}_t \begin{bmatrix} \sum_{t=1}^{T} \hat{\phi}_t^\top \hat{\phi}_t \end{bmatrix}^{-1} \sum_{t=1}^{T} \hat{\phi}_t \hat{\phi}_t^\top.
\]

(A8)

An asymptotically valid LM statistic for testing (A4) is then available as \( T \cdot R^2_U \) from a regression of \( 1 \) on \( \tilde{\eta}_t \), where \( R^2_U \) denotes the unccentred r-squared from this regression. Note, \( (\hat{\eta}_t^\top \hat{\eta}_t) \nabla_\theta r(\phi) = \hat{\eta}_t^\top (\hat{\eta}_t^\top \nabla_\theta r(\phi)) = \hat{\eta}_t^\top 0 = 0 \), and the regression of \( 1 \) on \( \tilde{\eta}_t \) contains perfect multicollinearity. Therefore, in calculating \( R^2_U \), the corresponding \( q = p - m \) redundant elements can be omitted from the regression.

While the preceding discussion assumed that the model correctly parameterizes both the conditional mean and the conditional variance functions, it is possible to show that the asymptotic covariance matrix in (A1), and the same robust LM procedure as outlined above, remain valid under fairly general conditions, when the conditional mean is correctly specified, but the maintained assumption of conditional homoskedasticity, i.e. \( \sigma_t^2(\theta) = \sigma \) for all \( t \), is violated. In that situation, the covariance matrix in (A1) reduces to the well known estimator in White (1982).
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