Payoff Uncertainty, Bargaining Power, and the Strategic Sequencing of Bilateral Negotiations*

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Abstract

This paper investigates the sequencing choice of a buyer who negotiates with the sellers of two complementary objects with uncertain payoffs. We show that the buyer cares about the sequence only when equilibrium trade can be inefficient. In this case, the buyer begins with the weaker seller if the sellers have diverse bargaining powers. If, however, both sellers are strong bargainers, then the buyer begins with the stronger of the two. For either choice, the buyer's sequencing is likely to increase the social surplus. We also show that the buyer may find it optimal to raise her own cost of acquiring objects by committing to a minimum purchase price or outsourcing. The first- and second-mover advantages for the sellers are also identified.

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1 Introduction

Bargaining settings in which a buyer deals with different sellers to acquire complementary objects are ubiquitous. Examples include a shopping mall developer negotiating with several landowners to assemble parcels of land; a large airline carrier subcontracting with independent regional carriers to boost its demand for a major route; a venture capitalist buying

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up equity stakes from various high-tech start-ups; an academic department recruiting multiple faculty members with matching skills; and a home-owner facing different contractors for related parts of a large project. A salient feature of such settings is that the buyer often negotiates with the sellers separately – perhaps it is too costly to bring the sellers together, or perhaps each seller carries proprietary information. Thus, a key strategic choice for the buyer is the sequence in which these bilateral negotiations are conducted. In this paper, we study this choice and its social efficiency consequences. In doing so, we identify an incentive for the buyer to raise her *own* cost of acquiring the goods and examine its implications for her procurement policy.

Our model consists of two sellers who supply complementary objects and a buyer with unit demands.[1] While the buyer's joint valuation is commonly known, her stand-alone valuations are initially uncertain.[2] The buyer privately learns her valuation for each good upon meeting with its seller, and their bilateral negotiation generates a publicly observed "posted price" that the buyer must pay in case of a purchase.[3] In each negotiation, nature selects one party to fix the posted price by tossing a biased coin that reflects the seller's bargaining power relative to the buyer's. After receiving the price offers and ascertaining all her valuations, the buyer makes her purchasing decision.

As a benchmark, we first establish that if all buyer buyer's valuations were common knowledge, then the buyer would be indifferent to the sequence. This is because in this case, the sellers' equilibrium prices would not depend on their bargaining powers. In particular, regardless of who makes the offer in the second negotiation, the leading seller would post a price equal to the buyer's marginal value for his unit – either to induce a joint purchase by the buyer or price coordination by the follower. We show that the independence of the sellers' prices from their bargaining powers, and thus the buyer's indifference to the sequence, would hold more generally if each seller symmetrically learned the buyer's valuation in their meeting.

The buyer, however, cares about the sequence if, as in the base model, she becomes pri-

[1]We briefly discuss the case of substitutes in the Conclusion.

[2]For example, a developer may be uncertain about the profitability of a smaller shopping mall built on a subset of targeted parcels; a large airline carrier may face demand uncertainty for its route without scheduled access to regional airports; an academic department may be uncertain about the individual faculty contributions to the department; and a home-owner may be uncertain about the use of a backyard porch without landscaping. It is, however, conceivable that the buyer will resolve her payoff uncertainty as she meets with the sellers and learn about the objects.

[3]The posted-price mechanism is commonly used in practice. We elaborate on this and other modeling assumptions in the next section.

vately informed of her stand-alone valuations. Specifically, for strong but imperfect complements, we find that the buyer optimally begins with the *weak* seller if one seller is weak and the other is a strong bargainer. If, on the other hand, both sellers are strong bargainers, then the buyer begins with the *stronger* of the two. To see why, note that in the last negotiation, the buyer optimally proposes to pay the seller's marginal cost. Given this, the leading seller prices aggressively if he is followed by a weak seller who is unlikely to propose against the buyer and capture any surplus. To curb this behavior, the buyer begins with the weak seller. When both sellers are strong bargainers, the buyer's concern for aggressive pricing shifts to the last seller. By leaving the weaker of the two sellers to be the last, the buyer minimizes the likelihood of a high price response in the second negotiation in the event that the first one ends in her favor. In either case, we show that the buyer's sequencing (weakly) improves the social surplus except when the sellers are sufficiently uneven bargainers. Intuitively, the buyer's sequencing helps reduce the sellers' prices, which, in turn, facilitate the joint (and efficient) purchase of the complementary goods. Her sequencing may not be aligned with the social interest when the buyer herself posts a high price in the first negotiation to generate concessions from a strong seller in the second.

In the presence of asymmetrically informed parties, it is not surprising that the negotiations can result in an inefficient trade. The buyer may, however, adopt some (costly) procurement policies to increase efficiency so long as she grabs a reasonable share from the increased surplus. We show that one such policy for the buyer is to commit to posting a minimum price that exceeds the marginal cost even in the last negotiation. While directly reducing her payoff, a minimum price policy benefits the buyer by discouraging the leading seller from targeting the full surplus. That is, the buyer intentionally raises her own procurement cost to soften the sellers' pricing.[4] The same incentive can also lead to a "strategic" outsourcing policy. Specifically, we show that the buyer who can make an input at the seller's marginal cost may choose to outsource it along with the other input. In the same vein, we also show that the buyer is sometimes better off dealing with more powerful sellers. Again, although more powerful sellers diminish the buyer's payoff for fixed prices, such sellers are also more likely to moderate their supply prices due to coordination concerns.

While, as the central agent who initiates negotiations, our main focus is on the buyer's sequence preference, we also consider the sellers' preference, as this may inform us of their

[4]This is a reminiscent of, but quite distinct from, the "handicapping" principle in procurement auctions where the buyer commits to purchasing a single good from the high-cost supplier at times to induce a more intense supplier competition (e.g., McAfee and McMillan 1989, and Lewis and Yildirim 2002).

incentives to actively solicit the buyer's business and even bid for the right to negotiate at the desired order. We discover that contrary to the standard IO theory that establishes a first-mover advantage for price-setting duopolists selling complementary goods (e.g., Gal-Or 1985, and Dowrick 1986), a second-mover advantage can also emerge in our model with a powerful buyer. The reason is that a powerful buyer is highly likely to secure a low price from the first negotiation, which leaves a large surplus to the second negotiation.

Related Literature. There is an emerging literature that investigates the optimal negotiation sequence but does so with commonly known valuations. Marx and Shaffer (2007) show that if price contracts can be contingent on *all* purchasing and non-purchasing decisions, then the last seller will be left with no profit, giving the buyer strict preference to begin with the weaker seller in order to claim a larger portion of the total surplus. They also show that absent contingent contracts (as in our model), the buyer is indifferent to the sequence.[5] Xiao (2010) studies a complementary-goods setting with noncontingent contracts but with a pay-as-you-go scheme. He too finds that the buyer is better off starting with the weaker seller, though only to alleviate a holdup problem due to sunk payments. Such a problem does not arise in our setting because of posted prices. Li (2010) studies an infinite-horizon random-offer model of complementary goods and proves that any sequencing is sustainable in equilibrium.[6] In contrast, our model yields a unique equilibrium and a strict sequencing preference. In a more specific application, Noe and Wang (2000) establish that a financially distressed firm would be neutral to the debt renegotiation sequence if each creditor's nominal claim is larger than the firm's value. This neutrality result is in line with ours for commonly known valuations because, as with the sellers in our model, creditors with very high claims need to negotiate with the firm to receive any payment.[7] In a labor union-multiple firms framework, Marshall and Merlo (2004) examine "pattern bargaining" where the buyer uses the contract agreed upon

[5]As Marx and Shaffer also demonstrate, equilibrium contingent contracts often involve below-cost pricing and commitment to a non-purchase payment, also called a "breakup fee". In this paper, we consider simple (noncontingent) price contracts that seem more practical in many real examples and that help us isolate the effect of payoff uncertainty. We should, however, point out that with commonly known valuations, our model would yield equivalent results to those of Marx and Shaffer's if we allowed for contingent contracts. The formal details are available upon request.

[6]Both Li and Xiao build on Cai (2000) who assumes homogenous sellers. See also Horn and Wolinsky (1988), and Stole and Zwiebel (1996) who assume a fixed order of negotiations.

[7]If, however, creditors hold very uneven claims, then Noe and Wang find that the firm would optimally start with the larger creditor. This result is not directly comparable to ours because unlike our sellers, the creditors can refuse to bargain with the firm and still recover their original claims as long as some others make concessions to prevent firm's bankruptcy. That is, in debt restructuring, creditors may possess different reservation payoffs due to their debt claims whereas we assume equal reservation payoffs for the sellers to isolate the effects of payoff uncertainty.

in the first negotiation as a starting point of the second negotiation (also see Banerji 2002). In their case with non-pattern bargaining, the buyer does not, however, care about the sequence. A similar indifference result is obtained by Moresi et al. (2010) in a fairly general model of bilateral negotiations. With commonly known valuations, our model would also produce the buyer's indifference to the sequence as we demonstrate in Section 3.

Our paper is also related to Krasteva and Yildirim (2012), and Noe and Wang (2004) who each compare the outcomes of publicly and confidentially conducted bilateral negotiations by assuming no payoff uncertainty. These authors also discover that the buyer is indifferent to the sequence under both types of negotiations, though she may strictly randomize over the sequence under confidentiality and pay-as-you-go schemes. We abstract from confidentiality here and assume posted prices. Finally, our paper complements the studies of the sequencing through sellers' bidding for positions, e.g., Arbatskaya (2007), and Marx and Shaffer (2010). While these papers uncover either a first- or second-mover advantage for the sellers, our setting features both advantages depending on the buyer's bargaining power.

The remainder of the paper is organized as follows. We set up the model in the next section, followed by the analysis of two benchmark cases in Section 3. In Section 4, we provide the full equilibrium characterization of our base model and then address the buyer's optimal sequencing choice in Section 5. In Section 6, we show how the buyer can be better off adopting some costly procurement policies. We investigate the first- and second-mover advantages for the sellers in Section 7 and conclude in Section 8. The proofs of all formal results are relegated to an appendix.

2 The Model

There are three risk-neutral parties: one buyer (b) and two sellers (s_i, $i = 1, 2$). Each seller costlessly provides a complementary good for which the buyer has a unit demand. It is commonly known at the outset that the buyer possesses a joint value normalized to 1, while her stand-alone value for good i, v_i, is an independent draw from a Bernoulli distribution where $\Pr\{v_i = 0\} = q_i \in [0, 1]$ and $\Pr\{v_i = \frac{1}{2}\} = 1 - q_i$. In particular, with probability $q_1 q_2$, she views goods to be perfect complements, whereas, with probability $(1 - q_1)(1 - q_2)$, she views them to be unrelated or modular.[8] We assume that the buyer privately learns v_i as she meets

[8]Since our qualitative results do not change under a more general support $0 \leq v^L < v^H \leq \frac{1}{2}$, we set $v^L = 0$ and $v^H = \frac{1}{2}$ in the text to ease exposition.

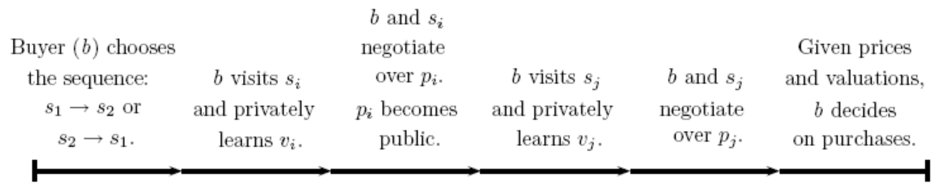

Figure 1: Timing and information structure

with seller i to negotiate.[9]

The buyer negotiates with the sellers sequentially and only once. The price for good i is determined between the buyer and seller i through a one-shot random-proposer bargaining. Let $\sigma_i \in \{b, s_i\}$ denote the player who makes the offer such that $\sigma_i = s_i$ with probability $\alpha_i \in (0,1)$, and $\sigma_i = b$ with probability $1 - \alpha_i$, where α_i measures seller i's bargaining power relative to the buyer's.[10] We assume that σ_1 and σ_2 are independently distributed, and that the realization of σ_i is observed only by the buyer and seller i during their negotiation.

The timing and information structure of our negotiation game unfolds as described by Figure 1. First, the buyer publicly chooses the sequence, $s_i \to s_j$. Next, the buyer approaches the first seller, s_i, and privately observes her valuation v_i. Then, the buyer and s_i bargain over the posted-price p_i, which becomes publicly observable. The buyer, then, meets with s_j and privately learns v_j. Subsequently, the buyer and s_j bargain over the posted-price p_j. Having obtained the prices p_i and p_j, and ascertained her valuations v_i and v_j in the process, the buyer decides which goods to purchase (if any). Our solution concept is perfect Bayesian equilibrium throughout.

Note that given the complementarity, trade is (socially) efficient if and only if the buyer acquires both goods with probability 1. Thus, we call any equilibrium *inefficient* if it involves less than joint purchase with a positive probability. In case of indifference, we assume that all players break ties in favor of efficiency, i.e., purchasing and selling more units. Before proceeding to the analysis, we briefly discuss some of the modeling assumptions.

[9]In this regard, we have in mind environments in which information is too costly for the buyer to acquire independently. For instance, an airline company can truly learn the capacity and quality of a regional carrier from its representatives; an employer often has to interview a job candidate to determine the match value; a home-owner frequently needs to consult with a contractor for a customized project; and a real-estate developer may require access to the land's construction history from its owner.

[10]For instance, α_i may proxy a seller's likelihood of having other customers at the time or his urgent need for cash, and a landowner's likelihood of having an alternative use for his land. It is also conceivable that α_i may simply reflect the intrinsic bargaining ability of the seller vis-à-vis the buyer.

2.1 Discussion of the Assumptions

We keep each buyer-seller bargaining simple to better focus on sequencing; nevertheless, our one-shot bargaining can be a good approximation of applications in which the buyer has a short-time period to acquire the goods, or else the trade opportunity is lost. Such reduced form bargaining has also been adopted extensively in the literature, e.g., Marx and Shaffer (2007, 2010), Noe and Wang (2000, 2004), and Segal (1999). Our assumption that negotiations yield posted prices implies that the scope of the second negotiation will depend on the terms of the agreement reached in the first, which seems quite realistic. For example, when a venture capitalist tries to buy up two complementary businesses, the amount of funds that can be raised to buy the second firm will be affected by the control transfer agreement reached with the first firm. Similarly, in land acquisitions, the buyer's negotiated option contract on a property is likely to affect subsequent land negotiations.[11] In other applications, the prevalence of posted prices can be attributed to certain consumer rights. For instance, the Federal Trade Commission's "cooling-off" rule allow consumers to cancel a contract or return a purchase within a fixed time period, effectively extending their decision deadline.[12,13]

We also assume that the negotiated prices are publicly observable. Note that with complementary goods, the buyer would have an incentive to disclose a high price from the first negotiation so as to induce the seller's accommodation in the second negotiation. But, such a "monotonic" incentive would lead to the full price disclosure, much like in the literature on signaling a verifiable quality, e.g., Grossman (1981). In this regard, the key assumption is that the price information is "hard," i.e., it cannot be forged, which seems reasonable as posted prices are often provided in the form of a written contract. Note also that if purchases are made on behalf of the government, then, in many countries, the buyer will be subject to "sunshine" laws that typically allow public access to her transaction records (e.g., Berg et al. (2005), pp. 42-44). Finally, we assume that the sellers know the sequence. In this respect, we envision environments where each buyer-seller meeting is highly visible or publicized, or it can be easily inferred by the sellers from the calendar time.

We begin our analysis with two benchmark cases, one in which there is no uncertainty

[11]Option contracts are commonly used in the real-estate market (Poorvu 1999, pp. 151-3).

[12]See http://www.ftc.gov/bcp/edu/pubs/consumer/products/pro03.shtm.

[13]Note that the buyer could also decide on a purchase after each negotiation. Such a pay-as-you-go scheme would, however, not be desirable by the buyer who is averse to any loss because the sum of the sellers' equilibrium prices in this case would exceed the buyer's valuation for the entire project. A formal proof of this observation is available upon request.

about the buyer's valuations and the other in which each bilateral negotiation is performed under symmetric information. Our main observation in each case is that the buyer would be indifferent to the negotiation sequence.

3 Benchmarks

Commonly known valuations. Suppose that there is no uncertainty in the buyer's stand-alone valuations, i.e., $q_i, q_j \in \{0, 1\}^2$. Without loss of generality, let the buyer visit seller i first. Then, in the unique equilibrium, the buyer offers sellers' reservation, 0, in each negotiation and the sellers post these coordinating prices: $p_i^*(s_i) = 1 - v_j$, $p_j^*(s_j|1 - v_j) = v_j$, and $p_j^*(s_j|0) = 1 - v_i$. To understand, note that since $1 - v_i - v_j \geq 0$ by complementarity, a price $p_i(s_i) < v_i$ cannot be optimal for seller i as it can be slightly increased without risking a sale. Thus, $p_i(s_i) \geq v_i$, and seller i realizes a profit only when both goods are purchased. Moreover, in anticipation of the buyer posting the price in the second negotiation (occurring with probability $1 - \alpha_j$), seller i would optimally set $p_i(s_i) = 1 - v_j$, which is (weakly) greater than v_i due to complementarity. The optimal price is the same if seller j is expected to be the proposer in the second negotiation since he will choose to coordinate with seller i by offering $1 - p_i(s_i)$ rather than setting his monopoly price v_j. Thus, irrespective of who makes the offer in the second negotiation, seller i posts the price $p_i^*(s_i) = 1 - v_j$. Using the equilibrium prices, the buyer's expected payoff from the sequence $s_i \to s_j$ is found to be

$$\pi_{ij}(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_j + (1 - \alpha_i)\alpha_j v_i.$$

Re-labeling the sellers, it is evident that $\pi_{ij}(b) = \pi_{ji}(b)$. That is, the buyer is indifferent to the sequence irrespective of bargaining powers and stand-alone valuations. Since prices are coordinated in equilibrium, a joint purchase results with probability 1, which is efficient.

Symmetric information. Suppose now that the buyer's stand-alone valuations are uncertain, but, unlike in the base model, each seller also learns the buyer's valuation for his good prior to bargaining. In this setting, the buyer will still offer 0 in each negotiation.[14] Following a buyer offer in the first negotiation, seller j selects between two options in the second: (1)

[14]Compared to the case of known valuations, it may be less obvious in this benchmark why the buyer would always offer 0 in the first negotiation since, given the residual uncertainty for the second seller, the buyer could post a positive price to induce accommodation by the second seller. This, however, is not optimal for her since, being informed of the buyer's stand-alone valuation, the second seller accommodates only if his good has no stand-alone value. The buyer's incentive to post a positive price will change when she is privately informed in each negotiation as in the base model.

post a price $\frac{1}{2}$ and guarantee a sale, or (2) demand the entire surplus 1 and sell only when the buyer has no value for good i (with probability q_i). It is thus optimal for seller j to post $p_j^*(s_j|0) = \frac{1}{2}$ if $q_i \leq \frac{1}{2}$, and $p_j^*(s_j|0) = 1$ if $q_i > \frac{1}{2}$. A symmetric argument reveals that if seller i anticipated price posting by the buyer in the second negotiation, he would optimally set:

$$p_i^*(s_i) = \begin{cases} \frac{1}{2}, & \textit{if} \quad q_j \leq \frac{1}{2} \\ 1, & \textit{if} \quad q_j > \frac{1}{2}. \end{cases}$$

Interestingly, $p_i^*(s_i)$ would be optimal for seller i even if seller j were anticipated to make the offer in the second negotiation. To see this, note that as with the known valuations case, seller i would choose a coordinating price $1 - v_j$ except that v_j is now uncertain. This means that seller i will either post a price $\frac{1}{2}$ and sell regardless of v_j realization, or post a price 1 and sell only when $v_j = 0$ (with probability q_j). But this trade-off yields $p_i^*(s_i)$. To this price offer, seller j responds by:

$$p_j^*(s_j|p_i^*(s_i)) = \begin{cases} \frac{1}{2}, & \textit{if} \quad q_j \leq \frac{1}{2} \quad \textit{or} \quad v_j = \frac{1}{2} \\ 0, & \textit{if} \quad q_j > \frac{1}{2} \quad \textit{and} \quad v_j = 0. \end{cases}$$

By construction, these buyer and sellers' prices form the unique equilibrium. Using them, the buyer's expected payoff from the sequence $s_i \to s_j$ is given by

$$\pi_{ij}(b) = (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)\widehat{v}_j \times +(1-\alpha_i)\alpha_j\widehat{v}_i,$$

where $\widehat{v}_k = \begin{cases} \frac{1}{2}, & \textit{if} \quad q_k \leq \frac{1}{2} \\ \frac{1-q_k}{2}, & \textit{if} \quad q_k > \frac{1}{2} \end{cases}$.

Since, re-labeling the sellers, we have $\pi_{ij}(b) = \pi_{ji}(b)$, the buyer is indifferent to the sequence in this benchmark, too. Due to the residual uncertainty in valuations, however, an inefficient trade can occur. For instance, if $q_i \in (\frac{1}{2}, 1)$ and $q_j = 1$, then with probability $(1-\alpha_i)\alpha_j(1-q_i)$, the buyer purchases only good i. It is readily verified that efficiency obtains if and only if $q_k \notin (\frac{1}{2}, 1)$ for $k = i, j$.

Armed with the insights from these benchmarks, we next characterize the equilibrium of our base model.

4 Equilibrium Characterization

Recall that in our base model, only the buyer learns her stand-alone valuation before each bargaining. This asymmetric information between the buyer and the sellers leads to more involved equilibrium prices than those in the benchmarks; so we present them in two lemmas depending on the proposer in the first negotiation.

Lemma 1 (Buyer proposing). *Suppose that the negotiation sequence is $s_i \to s_j$. In the unique equilibrium,*

(a) *the buyer offers to pay the marginal cost, 0, in each negotiation except when $q_i > \frac{1}{2}$ and $\alpha_j > \frac{1+q_j^2}{1+q_j}$, in which case a low value buyer mixes in the first negotiation as follows:*

$$p_i^*(b|v_i = 0) = \begin{cases} 0, & \text{with prob. } \frac{1-q_i}{q_i} \\ \frac{1+q_j}{2}, & \text{with prob. } \frac{2q_i-1}{q_i}; \end{cases} \tag{1}$$

(b) *seller j responds to the buyer offers by: $p_j^*(s_j|\frac{1+q_j}{2}) = \frac{1-q_j}{2}$; and $p_j^*(s_j|0) = \frac{1}{2}$ if $q_i \leq \frac{1}{2}$, and*

$$p_j^*(s_j|0) = \begin{cases} \frac{1}{2}, & \text{with prob. } \theta_j^* \\ 1, & \text{with prob. } 1-\theta_j^* \end{cases} \tag{2}$$

if $q_i > \frac{1}{2}$, where $\theta_j^* = \frac{(1+q_j)}{\alpha_j} \max\{\alpha_j - \frac{1+q_j^2}{1+q_j}, 0\}$.

It is intuitive that in the last negotiation, the buyer will make a marginal cost offer, 0, to seller j. In the first negotiation, a buyer with high valuation will also make a 0 offer to seller i in order to maximize her outside option against seller j. A buyer with low valuation can mimic this 0 price, but this strategy will be profitable only if it is not met by seller j's aggressive response aimed at capturing the full surplus. Thus, if seller j is very powerful, a buyer with low valuation may offer a high positive price, $\frac{1+q_j}{2}$, in the first negotiation to curb seller j's response.[15] However, since, upon observing a 0 price, seller j may infer a high valuation for good i and reduce his price, a low value buyer will mix between the high and low prices as recorded in (1). Interestingly, this implies that in the first negotiation, the higher price is associated with the lower value buyer. The mixing by the buyer, in turn, causes seller j to mix upon observing a 0 price from the first negotiation. We complete the equilibrium characterization by the following lemma where seller i posts the price in the first negotiation (see also Figure 2).

Lemma 2 (Seller proposing). *Suppose that the negotiation sequence is $s_i \to s_j$. If seller i makes the offer in the first negotiation, then* the *sellers' unique equilibrium prices are given by:*

$$\left(p_i^*(s_i), p_j^*(s_j|p_i^*(s_i))\right) = \begin{cases} \left(1, \frac{1}{2}\right), & if \quad \alpha_j < \widehat{\alpha}(q_j) \\ \left(\frac{1+q_j}{2}, \frac{1-q_j}{2}\right), & if \quad \alpha_j \geq \widehat{\alpha}(q_j) \text{ and } q_j > \frac{\sqrt{5}-1}{2} \\ \left(\frac{1}{2}, \frac{1}{2}\right), & if \quad \alpha_j \geq \widehat{\alpha}(q_j) \text{ and } q_j \leq \frac{\sqrt{5}-1}{2}, \end{cases} \tag{3}$$

[15] Note that in the symmetric information benchmark, a price of $\frac{1+q_j}{2}$ would lead seller j to reduce his price to $\frac{1-q_j}{2}$ only if $v_j = 0$.

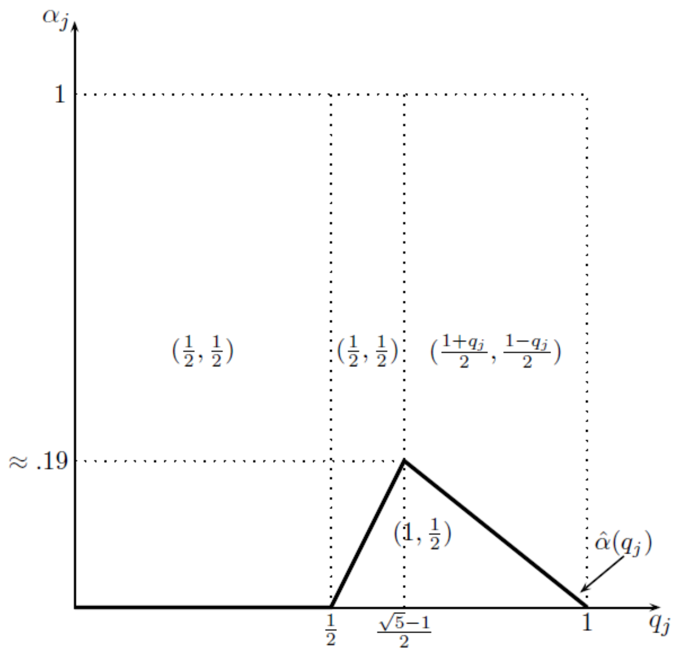

Figure 2: Sellers' equilibrium prices

where

$$\widehat{\alpha}(q_j) \equiv \begin{cases} 0, & if \quad q_j \leq \frac{1}{2} \\ 1 - \frac{1}{2q_j}, & if \quad \frac{1}{2} < q_j \leq \frac{\sqrt{5}-1}{2} \\ \frac{1-q_j}{2}, & if \quad q_j > \frac{\sqrt{5}-1}{2}. \end{cases} \tag{4}$$

To understand Lemma 2, note that, as in the benchmarks, seller i will never post a price below $\frac{1}{2}$ because any such price can be slightly increased without risking a joint sale. A price above $\frac{1}{2}$, however, requires coordination by seller j. All else equal, if seller j grows powerful, seller i needs to lower his price to encourage this coordination; otherwise, since the buyer always offers 0 in the second negotiation, seller i will ignore the coordination problem and instead demand the entire surplus 1 (inside the triangle region in Figure 2). The effect of seller j's complementarity, q_j, is less obvious. On the one hand, an increase in complementarity reduces the chance of a stand-alone purchase of good j, leading seller i to demand the entire surplus; on the other hand, it lowers seller i's cost of inducing accommodation by seller j, discouraging such aggressive pricing. As depicted in Figure 2, when seller j is sufficiently powerful, the coordination incentive binds for seller i, and thus an increase in seller j complementarity lets him raise his price. When seller j is weak, however, the two competing effects of complementarity produces a non-monotone change in seller i's price, peaking at some intermediate region where his coordination incentive disappears.

Given these equilibrium prices, we are now ready to explore the buyer's sequencing choice.

5 Strategic Sequencing

A key observation from the equilibrium characterization is that, unlike in the benchmarks, prices depend on the sellers' bargaining powers. This means that the buyer can influence them by her sequencing decision. We first note that the sequencing is inconsequential to the buyer if trade is efficient.

Proposition 1. (Efficiency) *Let $q_1 = q_2 = q$. Then,* the *equilibrium trade is efficient irrespective of the sequence if and only if $q \notin (\frac{1}{2}, 1)$, or equivalently $\widehat{\alpha}(q) = 0$. In addition, if $q \notin (\frac{1}{2}, 1)$, then the buyer is indifferent to the sequencing.*

With no payoff uncertainty, $q \in \{0, 1\}$, we already know from the first benchmark that the equilibrium trade is efficient. Proposition 1 adds that efficiency continues to hold when goods are weak complements, i.e., $q \leq \frac{1}{2}$, so that the equilibrium seller prices stay at their low level, $\frac{1}{2}$. Since, under an efficient trade, the extra surplus due to complementarity is captured by the sellers unless the buyer proposes in both negotiations, the buyer is indifferent to the sequence. Note that the efficiency condition in Proposition 1 is more stringent than the one for the symmetric information benchmark as it assumes (stochastically) identical goods. As such, efficiency is "harder" to obtain when the buyer is privately informed. The reason is that with nonidentical goods, a privately informed buyer may be unlikely to purchase from the first seller by posting a high price (see Lemma 1).[16]

Proposition 1 implies that the sequencing matters to the buyer only if the equilibrium trade involves inefficiency for at least one sequence. The following result shows the buyer's sequence choice based on the sellers' bargaining powers.

Proposition 2. (Bargaining Powers) *Let $q_1 = q_2 = q \in (\frac{1}{2}, 1)$ and $\alpha_1 < \alpha_2$. Then,*

[16] For instance, if $q_1 = 1$ and $q_2 \leq \frac{1}{2}$, then efficiency obtains for each sequence under the symmetric information benchmark. But, under private information, inefficiency can result for the sequence $s_1 \to s_2$ if $\alpha_2 > \frac{1+q_2^2}{1+q_2}$, because, by Lemma 1, the buyer offers $\frac{1+q_2}{2}$ in the first negotiation. In this case, with probability $(1-\alpha_1)\alpha_2(1-q_2)$, the buyer purchases only good 2.

(a) *the buyer*

$$\begin{cases} \textit{is indifferent to the sequence,} & if \quad \alpha_1 < \alpha_2 < \widehat{\alpha}(q) \\ \textit{strictly prefers the sequence } s_1 \rightarrow s_2, & if \quad \alpha_1 < \widehat{\alpha}(q) \leq \alpha_2 \\ \textit{strictly prefers the sequence } s_2 \rightarrow s_1, & if \quad \widehat{\alpha}(q) \leq \alpha_1 < \alpha_2; \end{cases}$$

(b) *the buyer's sequencing (weakly) improves ex ante social surplus except when* $\alpha_1 < \widetilde{\alpha}(\alpha_2, q)$ *and* $\alpha_2 \in (\frac{1+q^2}{1+q}, \frac{2q+q^2}{1+q})$, *where* $\widetilde{\alpha}(\alpha_2, q)$ *is a unique cutoff satisfying:* $\widetilde{\alpha}(\alpha_2, q) < \widehat{\alpha}(q) < \frac{1+q^2}{1+q}$.

According to part (a), when goods are strong but imperfect complements, the buyer is indifferent to the sequence if both sellers are weak bargainers, i.e., $\alpha_1 < \alpha_2 < \widehat{\alpha}(q)$. This indifference, however, is not due to efficient trade; rather it is due to the fact that switching the order would not alter the sellers' pricing behavior. In particular, the leading seller would post an aggressive price of 1 given that the follower is unlikely to make an offer against the buyer. This implies that if the buyer could increase the leader's concern for price coordination by switching the order, she would choose to do so. This is possible if the sellers' bargaining powers are sufficiently diverse in the sense that $\alpha_1 < \widehat{\alpha}(q) \leq \alpha_2$. In this case, the buyer strictly prefers to start negotiations with seller 1 because, being followed by a strong rival, seller 1 has an equilibrium incentive to coordinate prices by lowering his own. If, on the other hand, both sellers are sufficiently powerful such that $\widehat{\alpha}(q) \leq \alpha_1 < \alpha_2$, it is optimal for the buyer to start with seller 2 instead. Notice that with two sufficiently strong sellers, price coordination between them occurs in equilibrium irrespective of the sequence. Hence, the buyer's main objective in this case is to prevent an aggressive price response in the second negotiation in the event that she receives a favorable offer in the first.

According to part (b), even though the buyer is not maximizing the social surplus per se, her sequencing choice (weakly) increases it except when sellers' bargaining powers are sufficiently diverse. Note that the sequencing is likely to improve the social surplus if it helps lower the equilibrium prices to facilitate a joint purchase. Clearly, the buyer's sequencing is intended to decrease the sellers' prices. Thus, if the sequencing also decreases the buyer's own price in the first negotiation, then the surplus increases. This is the case when the sellers' bargaining powers are not too diverse, since then the buyer posts a marginal-cost price regardless of the sequence. Otherwise, the buyer optimally starts with the weak seller and makes a high price offer herself upon realizing a low valuation. In this case, the social efficiency would favor starting with the strong seller instead in order for the buyer to post a marginal-cost price in the first negotiation. The observation in part (b) is also consistent with Proposition 1 above: since the maximum social surplus is obtained irrespective of the sequence, the buyer is indifferent to the sequence.

In addition to the sellers' bargaining powers, the buyer's sequence may also be dictated by the degree of uncertainty across the objects. Proposition 3 offers insights in this direction.

Proposition 3. **(Payoff Uncertainty)** *Suppose that $\alpha_1 = \alpha_2 = \alpha$, and that $q_1 \notin (\frac{1}{2}, 1)$ and $q_2 \in (\frac{1}{2}, 1)$. Then, the buyer strictly prefers to approach seller 1 first if $\alpha \geq \widehat{\alpha}(q_2)$, but she is indifferent to the sequence if $\alpha < \widehat{\alpha}(q_2)$.*

To understand Proposition 3, recall that the leading seller always posts a price exceeding the buyer's stand-alone valuation, and that powerful sellers are likely to coordinate their prices in equilibrium. Thus, with powerful sellers, the buyer is likely to purchase only from the follower. To ensure the lowest price by this seller, if $q_1 \leq \frac{1}{2} < q_2$, the buyer first visits the (stochastically) higher value seller 1 whose high price induces a low coordinating price by seller 2. The same sequence is also optimal for the buyer if she has no value for good 1 in order to have the option of purchasing from the follower.

6 Benefits of Buyer Concessions

An important implication of our model is that equilibrium trade can be inefficient. This suggests that the buyer may adopt some (costly) procurement policies to improve efficiency as long as they produce more favorable supply prices. In what follows, we consider three such policies: remaining a weak buyer, committing to a minimum price, and strategic outsourcing.

6.1 The Power of Being a Weak Buyer

As observed above, the leading seller softens his pricing behavior only if followed by a powerful rival. That is, all else equal, the more powerful the follower, the less aggressive the leader. This price-bargaining power trade off raises an obvious question: can the buyer be better off having *more* powerful sellers, or equivalently, being a *weaker* bargainer herself? The answer to this question can be affirmative, as we demonstrate in

Proposition 4. *Suppose $q_1 = q_2 = q$, and that $\alpha^L = (\alpha_1, \alpha_2^L)$ and $\alpha^H = (\alpha_1, \alpha_2^H)$ are two bargaining power profiles where $\alpha_2^L < \alpha_2^H$.*

(a) *If, in addition,* $q \in (\frac{1}{2}, 1)$, $\alpha_1 \neq \widehat{\alpha}(q)$, $\alpha_2^L = \widehat{\alpha}(q) - \Delta$, *and* $\alpha_2^H = \widehat{\alpha}(q) + \Delta$, *then there is some* $\overline{\Delta} > 0$ *such that the buyer is strictly better off under* α^H *than under* α^L *for all* $\Delta \in (0, \overline{\Delta})$.

(b) *If, on the other hand,* $q \notin (\frac{1}{2}, 1)$ *or* $\alpha_2^L < \alpha_2^H < \widehat{\alpha}(q)$, *then the buyer is strictly worse off under* α^H *than under* α^L.

Part (a) is best understood by considering a weak seller 1, i.e., $\alpha_1 < \widehat{\alpha}(q)$. Note that, by Proposition 1, equilibrium trade is inefficient for strong but imperfect complements. If seller 2 is also weak, Proposition 2 implies that the buyer cannot alleviate this inefficiency through her sequencing. If, however, one seller is powerful, she can engender lower equilibrium prices by first visiting the weak seller. Thus, while, for fixed prices, a powerful seller will only have a *negative* (direct) effect on the buyer's payoff, he may also have a *positive* (strategic) effect on her payoff through pricing. Part (a) demonstrates that when the seller's bargaining power is not too high, the strategic effect dominates. That is, the buyer may prefer to deal with a more powerful seller, or equivalently, be a weaker bargainer herself. Part (b) follows because, under each condition, the buyer cannot change the sellers' equilibrium prices by her sequencing and thus a stronger seller can only diminish her payoff. This logic would also explain why the buyer would prefer weaker sellers in the benchmark cases presented in Section 3.

The message of Proposition 4 is that it may sometimes be in the buyer's best interest to limit her own bargaining power vis-à-vis the sellers. If this power comes from forming buyer alliances and cooperatives, our result says that there may be strategic reasons for the buyer to cap the size of the alliance. Our prediction for the adverse effect of the "buyer power" on the buyer herself is consistent with those in the literature, e.g., Chipty and Snyder (1999), Horn and Wolinsky (1988), and Inderst and Wey (2003), though our reasoning is quite different.[17] By focusing on efficient bilateral negotiations (often utilizing the Nash Bargaining Solution), these papers show that increased buyer power through mergers is not necessarily beneficial to the buyer, and its benefit crucially depends on the curvature of the value created by the mergers. In contrast, our comparative static result in part (a) underlines the impact of buyer power on the potential for inefficient bargaining with the suppliers while keeping the surplus fixed. In fact, when bargaining yields efficiency, i.e., $q \notin (\frac{1}{2}, 1)$, part (b) of Proposition 4 implies that the buyer is better off having more power.

[17] There is a relatively vast literature on *buyer power*, which is ably surveyed by Inderst and Mazzarotto (2009). Consistent with our model, these authors define buyer power as "the bargaining strength that a buyer has with respect to the suppliers with whom it trades."

6.2 Minimum Purchase Price

In our model, one reason behind the leading seller's aggressive pricing is the buyer's marginal cost offer, 0, in the second negotiation. In particular, when the buyer is highly likely to propose in the second negotiation, the leader disregards price coordination with the follower and targets the entire surplus of 1 instead, which often creates inefficiency. This suggests that it may be possible for the buyer to restore the leader's coordination incentive and grab additional surplus by committing to pay a minimum price.

Proposition 5. *Let $q_1 = q_2 = q$ and $\alpha_1 = \alpha_2 = \alpha$. Suppose that prior to negotiations, the buyer commits to pay at least $w \geq 0$ for each unit she purchases. Then,*

(a) *for $q \in (\frac{1}{2}, 1)$, there exists $\alpha^c(q) \in (0, \widehat{\alpha}(q))$ such that the buyer optimally sets $w > 0$ for all $\alpha \in [\alpha^c(q), \widehat{\alpha}(q))$;*

(b) *for $q \notin (\frac{1}{2}, 1)$, the buyer optimally sets $w = 0$ for all α.*

Part (a) indicates that for strong but imperfect complements, the buyer may optimally commit to a positive price even when she makes the offers. As noted above, to be beneficial to the buyer, any such commitment must moderate the sellers' prices. This is most easily seen with the weak sellers who tend to demand the full surplus. When the buyer posts a minimum price $w > 0$ per unit, the leader knows that his available surplus is reduced to $1 - w$, making a non-coordinating price strategy less attractive. By optimally setting $w > 0$, the buyer engenders a coordinating equilibrium with lower supply prices. Note that an upfront commitment to $w > 0$ is crucial here, because, once the leader decreases his price offer, the buyer has a strict incentive to decrease hers to 0 in the second negotiation whenever she proposes. Part (b) implies that the buyer cannot gain from posting a minimum price when equilibrium trade is efficient so that the surplus cannot grow further.[18]

It is worth noting that a minimum price policy is optimal for a powerful buyer given that $\widehat{\alpha}(q) < 0.2$. Hence, our result in part (a) might suggest that even without quality or ethical concerns, participating in a "fair trade" agreement that sets a minimum negotiation price can be in the best interest of powerful buyers.[19] In the same vein, large employers might favor

[18] By the same token, it follows that the buyer would set $w = 0$ under the known valuations' benchmark while she may set $w > 0$ under the symmetric information bechmark due to the potential inefficiency.

[19] According to the Fairtrade Foundation of the UK, the minimum price set by the Fairtrade Labelling Organizations International "...is not a fixed price, but should be seen as the lowest possible starting point for price negotiations between producer and purchaser." See <www.fairtrade.org.uk>.

minimum wage regulations when hiring new employees.

6.3 Strategic Outsourcing

A critical decision for many industrial buyers is whether to make inputs internally or outsource them from independent suppliers. Conventional wisdom suggests that an input should be made in-house if its internal cost of production is less than the price charged by the outside supplier. This simple criterion would apply if a single input were required for a final product.[20] However, if two complementary inputs are required, as in the present setting, the following result shows that the buyer may optimally outsource an input even if it could be costlessly provided in-house.

Proposition 6. *Let $\alpha_1 = \alpha_2 = \alpha$, and suppose that the buyer can costlessly make input 1 in-house, which has no stand-alone value, $q_1 = 1$, but she needs to outsource input 2 with $q_2 \in (\frac{\sqrt{5}-1}{2}, 1)$. Then, the buyer is strictly better off outsourcing both inputs than only input 2 if and only if $\alpha > \frac{1+q_2^2}{1+q_2}$.*

Proposition 6 says that when inputs are strong complements *and* suppliers are powerful bargainers, the "naive" decision of making the zero-cost input in-house while outsourcing the other cannot be optimal for the buyer. In particular, it is strictly better for the buyer to outsource both inputs in this case. The reason is twofold. First, given the high degree of complementarity, the surplus generated by the internal production of input 1 at zero cost is likely to be shared with supplier 2 at the negotiation. Second, we know from Lemma 2 that two powerful suppliers would have the greatest incentives to coordinate and lower their prices. From Lemma 1 and Proposition 3, it is interesting to note that when outsourcing both inputs, the buyer first negotiates for input 1 which has no stand-alone value and offers a high price of $\frac{1+q_2}{2}$ (as does seller 1) in order to induce seller 2 to reduce his price to $\frac{1-q_2}{2}$. Thus, it is readily verified that the buyer ends up acquiring both inputs if she also realizes a low stand-alone value for input 2, and acquiring only input 2 otherwise.

Two observations are in order. First, when goods are strong complements and suppliers are powerful bargainers, the buyer is likely to follow an *all-or-nothing* sourcing strategy. The optimal decision will, however, depend on suppliers' bargaining powers, complementarity,

[20]The make-or-buy decision can, of course, be complicated by various other factors that we ignore here such as asset specificity and incomplete contracts (see, e.g., Williamson (2005) for a recent survey).

and the total internal cost of production.[21]Second, our finding that the buyer may outsource even without a cost disadvantage complements other strategic explanations for the same phenomenon. For instance, Arya et al. (2008) have demonstrated that a retail competitor may pay a premium to outsource production to a common supplier in order to raise its rivals' costs through unfavorable supply deals. In contrast, in our model, a single firm outsources production to raise its *own* cost for an input in order to receive a favorable deal from the other supplier of a complementary input.

7 First- and Second-Mover Advantages for Sellers

So far, as the central agent who initiates the negotiations, we have focused on the buyer's preferred sequence. It is, however, conceivable that the sellers may also have a preferred sequence. In particular, if the sellers expect a higher profit from being the first to negotiate with the buyer than being the second, then they may actively solicit the buyer's business by offering a discount for the right to be the first. If, on the other hand, the sellers expect a greater profit from being the second to negotiate, then no such solicitation should take place. Denoting by $\pi^l(s_i)$ and $\pi^f(s_i)$ seller i's expected profits from being the leader and the follower in the negotiations, respectively, we obtain

Proposition 7. *Let $q_1 = q_2 = q$ and $\alpha_1 = \alpha_2 = \alpha$. Then,*

(a) *for high complementarity, $q > \frac{1}{2}$, we have $sign[\pi^l(s_i) - \pi^f(s_i)] = sign[\alpha - \overline{\alpha}(q)]$, where $\overline{\alpha}(q) \geq \widehat{\alpha}(q)$ is a unique cutoff;*

(b) *for low complementarity, $q \leq \frac{1}{2}$, however, we have $\pi^l(s_i) - \pi^f(s_i) = 0$.*

Proposition 7 says that for high complementarity, $q > \frac{1}{2}$, there is a first-mover advantage if the sellers are sufficiently powerful, and a second-mover advantage otherwise. Consider, for instance, the case of perfect complements, $q = 1$. In equilibrium, the leader demands the entire surplus, 1, to which the follower responds by posting the lowest price, 0; and symmetrically, the follower demands the entire surplus if the buyer posts the price, 0, in the first negotiation. Thus, seller i's respective payoffs from being the leader and being the follower are $\pi^l(s_i) = \alpha$ and $\pi^f(s_i) = \alpha(1-\alpha)$, implying a first-mover advantage. When complementarity

[21]Note that we intentionally assumed in Proposition 6 that the cost of producing input 1 is zero; but the internal cost of both inputs can be some $K \geq 0$, leaving a surplus of $1 - K$ in case of an all in-house production.

is high but imperfect, the first-mover advantage continues to exist for sufficiently powerful sellers since the leader can still claim more of the surplus without worrying about coordination. Otherwise, for weak sellers, a second-mover advantage emerges because the follower can tailor his price to a likely buyer offer from the first negotiation. Proposition 7 also says that for low complementarity, the sellers enjoy no first- or second-mover advantage. This is obviously true when goods are modular, $q = 0$, and it remains true for low complementarity that does not affect equilibrium prices.

It is worth comparing our observations from Proposition 7 with those from the standard duopoly theory in which the sellers are price-setters, i.e, $\alpha \to 1$. For complementary products, the IO literature has observed the presence of a first-mover advantage for duopolists (e.g., Gal-Or, 1985; and Dowrick, 1986). This is in line with our result when $q > \frac{1}{2}$ and $\alpha > \overline{\alpha}(q)$. However, a switch to the second-mover advantage occurs in our model as the buyer becomes more powerful so that she is no longer a price-taker.[22]

8 Conclusion

Unlike the standard consumer theory, the buyer is not a simple price-taker in many real examples; rather she is a powerful agent who actively negotiates with the sellers. With multiple sellers, a strategic decision for the buyer is in what order to approach them. In this paper, we have focused on the sellers of complementary goods and included the possibility of uncertain buyer valuations. Two key results have emerged. First, when equilibrium trade is inefficient, the buyer can have a strict sequence preference based on the sellers' bargaining powers. Second, the buyer may sometimes want to raise her own cost of acquiring the objects. In particular, we find that the buyer may commit to giving up some of her surplus up front to obtain more favorable prices later. This commitment can manifest itself in the form of a minimum purchase price or strategic outsourcing.

In closing, we briefly discuss the case of substitute goods and a future research avenue. Let stand-alone valuations be such that $v_i = 1$ with probability q and $v_i = \frac{1}{2}$ with probability $1 - q$ so that goods are perfect substitutes with probability q^2 and modular with probability $(1 - q)^2$. As in our base model, assume that the buyer privately learns her valuations as

[22] There is an extensive IO literature identifying a second-mover advantage by enriching the standard duopoly model. Most related to our work are the papers with demand uncertainty where the strategic action of the leader has a signaling value, which is not the case in ours. See, e.g., Daughety and Reinganum (1994), Gal-Or (1987), and Mailath (1987).

she visits the sellers. Then, it can be shown that equilibrium prices are unique and the buyer always posts a price of 0 (see the supplementary material). More interestingly, if $\alpha_1 < \frac{1}{2q} < \alpha_2$, then the buyer strictly prefers to start with the stronger seller, 2. The reason is that unlike with the complements, the leading seller of the substitutes lowers his price if the buyer is more likely to propose and receive a low price in the second negotiation. This observation suggests that for substitute goods, it is unlikely for the buyer to raise her own procurement cost, say through posting a minimum purchase price. Note that similar to the case of complements, the source of the buyer's strict sequencing preference for substitutes is her private information. Otherwise, with commonly known valuations, it is readily verified that the sellers will post modified Bertrand prices: $p_i^*(s_i) = 1 - v_j$ and $p_j^*(s_j|1 - v_j) = p_j^*(s_j|0) = 1 - v_i$, which will, in turn, yield the following expected payoff to the buyer:

$$\pi_{ij}(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_j + (1 - \alpha_i)\alpha_j v_i + \alpha_i\alpha_j(v_i + v_j - 1).$$

Clearly, $\pi_{ij}(b) = \pi_{ji}(b)$ and thus the buyer is indifferent to the sequence.

We believe that a promising avenue for future research is to relax our assumption that the buyer cannot gather information herself. This may be especially pertinent to applications where goods are relatively standard. For instance, it may not be too costly for some employers to find out about job candidates' skills through their websites before scheduling a meeting. In such a case, it would be interesting to determine the buyer's incentive to invest in this information prior to negotiations given that her sequencing choice can (partially) signal her valuations to the candidates.

Appendix

Proof of Lemma 1. Suppose that the buyer negotiates with s_i first, and that p_i is the resulting price. When negotiating with s_j, the buyer will propose the lowest price accepted by s_j, namely $P_j(b|p_i) = 0$. To derive s_j's best price response $P_j(s_j|p_i)$, let $\bar{q}_i(p_i)$ denote s_j's belief that $v_i = 0$ conditional on p_i. Note that for any p_i, s_j realizes a sale if and only if

$$\max\{1 - p_i - P_j(s_j|p_i), v_j - P_j(s_j|p_i)\} \geq \max\{v_i - p_i, 0\}. \tag{A-1}$$

where the weak inequality follows from our tie-breaking rule. We exhaust two possibilities for p_i. Consider $p_i \leq \frac{1}{2}$. Then, $\max\{1 - p_i - P_j(s_j|p_i), v_j - P_j(s_j|p_i)\} = 1 - p_i - P_j(s_j|p_i)$. This means that s_j can either post a price of $1 - p_i$, selling his good only when $v_i = 0$, or post a price of $\frac{1}{2}$, selling his good with certainty. Comparing his respective payoffs, $\bar{q}_i(p_i)(1 - p_i)$ and $\frac{1}{2}$, from these two pricing options, it follows that $P_j(s_j|p_i) = 1 - p_i$ if $p_i \leq 1 - \frac{1}{2\bar{q}_i(p_i)}$, and $P_j(s_j|p_i) = \frac{1}{2}$ if $1 - \frac{1}{2\bar{q}_i(p_i)} \leq p_i \leq \frac{1}{2}$. Next, consider $p_i > \frac{1}{2}$. Then, since $v_i \leq \frac{1}{2}$, good i is purchased only if the buyer acquires both units. Given this, s_j can either set a coordinating price of $1 - p_i$ and sell his unit with certainty, or he can set a price of $\frac{1}{2}$ and sell only his own unit when $v_j = \frac{1}{2}$. Comparing the respective payoffs, $1 - p_i$ and $(1 - q_j)\frac{1}{2}$, it follows that $P_j(s_j|p_i) = 1 - p_i$ if $\frac{1}{2} < p_i \leq \frac{1+q_j}{2}$ (tie-breaking in favor of efficiency), and $P_j(s_j|p_i) = \frac{1}{2}$ if $p_i > \frac{1+q_j}{2}$. To summarize,

$$P_j(s_j|p_i) = \begin{cases} 1 - p_i, & if \quad 0 \leq p_i \leq 1 - \frac{1}{2\bar{q}_i(p_i)} \\ \frac{1}{2}, & if \quad 1 - \frac{1}{2\bar{q}_i(p_i)} \leq p_i \leq \frac{1}{2} \\ 1 - p_i, & if \quad \frac{1}{2} < p_i \leq \frac{1+q_j}{2} \\ \frac{1}{2}, & if \quad p_i > \frac{1+q_j}{2}. \end{cases} \tag{A-2}$$

We now turn to the first negotiation and suppose that the buyer makes the offer. Consider a high valuation buyer, $v_i = \frac{1}{2}$. Then, $p_i(b|v_i = \frac{1}{2}) = 0$ is the buyer's unique best response to $P_j(s_j|p_i)$. To see this, note that given (A-2) she receives an expected payoff $(1 - \alpha_j) + \alpha_j \frac{1}{2}$ by posting $p_i(b|v_i = \frac{1}{2}) = 0$. A price of $\tilde{p}_i \in (0, \frac{1}{2}]$ performs strictly worse than 0 since it yields an expected payoff $(1 - \alpha_j)(1 - \tilde{p}_i) + \alpha_j(\frac{1}{2} - \tilde{p}_i)$. This is also true for $\tilde{p}_i > \frac{1}{2}$ since it results in a payoff of $(1 - \alpha_j)\max\{1 - \tilde{p}_i, v_j\} + \alpha_j \max\{1 - \tilde{p}_i - P_j(s_j|\tilde{p}_i), v_j - P_j(s_j|\tilde{p}_i)\} < (1 - \alpha_j) + \alpha_j \frac{1}{2}$.

Next, consider a low valuation buyer, $v_i = 0$. We show that the only equilibrium candidates are $p_i(b|v_i = 0) = 0$ and $\frac{1+q_j}{2}$. Let $\gamma_i(p_i|0)$ be the equilibrium probability that the buyer with $v_i = 0$ offers p_i. We first rule out $\gamma_i(p_i|0) > 0$ for $p_i \in (0, \frac{1}{2})$. A price in this range results in $\bar{q}_i(p_i) = 1$. To see this, note that s_i would never make a price offer in this range since he will have a profitable deviation to $\frac{1}{2}$, accepted for sure based on condition (A-1). Therefore, $p_i \in (0, \frac{1}{2})$ results in buyer's payoff $(1-\alpha_j)(1-p_i)$, which is less than the payoff $(1-\alpha_j)$ under price 0. Next, we rule out $\gamma_i(p_i|0) > 0$ for $p_i \in [\frac{1}{2}, \frac{1+q_j}{2}) \cup (\frac{1+q_j}{2}, 1]$. Note that in this price region, s_j's belief $\bar{q}_i(p_i)$ is not payoff-relevant for the buyer since she does not purchase good i alone. The buyer's payoff from offering $p_i > \frac{1+q_j}{2}$ is $(1-\alpha_j)[\frac{1-q_j}{2} + q_j(1-p_i)]$, which is again less than her payoff under price 0. The buyer's payoff from offering $p_i \in [\frac{1}{2}, \frac{1+q_j}{2}]$ is $(1-\alpha_j)[\frac{1-q_j}{2} + q_j(1-p_i)] + \alpha_j(1-q_j)[\frac{1}{2} - (1-p_i)] = \frac{1-q_j}{2} + (q_j - \alpha_j)(1-p_i)$. If $q_j > \alpha_j$, then this payoff is decreasing in p_i, implying that any $p_i \in (\frac{1}{2}, \frac{1+q_j}{2}]$ will have a profitable deviation to $\frac{1}{2}$, since $\frac{1-q_j}{2} + (q_j-\alpha_j)(1-p_i) < \frac{1-q_j}{2} + (q_j-\alpha_j)\frac{1}{2} = \frac{1-\alpha_j}{2}$. Moreover, a price $p_i = \frac{1}{2}$ with a payoff of $\frac{1-\alpha_j}{2}$ is worse than $p_i = 0$ with a payoff of $1-\alpha_j$. If, on the other hand, $q_j < \alpha_j$, the payoff is increasing in p_i, which means that any price $p_i \in [\frac{1}{2}, \frac{1+q_j}{2})$ is worse than $p_i = \frac{1+q_j}{2}$. Thus, $p_i = \frac{1+q_j}{2}$ is the only equilibrium candidate in $[\frac{1}{2}, \frac{1+q_j}{2}]$, which generates the buyer payoff $\pi_i(b|\frac{1+q_j}{2}) = \frac{1-q_j}{2}[1+q_j-\alpha_j]$.

To complete the pricing for the low valuation buyer, note that $\gamma_i(0|0) = 1 - \gamma_i(\frac{1+q_j}{2}|0)$ since $p_i(b|v_i = 0) = 0$ and $\frac{1+q_j}{2}$ are the two possible prices. Moreover, by Bayesian updating, $\bar{q}_i(0) = \frac{\gamma_i(0|0)q_i}{\gamma_i(0|0)q_i + 1 - q_i} \leq q_i$. For $q_i \leq \frac{1}{2}$, we have $\pi_i(b|0) > \pi_i(b|\frac{1+q_j}{2})$, implying that $\gamma_i(0|0) = 1$ and $\gamma_i(\frac{1+q_j}{2}|0) = 0$. For $q_i > \frac{1}{2}$, it is straightforward to verify that $\gamma_i(0|0) = 1$ for $\alpha_j \leq \frac{1+q_j^2}{1+q_j}$. For $\alpha_j > \frac{1+q_j^2}{1+q_j}$, $\gamma_i(0|0) = 1$ has a profitable deviation to $\gamma_i(0|0) = 0$. However, $\gamma_i(0|0) = 0$ results in $\bar{q}_i(0) = 0$ for any $q_i < 1$. Thus, it follows that $\gamma_i(0|0) = 0$ has a profitable deviation to $\gamma_i(0|0) = 1$. This implies that for $\alpha_j > \frac{1+q_j^2}{1+q_j}$ and $q_i \in (\frac{1}{2}, 1)$, the only possible equilibrium is for the buyer to mix between the two prices. The resulting equilibrium payoff is

$$\pi_i(b|\frac{1+q_j}{2}) = \pi_i(b|0) = (1-\alpha_j) + \alpha_j \times \begin{cases} \frac{1}{2}, & if \quad \bar{q}_i(0) < \frac{1}{2} \\ \frac{1}{2}\theta_j, & if \quad \bar{q}_i(0) = \frac{1}{2} \\ 0, & if \quad \bar{q}_i(0) > \frac{1}{2} \end{cases}$$

where $\theta_j = \Pr(P_j(s_j|0) = \frac{1}{2}|\bar{q}_i(0) = \frac{1}{2})$. Note that the two payoffs are equal if and only if $\bar{q}_i(0) = \frac{1}{2}$ and $\theta_j^* = \frac{\alpha_j(1+q_j)-(1+q_j^2)}{\alpha_j}$. $\bar{q}_i(0) = \frac{1}{2}$ implies that $\gamma_i^*(0|0) = \frac{1-q_i}{q_i}$. For $q_i = 1$ and $\alpha_j > \frac{1+q_j^2}{1+q_j}$, $\bar{q}_i(0) = 1$ for any $\gamma_i^*(0|0)$ implying that the equilibrium involves $\gamma_i^*(0|0) = 0$, proving part (a). Part (b) follows easily from the buyer's equilibrium price and s_j's best response in (A-2). The uniqueness of these equilibrium prices follows by construction and by our tie-breaking rule. ■

Proof of Lemma 2. Suppose that the buyer negotiates with s_i first, and s_i makes the offer.

Clearly, it cannot be that $p_i(s_i) < \frac{1}{2}$ because by (A-1) s_i can slightly increase the price while still guaranteeing a joint purchase. Thus, $p_i(s_i) \geq \frac{1}{2}$. Note that $p_i(s_i) = \frac{1+q_j}{2}$ is optimal in $(\frac{1}{2}, \frac{1+q_j}{2}]$, since, by (A-2), $P_j(s_j|p_i) = 1 - p_i < \frac{1}{2}$, and s_i makes a sale with the same probability q_j, i.e., if $v_j = 0$. Note also that $p_i(s_i) = 1$ is optimal in $(\frac{1+q_j}{2}, 1]$, since s_i makes a sale with the same probability $(1-\alpha_j)q_j$, i.e., if the buyer is the proposer for good j and $v_j = 0$. In sum, s_i chooses a price among candidates, $\frac{1}{2}$, $\frac{1+q_j}{2}$, and 1, yielding expected payoffs, $\frac{1}{2}$, $\frac{1+q_j}{2}q_j$, and $(1-\alpha_j)q_j$, respectively. Defining $\widehat{\alpha}(q_j)$ as in (4), it is clear that if $\alpha_j < \widehat{\alpha}(q_j)$, then $p_i^*(s_i) = 1$, resulting in $P_j(s_j|p_i^*(s_i)) = \frac{1}{2}$. If, however, $\alpha_j \geq \widehat{\alpha}(q_j)$, then s_i compares payoffs $\frac{1}{2}$ and $\frac{1+q_j}{2}q_j$, resulting in

$$p_i^*(s_i) = \begin{cases} \frac{1}{2}, & if \quad q_j \leq \frac{\sqrt{5}-1}{2} \\ \frac{1+q_j}{2}, & if \quad q_j > \frac{\sqrt{5}-1}{2} \end{cases} \text{ and } P_j(s_j|p_i^*(s_i)) = \begin{cases} \frac{1}{2}, & if \quad q_j \leq \frac{\sqrt{5}-1}{2} \\ \frac{1-q_j}{2}, & if \quad q_j > \frac{\sqrt{5}-1}{2} \end{cases}.$$

The uniqueness of these equilibrium prices follows by construction and by our tie-breaking rule. ■

Proof of Proposition 1. We first derive the general expressions for the social surplus and buyer's payoff for a given sequence $s_i \to s_j$. Let SS_{ij} denote the equilibrium social (joint) surplus from this sequence. Using the equilibrium prices from Lemma 1 and 2, we find:

- If $q_i \leq \frac{1}{2}$ or $\alpha_j \leq \frac{1+q_j^2}{1+q_j}$,

$$\begin{aligned} SS_{ij} &= (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)\left[q_j + (1-q_j)\left(\frac{1}{2} + \frac{1}{2}\mathbf{1}(p_i^*(s_i) \leq \frac{1}{2})\right)\right] \\ &\quad + (1-\alpha_i)\alpha_j\left[q_i + (1-q_i)\left(\frac{1}{2} + \frac{1}{2}\mathbf{1}(p_j^*(s_j|0) \leq \frac{1}{2})\right)\right] \\ &\quad + \alpha_i\alpha_j\left[q_j\mathbf{1}(p_i^*(s_i) + p_j^*(s_j|p_i^*(s_i)) \leq 1) + (1-q_j)\left(\frac{1}{2} + \frac{1}{2}\mathbf{1}(p_i^*(s_i) \leq \frac{1}{2})\right)\right]. \end{aligned} \tag{A-3}$$

- If $q_i > \frac{1}{2}$ and $\alpha_j > \frac{1+q_j^2}{1+q_j}$,

$$\begin{aligned} SS_{ij} &= (1-\alpha_i)(1-\alpha_j)[1 - \frac{q_i(1-q_j)(1-\gamma_i^*(0|0))}{2}] \\ &\quad + \alpha_i(1-\alpha_j)\left[q_j + (1-q_j)\left(\frac{1}{2} + \frac{1}{2}\mathbf{1}(p_i^*(s_i) \leq \frac{1}{2})\right)\right] \\ &\quad + (1-\alpha_i)\alpha_j\left[1 - \frac{(1-q_i)(1-\theta_j^*)}{2} - \frac{q_i(1-q_j)(1-\gamma_i^*(0|0))}{2}\right] \\ &\quad + \alpha_i\alpha_j\left[q_j\mathbf{1}(p_i^*(s_i) + p_j^*(s_j|p_i^*(s_i)) \leq 1) + (1-q_j)\left(\frac{1}{2} + \frac{1}{2}\mathbf{1}(p_i^*(s_i) \leq \frac{1}{2})\right)\right], \end{aligned} \tag{A-4}$$

where $\mathbf{1}(\cdot)$ stands for the indicator function. The first term in SS_{ij} accounts for the event that the buyer is the proposer in both negotiations. If $q_i \leq \frac{1}{2}$ or $\alpha_j \leq \frac{1+q_j^2}{1+q_j}$, then the buyer purchases both goods with certainty. Otherwise, she purchases only j with probability $1-\gamma_i^*(0|0)$ if $v_i = 0$, $v_j = \frac{1}{2}$ resulting in a surplus of $\frac{1}{2}$. The second term accounts for the possibility of s_i being the proposer in the first negotiation and the buyer in the second, in which case the buyer purchases both goods whenever $v_j = 0$ and purchases only good j if $v_j = \frac{1}{2}$ and $p_i^*(s_i) > \frac{1}{2}$. The third term accounts for the possibility of s_j being the proposer in the second negotiation and the buyer in the first. Then, if $q_i \leq \frac{1}{2}$ or $\alpha_j \leq \frac{1+q_j^2}{1+q_j}$ the buyer offers 0 in the first negotiation, acquires both goods if $v_i = 0$ and acquires only good i if $v_i = \frac{1}{2}$ and $p_j^*(s_j|0) > \frac{1}{2}$. If $q_i > \frac{1}{2}$ and $\alpha_j > \frac{1+q_j^2}{1+q_j}$, she reject s_j's offer if $v_i = \frac{1}{2}$ and s_j responded with $p(s_j|0) = 1$, captured by the first negative term in equation (A-4). The buyer will not purchase from s_i if $v_i = 0$, $v_j = \frac{1}{2}$, and the buyer made a high price offer of $\frac{1+q_j}{2}$ in the first negotiation, captured by the second negative term. Finally, the last term accounts for the sellers making price offers in each negotiation. Since $p_i^*(s_i) \geq \frac{1}{2}$, for $v_j = 0$, the buyer will acquire both goods if the sum of the prices does not exceed 1. For $v_j = \frac{1}{2}$, the buyer will purchase good j only if $p_i^*(s_i) > \frac{1}{2}$.

We can similarly write the buyer's equilibrium payoff $\pi_{ij}(b)$ from sequence $s_i \to s_j$.

- If $q_i \leq \frac{1}{2}$ or $\alpha_j \leq \frac{1+q_j^2}{1+q_j}$,

$$\begin{aligned}\pi_{ij}(b) \quad &= \quad (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)\left[q_j(1-p_i^*(s_i)) + (1-q_j)\frac{1}{2}\right] \\ &\quad +\alpha_j(1-\alpha_i)\left[q_i(1-p_j^*(s_j|0)) + (1-q_i)\frac{1}{2}\right] + \alpha_i\alpha_j(1-q_j)\left[\frac{1}{2} - p_j^*(s_j|p_i^*(s_i))\right].\end{aligned} \tag{A-5}$$

- If $q_i > \frac{1}{2}$ and $\alpha_j > \frac{1+q_j^2}{1+q_j}$,

$$\begin{aligned}\pi_{ij}(b) \quad &= \quad (1-\alpha_i)(1-\alpha_j)\left[1-q_i+q_i\left(\frac{1-q_j}{2}+q_j\frac{1-q_j}{2}\right)\right] \\ &\quad +\alpha_i(1-\alpha_j)\left[q_j(1-p_i^*(s_i)) + (1-q_j)\frac{1}{2}\right] + \\ &\quad +\alpha_j(1-\alpha_i)\left[\frac{1-q_i}{2} + \frac{q_i(1-q_j)q_j}{2}\right] + \alpha_i\alpha_j(1-q_j)\left[\frac{1}{2} - p_j^*(s_j|p_i^*(s_i))\right].\end{aligned} \tag{A-6}$$

For $q_i > \frac{1}{2}$ and $\alpha_j > \frac{1+q_j^2}{1+q_j}$, the above payoff is simply the payoff resulting from the buyer offering $p_i(b|v_i = 0) = \frac{1+q_j}{2}$ since in the mixed strategy equilibrium the buyer is indifferent between $p_i(b|v_i = 0) = \frac{1+q_j}{2}$ and $p_i(b|v_i = 0) = 0$.

To complete the proof, recall that it is efficient for the buyer to make a joint purchase with certainty. Thus, the maximum social surplus is $SS = 1$. Let $q_i = q$. Then for $q \leq \frac{1}{2}$, it is easily verified that $SS_{ij} = SS_{ji} = 1$ since by Lemma 1 and 2, $p_i^*(s_i) \leq \frac{1}{2}$, $p_j^*(s_j|p_i^*(\sigma_i)) \leq \frac{1}{2}$. Similarly, for $q = 1$, $\alpha_i \leq \frac{1+q^2}{1+q}$ for all i and equation (A-3) reveals that $SS_{ij} = SS_{ji} = 1$. Next, consider $q \in (\frac{1}{2}, 1)$. Then, if $\alpha_j \leq \frac{1+q^2}{1+q}$, $p_j^*(s_j|0) = 1$ and thus $SS_{ij} < 1$. If, instead, $\alpha_j > \frac{1+q^2}{1+q}$, $\gamma_i^*(0|0) < 1$ implying that $SS_{ij} < 1$. As for the buyer, if $q \leq \frac{1}{2}$, (A-5) reveals that $\pi_{12}(b) = \pi_{21}(b) = (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)\frac{1}{2} + \alpha_j(1-\alpha_i)\frac{1}{2}$. In addition, if $q = 1$, then $\pi_{12}(b) = \pi_{21}(b) = (1-\alpha_i)(1-\alpha_j)$. Therefore, the buyer is indifferent to the sequence if $q \notin (\frac{1}{2}, 1)$. ■

Proof of Proposition 2. Let $q_1 = q_2 = q \in (\frac{1}{2}, 1)$, and $\alpha_1 < \alpha_2$. We prove both parts together. Suppose that $\alpha_1 < \alpha_2 < \widehat{\alpha}(q)$. Then, from the equilibrium prices from Lemma 1 and 2, and eq.(A-5), it follows that $\pi_{12}(b) - \pi_{21}(b) = 0$. Moreover, from equation (A-3), $SS_{12} - SS_{21} = 0$.

Next, suppose that $\alpha_1 < \widehat{\alpha}(q) \leq \alpha_2$. Then, if $\alpha_2 \leq \frac{1+q^2}{1+q}$, Lemma 2 and (A-5) yield $\pi_{12}(b) - \pi_{21}(b) = \alpha_1(1-\alpha_2)q(1-p_1^*(s_1)) + \alpha_1\alpha_2(1-q)[p_1^*(s_1) - \frac{1}{2}] > 0$ since $p_1^*(s_1) \geq \frac{1}{2}$. Therefore, the buyer is strictly better off approaching s_1 first. Moreover, from (A-3), $SS_{12} - SS_{21} = \alpha_1(1-\alpha_2)\frac{1}{2}\mathbf{1}(p_1^*(s_1) \leq \frac{1}{2}) + \alpha_1\alpha_2[q + (1-q)\frac{1}{2}\mathbf{1}(p_1^*(s_1) \leq \frac{1}{2})] > 0$. If, instead, $\alpha_2 > \frac{1+q^2}{1+q}$, then $\pi_{12}(b) - \pi_{21}(b) = (1-\alpha_1)\frac{q}{2}[\alpha_2(1+q) - (1+q^2)] + \alpha_1(1-\alpha_2)q(1-p_1^*(s_1)) + \alpha_1\alpha_2(1-q)\left[\frac{1}{2} - p_2^*(s_2|s_1)\right] > 0$. Simplifying SS_{12} given by equation (A-4) yields,

$$\begin{aligned} SS_{12} &= (1-\alpha_1)(1-\alpha_2) + \alpha_1(1-\alpha_2) \times \begin{cases} 1, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ \frac{1+q}{2}, & if \quad q > \frac{\sqrt{5}-1}{2} \end{cases} + \alpha_2(1-\alpha_1)\frac{1+q}{2} \qquad \text{(A-7)} \\ &\quad + \alpha_1\alpha_2 \times \begin{cases} 1, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ \frac{1+q}{2}, & if \quad q > \frac{\sqrt{5}-1}{2} \end{cases} + (1-\alpha_1)\frac{1-q}{2}(\alpha_2(1+q) - (2q+q^2)), \end{aligned}$$

whose last term captures the additional surplus generated by the buyer's mixing, which can also be expressed as $(1-\alpha_1)\alpha_2(1-q)\theta_2^*\frac{1}{2} - (1-\alpha_1)q(1-q)[1-\gamma^*(0|0)]\frac{1}{2}$. Note that the first term of this expression represents the gain in the surplus due to s_2's posting a lower price of $\frac{1}{2}$with probability θ_2^*, while the second term represents the loss in the surplus due to the higher likelihood of the buyer not purchasing good 1. Simple algebra shows that $SS_{12} - SS_{21} =$

$\alpha_1\alpha_2 q + (1-\alpha_1)\frac{1-q}{2}(\alpha_2(1+q)-(2q+q^2)) + \alpha_1 \times \begin{cases} \frac{1-q}{2}, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ 0, & if \quad q > \frac{\sqrt{5}-1}{2} \end{cases}$. If $\alpha_2 \geq \frac{2q+q^2}{1+q}$, then clearly $SS_{12} - SS_{21} > 0$. Otherwise, if $\alpha_2 < \frac{2q+q^2}{1+q}$, then $SS_{12} - SS_{21} > 0$ if and only if $\alpha_1 > \widetilde{\alpha}(\alpha_2, q)$, where

$$\widetilde{\alpha}(\alpha_2, q) = \begin{cases} \frac{(1-q)[(2q-q^2)-\alpha_2(1+q)]}{(1-q)[(2q-q^2)-\alpha_2(1+q)+1]+2\alpha_2 q}, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ \frac{(1-q)[(2q-q^2)-\alpha_2(1+q)]}{(1-q)[(2q-q^2)-\alpha_2(1+q)]+2\alpha_2 q}, & if \quad q > \frac{\sqrt{5}-1}{2}. \end{cases}$$

To see that $\widetilde{\alpha}(\alpha_2, q) < \widehat{\alpha}(q)$, note that $\widetilde{\alpha}(\alpha_2, q)$ is decreasing in α_2. Moreover, it is straightforward to verify that $\widetilde{\alpha}(\frac{1+q^2}{1+q}, q) = \begin{cases} \frac{(2q-1)(1-q^2)}{q^2+4q-1}, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ \frac{(2q-1)(1-q^2)}{4q}, & if \quad q > \frac{\sqrt{5}-1}{2}, \end{cases} < \widehat{\alpha}(q)$, which implies that $\widetilde{\alpha}(\alpha_2, q) < \widehat{\alpha}(q)$ for all $\alpha_2 \in (\frac{1+q^2}{1+q}, \frac{2q+q^2}{1+q})$.

Finally, suppose that $\widehat{\alpha}(q) \leq \alpha_1 < \alpha_2$. Then, by Lemma 2, $p_1^*(s_1) = p_2^*(s_2)$ and $P_1(s_1|p_2^*(s_2)) = P_2(s_2|p_1^*(s_1))$. Since $\widehat{\alpha}(q) < \frac{1+q^2}{1+q}$ for all q, we consider three cases:

$\underline{\widehat{\alpha}(q) \leq \alpha_1 < \alpha_2 \leq \frac{1+q^2}{1+q}}$: Using (A-5), we find that $\pi_{12}(b) - \pi_{21}(b) = (\alpha_1 - \alpha_2)q\,[1 - p_1^*(s_1)] < 0$. Thus, the buyer will choose to negotiate with s_2 first. Moreover, from (A-3), $SS_{12} - SS_{21} = (\alpha_1 - \alpha_2)\frac{1}{2}\mathbf{1}(p_1^*(s_1) \leq \frac{1}{2}) \leq 0$.

$\underline{\widehat{\alpha}(q) \leq \alpha_1 \leq \frac{1+q^2}{1+q} < \alpha_2}$: From (A-6) and (A-5) respectively,

$$\begin{aligned} \pi_{12}(b) &= (1-\alpha_1)(1-\alpha_2)[1-q+q\frac{1-q^2}{2}] + \alpha_1(1-\alpha_2)[q(1-p_1^*(s_1)) + \frac{1-q}{2}] \\ &\quad + \alpha_2(1-\alpha_1)\frac{1-q}{2}(1+q^2) + \alpha_1\alpha_2(1-q)(\frac{1}{2} - p_2^*(s_2|s_1)) \end{aligned}$$

and

$$\begin{aligned} \pi_{21}(b) &= (1-\alpha_1)(1-\alpha_2) + \alpha_2(1-\alpha_1)[q(1-p_1^*(s_1)) + \frac{1-q}{2}] \\ &\quad + \alpha_1(1-\alpha_2)\frac{1-q}{2} + \alpha_1\alpha_2(\frac{1}{2} - p_2^*(s_2|s_1)) \end{aligned}$$

Hence,

$$\pi_{12}(b) - \pi_{21}(b) = (1-\alpha_1)\frac{q}{2}[1+q^2-\alpha_2(1+q)] + (\alpha_2-\alpha_1)q(1-p_1^*(s_1)).$$

While the first term is negative because $\alpha_2 > \frac{1+q^2}{1+q}$, the second term is positive. Note that $\pi_{12}(b) - \pi_{21}(b) \geq (1-\alpha_1)\frac{q}{2}[1+q^2-\alpha_2(1+q)] + (\alpha_2-\alpha_1)q\frac{1-q}{2} = \Omega(\alpha_2)$ since $p_1^*(s_1) \leq \frac{1+q}{2}$. Therefore, it is sufficient to show that $\Omega(\alpha_2) > 0$ for all $\alpha_2 \geq \frac{1+q^2}{1+q}$. This is indeed true for $\alpha_2 = \frac{1+q^2}{1+q}$. Moreover, $\frac{d\Omega}{d\alpha_2} = \frac{q}{2}(1+q)[\alpha_1 - \frac{2q}{1+q}]$. If $\alpha_1 \geq \frac{2q}{1+q}$, then $\Omega(\alpha_2) > 0$ follows. If

$\alpha_1 < \frac{2q}{1+q}$, $\Omega(\alpha_2)$ is continuous and decreasing in α_2. Since $\Omega(1) = (1-\alpha_1)q\frac{(1-q)^2}{2} > 0$, this implies that $\Omega(\alpha_2) > 0$ for all $\alpha_2 \geq \frac{1+q^2}{1+q}$.

In terms of the social surplus, SS_{21} given by equation (A-3) and S_{12} given by equation (A-7) results in $SS_{21} - SS_{12} = -(1-\alpha_1)\frac{1-q}{2}(\alpha_2(1+q) - (2q+q^2)) + (\alpha_2 - \alpha_1) \times \begin{cases} \frac{1-q}{2}, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ 0, & if \quad q > \frac{\sqrt{5}-1}{2} \end{cases}$. For $q > \frac{\sqrt{5}-1}{2}$, we have $\alpha_2 \leq 1 < \frac{2q+q^2}{1+q}$, which implies that $S_{21} - S_{12} > 0$. For $q \leq \frac{\sqrt{5}-1}{2}$, $SS_{21} - SS_{12} > 0$ for $\alpha_2 \leq \frac{2q+q^2}{1+q}$. If $\alpha_2 > \frac{2q+q^2}{1+q}$, then note that $\frac{\partial(S_{21}-S_{12})}{\partial\alpha_2} = \frac{1-q}{2} - (1-\alpha_1)\frac{1-q^2}{2}$. For $\alpha_1 \geq \frac{q}{1+q}$, it follows that $\frac{d(S_{21}-S_{12})}{d\alpha_2} > 0$. Since $SS_{21} - SS_{12} \geq 0$ for $\alpha_2 = \frac{2q+q^2}{1+q}$, this implies that $SS_{21} - SS_{12} > 0$ for all $\alpha_2 > \frac{2q+q^2}{1+q}$. For $\alpha_1 < \frac{q}{1+q}$, it follows that $\frac{\partial(S_{21}-S_{12})}{\partial\alpha_2} < 0$. Then, $SS_{21} - SS_{12}$ reaches a minimum at $\alpha_2 = 1$. Note that $S_{21}(\alpha_2 = 1) - SS_{12}(\alpha_2 = 1) = (1-\alpha_1)q\frac{1-q^2}{2} > 0$, which implies that $S_{21} - SS_{12} > 0$ for all $\alpha_2 > \frac{2q+q^2}{1+q}$.

$\underline{\frac{1+q^2}{1+q} < \alpha_1 < \alpha_2}$: Then, from (A-6), $\pi_{21}(b) - \pi_{12}(b) = (\alpha_2 - \alpha_1)[q(1-p_1^*(s_1)) - q^2\frac{1-q}{2}] \geq (\alpha_2 - \alpha_1)q\frac{(1-q)^2}{2} > 0$ since $p_1^*(s_1) \leq \frac{1+q}{2}$. Moreover, by equation (A-7) $SS_{21} - SS_{12} = (\alpha_2 - \alpha_1)q\frac{1-q^2}{2} - (\alpha_2 - \alpha_1) \times \begin{cases} 0, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ \frac{1-q}{2}, & if \quad q > \frac{\sqrt{5}-1}{2} \end{cases} > 0$. ■

Proof of Proposition 3. Suppose that $\alpha_1 = \alpha_2 = \alpha$, and that $q_1 \notin (\frac{1}{2}, 1)$ and $q_2 \in (\frac{1}{2}, 1)$. From Lemma 2, $\widehat{\alpha}(q_1) = 0$ and $\widehat{\alpha}(q_2) > 0$. If $\alpha < \widehat{\alpha}(q_2)$, then, since $\widehat{\alpha}(q_2) < \frac{1+q_2^2}{1+q_2}$, eq.(A-5) implies that $\pi_{12}(b) - \pi_{21}(b) = 0$. If $\alpha > \widehat{\alpha}(q_2)$, we consider the following parameter regions.

$\underline{q_1 \leq \frac{1}{2}}$: Then, by (A-5), we have

$$\begin{aligned} \pi_{12}(b) &= (1-\alpha)^2 + \alpha(1-\alpha) \times \begin{cases} \frac{1}{2}, & if \quad q_2 \leq \frac{\sqrt{5}-1}{2} \\ \frac{1-q_2^2}{2}, & if \quad q_2 > \frac{\sqrt{5}-1}{2} \end{cases} + (1-\alpha)\alpha\frac{1}{2} \\ &+ \alpha^2 \times \begin{cases} 0, & if \quad q_2 \leq \frac{\sqrt{5}-1}{2} \\ \frac{(1-q_2)q_2}{2}, & if \quad q_2 > \frac{\sqrt{5}-1}{2} \end{cases}. \end{aligned}$$

If $\alpha \leq \frac{1+q_1^2}{1+q_1}$, then $\pi_{21}(b) = (1-\alpha)^2 + \alpha(1-\alpha)\frac{1}{2} + \alpha(1-\alpha)\frac{1-q_2}{2}$, and clearly, $\pi_{12}(b) > \pi_{21}(b)$. If, however, $\alpha > \frac{1+q_1^2}{1+q_1}$, then by (A-6) $\pi_{21}(b) = (1-\alpha)^2(1 - q_2 + q_2\frac{1-q_1^2}{2}) + \alpha(1-\alpha)\frac{1}{2} + \alpha(1-\alpha)(\frac{1-q_2}{2} + q_2(1-q_1)\frac{q_1}{2})$. For $q_2 \leq \frac{\sqrt{5}-1}{2}$, $\pi_{12}(b) - \pi_{21}(b) = (1-\alpha)\frac{q_2}{2}[(1-\alpha)(1+q_1^2) + \alpha(1 - q_1(1-q_1))] > 0$. For $q_2 > \frac{\sqrt{5}-1}{2}$, $\pi_{12}(b) - \pi_{21}(b) = \frac{q_2}{2}\{(1-\alpha)[1 + q_1^2 - \alpha(1+q_1)] + \alpha(1-q_2)\}$. Note that at $\alpha = \frac{1+q_1^2}{1+q_1}$, this difference is positive. Moreover, $\frac{\partial(\pi_{12}(b)-\pi_{21}(b))}{\partial\alpha} = -(1 + q_1^2 + q_1 + q_2) + 2\alpha(1+q_1)$, which is positive for $\alpha > \frac{1+q_1+q_1^2+q_2}{2(1+q_1)} = \tilde{\alpha}$. If $q_2 \leq 1 - q_1(1-q_1)$, then $\tilde{\alpha} < \frac{1+q_1^2}{1+q_1}$, implying that $\pi_{12}(b) - \pi_{21}(b) > 0$ for all $\alpha > \frac{1+q_1^2}{1+q_1}$. If $q_2 > 1 - q_1(1-q_1)$, then $\pi_{12}(b) - \pi_{21}(b)$ reaches its unique minimum at $\alpha = \tilde{\alpha}$ in $(\frac{1+q_1^2}{1+q_1}, 1]$. Simple algebra shows that

$\pi_{12}(b|\tilde{\alpha}) - \pi_{21}(b|\tilde{\alpha}) = \frac{q_2}{8(1+q_1)}([(1-q_2) - q_1(1-q_1)]^2 + 2(1-q_2^2)) > 0$, which implies that $\pi_{12}(b) - \pi_{21}(b) > 0$ for all $\alpha \in (\frac{1+q_1^2}{1+q_1}, 1]$.

$\underline{q_1 = 1}$: Then, by eq.(A-5), $\pi_{21}(b) = (1-\alpha)^2 + \alpha(1-\alpha)\frac{1-q_2}{2}$. If $\alpha \leq \frac{1+q_2^2}{1+q_2}$, we also have

$$\pi_{12}(b) = (1-\alpha)^2 + \alpha(1-\alpha) \times \begin{cases} \frac{1}{2}, & if \quad q_2 \leq \frac{\sqrt{5}-1}{2} \\ \frac{1-q_2^2}{2}, & if \quad q_2 > \frac{\sqrt{5}-1}{2} \end{cases} + \alpha^2 \times \begin{cases} 0, & if \quad q_2 \leq \frac{\sqrt{5}-1}{2} \\ \frac{(1-q_2)q_2}{2}, & if \quad q_2 > \frac{\sqrt{5}-1}{2}. \end{cases}$$

Clearly, $\pi_{12}(b) > \pi_{21}(b)$.

If, on the other hand, $\alpha > \frac{1+q_2^2}{1+q_2}$, then

$$\begin{aligned} \pi_{12}(b) &= (1-\alpha)^2\frac{1-q_2^2}{2} + \alpha(1-\alpha) \times \begin{cases} \frac{1}{2}, & if \quad q_2 \leq \frac{\sqrt{5}-1}{2} \\ \frac{1-q_2^2}{2}, & if \quad q_2 > \frac{\sqrt{5}-1}{2} \end{cases} \\ &\quad + \alpha(1-\alpha)\frac{q_2(1-q_2)}{2} + \alpha^2 \times \begin{cases} 0 & if \quad q_2 \leq \frac{\sqrt{5}-1}{2} \\ \frac{(1-q_2)q_2}{2} & if \quad q_2 > \frac{\sqrt{5}-1}{2} \end{cases}. \end{aligned}$$

For $q_2 \leq \frac{\sqrt{5}-1}{2}$, we have $\pi_{12}(b) - \pi_{21}(b) = \frac{1-\alpha}{2}[-(1+q_2^2) + \alpha(1+2q_2)] > 0$, and for $q_2 > \frac{\sqrt{5}-1}{2}$, we have $\pi_{12}(b) - \pi_{21}(b) = (1-\alpha)\frac{\alpha(1+q_2)-(1+q_2^2)}{2} + \alpha\frac{(1-q_2)q_2}{2} > 0$. ■

Proof of Proposition 4. To prove part (a), suppose that $q_1 = q_2 = q \in (\frac{1}{2}, 1)$ and that $\alpha_2^L = \widehat{\alpha}(q) - \Delta$, and $\alpha_2^H = \widehat{\alpha}(q) + \Delta$. We distinguish three cases for α_1:

$\underline{\alpha_1 < \widehat{\alpha}(q)}$: From Proposition 2, the buyer is indifferent to the sequence under $\alpha^L = (\alpha_1, \alpha_2^L)$, and using Lemma 1 and 2, the buyer's payoff is found to be:

$$\pi_{12}(b|\alpha^L) = \pi_{21}(b|\alpha^L) = (1-\alpha_1)(1-\alpha_2^L) + \alpha_1(1-\alpha_2^L)(1-q)\frac{1}{2} + (1-\alpha_1)\alpha_2^L(1-q)\frac{1}{2}.$$

Under $\alpha^H = (\alpha_1, \alpha_2^H)$, Proposition 2 implies that the buyer optimally visits s_1 first. From (A-5) and (A-6), it is straightforward to verify that $\pi_{12}(b, \alpha^H)$ is continuous and decreasing in α_2^H. Let us conjecture that $\widehat{\alpha}(q) + \overline{\Delta} < \frac{1+q^2}{1+q}$. Then, we have the following expected payoff:

$$\begin{aligned} \pi_{12}(b|\alpha^H) &= (1-\alpha_1)(1-\alpha_2^H) + \alpha_1(1-\alpha_2^H) \begin{cases} \frac{1}{2}, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ \frac{1-q^2}{2}, & if \quad q > \frac{\sqrt{5}-1}{2} \end{cases} \\ &\quad + (1-\alpha_1)\alpha_2^H\frac{1-q}{2} + \alpha_1\alpha_2^H \begin{cases} 0, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ q\frac{1-q}{2}, & if \quad q > \frac{\sqrt{5}-1}{2} \end{cases} \end{aligned}$$

Simple algebra shows that $\pi_{12}(b|\alpha^L) = \pi_{21}(b|\alpha^L) < \pi_{12}(b|\alpha^H)$ for any $\Delta \in (0, \overline{\Delta})$, where

$$\overline{\Delta} = \begin{cases} \frac{\alpha_1}{4+2q(2-3\alpha_1)}, & if \quad q \in (\frac{1}{2}, \frac{\sqrt{5}-1}{2}] \\ \frac{\alpha_1 q(1-q)}{2+2q(1-2\alpha_1)}, & if \quad q \in (\frac{\sqrt{5}-1}{2}, 1). \end{cases}$$

Clearly, $\overline{\Delta} > 0$. Moreover, it is readily verified that $\widehat{\alpha}(q) - \overline{\Delta} > 0$ and $\widehat{\alpha}(q) + \overline{\Delta} < \frac{1+q^2}{1+q}$, which confirms our initial conjecture.

$\underline{\alpha_1 \in (\widehat{\alpha}(q), \frac{1+q^2}{1+q}]}$: Under α^L, the buyer visits s_2 first. Suppose that $\alpha_2^H < \alpha_1$, in which case the buyer optimally visits s_1 first. Then, $q \in (\frac{1}{2}, \frac{\sqrt{5}-1}{2}]$ implies that

$$\pi_{21}(b|\alpha^L) = (1-\alpha_1)(1-\alpha_2^L) + \alpha_1(1-\alpha_2^L)(1-q)\frac{1}{2} + (1-\alpha_1)\alpha_2^L\frac{1}{2},$$

and

$$\pi_{12}(b|\alpha^H) = (1-\alpha_1)(1-\alpha_2^H) + \alpha_1(1-\alpha_2^H)\frac{1}{2} + (1-\alpha_1)\alpha_2^H(1-q)\frac{1}{2}.$$

Comparing the two payoffs, it follows that $\pi_{21}(b|\alpha^L) < \pi_{12}(b|\alpha^H)$ for any $\Delta \in (0, \overline{\Delta})$, where $\overline{\Delta} = \frac{q(\alpha_1 - \hat{\alpha}(q))}{2+q(1-2\alpha_1)} > 0$. It is easily shown that $\hat{\alpha}(q) - \overline{\Delta} > 0$ and $\hat{\alpha}(q) + \overline{\Delta} < \alpha_1$ for $\alpha_1 > \widehat{\alpha}(q)$.

Next, $q \in (\frac{\sqrt{5}-1}{2}, 1)$ implies that

$$\pi_{21}(b|\alpha^L) = (1-\alpha_1)(1-\alpha_2^L) + \alpha_1(1-\alpha_2^L)(1-q)\frac{1}{2} + (1-\alpha_1)\alpha_2^L\frac{1-q^2}{2} + \alpha_1\alpha_2^L(1-q)q\frac{1}{2},$$

and

$$\pi_{12}(b|\alpha^H) = (1-\alpha_1)(1-\alpha_2^H) + \alpha_1(1-\alpha_2^H)(1-q^2)\frac{1}{2} + (1-\alpha_1)\alpha_2^H(1-q)\frac{1}{2} + \alpha_1\alpha_2^H(1-q)q\frac{1}{2}.$$

Comparing the two payoffs, it follows that $\pi_{21}(b|\alpha^L) < \pi_{12}(b|\alpha^H)$ for any $\Delta \in (0, \overline{\Delta})$, where $\overline{\Delta} = \frac{q(1-q)(\alpha_1 - \hat{\alpha}(q))}{2+q^2+q(1-4\alpha_1)} > 0$, satisfying $\hat{\alpha}(q) - \overline{\Delta} > 0$ and $\hat{\alpha}(q) + \overline{\Delta} < \alpha_1$ for $\alpha_1 > \widehat{\alpha}(q)$.

$\underline{\alpha_1 > \frac{1+q^2}{1+q}}$: Under α^L, the buyer visits s_2 first. Suppose that $\alpha_2^H < \alpha_1$, in which case the buyer visits s_1 first. As in the previous cases, let us conjecture that $\widehat{\alpha}(q) + \overline{\Delta} \leq \frac{1+q^2}{1+q}$. Then, for $q \in (\frac{1}{2}, \frac{\sqrt{5}-1}{2}]$,

$$\pi_{21}(b|\alpha^L) = (1-\alpha_1)(1-\alpha_2^L)\frac{1-q}{2}\left[2+q(1+q)\right] + \alpha_1(1-\alpha_2^L)\frac{1-q}{2}(1+q^2) + (1-\alpha_1)\alpha_2^L\frac{1}{2},$$

and

$$\pi_{12}(b|\alpha^H) = (1-\alpha_1)(1-\alpha_2^H) + \alpha_1(1-\alpha_2^H)\frac{1}{2} + (1-\alpha_1)\alpha_2^H(1-q)\frac{1}{2}.$$

Comparing the two payoffs, it follows that $\pi_{21}(b|\alpha^L) < \pi_{12}(b|\alpha^H)$ for any $\Delta < \overline{\Delta}$, where $\overline{\Delta} = \frac{q+q^3-\widehat{\alpha}(q)(2q+q^3)-q\alpha_1(q-\widehat{\alpha}(q)(1+q))}{2-q\alpha_1(1-q)-q^3}$. Note that at $\frac{\partial\overline{\Delta}}{\partial\alpha_1} = -\frac{q}{(2-q\alpha_1(1-q)-q^3)^2}[(1+q(1-q))q - (1+q^2(1-q))2\widehat{\alpha}(q)] < 0$ since $\widehat{\alpha}(q) = 1 - \frac{1}{2q} < \frac{q}{2}$. Therefore, since $\overline{\Delta}(\alpha_1 = 1) = q\,(1-\alpha)\,\frac{1-q+q^2}{2-q+q^2-q^3} > 0$, $\overline{\Delta}(\alpha_1) > 0$ for all $\alpha_1 > \frac{1+q^2}{1+q}$. It is readily verified that $\hat{\alpha}(q) + \overline{\Delta} \leq \frac{1+q^2}{1+q}$.

For $q \in (\frac{\sqrt{5}-1}{2}, 1)$,

$$\begin{aligned}\pi_{21}(b|\alpha^L) &= (1-\alpha_1)(1-\alpha_2^L)\frac{1-q}{2}\left[2+q(1+q)\right]+\alpha_1(1-\alpha_2^L)\frac{1-q}{2}(1+q^2)+\\ &\quad +(1-\alpha_1)\alpha_2^L\frac{1-q^2}{2}+\alpha_1\alpha_2^L(1-q)q\frac{1}{2},\end{aligned}$$

and

$$\pi_{12}(b|\alpha^H) = (1-\alpha_1)(1-\alpha_2^H)+\alpha_1(1-\alpha_2^H)(1-q^2)\frac{1}{2}+(1-\alpha_1)\alpha_2^H(1-q)\frac{1}{2}+\alpha_1\alpha_2^H(1-q)q\frac{1}{2}.$$

Comparing the two payoffs, it follows that $\pi_{21}(b|\alpha^L) < \pi_{12}(b|\alpha^H)$ for any $\Delta \in (0, \overline{\Delta})$, where $\overline{\Delta} = \frac{q\left(1+q^2-\widehat{\alpha}(q)(2-q+q^2)-\alpha_1(2q-\widehat{\alpha}(q)(1+q)\right)}{2+q^2-q^3-\alpha_1 q(3-q)}$. Note that $\frac{\partial\overline{\Delta}}{\partial\alpha_1} = -\frac{q(1-q)[q(1+2q+q^2)-2\widehat{\alpha}(q)(1-q+2q^2)]}{(2+q^2-q^3-\alpha_1 q(3-q))^2} < 0$ since $\widehat{\alpha}(q) = \frac{1-q}{2} < \frac{q}{2}$. Therefore, since $\overline{\Delta}(\alpha_1 = 1) = q\,(1-\alpha)\,\frac{1-q}{2-q+q^2} > 0$, $\overline{\Delta}(\alpha_1) > 0$ for all $\alpha_1 > \frac{1+q^2}{1+q}$. It is readily verified that $\hat{\alpha}(q) + \overline{\Delta} \leq \frac{1+q^2}{1+q}$, completing the proof of part (a).

To prove part (b), note that whenever $q \notin (\frac{1}{2}, 1)$ or $\alpha_2^L < \alpha_2^H < \widehat{\alpha}(q)$, Proposition 1 and 2 imply that the buyer weakly prefers negotiating with s_2 first. For $q \leq \frac{1}{2}$ or $\alpha_1 \leq \frac{1+q^2}{1+q}$, her payoff is given by (A-5) with $i = 2$ and $j = 1$. Then, $\frac{\pi_{21}(b)}{d\alpha_2} = -(1-\alpha_1)[\frac{1}{2} + q(p_2^*(s_2) - \frac{1}{2})] - \alpha_1[q(1-p_1^*(s_1|0)) + p_1^*(s_1|p_2^*(s_2))] < 0$ since by Lemma 1 $p_1^*(s_1|0) \leq 1$ and by Lemma 2 $p_2^*(s_2) \geq \frac{1}{2}$. For $q > \frac{1}{2}$ and $\alpha_1 > \frac{1+q^2}{1+q}$, the buyer's payoff is given by (A-6) with $i = 2$ and $j = 1$. Then, $\frac{\pi_{21}(b)}{d\alpha_2} = -(1-\alpha_1)[\frac{1-q-q^3}{2} + q(p_2^*(s_2) - \frac{1}{2})] - \alpha_1[q^2\frac{1-q}{2} + p_1^*(s_1|p_2^*(s_2))] < 0$ since for $q \in (\frac{1}{2}, \frac{\sqrt{5}-1}{2}]$, $\frac{1-q-q^3}{2} > 0$ and for $q > \frac{\sqrt{5}-1}{2}$, by Lemma 2 $p_2^*(s_2) = \frac{1+q}{2}$ resulting in $\frac{1-q-q^3}{2} + q(p_2^*(s_2) - \frac{1}{2}) = \frac{(1-q)(1+q^2)}{2}$. Therefore, the buyer is strictly worse off under α_2^H compared to α_2^L whenever $q \notin (\frac{1}{2}, 1)$ or $\alpha_2^L < \alpha_2^H < \widehat{\alpha}(q)$. ■

Proof of Proposition 5. Suppose $q_1 = q_2 = q$, $\alpha_1 = \alpha_2 = \alpha$, and that prior to negotiations, the buyer commits to paying at least $w \geq 0$ for each unit she purchases. Without loss of generality, let the negotiation sequence be $s_i \to s_j$. To prove part (a), assume $q \in (\frac{1}{2}, 1)$ and restrict attention to $w \in [0, \frac{1-q}{2})$. Clearly, in the second negotiation, the buyer will optimally offer $p_j(b) = w$. Given (A-2), s_i will choose $p_i(s_i) \in \{\frac{1}{2}, \frac{1+q}{2}\}$ if he anticipates s_j to make the offer in the second negotiation. If, instead, s_i anticipates the buyer to make the offer, then he may realize a sale by setting $p_i(s_i) = 1 - w$. Thus, s_i has three price candidates: $p_i(s_i) \in \{\frac{1}{2}, \frac{1+q}{2}, 1-w\}$. The price of $\frac{1}{2}$ is accepted with certainty. If $w < \frac{1-q}{2}$, then the price $\frac{1+q}{2}$ is accepted with probability q, while $1-w$ is accepted with probability $(1-\alpha)q$. As a result, equilibrium prices are

$$(p_i^*(s_i), p_j^*(s_j|\cdot)) = \begin{cases} (1-w, \frac{1}{2}), & \textit{if} \quad \alpha < \hat{\alpha}(q,w) \\ (\frac{1+q}{2}, \frac{1-q}{2}), & \textit{if} \quad \alpha \geq \hat{\alpha}(q,w) \textit{ and } q > \frac{\sqrt{5}-1}{2} \\ (\frac{1}{2}, \frac{1}{2}), & \textit{if} \quad \alpha \geq \hat{\alpha}(q,w) \textit{ and } q \leq \frac{\sqrt{5}-1}{2}, \end{cases}$$

where $\hat{\alpha}(q,w) = \begin{cases} 0, & \textit{if} \quad q \leq \frac{1}{2(1-w)} \\ 1 - \frac{1}{2q(1-w)}, & \textit{if} \quad \frac{1}{2(1-w)} < q \leq \frac{\sqrt{5}-1}{2} \\ 1 - \frac{1+q}{2(1-w)}, & \textit{if} \quad q > \frac{\sqrt{5}-1}{2} \end{cases}$.

Note that $\hat{\alpha}(q,0)$ is equivalent to (4). Note also that $w < \frac{1-q}{2}$ and $q \in (\frac{1}{2}, 1)$ imply that $\frac{1}{2(1-w)} < \frac{\sqrt{5}-1}{2}$. Consider now the buyer's pricing in the first negotiation. Similar to the proof of Lemma 1, the buyer's optimal price is $p_i(b|v_i = \frac{1}{2}) = w$ and $p_i(b|v_i = 0) \in \{w, \frac{1+q}{2}\}$. Observe that

$$\pi(b|v_i = 0, p_i = w) = (1-\alpha)(1-2w) + \alpha \times \begin{cases} \frac{1}{2} - w & \text{if} \quad \bar{q}(w) \leq \frac{1}{2(1-w)} \\ 0 & \text{if} \quad \bar{q}(w) > \frac{1}{2(1-w)}, \end{cases}$$

where $\bar{q}(w)$ is s_j's belief that $v_i = 0$ given w, and that

$$\pi(b|v_i = 0, p_i = \frac{1+q}{2}) = (1-\alpha)\left[(1-q)(\frac{1}{2} - w) + q(\frac{1-q}{2} - w)\right] + \alpha(1-q)\frac{q}{2}.$$

Comparing these two payoffs, the equilibrium price by the low valuation buyer is as follows:

- If $w < \min\{1 - \frac{1}{2q}, \frac{1-q}{2}\}$, then $p_i^*(b|v_i = 0) = w$ for $\alpha \leq \frac{1+q^2-2w}{1+q-2w}$; and $p_i^*(b) = \begin{cases} w & \text{with prob.} \quad \gamma(w|0) \\ \frac{1+q}{2} & \text{with prob.} \quad 1-\gamma(w|0) \end{cases}$ for $\alpha > \frac{1+q^2-2w}{1+q-2w}$, where $\gamma(w|0) = \frac{1-q}{q(1-2w)}$.
- If $w \in [1 - \frac{1}{2q}, \frac{1-q}{2})$, then $p_i^*(b|v_i = 0) = w$.

Note that $\frac{1+q^2-2w}{1+q-2w} \geq q$ and $\widehat{\alpha}(q,0) \leq q$ for all q. Therefore, $\alpha < \widehat{\alpha}(q,0)$ implies that $p_i^*(b|v_i = 0) = w$. For $\alpha < \widehat{\alpha}(q,0)$, the equilibrium will be one of noncoordination under $w = 0$ with buyer's payoff given by

$$\pi(b|w=0) = (1-\alpha)^2 + \alpha(1-\alpha)(1-q)\frac{1}{2} + \alpha(1-\alpha)(1-q)\frac{1}{2} = (1-\alpha)(1-q\alpha).$$

Let $\underline{w}(\alpha, q)$ denote the minimum price that induces coordination in equilibrium. We want to show that there exists $\alpha^c(q) \in (0, \widehat{\alpha}(q))$ such that $\pi(b|w = \underline{w}(\alpha,q)) > \pi(b|w = 0)$ for all $\alpha \in [\alpha^c(q), \widehat{\alpha}(q))$. There are two regions to consider:

- $q \in (\frac{1}{2}, \frac{\sqrt{5}-1}{2}]$: Then, $\underline{w}(\alpha, q) = 1 - \frac{1}{2q(1-\alpha)} < \frac{1-q}{2}$ since $1 - \frac{1}{2q(1-\alpha)} < 1 - \frac{1}{2q} \leq \frac{1-q}{2}$ for $q < \frac{\sqrt{5}-1}{2}$. In this case, $(p_i^*(s_i), p_j^*(s_j|p_i^*(s_i))) = (\frac{1}{2}, \frac{1}{2})$ and $p_j^*(s_j|0) = 1 - w$. Thus, the buyer's payoff is

$$\begin{aligned}\pi[b|w \geq \underline{w}(\alpha,q)] &= (1-\alpha)^2(1-2w) + \alpha(1-\alpha)(\frac{1}{2} - w) + \alpha(1-\alpha)(1-q)(\frac{1}{2} - w) \\ &= \frac{(1-\alpha)(2-q\alpha)(1-2w)}{2} .\end{aligned}$$

  Comparing the two payoffs, it follows that $\pi[b|w \geq \underline{w}(\alpha,q)] > \pi(b|w = 0)$ for $w < \frac{\alpha q}{2(2-q\alpha)}$. Note that $\frac{\alpha q}{2(2-q\alpha)}$ is increasing in α and $\underline{w}(\alpha,q)$ is decreasing in α. Moreover, $\underline{w}(\hat{\alpha}(q,0),q) = 0$. Therefore, there exists $\alpha^c(q)$ such that for $\alpha \in (\alpha^c(q), \hat{\alpha}(q,0))$, $\underline{w}(\alpha,q) \leq \frac{\alpha q}{2(2-q\alpha)}$ and $\pi[b|w = \underline{w}(\alpha,q)] > \pi(b|w = 0)$.

- $q \in (\frac{\sqrt{5}-1}{2}, 1)$: Then, $\underline{w}(\alpha,q) = 1 - \frac{1+q}{2(1-\alpha)} < \frac{(1-q)}{2}$. Furthermore, $(p_i^*(s_i), p_j^*(s_j|p_i^*(s_i))) = (\frac{1+q}{2}, \frac{1-q}{2})$ and $p_j^*(s_j|0) = 1 - w$. Then,

$$\begin{aligned}\pi(b|w \geq \underline{w}(\alpha,q)) &= (1-\alpha)^2(1-2w) + \alpha(1-\alpha)\left[\frac{(1-q)^2}{2} - w\right] \\ &\quad + \alpha(1-\alpha)(1-q)(\frac{1}{2} - w) + \alpha^2 q(1-q)\frac{1}{2} .\end{aligned}$$

Trivial algebra reveals that $\pi(b|w \geq \underline{w}(\alpha,q)) > \pi(b|w = 0)$ for $w < \frac{\alpha(1-q)q}{2(1-\alpha)(2-q\alpha)}$. As in the previous case, $\frac{\alpha(1-q)q}{2(1-\alpha)(2-q\alpha)}$ is increasing in α and $\underline{w}(\alpha,q)$ is decreasing in α with $\underline{w}(\hat{\alpha}(q,0),q) = 0$. Therefore, there exists $\alpha^c(q)$ such that for $\alpha \in (\alpha^c(q), \hat{\alpha}(q,0))$, $\underline{w}(\alpha,q) \leq \frac{\alpha(1-q)q}{2(1-\alpha)(2-q\alpha)}$ and $\pi(b|w = \underline{w}(\alpha,q)) > \pi(b|w = 0)$, proving part (a).

To prove part (b), we first note that $w \geq \frac{1}{2}$ cannot be optimal for the buyer as it performs worse than $w = 0$. Consider $w \in [0, \frac{1}{2})$. For $q \leq \frac{1}{2}$, this implies that $(p_i^*(s_i), p_j^*(s_j|p_i^*(s_i))) = (\frac{1}{2}, \frac{1}{2})$, $p_i^*(b) = p_j^*(b) = w$ by the equilibrium characterization above, and $p_j^*(s_j|w) = \frac{1}{2}$ by (A-2). The buyer's payoff is $\pi(b|w) = (1-\alpha)^2(1-2w) + 2\alpha(1-\alpha)(\frac{1}{2} - w)$, and it is strictly *decreasing* in w. For $q = 1$, we obtain $(p_i^*(s_i), p_j^*(s_j|p_i^*(s_i))) = \begin{cases} (1-w, w) & if \quad w \leq 1-\alpha \\ (1,0) & if \quad w > 1-\alpha \end{cases}$, $p_i^*(b) = p_j^*(b) = w$ and $p_j^*(s_j|w) = \begin{cases} 1-w & if \quad w \leq 1-\alpha \\ 1 & if \quad w > 1-\alpha \end{cases}$. The buyer's payoff is $\pi(b|w) = (1-\alpha)^2(1-2w)$, which is also strictly *decreasing* in w. Thus, for $q \notin (\frac{1}{2}, 1)$, the buyer's optimal choice is $w = 0$. ■

Proof of Proposition 6. Let $\alpha_1 = \alpha_2 = \alpha$, and suppose that $q_1 = 1$ and $q_2 \in (\frac{\sqrt{5}-1}{2}, 1)$. If the buyer makes input 1 and outsources only input 2, then $p_2^*(s_2) = 1$, in which case the buyer's expected payoff is $\pi^{Make}(b) = 1 - \alpha$. Suppose that she outsources both inputs. Then, by Proposition 3, the buyer first negotiates for input 1 if $\alpha > \widehat{\alpha}(q)$, and she is indifferent to the sequence if $\alpha \leq \widehat{\alpha}(q)$. Note that $\alpha \leq \widehat{\alpha}(q)$ implies that $\pi^{Out}(b) = (1-\alpha)^2 + \alpha(1-\alpha)\frac{1-q_2}{2}$, and thus $\pi^{Out}(b) - \pi^{Make}(b) = -\alpha(1-\alpha)\frac{1+q_2}{2} < 0$. Note also that $\widehat{\alpha}(q) < \alpha \leq \frac{1+q_2^2}{1+q_2}$ leads to $\pi^{Out}(b) = (1-\alpha)^2 + \alpha(1-\alpha)\frac{1-q_2^2}{2} + \alpha^2(1-q_2)\frac{q_2}{2}$, and in turn, $\pi^{Out}(b) - \pi^{Make}(b) = \frac{\alpha}{2}(1+q_2)\left(\alpha - \frac{1+q_2^2}{1+q_2}\right) \leq 0$. Finally, $\alpha > \frac{1+q_2^2}{1+q_2}$ implies that $\pi^{Out}(b) = (1-\alpha)^2\frac{1-q_2^2}{2} + \alpha(1-\alpha)\frac{1-q_2^2}{2} + \alpha(1-\alpha)\frac{q_2(1-q_2)}{2} + \alpha^2(1-q_2)\frac{q_2}{2}$, and thus $\pi^{Out}(b) - \pi^{Make}(b) = \frac{1+q_2}{2}\left(\alpha - \frac{1+q_2^2}{1+q_2}\right) > 0$. In sum, $\pi^{Out}(b) - \pi^{Make}(b) > 0$ if and only if $\alpha > \frac{1+q_2^2}{1+q_2}$. ■

Proof of Proposition 7. Let $q_1 = q_2 = q$ and $\alpha_1 = \alpha_2 = \alpha$. Using the equilibrium prices in Lemma 1 and 2, we prove each part in turn.

$\underline{q > \frac{1}{2}}$: If $\alpha < \widehat{\alpha}(q)$, which means $q \neq 1$, then $\pi^l(s_i) = \alpha(1-\alpha)q$ and $\pi^f(s_i) = \alpha[\alpha\frac{1-q}{2} + (1-\alpha)q]$, resulting in $\pi^l(s_i) - \pi^f(s_i) = -\alpha^2\frac{1-q}{2} < 0$. If $\alpha \in [\widehat{\alpha}(q), \frac{1+q^2}{1+q}]$, then, for $q \leq \frac{\sqrt{5}-1}{2}$, we have $\pi^l(s_i) = \frac{\alpha}{2}$, $\pi^f(s_i) = \frac{\alpha^2}{2} + \alpha(1-\alpha)q$, and thus $\pi^l(s_i) - \pi^f(s_i) = \alpha(1-\alpha)\frac{1-2q}{2} < 0$, while for $q > \frac{\sqrt{5}-1}{2}$, we have $\pi^l(s_i) = \alpha q\frac{1+q}{2}$, $\pi^f(s_i) = \alpha^2\frac{1-q}{2} + \alpha(1-\alpha)q$, and thus $\pi^l(s_i) - \pi^f(s_i) = \frac{\alpha}{2}(3q-1)[\alpha - \overline{\alpha}(q)] > 0$, where $\overline{\alpha}(q) = \frac{q(1-q)}{3q-1}$. Finally, if $\alpha > \frac{1+q^2}{1+q}$, then: for $q \leq \frac{\sqrt{5}-1}{2}$, we have

$$\pi^l(s_i) = \frac{\alpha}{2} + (1-\alpha)(2q-1)q\frac{1+q}{2} \text{ and } \pi^f(s_i) = \frac{\alpha^2}{2} + \alpha(1-\alpha)(1-q)(\frac{2q-1}{2}),$$

resulting in $\pi^l(s_i) - \pi^f(s_i) = (1-\alpha)\frac{2q-1}{2}q[\alpha + q(1+q)] > 0$. For $q > \frac{\sqrt{5}-1}{2}$,

$$\pi^l(s_i) = \alpha q\frac{1+q}{2} + (1-\alpha)(2q-1)q\frac{1+q}{2} \text{ and } \pi^f(s_i) = \alpha^2\frac{1-q}{2} + \alpha(1-\alpha)(1-q)(\frac{2q-1}{2}),$$

resulting in $\pi^l(s_i) - \pi^f(s_i) > (1-\alpha)\frac{2q-1}{2}q[\alpha + q(1+q)] > 0$ because $q\frac{1+q}{2} > \frac{1}{2}$ and $\frac{1-q}{2} < \frac{1}{2}$. In sum, for $q > \frac{1}{2}$, we have $sign[\pi^l(s_i) - \pi^f(s_i)] = sign[\alpha - \overline{\alpha}(q)]$, where $\overline{\alpha}(q) = \begin{cases} \frac{1+q^2}{1+q}, & if \quad q \leq \frac{\sqrt{5}-1}{2} \\ \frac{q(1-q)}{3q-1}, & if \quad q > \frac{\sqrt{5}-1}{2} \end{cases}$.
It is easy to verify that $\overline{\alpha}(q) \geq \widehat{\alpha}(q)$, with strict inequality for $q \neq 1$.

$\underline{q \leq \frac{1}{2}}$: Then, we have $\pi^l(s_i) = \pi^f(s_i) = \frac{\alpha}{2}$, revealing that $\pi^l(s_i) - \pi^f(s_i) = 0$. ■

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