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Piecewise procurement of a large-scale project

Huseyin Yildirim*

Department of Economics, Duke University, Durham, NC 27708, USA

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Abstract

This paper studies the optimal piecewise procurement of a large-scale project. In the unique Markov Perfect Equilibrium (MPE) of the dynamic procurement game, it is found that (1) unlike the static setting, the procurer's optimal strategy depends on the number of suppliers and more importantly, it is nonmonotonic. As one more supplier participates in the procurement auction, the procurer softens competition in the initial stages by including more cost "types" while increasing competition in the mature stages; (2) this, in turn, implies that existing suppliers might favor participation of additional suppliers; (3) absent scheduling and/or resource constraints, the procurer prefers to procure the project as one piece if the suppliers' technology exhibits constant or increasing returns, and no additional suppliers are enticed to bid; and (4) the optimal dynamic mechanism can be easily implemented via a sequence of dominant strategy auctions of the Vickrey type.

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1. Introduction

Understanding the nature of government procurement processes has significant potential benefits as government procurements of goods and services in many countries

* Tel.: +1 919 660 1805; fax: +1 919 684 8974.

E-mail address: yildirh@econ.duke.edu.

constitute more than 10% of national income (see, e.g., [Hoekman and Mavroidis, 1997](#); [McAfee and McMillan, 1988](#)). Governments and public agencies contract out to private firms a variety of large-scale projects such as building road networks, constructing schools and multifacility sport complexes, and renovating and repairing multibuilding historic sites and university campuses, as well as a variety of small-scale projects such as snow clearing, garbage collection, and opinion polling. This paper studies the optimal procurement of a large-scale project through competitive bidding.¹

Unlike small-scale projects, large-scale projects often need to be divided into small pieces or subprojects and procured in a predetermined order. For instance, most of the major reconstruction and modernization projects in developing countries, which are supported by the World Bank, consist of subprojects.² Similarly, schools periodically undergo major repair and renovation work encompassing multiple buildings.

There are several institutional and practical reasons for procuring a large-scale project in a piecemeal fashion. First, current governments may not commit resources that are to be available to future governments. Second, there may arise significant scheduling constraints in undertaking large-scale projects, e.g., road networks and repair of college buildings on campuses, that require shutting down essential activities. Third, there may be too few firms with sufficient resources to undertake the entire project. This is especially true in the World Bank projects for which the Bank preferably relies on domestic firms in developing countries to revive their economies. Last, but not least, supply technologies may exhibit decreasing returns to scale due to, e.g., capacity constraints.

To understand the procurement of a large-scale project, several important theoretical and practical questions need to be addressed. From the procurer's perspective, how would she optimally design a sequence of procurement auctions to minimize costs, especially when she cannot commit to the terms of future trades? Would she be able to implement the optimal outcome via simple auctions? Absent the concerns alluded to above, what would the optimal division of a large-scale project be? From the suppliers' perspective, would they facilitate the progress of the project by offering price discounts early on, knowing that doing so will leave fewer profit opportunities? Would the existing firms actively search for new firms along with the procurer to bid for the project?

To address these and other related issues, I construct a simple dynamic procurement model in which the procurer owns a multistage project. Each stage corresponds to a subproject. There are n firms, each of which is capable of doing all subprojects (possibly through their own network of subcontractors). I assume firms' current costs are privately known and vary over time. These costs can reflect, for instance, firms' labor, fuel and raw material costs, or simply their opportunity costs fluctuating with their access to input markets. In each period, the procurer designs an optimal procurement auction for the current subproject that determines the payment and allocation functions based on firms'

¹ While the primary focus is on public contracts, similar issues discussed here arise in private sector procurements as well.

² After the war, the Federation of Bosnia and Herzegovina with the support of the World Bank launched a major reconstruction and recovery project in 1997. The project has been divided into more than 400 subprojects scheduled according to their urgency. These subprojects have been procured through competitive bidding with the Bank's oversight. For more on this project, visit www.worldbank.org.ba.

cost reports. I assume that the procurer has limited commitment power and thus can only commit to the terms of the current auction.³ I also assume that both the procurer and firms base their strategies only on the state of the project. Thus, the Markov Perfect Equilibrium (MPE) concept is used throughout. While this rules out the interesting aspects of reputation building and collusion, it serves the purpose of this paper, as I wish to investigate the effects of the physical progress of the project on parties' relationships.

The complicated interaction between the procurer and firms can be simplified by using dynamic programming arguments. I show that there exists a unique MPE in which the procurer follows a cutoff strategy and, as in a static setting, selects the lowest cost supplier for each task (by paying, of course, appropriate rents). Unlike in the static setting, however, the cutoff in the dynamic setting is endogenously determined for each task. I find that the procurer increases the cutoff with the progress of the project, and hence, not surprisingly, facilitates the progress of the project. Somewhat surprisingly, however, I also find that firms too facilitate the progress by giving early discounts, despite the fact that each step forward will leave fewer profit opportunities for them, and that they sometimes incur losses for which the procurer cannot commit (even implicitly) to compensating in the future. The intuition is that, by increasing the cutoff and thus including more cost "types" along with the progress of the project, the procurer enables firms to earn *progressively* greater information rents that become the "prizes" to win.

As for any optimal mechanism analysis, the practical value, of course, comes from its implementation through some well-known auctions. I demonstrate that the dynamic procurement mechanism identified in this paper can be easily implemented via a sequence of dominant strategy auctions of the Vickrey type. In particular, the procurer sets an entry fee for each auction that is equal to the change in each firm's expected profits from the completion of the current subproject.

Next, I investigate two comparative statics. The first one deals with how the participation of one more firm in the competition affects the procurement design and parties' expected value of the project. Unlike the static setting in which the cutoff is independent of the number of bidders, e.g., [Laffont and Tirole \(1987\)](#), [McAfee and McMillan \(1987\)](#), and [Riordan and Sappington \(1987\)](#), the procurer modifies her optimal strategy in the dynamic setting in response to one more bidder. The procurer *increases* the cutoff with one more firm in the initial stages of the project while reducing it in mature stages, implying that there is a nonmonotonic relationship between the number of bidders and the procurer's optimal strategy. Intuitively, by increasing the cutoff early on, the procurer softens the competition to ensure the progress of the project. From the existing firms' point of view, this implies an additional firm has two opposing effects on their expected profits: First, all else equal, an additional firm reduces the chances of being the winner, but second, it also leads the procurer to raise the cutoff, thereby leaving greater information rents. I show that if there are sufficiently many subprojects ahead, the increase in rents can more than offset the negative competition effect, and thus, existing firms might indeed *benefit* from more entry into the competition. An interesting implication of this finding is that, for large projects, existing firms might actively search for new bidders

³ Government procurement agents usually face legal and administrative constraints preventing them from committing resources for their successors.

along with the procurer. This observation is important because [Bulow and Klemperer \(1996\)](#) argue that attracting a new bidder is more valuable for the procurer (the seller in their setup) than having more bargaining power. Despite the nonmonotonicity in the optimal strategy, the procurer always prefers more participation into the procurement.

The second comparative static asks whether the buyer likes to deal with long- or short-sighted firms. While long-sighted firms are willing to offer severe price discounts in the initial states of the project to facilitate its progress, they tend to raise their prices in states near completion when the buyer's value of the project is high. Thus, I find that the procurer is better off dealing with long-sighted firms only if the project is sufficiently large in order for initial price discounts to overcome future price increases. Otherwise, small-scale projects are supplied at a lower expected cost by short-sighted suppliers.

Absent resource and scheduling constraints alluded to above, an important control variable for the procurer is to divide a large-scale project into subprojects, if doing so would be optimal. I identify two opposing incentives for the procurer to divide the project: First, to do so, the procurer takes advantage of repeated competition, but second, the procurer suffers from further delay in completing the project. I then show, under very general conditions, that if the production technology exhibits constant or increasing returns to scale, and no new (possibly smaller) supplier is enticed to bid, then the procurer is better off procuring the project as one piece. In light of this finding, I demonstrate by an example that the procurer may divide the project into pieces when there are significant decreasing returns to scale and/or a sufficient number of additional bidders are enticed to bid.

The rest of the paper is organized as follows. Section 2 sets up the model and characterizes the equilibrium. Section 3 introduces a first-best benchmark with complete information. Section 4 shows how the total surplus is shared between the procurer and suppliers, and demonstrates the implementation of the optimal mechanism via simple auctions. The two comparative statics with respect to the number of suppliers and the discount factors are performed in Section 5. The issue of the project division is addressed in Section 6, and finally, Section 7 notes concluding remarks. The proofs of all formal results appear in an Appendix.

Related Work Papers by [Compte and Jehiel \(2002\)](#), [Laffont and Tirole \(1987\)](#), [Manelli and Vincent \(1995\)](#), [McAfee and McMillan \(1987\)](#), and [Riordan and Sappington \(1987\)](#) investigate optimal procurement auctions for one-time lump-sum purchases only. Thus, repeated competition along with the progress of the project does not arise. [Gale et al. \(2000\)](#) study the possibility of subcontracting between suppliers when buyers are nonstrategic. [Laffont and Tirole \(1988\)](#) and [Luton and McAfee \(1986\)](#) consider sequential auctions for the repeat purchases of the same good. Unlike the present work, they assume the procurer can commit to a long-term contract. [Lewis and Yildirim \(2002a,b\)](#) focus on a repeated procurement model in which the same good is purchased and, much like in the present setting, the procurer cannot commit to long-term contracts. However, they focus on the effects of learning by doing on the part of suppliers, and do not consider large-scale projects.⁴

⁴ Papers by [Lewis \(1983\)](#) and [Krishna \(1993\)](#) examine a sequential procurement competition between the incumbent and potential entrants. Although their focus is different, they also recognize how firms' valuations of each unit of good are affected by the anticipation of the future auctions.

Anton and Yao (1987) and Rob (1986) address the question of how a procurer should optimally divide a large-scale project into two stages where the first stage is awarded to a sole source in the presence of learning-by-doing. Here, I abstract from the learning aspect and focus instead on the continuing competition among the same firms, along with the progress of the project involving (possibly) more than two stages.

Perhaps, Lewis (1986) is the closest paper in spirit to the present work. As does the present research, Lewis presents a piecewise procurement model of large-scale projects. However, his focus is on rationalizing the frequent cost overruns in a long-term bilateral relationship between a procurer and a supplier, where neither party has long-term commitment ability and the supplier has better information about the type of project than does the procurer.

Finally, this paper also relates to the literature on sequential auctions where a seller repeatedly auctions off single- or multi-units of the same good. Papers by Ashenfelter (1989), Bernhardt and Scoones (1994), Caillaud and Mezzetti (2004), Gale and Hausch (1994), McAfee and Vincent (1993), and Jeitschko (1999), among others, consider sequential sales of multiple units of the same good. Unlike the present setting, the auctioneer in these papers values all units the same so that no complementarity between units arises. McAfee and Vincent (1997) analyze a case where the seller lacks commitment not to reacquire the same unit if it is not sold in previous periods. While the procurer can reacquire the same subproject in my setting too, unlike McAfee and Vincent, costs are drawn anew in each period and there are multiple subprojects to be auctioned off.

2. A model of sequential procurement

There are $n+1$ risk-neutral agents in the model: one buyer (she) and n suppliers. The buyer owns a multistage project, e.g., a multistage road network, a multifacility sport complex, or a multibuilding campus renovation project.⁵ The project consists of $m \in \{1, 2, \dots\}$ subprojects that are indexed such that subproject ρ^j , $j \in \{1, \dots, m\}$ is the one after which $j-1$ more subprojects remain. The buyer receives a value $V > 0$ once the project is completed. While completing each subproject might yield some intermediate payoff, for simplicity, I normalize these payoffs to zero to better focus on the dynamics of the procurement.⁶

Although each supplier is capable of undertaking all subprojects,⁷ the buyer auctions off each subproject that takes only one period to complete.⁸ I assume that the procurer has

⁵ In what follows, I use the procurer and buyer interchangeably.

⁶ To be more specific, one can let v_j be the cumulative payoff up to subproject ρ^j , where v_j is nonincreasing in j and $v_0 \equiv V > v_1$. If the marginal return of each subproject is nondecreasing, i.e., $v_{j-1} - v_j \geq v_j - v_{j+1}$, the results will go through. Note that such a “convex” cumulative payoff property is likely to hold in most large-scale projects such as a road network construction for which each completed subproject increases the payoff from the previous ones.

⁷ This should be interpreted in a broader sense. In particular, suppliers themselves might have several subcontractors capable of doing different parts of the project.

⁸ In general, one can assume the completion time to be $t \geq 1$ periods. This can be incorporated into the analysis without changing the main point of the paper.

limited commitment power. In particular, she can only commit to the terms of the current procurement contract and offers a (possibly) new contract in future periods. Each supplier privately draws a cost of undertaking the subproject, c_i each period from a twice-differentiable distribution function, $F(c_i)$, which is independently and identically distributed (IID) over time and across suppliers.^{9,10} The support of distribution is $[0, \bar{c}]$ with $\bar{c} > 0$ and $F'(c_i) = f(c_i) > 0$. For future reference, let $h(c_i) \equiv c_i + [F(c_i)]/[f(c_i)]$ denote the “virtual” cost entailing the cost of undertaking the task plus a term for information rents. As is standard in the procurement literature, I ensure that the virtual cost is nondecreasing:

Assumption 1.

$$\frac{d}{dc_i} \left[\frac{F(c_i)}{f(c_i)} \right] \geq 0.$$

The assumption that firms’ costs are distributed IID over time deserves further comment. In reality, a firm’s cost of undertaking a subproject would have (at least) two components: One task-specific component capturing the unanticipated difficulties of the task ahead and one firm-specific component capturing the unanticipated volatility of input prices used in production and/or of the firm’s outside opportunities. Because each firm can have its own access to inputs such as labor, fuel, and raw materials, input prices might vary across firms. While it is reasonable to assume that the task-specific cost would remain the same over time until the subproject in question is completed, the firm-specific cost can change over time even for the same subproject. Which type of cost is more important in shaping the procurer’s and firms’ strategies will surely depend on the type of the large-scale project in hand. By focusing on the fluctuations in firm-specific costs only, I am implicitly assuming a relatively well-defined project with routine tasks whose levels of difficulty can be ascertained in advance. This seems consistent with the projects in developing countries in which input markets are not well developed and therefore input prices are likely to be less certain than the job itself. The same would not be true, however, if the project consisted of relatively innovative tasks.

The interaction between the buyer and suppliers is modeled as an infinite horizon Markov game where parties base their strategies only on the state of the project (and on current costs for suppliers) and Markov Perfect Equilibrium (MPE) is the equilibrium

⁹ The IID assumption rules out the potentially interesting issues of strategic learning and signalling among suppliers. I make the IID assumption for two reasons. First, it helps isolate the impact of the progress of the project on parties’ relationship without clouding the model with these additional features. Second, it simplifies the analysis greatly, especially because the procurer lacks long-term commitment in my model. See, for instance, [Laffont and Tirole \(1993, ch. 9\)](#) for a review and difficulties of repeated procurement with correlated costs in bilateral relationships between the procurer and a single supplier. I should probably note that when suppliers’ costs are correlated (even slightly), the procurer can optimally design an auction a la [Cremer and McLean \(1988\)](#) and achieve the first-best described in Section 3, although such a mechanism requires rather strong assumptions such as the ability of imposing large penalties on suppliers.

¹⁰ The results of this paper are robust to the extension in which firms are allowed to exert cost reducing efforts a la [Laffont and Tirole \(1993\)](#). The details of this extension are available from the author upon request.

concept throughout the paper. To be precise, the timing and information structure of this dynamic procurement model is as follows:

- In each period, upon observing the state (ρ^j) of the project, the procurer offers a contract $\{P_i(\rho^j, \hat{c}), \lambda_i(\rho^j, \hat{c})\}$ to supplier i , where $P_i(\rho^j, \hat{c})$ and $\lambda_i(\rho^j, \hat{c})$ denote the payment and the probability of awarding the subproject based on the state of the project and suppliers' cost reports, $\hat{c} \equiv (\hat{c}_1, \dots, \hat{c}_n)$.
- Each supplier privately observes its current cost and decides whether or not to participate in the current procurement auction. (In equilibrium, all suppliers agree to participate.) Suppliers' outside opportunities in each period are normalized to 0.
- Finally, suppliers *simultaneously* report their costs to the buyer. If one is awarded the task, then the project moves to the next task, ρ^{j-1} . However, it is possible that the buyer would find costs too high and decide to wait for one period.¹¹ This ongoing procurement process starts next period with the revised state and with the exception that suppliers draw new costs.

Let $B(\rho^j)$ and $S_i(\rho^j)$ be the expected present value for the buyer and for supplier i respectively from participating in the current and future procurements given the current state, ρ^j . All parties discount future returns and costs by $\delta \in [0, 1)$.

Due to the Revelation Principle (see, e.g., Myerson, 1979), when determining the optimal current contract, the procurer can restrict attention to truth-telling direct mechanisms.¹² Thus, it suffices for her to take two constraints into account for each supplier: First, the contract must induce supplier i to truthfully report his cost, c_i , given that others also do so. That is, the contract $\{P_i(\rho^j, \hat{c}_i), \lambda_i(\rho^j, \hat{c}_i)\}$ must satisfy

$$S_i(\rho^j, c_i) = \max_{\hat{c}_i} S_i(\rho^j, \hat{c}_i) \equiv P_i(\rho^j, \hat{c}_i) - \lambda_i(\rho^j, \hat{c}_i)c_i + \delta S_i(\rho^j) + \delta \sum_{k=1}^n \lambda_k(\rho^j, \hat{c}_i) \Delta S_i(\rho^j) \tag{ICi}$$

where $S_i(\rho^j, c_i) \equiv E_{c_{-i}}[S_i(\rho^j, \mathbf{c})]$, $S_i(\rho^j) \equiv E_{c_i}[S_i(\rho^j, c_i)]$, $P_i(\rho^j, c_i) \equiv E_{c_{-i}}[P_i(\rho^j, \mathbf{c})]$, and $\lambda_i(\rho^j, c_i) \equiv E_{c_{-i}}[\lambda_i(\rho^j, \mathbf{c})]$ with E_c being the expectation operator with respect to c , and $\mathbf{c}_{-i} \equiv (c_1, \dots, c_{i-1}, c_{i+1}, \dots, c_n)$. The Δ is the usual difference operator between states ρ^{j-1} and ρ^j throughout.

Eq. (ICi) indicates that supplier i decides on his cost report by maximizing his discounted expected value. This value is comprised of the expected current and future profits. The current profit is the expected payment, $P_i(\rho^j, \hat{c}_i)$, minus the expected cost of

¹¹ Note that the same subproject may be put up for bids if it fails to be awarded in previous periods. See, for example, McAfee and Vincent (1997) on a similar point with one object.

¹² Note that, in general, when suppliers' costs are correlated over time and the procurer cannot commit to a long-term contract for the entire relationship, the Revelation Principle in its "standard" form does not apply. In particular, suppliers become reluctant to reveal their types early on, and thus substantial pooling in equilibrium may occur (e.g., Laffont and Tirole, 1993, ch. 9). However, because costs are assumed to be IID over time in my model, current costs do not reveal any information about future costs, and thus the standard Revelation Principle can be used.

undertaking the subproject, $\lambda_i(\rho^j, \tilde{c}_i)c_i$. The future expected returns are the discounted expected surplus, $\delta S_i(\rho^j)$, and the increase in the expected continuation value, $\delta \sum_{k=1}^n \lambda_k(\rho^j, \tilde{c}_i) \Delta S_i(\rho^j)$ when the subproject is assigned to a supplier. Note that when finding the continuation value, supplier i considers the probability that the project will be assigned to *some* supplier, not only to himself. As we will see below, this will create a dynamic externality among suppliers. Applying the Envelope Theorem on (IC_i) , we obtain

$$\frac{dS_i(\rho^j, c_i)}{dc_i} = -\lambda_i(\rho^j, c_i) \tag{1}$$

That is, supplier i 's expected discounted profits decrease in his current cost at the rate of his probability of winning.

Second, the contract must yield supplier i expected discounted returns at least as high as his expected returns when he rejects the current contract. That is, the contract must satisfy the following participation constraint:

$$S_i(\rho^j, c_i) \geq \delta R_i(\rho^j) \tag{IRi}$$

where $R_i(\rho^j)$ is supplier i 's expected discounted value if he rejects the current contract.

Combining (IC_i) , (IR_i) , and (1), the following lemma characterizes the implementable contracts.

Lemma 1. *For any procurement allocation satisfying (IC) and (IR), $\lambda_i(\rho^j, c_i)$ is nonincreasing in c_i and the payment and suppliers' surplus are given respectively by:*

$$P_i(\rho^j, c_i) = \delta [R_i(\rho^j) - S_i(\rho^j)] + \lambda_i(\rho^j, c_i)c_i + \int_{c_i}^{\tilde{c}} \lambda_i(\rho^j, \tilde{c}_i) d\tilde{c}_i - \delta \sum_{k=1}^n \lambda_k(\rho^j, \tilde{c}_i) \Delta S_i(\rho^j) \tag{2}$$

$$S_i(\rho^j) = \delta R_i(\rho^j) + E_{c_i} \left[\lambda_i(\rho^j, c_i) \frac{F(c_i)}{f(c_i)} \right] \tag{3}$$

Proof. All proofs are relegated to an Appendix. \square

The intuition behind (2) and (3) will be more transparent below once the equilibrium $R_i(\rho^j)$ is characterized.

Having described the suppliers' behavior, I now specify the procurer's program and determine her optimal strategy. Note that once the project is completed, the buyer optimally severs the relationship with suppliers and enjoys the benefit V . This implies $B(\rho^0) = V$ and $S_i(\rho^0) = 0$, where ρ^0 denotes the completion stage. In the previous stages of the project, however, the buyer solves the following dynamic program to determine the optimal contract:

$$B(\rho^j) = \max_{\{\lambda_i(\cdot), P_i(\cdot)\}} \delta B(\rho^j) + E_c \sum_{i=1}^n [-P_i(\rho^j, c) + \delta \lambda_i(\rho^j, c) \Delta B(\rho^j)] \text{ subject to } (IR_i) \text{ and } (IC_i) \text{ for all } i. \tag{4}$$

According to (4), the buyer maximizes her expected present value in the relationship by choosing the contract $\{P_i(\rho^j, \mathbf{c}), \lambda_i(\rho^j, \mathbf{c})\}$ for supplier i . This value consists of the discounted expected value, $\delta B(\rho^j)$, and the current benefit, normalized to 0, minus payments to the suppliers, and plus the increase in the discounted expected future value when the project is assigned to firm i , $\delta \lambda_i(\rho^j, \mathbf{c}) \Delta B(\rho^j)$.

Inserting payments from (2) into (4) reduces the buyer’s program to:

$$(1 - \delta)B(\rho^j) = \max_{\{\lambda_i(\cdot)\}} \sum_{i=1}^n E_c \lambda_i(\rho^j, \mathbf{c}) [-h(c_i) + \delta \Delta W(\rho^j)] - \sum_{i=1}^n \delta [R_i(\rho^j) - S_i(\rho^j)] \tag{5}$$

where we define the total surplus as $W(\rho^j) \equiv B(\rho^j) + \sum_{k=1}^n S_k(\rho^j)$.

It is clear from (5) that the procurer adopts the following selection strategy:

$$\lambda_i(\rho^j, c) = \begin{cases} 1, & \text{if } c_i \leq \min \{c(\rho^j), \min_{j \neq i} \{c_j\}\} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where we define the cutoff $c(\rho^j)$ such that $h(c(\rho^j)) \equiv \delta \Delta W(\rho^j)$.

According to (6), the buyer awards subproject ρ^j to the lowest cost supplier unless this cost is too high. This is, of course, due to the IID cost assumption that keeps suppliers ex ante symmetric with respect to future subprojects, as well as the monotonicity condition in Assumption 1.¹³ Thus, the optimal strategy is similar to that in a static model with two important differences: First, the cutpoint is now endogenously determined in each stage and increases as the project matures. Second, unlike in a static model, the cutpoint is a function of the number of suppliers. These remarks will become clearer in the ensuing analysis.

Determining the unique equilibrium of this dynamic procurement game amounts to determining the unique sequence of $\{c(\rho^j)\}$. Let $G(c; n) \equiv 1 - [1 - F(c)]^n$ be the distribution function of $\min\{c_1, \dots, c_n\}$ and define the following function:

$$\Phi(x; \delta) = \frac{1 - \delta}{\delta} h(x) + \frac{F(x)}{f(x)} G(x; n) + \int_0^x G(c; n) dc, \quad \text{for } x > 0. \tag{7}$$

Lemma 2.

- (a) $\Phi(0; \delta) = 0$, $\Phi'(x; \delta) > 0$ for all $x > 0$ and $\Phi'(0; \delta) = (1 - \delta) / \delta h'(0) \geq 0$.
- (b) For $j \geq 1$, $\overline{W}(\rho^{j-1}) = \Phi(c(\rho^j); \delta)$, $\overline{W}(\rho^j) = \Phi(c(\rho^j); 1)$ where $\overline{W}(\rho^j) = (1 - \delta) \overline{W}(\rho^j)$.

Lemma 2 provides the properties of Φ , which will be important in proving our results and it shows how the backward induction works to find $\{c(\rho^j)\}$. Fig. 1 further demonstrates

¹³ In particular, if winning the current subproject created a systematic efficiency gap between the winner and losers because of, say, learning-by-doing, then the procurer would sometimes optimally favor the inefficient supplier to balance future competition. This is the standard “handicapping” result in optimal auctions with unevenly matched bidders. See, for instance, Fudenberg and Tirole (1991, ch. 7) and McAfee and McMillan (1988, ch. 9) for a static setting, and Lewis and Yildirim (2002b) for a dynamic one.

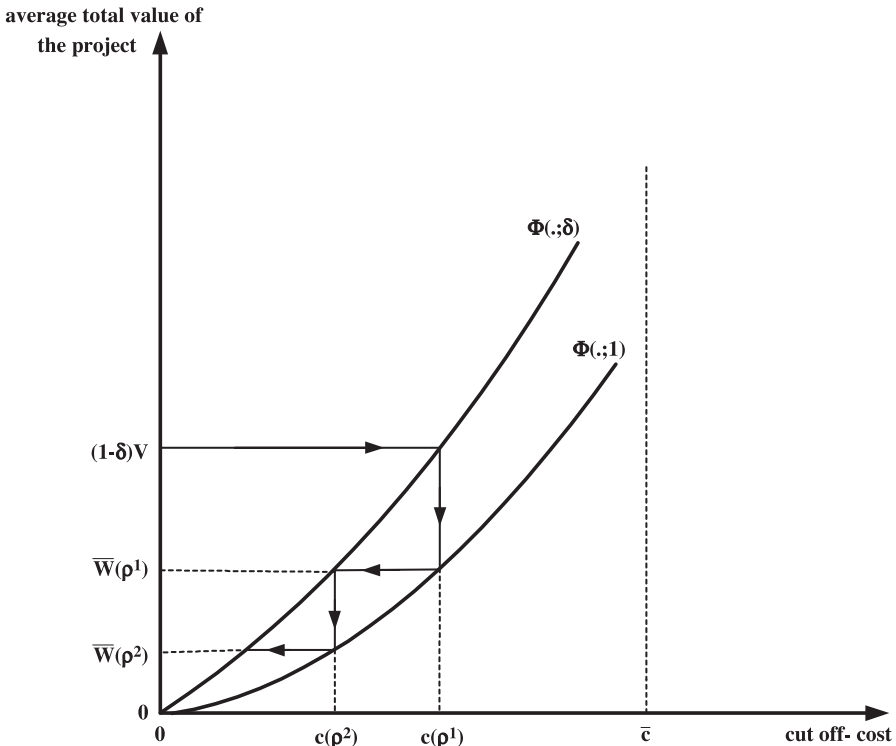


Fig. 1. Equilibrium dynamics and backward induction.

these findings in a graph. Note that $c(\rho^j)$ is independent of the number of subprojects, m , due to the backward induction from the final stage. For the clarity of exposition below, I assume that the upper bound of cost distribution is large enough that the procurer finds it optimal to exclude some cost types in each state.¹⁴ More formally, I make

Assumption 2.

$$(1 - \delta)V < \Phi(\bar{c}; \delta)$$

The following proposition characterizes the equilibrium:

Proposition 1. *There exists a unique MPE of the dynamic procurement game, and it has these properties: For $j \geq 1$,*

- (a) $c(\rho^j) > 0$, and $c(\rho^j)$ is decreasing in j .
- (b) Total surplus, $W(\rho^j)$, increases at an increasing rate as the project moves forward. That is, $\Delta W(\rho^j) > 0$ and $\Delta W(\rho^{j-1}) > \Delta W(\rho^j)$.

¹⁴ Otherwise, I would simply keep track of uninteresting cases in which $c(\rho^j) = \bar{c}$, and in equilibrium, the task is assigned to a supplier with certainty.

According to part (a) of Proposition 1, because waiting is costly, i.e., $\delta < 1$, the buyer chooses a strictly positive cutoff cost. Otherwise, she would just set the cutoff to θ , and wait until some supplier draws such a cost. Part (a) also reveals that the cutoff increases as the project moves forward. This means that the average waiting time in state ρ^j given by $w(\rho^j) = \frac{1}{G(c(\rho^j);n)}$ shrinks with the progress of the project. Thus, although subprojects are ex ante identical, on average, initial subprojects are completed in relatively longer periods than the later ones. Part (b) implies that the total surplus grows at an increasing rate with the progress of the project. This observation will be helpful in determining the optimal division of the project in Section 6.

3. Benchmark: complete information

Before proceeding further into the analysis of the competitive bidding model, it is useful to review a first-best case where suppliers' current costs are common knowledge. The analysis of this case is similar to that above except now no incentive compatibility constraints are imposed on the optimal contract. Formally, the procurer solves the following dynamic program:

$$B^*(\rho^j) = \max_{\{\lambda_i^*(\cdot), P_i^*(\cdot)\}} \delta B^*(\rho^j) + E_c \sum_{i=1}^n [-P_i^*(\rho^j, c) + \delta \lambda_i^*(\rho^j, c) \Delta B^*(\rho^j)] \text{ subject to } S_i^*(\rho^j, c_i) \geq \delta R_i^*(\rho^j) \tag{8}$$

Because the buyer's objective requires that she leave the least surplus to the suppliers, she sets the payments so that the constraint binds for all realizations of costs. Inserting these payments into (8) reduces the program to:

$$(1 - \delta)B^*(\rho^j) = \max_{\{\lambda_i^*(\cdot)\}} \sum_{i=1}^n E_c \lambda_i^*(\rho^j, \mathbf{c}) [-c_i + \delta \Delta W^*(\rho^j)] - \sum_{i=1}^n \delta [R_i^*(\rho^j) - S_i^*(\rho^j)] \tag{9}$$

The following result characterizes the solution to the first-best:

Proposition 2. *There exists a unique MPE of the dynamic procurement game with complete information, and it has these properties: For $j \geq 1$,*

- (a) $\lambda_i^*(\rho^j, \mathbf{c}) = \begin{cases} 1, & \text{if } c_i \leq \min \{c^*(\rho^j), \min_{j \neq i} \{c_j\}\} \\ 0, & \text{otherwise} \end{cases}$.
- (b) $c(\rho^j) < c^*(\rho^j)$.
- (c) $S_i^*(\rho^j) = 0$.

According to part (a) of Proposition 2, the procurer selects the lowest cost supplier in each procurement. By comparing $\lambda_i^*(\rho^j, \mathbf{c})$ and (6), this implies that there is no inefficiency in terms of the identity of the winning supplier. However, in order to restrict information rents to the bidders, the procurer assigns the project too infrequently in the presence of incomplete information, as recorded in part (b). This means the project

progresses too slowly in the competitive bidding from a social standpoint. Finally, part (c) reveals that because the procurer can perfectly observe suppliers' costs in the benchmark case, she leaves no rents to them.

4. Surplus sharing and implementation of the optimal mechanism

In the competitive bidding model introduced in Section 2, suppliers are privately informed about their production costs, which allows them to capture information rents. In this section, I first investigate how these rents evolve with the progress of the project and how the buyer designs the optimal mechanism to reduce them. I then demonstrate the implementation of the optimal mechanism through a sequence of simple auctions.

4.1. Surplus sharing

In equilibrium, part (b) of Proposition 1 indicates that the total surplus to be shared increases as the project moves forward. Hence, the optimal threat for the buyer is to exclude all suppliers from the current bidding if one of them refuses to participate. Formally, in what follows, I set¹⁵

$$R_i(\rho^j) = S_i(\rho^j) \quad (10)$$

In light of Proposition 1, it is intuitive that because the procurer chooses the terms of the procurement contracts, she should be better off with the progress of the project. Yet, it is not clear if the same is true for suppliers given that the progress means fewer profit opportunities. To determine parties' present values of the project in state ρ^j , I use (5), (6), and (10) to find:

$$B(\rho^j) = \frac{1}{1-\delta} \int_0^{c(\rho^j)} h'(c)G(c;n)dc \quad (11)$$

Similarly, I use (3) and (10) to determine supplier i 's present value:

$$S_i(\rho^j) = \frac{1}{1-\delta} \int_0^{c(\rho^j)} [1-F(c)]^{n-1}F(c)dc \quad (12)$$

¹⁵ Two comments are in order here. First, the buyer would be better off if she could impose a more severe threat of abandoning the project altogether, whenever a supplier refuses to participate, i.e., $R_i(\rho^j)=0$ for all j . However, given that the procurer can only commit to the current contract, such a threat is not feasible in the present setting. Second, the threat in (10) is suboptimal in the penultimate state, ρ^1 . This is because, whenever a supplier is awarded the last subproject, the project is completed, and thus the winner exerts a negative externality on others. This means the optimal threat on nonparticipants in this state is to award the project to another supplier with certainty, regardless of its cost. While incorporating this threat does not change the qualitative results in what follows, such a strong threat may not be feasible for a government agency either.

Armed with (11) and (12), Proposition 1 leads us to:

Proposition 3. For $j \geq 1$, both $B(\rho^j)$ and $S_i(\rho^j)$ are decreasing in j .

Proposition 3 implies that the expected surplus for both the procurer and the suppliers increases as the project progresses. To see how this increase affects suppliers' incentives, I use (10) and rewrite the equilibrium payment function previously stated in (2):

$$P_i(\rho^j, c_i) = \lambda_i(\rho^j, c_i)c_i + \int_{c_i}^{c(\rho^j)} \lambda_i(\rho^j, \tilde{c}_i)d\tilde{c}_i - \delta \sum_{k=1}^n \lambda_k(\rho^j, c_i)\Delta S_i(\rho^j) \quad (13)$$

According to the r.h.s. of (13), supplier i is reimbursed for his cost if he is awarded the subproject. In addition, he receives information rents as reflected in the second term. These rents increase with the cutpoint, $c(\rho^j)$, which in turn increases with the progress of the project. For short-sighted suppliers or suppliers that place no weight on future returns, the first two terms summarize the way payments are made. For long-sighted suppliers, however, the expected future gains or losses introduce the last component of compensation, which is affected by three factors: the discount factor, the expected probability that subproject ρ^j is assigned to some supplier, $\sum_{k=1}^n \lambda_k(\rho^j, c_i)$, and the expected change in supplier i 's surplus in state ρ^j , $\Delta S_i(\rho^j)$. Note that for the last stage of a large-scale project or, equivalently, for a one-stage project, waiting has a positive value to suppliers, i.e., $\Delta S_i(\rho^1) < 0$.¹⁶ That is, by agreeing to sell today, they forgo the last profit opportunity, and hence demand further compensation. In the previous stages, however, waiting has a negative value, i.e., $\Delta S_i(\rho^j) > 0$ for $j \geq 2$, so that this incentive is reversed. That is, as the last term in (13) indicates, suppliers are now willing to offer a discount in the amount of the future increase in their surplus. The procurer's ability to extract this surplus is due to the IID cost assumption that endows all parties with symmetric information about future supply costs. To see the impact of early discounts on suppliers' current profits, note that $\Pi_i(\rho^j, c_i) = P_i(\rho^j, c_i) - \lambda_i(\rho^j, c_i)c_i$:

$$\Pi_i(\rho^j, c_i) = \begin{cases} \int_{c_i}^{c(\rho^j)} \lambda_i(\rho^j, \tilde{c}_i)d\tilde{c}_i - \delta \Delta S_i(\rho^j), & \text{if } c_i \leq c(\rho^j) \\ -\delta G(c(\rho^j); n-1)\Delta S_i(\rho^j), & \text{if } c_i > c(\rho^j) \end{cases} \quad (14)$$

There are two noteworthy observations in (14) First, if a supplier draws a cost below the cutpoint, he expects to win the subproject with some probability. Yet, for costs close to the cutpoint, i.e., $c_i \approx c(\rho^j)$, the winner also expects to incur a loss with certainty, i.e., $\Pi_i(\rho^j, c_i) < 0$. Second, if a supplier draws a cost greater than the cutpoint, then he knows that he will not be awarded the subproject. However, according to (14), he is willing to pay the amount $\delta G(c(\rho^j); n-1)\Delta S_i(\rho^j)$ to the procurer.¹⁷ In fact, it is possible that all suppliers draw costs greater than the cutpoint and pay this amount, although ex post the subproject

¹⁶ Note that $\Delta S_i(\rho^1) = S_i(\rho^0) - S_i(\rho^1) = -S_i(\rho^1)$.

¹⁷ Note that for $n=1$, this payment is 0. This makes sense because the progress of the project then depends on a single supplier, and whenever he draws a cost greater than the cutoff, he expects the project to stall with certainty. In this case, the buyer cannot credibly extract a positive payment from the supplier based on a potential progress.

is not awarded to any supplier. This is so despite the fact that the procurer cannot write long-term contracts or build a reputation to guarantee a compensation in future periods. Two comments are in order here: First, the somewhat counterfactual result that even the losing suppliers make a payment to the procurer can be viewed as an entry fee to the auction (see the subsection below). Second, recall that the reason why a supplier cannot avoid paying the procurer by simply not participating is because doing so will induce the procurer to stall the project. We know, however, from Proposition 3 that suppliers expect to earn increasingly greater rents along with the progress of the project, as the buyer would then attach more value to it and be more generous.

The observation that losing suppliers are willing to make positive payments to the procurer is consistent with Jehiel et al. (1996). In a static auction setting, Jehiel et al. consider a seller who auctions off an object, such as a nuclear weapon, to maximize revenues. However, unlike a standard auction setting, each potential buyer exerts an identity-dependent negative externality on nonacquirers. They find that by optimally threatening each buyer with awarding the project to his “worst” fear, the seller extracts surplus also from agents who do not obtain the object, and that the seller is better off not selling the object at all, while obtaining some payments, if externalities are much larger than valuations. Although each stage game in my setting can also be considered as an “auction with externalities,” there are several differences from Jehiel et al.: First, externalities in my dynamic setting are determined endogenously in each stage rather than being fixed. Second, in light of Proposition 3, the winner imposes a positive externality on other suppliers by moving the project forward to a state where the buyer leaves greater rents.

4.2. Implementation by a Vickrey-type auction

As for any optimal mechanism, the practical value depends on its implementation via simple auctions. Here I demonstrate how the optimal mechanism described above can be implemented via a sequence of dominant strategy auctions of the Vickrey type.¹⁸ In each period, both the procurer and firms observe the current state of the project, ρ^j . Each firm also learns its private cost of undertaking the current task. Then, the procurer determines the cutoff, $c(\rho^j) = h^{-1}(\delta \Delta W(\rho^j))$, and commits to the following entry fee and payment structure, respectively:

$$\tilde{E}(c, \rho^j) = \begin{cases} \delta \Delta S(\rho^j), & \text{if } \min_i \{c_i\} \leq c(\rho^j) \\ 0, & \text{if } \min_i \{c_i\} > c(\rho^j) \end{cases},$$

$$\tilde{P}_i(c, \rho^j) = \begin{cases} \min\{c(\rho^j), \min_{j \neq i} \{c_j\}\}, & \text{if } c_i \leq \min\{c(\rho^j), \min_{j \neq i} \{c_j\}\} \\ 0, & \text{otherwise} \end{cases}.$$

Unlike the static Vickrey auction, the procurer also needs to discourage non-participation in the dynamic setting to correct the incentives for participating bidders. For instance, a bidder whose cost is above the threshold knows that he will not win the

¹⁸ I conjecture that modified versions of other simple auctions such as the first-price auction can implement the optimal mechanism as well.

current auction. Such a bidder will thus have a strict incentive not to participate if doing so enables him to avoid the entry fee. If this behavior is not prevented by the procurer, then the bidders whose costs are close to but less than the threshold might also find it optimal not to participate, given that their chances of winning are small. One way of correcting the incentives is for the procurer to commit to stalling the auction for one period and charging the participants in that period nothing, unless all bidders participate.

The intuition behind the entry fee is that whenever an admissible cost, i.e., below the cutoff, is reported by some firm, the subproject ρ^j is assigned with certainty. This ensures the progress of the project, and creates an additional surplus to all suppliers. The procurer taxes away this increase in surplus by the entry fee. Conversely, if no admissible cost is reported, no progress will be made in the current period, and thus the fee will be 0. The payment structure is also intuitive: As in a one-shot Vickrey auction, the winner is paid the second highest cost unless this cost is above the cutoff. It is easy to see that this auction induces truth-telling as a dominant strategy and selects the lowest-cost supplier. Furthermore, it yields the same expected rents to the suppliers and the procurement cost to the buyer, as the optimal mechanism.¹⁹

5. Comparative statics

In this section, I consider two important comparative statics: First, I determine how the procurer would change her optimal strategy if she faced more suppliers in each stage, and more importantly, I determine whether or not the procurer and suppliers would prefer greater competition. Second, I explore whether the procurer prefers to deal with long-sighted suppliers who care more about future returns.

5.1. The effects of the number of suppliers

To better see the effects of the number of suppliers in the dynamic setting, first consider its static counterpart. Suppose the buyer auctions off a project that yields a value $V > 0$ if it is completed in the current period. Otherwise, the project is abandoned and all parties receive 0 returns. This static setting is a special case of the dynamic setting with $B(\rho^j) = S_j(\rho^j) = 0$ for $j \geq 1$. Thus, the analysis above remains valid. In particular, the procurer follows the same optimal selection rule in (6) with the cutpoint c_0 satisfying:

$$h(c_0) = \delta V \quad (15)$$

The important observation here is that the cutoff in a static setting is independent of the number of suppliers, e.g., see [Laffont and Tirole \(1987\)](#), [McAfee and McMillan \(1987\)](#), and [Riordan and Sappington \(1987\)](#). However, when suppliers bid for a large-scale project repeatedly, an additional supplier changes the incentives for the existing suppliers and for

¹⁹ It is important to note that if the procurer could commit to long-term agreements, she could tax away all but the first period rents by requiring suppliers pay an entry fee that is equal to their expected profits from future auctions. Absent such a commitment power, however, the procurer is unlikely to implement this up-front fee, as she will be tempted to recharge suppliers after each auction, making them unwilling to pay the initial amount.

the procurer by changing their expectations about the progress of the project. These expectations affect the endogenous valuation of the project, which in turn affects the current auction design by the procurer. The following result records how the procurer optimally changes the auction design in each stage:

Proposition 4.

- (a) $c(\rho^j; n+1) < c(\rho^j; n)$,
 (b) For a sufficiently large m , there exists m_0 such that for $j > m_0$, $c(\rho^j; n+1) > c(\rho^j; n)$.

According to Proposition 4, the procurer optimally changes the selection rule in (6) in response to an additional supplier. The direction of this change depends crucially on the state of the project. Consider the last stage of a large-scale project or, equivalently, a one-stage project. Whereas one more supplier increases competition, the future value of the project at this stage remains fixed at V . Hence, the increased competition reduces the value of waiting for suppliers, which, in turn, leads the buyer to be more demanding by optimally lowering the cutoff cost. In the initial stages, however, the increased competition also raises the future value of the project, whereby the buyer optimally sets a higher cutoff to facilitate its progress.²⁰

What does this nonmonotonicity imply for parties' preferences toward greater competition? The following proposition answers this question:

Proposition 5.

- (a-i) $S_i(\rho^j; n+1) < S_i(\rho^j; n)$,
 (a-ii) For a sufficiently large m , there exists m_0 such that for $j > m_0$, $S_i(\rho^j; n+1) > S_i(\rho^j; n)$,
 (b) For $j \geq 1$, both $B(\rho^j; n)$ and $W(\rho^j; n)$ are increasing in n .

Parts (a-i) of Proposition 5 indicates that firm i 's expected surplus is nonmonotonic in the number of suppliers. In particular, each supplier would favor more suppliers in the initial stages of the project. This is in sharp contrast with the static setting in which suppliers are always worse off by the presence of additional suppliers—simply because this lowers the expected transfer from the buyer by reducing the probability of winning. While an additional supplier might reduce the likelihood of winning the current auction in the dynamic setting, it also ensures the progress of the project. As the project moves forward, its value to the buyer increases, leading her to become more willing to make greater payments. However, in order to receive these higher payments in mature states, each supplier would then prefer to be a monopoly against the buyer, as recorded in part (a-i).

As alluded to in the Introduction, one practical implication of part (a-ii) is that for sufficiently large projects, existing firms might have an incentive to actively search for

²⁰ The dependence of the cutoff value on the number of participants in an auction also arises in McAfee and Vincent (1997), who analyze a setting where the seller is unable to reauction an object, e.g., a fine wine, if it fails to sell in previous periods. They find that the cutoff value is monotonically decreasing in the number of buyers.

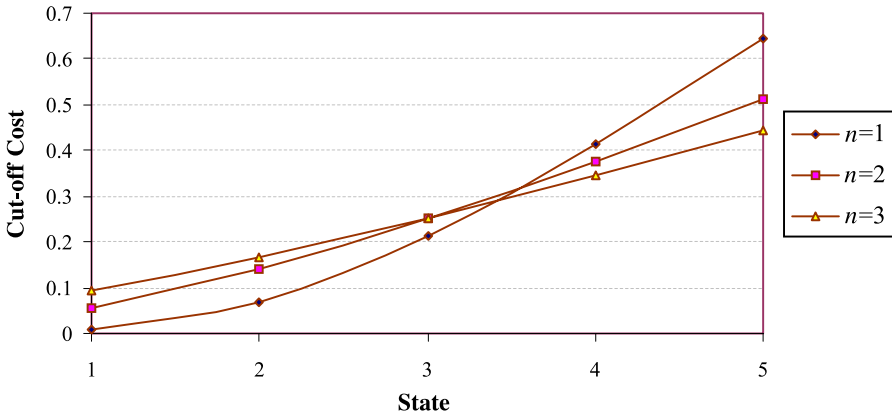


Fig. 2. Number of suppliers, project size, and the selection rule.

new participants in the auction, say by subsidizing their entry, along with the procurer. This is especially interesting in light of Bulow and Klemperer (1996) who argue that attracting one more bidder might be more valuable to the procurer than having more bargaining power. However, in their setting, it is only the auctioneer who has an incentive to search for new bidders.

According to part (b), the procurer prefers to have more suppliers in each stage of the project, as one would expect.

I further demonstrate the nonmonotonicity of $c(\rho^j; n)$ and $S_i(\rho^j; n)$ within an example as depicted in Figs. 2 and 3. In the example, there are five stages to complete the project. The project yields a value $V=3$ to the procurer upon its completion and suppliers' costs are distributed uniformly in $[0,4]$. Both the procurer and suppliers discount future returns by $\delta=0.9$.

As Fig. 2 illustrates, one more supplier leads the procurer to increase the cutoff in the initial three stages and reduce it in the remaining two stages. For the existing suppliers, an additional supplier has a direct effect of reducing the probability of winning. However,

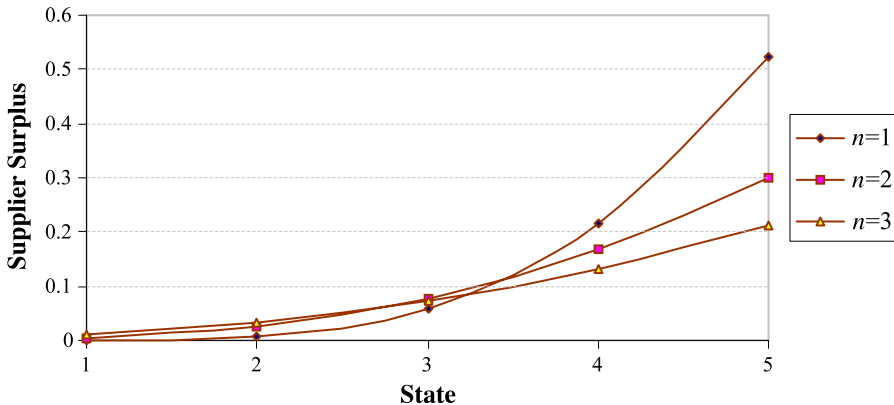


Fig. 3. Number of suppliers, project size, and supplier surplus.

since it also leads the procurer to increase the cutoff in the initial stages, the overall effect on suppliers' surplus is ambiguous. In fact, as Fig. 3 reveals, in the first two stages, each supplier favors one more entry into the competition. In the third stage, adding a third supplier reduces the surplus below a two-supplier competition. In the remaining two stages, because the procurer reduces the cutoff, both the direct and indirect effects move in the same direction and thus lower suppliers' surplus.

5.2. Supplier discounting and the project size

In this subsection, I ask the following question: Would the procurer be better off dealing with long-sighted or more patient suppliers? To answer this question, consider first a one-stage project. Recall that the winning supplier exerts a negative externality on others in the last stage, i.e., $\Delta S_i(\rho^1) < 0$. Because more patient suppliers with a higher discount factor place more weight on this negative change, they would demand higher payments than less patient suppliers.²¹ Thus, it is intuitive that for a one-stage project, the procurer would prefer suppliers with low discount factors. Consider now a large-scale project. In light of Proposition 3, suppliers' surplus grows as the project moves forward. While the impatient suppliers would place little weight on the future increase in surplus, more patient suppliers would internalize it and thus be willing to offer lower prices to facilitate the progress of the project. This means the procurer faces a trade-off when dealing with *patient* suppliers in a large-scale project: On the one hand, they offer lower prices in the initial stages of the project, benefiting the procurer. On the other hand, they raise the prices in the mature stages. Intuitively, which effect dominates should depend on the size of the project. In particular, for a sufficiently large project, the sum of initial price discounts should dominate the future price increases. Letting β and δ denote suppliers' and the procurer's discount factors respectively, the following result confirms this intuition:

Proposition 6. For $0 \leq \beta_0 < \beta_1 < 1$,

(a) $B(\rho^1; \beta_1) < B(\rho^1; \beta_0)$.

(b) For a sufficiently large m , there exists m_0 such that for $j > m_0$, $B(\rho^j; \beta_1) > B(\rho^j; \beta_0)$.

From the procurer's point of view, Proposition 6 highlights the important relationship between suppliers' discount factors and the size of the project. While she would prefer to deal with patient suppliers for sufficiently large projects, she would be better off dealing with impatient suppliers for small-scale projects. Of course, this raises the obvious question: Why doesn't the procurer deal with patient suppliers early on and enjoy price discounts, and then deal with the less patient ones to prevent the undue price increases in future stages? While the present analysis does not take the discount factor as a choice variable for the procurer, it is clear that even a patient supplier who realizes that he is very likely to be replaced in the future would act like a less-patient supplier and raise the price.

²¹ This is clearly seen in the payment function given by (13).

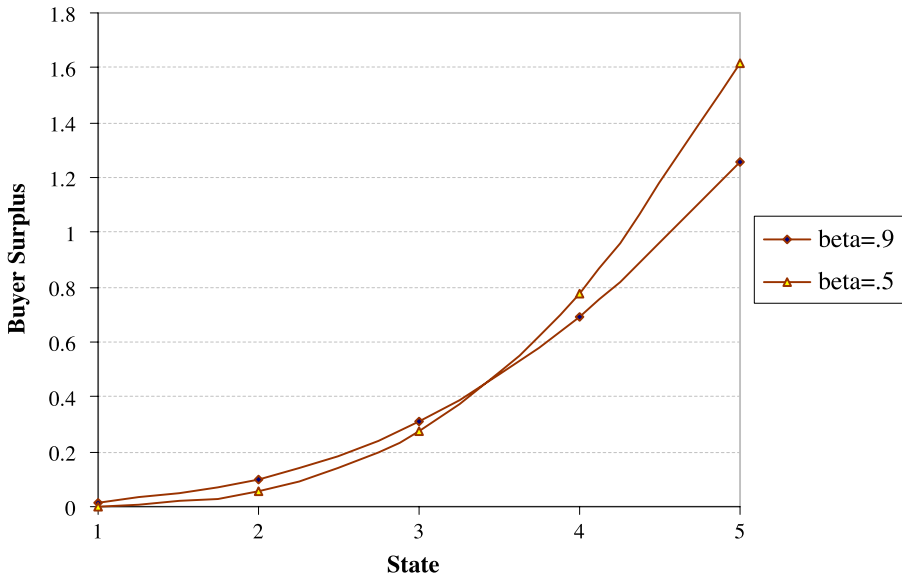


Fig. 4. Supplier discounting, project size, and buyer surplus.

To illustrate the result in Proposition 6, I reconsider the example introduced in Section 5.1 with two suppliers. Fig. 4 shows the buyer's expected value of the project when suppliers possess discount factors $\beta=0.9$ and $\beta=0.5$. It is clear that if the project has one or two subprojects, then the buyer is better off dealing with less patient suppliers. However, if the project is larger, more patient suppliers would yield a higher expected value to the buyer.

6. An extension: the optimal division of a large-scale project

In the analysis thus far, the division of the project is taken as given. This seems reasonable when the procurer cannot exceed a predetermined budget in each period and/or faces severe scheduling constraints, for example. Absent such concerns however, dividing a large project might still benefit the procurer in various ways. First, the procurer can take advantage of repeated competition. Second, suppliers with insufficient resources for the entire project might be enticed to participate. Third, there may be decreasing returns to scale in production.²²

To formally analyze the issue of project division, I consider a simple extension of the basic model in which the procurer first decides whether to auction off the project as a whole or divide it into subprojects. If it is divided, then subprojects are procured in a sequence, as

²² Papers by Anton and Yao (1987) and Rob (1986) focus on other reasons for why the procurer might stage a large project. Specifically, in their models, the first part of the project is contracted out with a sole source who improves its technology through learning-by-doing or R&D, and then the second part of the project is auctioned off among multiple firms. The main reason behind staging a project then is that the initial technological improvement is, at least partially, transferable to the future winner.

was previously analyzed. Suppose the project requires 1 unit of input. The unit price of input for supplier i is c_i . The assumptions on c_i are as in Section 2. Suppose that the procurer has the option to divide the project into $k \in \{1, 2, \dots\}$ equal subprojects, each of which requires $1/s(k)$ unit of input, where $s(1)=1$ and $s(k+1) \geq s(k)$. Upon dividing the project into pieces, whether or not the required input for each piece decreases proportionally is determined by the degree of scale economies in production. We say that there are constant (increasing) (decreasing) returns to scale if $s(tk) = (<)(>)ts(k)$ for all $t > 1$. The formal analysis of this case is exactly the same as in Section 2 except that supplier i 's cost for a subproject is $c_{ik} = c_i/s(k)$ rather than being c_i . The following proposition notes the main result of this section:

Proposition 7. *Suppose there are constant or increasing returns to scale. Moreover, suppose the number of firms remains unchanged by the division of the project. Then, the procurer strictly prefers to auction off the entire project without dividing it.*

To see the intuition behind the result and its proof, consider a project of value V and a technology with constant returns. Suppose the project is divided into two subprojects each of which requires $1/2$ unit input. To isolate the two opposing effects of the division, I make the following observation whose general form²³ is proved in the Appendix: From the procurer's perspective, completing a project of value V in two pieces each of which requires $1/2$ unit input, is equivalent to completing a project of value $2V$ in two pieces each of which requires 1 unit input. The fact that the final value of the project looks doubled shows the cost savings due to the repeated competition. The procurer now needs to compare two alternatives: Either procure a one-piece project with value V or procure a two-piece project with value $2V$, where each piece requires 1 unit input. Due to the sequential nature of the procurement, the latter alternative is equivalent to procuring the first piece of the project with a continuation value. Because part (a) of Proposition 1 implies that the total surplus increases at an increasing rate, this continuation value is strictly less than V for the total surplus and therefore for the buyer's surplus too.²⁴ Thus, procuring the project in two pieces is strictly less preferable to the procurer than procuring it as a whole.

A subsidiary intuition for Proposition 7 is gained by noting that a procurer implementing an optimal mechanism operates much like a monopsonist who reduces demand to cut costs. When the project is divided, the inability to commit to future contracts causes multiple demand reductions that work the same as the "double marginalization." This type of distortion, however, is avoided in a one-piece project, absent the diseconomies of scale in production.²⁵

The overall message of Proposition 7 is that for the division of a project to be beneficial to the procurer, there must be either decreasing returns to scale, e.g., capacity constraints, or an increase in the number of firms participating in the procurement competition.

²³ The general form of this observation (see the proof of Proposition 7) is that from the buyer's perspective, procuring a project of value V in one piece is equivalent to procuring a project of value $s(k)V$ in k pieces, where each piece requires 1 unit input.

²⁴ Mathematically, we essentially have an increasing and strictly convex function whose final value is $2V$. Then, it is clear that its value at the mid-point will be strictly less than V .

²⁵ I am grateful to a referee for this observation.

Continuing with the numerical example introduced in Section 5.1, the procurer would be better off if either the division attracted at least 3 more firms to the competition or there were significant scale diseconomies. To see the latter, suppose $s(k)=k^\alpha$, $\alpha>1$. In such a case, although the number of firms remains unchanged, the procurer would benefit from the division for $\alpha\geq 2.2$.

7. Concluding remarks

This paper presented a rich dynamic procurement model of how a buyer with limited commitment power procures a large-scale project in a piecewise fashion. While several new insights have emerged from the analysis, the model is simplistic in many ways and open to further extension. For one, as was mentioned in the Introduction, the winner of a subproject may gain experience and be a “stronger” bidder in future auctions. Following Lewis and Yildirim (2002a,b), this feature can easily be introduced. Yet, I believe new insights would emerge as to how the procurer would handicap the efficient bidder along with the progress of the project, whether or not the buyer will get locked into one supplier, and how this will affect the optimal division of the project.

Another interesting extension would allow for suppliers with different discounting. This is especially relevant when some suppliers are more likely to stay in business longer than others.²⁶ Based on the findings in Section 5.2, I conjecture that the procurer would set a high entry standard, e.g., a high entry fee, to deal only with patient suppliers when the project is sufficiently large. Otherwise, small firms will also be encouraged to participate.

Acknowledgement

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Appendix A

Proof of Lemma 1. Using the standard techniques, e.g., Fudenberg and Tirole (1991, ch. 7), it is easy to see that the second-order condition $\frac{\partial^2 S_i(\rho^j, c_i)}{\partial c_i^2} \leq 0$ for supplier i 's maximization in (IC _{i}) requires that $\lambda_i(\rho^j, c_i)$ be weakly decreasing in c_i . Moreover,

²⁶ Jofre-Bonet and Pesendorfer (2000) note this feature in their empirical analysis of California highway construction auctions.

because from (1), $S_i(\rho^j, c_i)$ is weakly decreasing in c_i , (IRi) is satisfied if and only if $S_i(\rho^j, \bar{c}) = \delta R_i(\rho^j)$. Using this boundary condition and integrating (1) over c_i yield

$$S_i(\rho^j, c_i) = \delta R_i(\rho^j) + \int_{c_i}^{\bar{c}} \lambda_i(\rho^j, \tilde{c}_i) d\tilde{c}_i \quad (\text{A1})$$

Combining (A1) and (ICi), we derive the payment function in (2). In addition, taking expectation of both sides of (A1) with respect to c_i , we find the expected value stated in (3) for supplier i . \square

Proof of Lemma 2. Differentiating $\Phi(x; \delta)$ with respect to x and using Assumption 1, part (a) easily follows.

To prove part (b), note that supplier i 's expected value can be rewritten as:

$$(1 - \delta)S_i(\rho^j) = \delta[R_i(\rho^j) - S_i(\rho^j)] + E_{c_i} \left[\lambda_i(\rho^j, c_i) \frac{F(c_i)}{f(c_i)} \right] \quad (\text{A2})$$

In equilibrium, summing (5) and (A2) for all i , we obtain

$$\bar{W}(\rho^j) = \sum_{i=1}^n E_c \lambda_i(\rho^j, c) [-c_i + \delta \Delta W(\rho^j)] \quad (\text{A3})$$

where $j \geq 1$ and $\bar{W}(\rho^j) \equiv (1 - \delta)\bar{W}(\rho^j)$.

Using the optimal selection rule in (6) and integrating the r.h.s. of (A3) by parts yield $\bar{W}(\rho^j) = \Phi(c(\rho^j); 1)$. Furthermore, $h(c(\rho^j)) = \delta \Delta W(\rho^j)$ implies that $\bar{W}(\rho^{j-1}) = \Phi(c(\rho^j); \delta)$ where $\Phi(\cdot)$ is as defined in (7). \square

Proof of Proposition 1. I use backward induction. Because $\Phi(0; \delta) = 0$, $\Phi(c; \delta) > (1 - \delta)V$ by Assumption 2, and $\Phi'(x; \delta) > 0$ for $x > 0$, there exists a unique $c(\rho^1) \in (0, \bar{c})$ that solves $\Phi(c(\rho^1); \delta) = \bar{W}(\rho^0) = (1 - \delta)V$. Now consider $j = 2$. By using similar arguments, one can conclude that there exists a unique $c(\rho^2) \in (0, \bar{c})$ that solves $\Phi(c(\rho^2); \delta) = \bar{W}(\rho^1) = \Phi(c(\rho^1); 1) < (1 - \delta)V$. Furthermore, because $\Phi(c(\rho^1); 1) < \Phi(c(\rho^1); \delta)$, we have $c(\rho^2) < c(\rho^1)$. To complete the induction argument, suppose, for some ρ^j , there exists a unique $c(\rho^j) \in (0, \bar{c})$. The exact arguments above imply the existence of a unique $c(\rho^{j+1}) \in (0, \bar{c})$ that solves $\Phi(c(\rho^{j+1}); \delta) = \bar{W}(\rho^j)$. This completes the proof of part (a).

To prove part (b), recall that $h(c(\rho^j)) = \delta \Delta W(\rho^j)$. Because $c(\rho^j) > 0$ and $h'(\cdot) > 0$, the result follows. \square

Proof of Proposition 2. We start with part (c). Note that, at the optimal solution, $S_i^*(\rho^j, c_i) = \delta R_i^*(\rho^j) = \delta S_i^*(\rho^j)$, which implies that $S_i^*(\rho^j) = 0$. Now note from (9) that the solution to the complete information case is the same as the incomplete information case with $h^*(x) \equiv x$. From here, the existence of a unique MPE and the part (a) immediately follow. To prove part (b), first observe that $\Phi(x) > \Phi^*(x)$ for all $x > 0$. Because $W(\rho^j) < W^*(\rho^j)$, this implies that $c(\rho^j) < c^*(\rho^j)$. \square

Proof of Proposition 3. Follows directly from part (a) of Proposition 1, and Eqs. (11) and (12). \square

Before proceeding to the proofs of Propositions 4 and 5, I first record the following useful result.

Lemma A1. $\lim_{m \rightarrow \infty} c(\rho^m) = 0$.

Proof. Note that the sequence $\{c(\rho^j)\}_{j=1}^{j=m}$ is decreasing and bounded below by 0. Thus, it converges to some $\alpha \geq 0$. Since every subsequence of a convergent sequence also converges to the same limit, it follows that $\lim_{m \rightarrow \infty} c(\rho^m) = \lim_{m \rightarrow \infty} c(\rho^{m+1})$. From the recursion described in Lemma 1 and by continuity of $\Phi(\cdot)$, this implies that $\Phi(x; \delta) = \Phi(x; 1)$, or $[(1 - \delta)/\delta]h(x) = 0$, whose unique solution is, $\alpha = 0$. \square

Proofs of Propositions 4 and 5. I first show that for $j \geq 1$, $W(\rho^j; n) < W(\rho^j; n+1)$. I proceed by induction. Suppose $W(\rho^1; n) \geq W(\rho^1; n+1)$. Then, because $W(\rho^0; n) = W(\rho^0; n+1) = V$, we have $\Delta W(\rho^1; n) \leq \Delta W(\rho^1; n+1)$. This implies $c(\rho^1; n) \leq c(\rho^1; n+1)$ due to $h(c(\rho^j)) = \delta \Delta W(\rho^j)$. However, since $\Phi(x; \delta)$ is increasing in both x and n , we must have $W(\rho^1; n) < W(\rho^1; n+1)$, a contradiction. Thus, $W(\rho^1; n) < W(\rho^1; n+1)$.

Now suppose $W(\rho^j; n) < W(\rho^j; n+1)$ for some $j \geq 1$. By way of contradiction, however, suppose also $W(\rho^{j+1}; n) \geq W(\rho^{j+1}; n+1)$. Since, from Proposition 1, $W(\rho^j; n)$ is increasing in ρ^j , we have

$$W(\rho^{j+1}; n+1) \leq W(\rho^{j+1}; n) < W(\rho^j; n) < W(\rho^j; n+1)$$

which, in turn, implies $\Delta W(\rho^{j+1}; n+1) > \Delta W(\rho^{j+1}; n+1)$. From $h(c(\rho^j)) = \delta \Delta W(\rho^j)$, this yields $c(\rho^{j+1}; n+1) > c(\rho^{j+1}; n)$. However, this means $W(\rho^{j+1}; n) < W(\rho^{j+1}; n+1)$, a contradiction. Thus, $W(\rho^{j+1}; n) < W(\rho^{j+1}; n+1)$.

Next, I prove Proposition 4.

Once again, the fact that $W(\rho^0; n) = W(\rho^0; n+1) = V$ and $W(\rho^1; n) < W(\rho^1; n+1)$ implies that $\Delta W(\rho^1; n+1) < \Delta W(\rho^1; n)$, which, together with $h(c(\rho^j)) = \delta \Delta W(\rho^j)$, yields $c(\rho^1; n+1) < c(\rho^1; n)$.

To prove part (b), first note that for x close to 0, the first-order Taylor expansion on $\Phi(x; \delta)$ yields

$$\begin{aligned} \Phi(x; \delta) &\approx \Phi(0; \delta) + \Phi'(0; \delta)x \\ &= \frac{1 - \delta}{\delta} h'(0)x \end{aligned} \tag{A4}$$

From Lemma A1, (A4) reveals that for sufficiently large m , there exists m_0 such that for $j > m_0$:

$$\bar{W}(\rho^j; n+1) \approx \frac{1 - \delta}{\delta} h'(0)c(\rho^j; n+1) \tag{A5}$$

$$\bar{W}(\rho^j; n) \approx \frac{1 - \delta}{\delta} h'(0)c(\rho^j; n) \tag{A6}$$

Given that $W(\rho^j; n)$ is strictly increasing in n , (A5) and (A6) imply that $c(\rho^j; n+1) > c(\rho^j; n)$. This completes the proof of Proposition 4.

Now, I complete the proof of Proposition 5.

Recall each supplier's surplus in (12):

$$S_i(\rho^j; n) = \frac{1}{1 - \delta} \int_0^{c(\rho^j; n)} [1 - F(c)]^{n-1} F(c) dc \tag{A7}$$

Part (a) of Proposition 4 and (A7) imply that $S_i(\rho^1;n+1) < S_i(\rho^1;n)$. To prove part (a-ii), define the function: $H(x) \equiv \int_0^x [1 - F(c)]^{n-1} F(c) dc$. For x close to 0, we have

$$\begin{aligned}
 H(x) &\approx H(0) + H'(0)x + H''(0) \frac{x^2}{2} \\
 &= f(0) \frac{x^2}{2}
 \end{aligned}
 \tag{A8}$$

by a second-order Taylor expansion. Using Lemma A1 and (A8), we can approximate $S_i(\rho^j;n)$ for sufficiently large j by:

$$S_i(\rho^j;n) \approx \frac{1}{1-\delta} f(0) \frac{c^2(\rho^j;n)}{2}
 \tag{A9}$$

From (A9) and part (b) of Proposition 4, we obtain the desired result in part (a-ii).

To show that $B(\rho^j;n)$ is increasing in n . Recall that $B(\rho^j;n) = \frac{1}{1-\delta} \int_0^{c(\rho^j;n)} h'(c) G(c;n) dc$. Note that since $c(\rho^j; \cdot)$ is a choice variable for the procurer, whenever there is one more firm, the procurer could simply choose $c(\rho^j;n+1) = c(\rho^j;n)$ and be better off. Because she chooses $c(\rho^j;n+1) \neq c(\rho^j;n)$, it must be that $B(\rho^j;n+1) > B(\rho^j;n)$. \square

Proof of Proposition 6. Let δ and β be the buyer’s and suppliers’ discount factors, respectively. Proceeding as in the previous case with the same discount factor, we see that the equilibrium sequence of $c(\rho^j)$ is determined by the following recursive equations:

$$\begin{aligned}
 B(\rho^0) &= V \text{ and } S_i(\rho^0) = 0 \\
 B(\rho^j) &= \frac{1}{1-\delta} \int_0^{c(\rho^j)} h'(c) G(c;n) dc \\
 S_i(\rho^j) &= \frac{1}{1-\beta} \int_0^{c(\rho^j)} [1 - F(c)]^{n-1} F(c) dc \\
 h(c(\rho^j)) &= \Delta \tilde{W}(\rho^j)
 \end{aligned}
 \tag{A10}$$

where $\tilde{W}(\rho^j) \equiv \delta B(\rho^j) + \beta \sum_{i=1}^n S_i(\rho^j)$.

Following the same steps in the Proof of Proposition 1, one can show from (A10) that there exists a unique MPE in this case as well. Moreover, the previous results in Propositions 1–5 hold. Now, to prove the result in Proposition 6, I first show that $\tilde{W}(\rho^j;\beta)$ is increasing in β . Take arbitrary β_0 and β_1 such that $\beta_0 < \beta_1$. Now suppose $\tilde{W}(\rho^1;\beta_1) \leq \tilde{W}(\rho^1;\beta_0)$. Because $\tilde{W}(\rho^0;\beta_1) = \tilde{W}(\rho^0;\beta_0) = \delta V$, (A10) implies that $c(\rho^1;\beta_0) \leq c(\rho^1;\beta_1)$, which in turn implies that $\tilde{W}(\rho^1;\beta_1) > \tilde{W}(\rho^1;\beta_0)$, a contradiction. Thus, $\tilde{W}(\rho^1;\beta_1) > \tilde{W}(\rho^1;\beta_0)$. To complete the induction argument, suppose for some $j \geq 1$ that $\tilde{W}(\rho^j;\beta_1) > \tilde{W}(\rho^j;\beta_0)$, but on the contrary, that $\tilde{W}(\rho^{j+1};\beta_1) \leq \tilde{W}(\rho^{j+1};\beta_0)$. Because $\tilde{W}(\rho^j)$ is decreasing in j , we have $\tilde{W}(\rho^{j+1};\beta_1) \leq \tilde{W}(\rho^{j+1};\beta_0) < \tilde{W}(\rho^j;\beta_0) < \tilde{W}(\rho^j;\beta_1)$. This means $c(\rho^{j+1};\beta_0) < c(\rho^{j+1};\beta_1)$. However, (A10) implies that $\tilde{W}(\rho^{j+1};\beta_1) > \tilde{W}(\rho^{j+1};\beta_0)$, a contradiction. Thus, $\tilde{W}(\rho^{j+1};\beta_1) > \tilde{W}(\rho^{j+1};\beta_0)$.

To prove part (a) of Proposition 6, note that because $\tilde{W}(\rho^1; \beta)$ is increasing in β , we have $c(\rho^1; \beta_1) < c(\rho^1; \beta_0)$. Thus, $B(\rho^1; \beta_1) < B(\rho^1; \beta_0)$. To prove part (b), note that Lemma A1 holds here as well. Thus, a second-order Taylor expansion on $\tilde{W}(\rho^j; \beta)$ for sufficiently small ρ^j yields:

$$\tilde{W}(\rho^j; \beta) \approx nf(0) \left[\frac{\delta}{1-\delta} h'(0) + \frac{\beta}{1-\beta} \right] \frac{c^2(\rho^j; \beta)}{2}$$

Because $\tilde{W}(\rho^j; \beta)$ is increasing in β , we have $c(\rho^j; \beta_0) > c(\rho^j; \beta_1)$, which implies that $B(\rho^j; \beta_1) > B(\rho^j; \beta_0)$. \square

Appendix B

I first note the following useful result:

Lemma B1. Consider the model in Section 2. Suppose two projects, X and Y , are of equal size but with different final payoffs, V_X and V_Y where $V_X < V_Y$. Then, for any $j \geq 1$, $B_X(\rho^j) < B_Y(\rho^j)$.

Proof of Lemma B1. Given (11), it suffices to show that for any $j \geq 1$, $c_X(\rho^j) < c_Y(\rho^j)$. Suppose, on the contrary, that $c_X(\rho^j) \geq c_Y(\rho^j)$. This means $W_X(\rho^j) \geq W_Y(\rho^j)$. Because $V_X < V_Y$, we have $c_X(\rho^1) < c_Y(\rho^1)$, a contradiction. Now suppose for some $j \geq 1$, $c_X(\rho^j) < c_Y(\rho^j)$, but, on the contrary, $c_X(\rho^{j+1}) \geq c_Y(\rho^{j+1})$. This implies that $W_Y(\rho^{j+1}) \leq W_X(\rho^{j+1}) < W_X(\rho^j) < W_Y(\rho^j)$, where the second inequality follows from Proposition 1. However, this implies $c_X(\rho^{j+1}) < c_Y(\rho^{j+1})$, a contradiction. \square

Proof of Proposition 7. Note first that the distribution and density functions of $c_{ik} = c_i/s(k)$ are given by $F_k(c_{ik}) = F(s(k)c_{ik})$ and $f_k(c_{ik}) = f(s(k)c_{ik})s(k)$, respectively. Second, to find the sequence of $\{c_k(\rho^j)\}$, we compute $\Phi_k(x_k; \delta)$ by substituting x with x_k and the distribution with $F_k(c_{ik})$ in (7) as:

$$\begin{aligned} \Phi_k(x_k; \delta) &= \frac{1-\delta}{\delta} h_k(x_k) + \frac{F_k(x_k)}{f_k(x_k)} G_k(x_k; n) + \int_0^{x_k} G_k(c; n) dc \\ &= \frac{1}{s(k)} \Phi(x; \delta) \end{aligned} \tag{B1}$$

where we make a change of variable by $x_k \equiv x/s(k)$. Thus, the following set of equations generates the unique sequence of $\{c(\rho^j)\}$ where $c_k(\rho^j) \equiv c(\rho^j)/s(k)$:

$$\begin{aligned} W_k(\rho^0) &= V; (1-\delta)W_k(\rho^{j+1}) = \frac{1}{s(k)} \Phi(c(\rho^j); \delta), \text{ and} \\ (1-\delta)W_k(\rho^j) &= \frac{1}{s(k)} \Phi(c(\rho^j); 1) \end{aligned} \tag{B2}$$

Letting $\tilde{W}_k(\rho^j) \equiv s(k)W_k(\rho^j)$, and $\tilde{B}_k(\rho^j) \equiv s(k)B_k(\rho^j)$, we can rewrite (B2) as:

$$\begin{aligned} \tilde{W}_k(\rho^0) &= s(k)V; (1-\delta)\tilde{W}_k(\rho^{j+1}) = \Phi(c(\rho^j); \delta), \text{ and} \\ (1-\delta)\tilde{W}_k(\rho^j) &= \Phi(c(\rho^j); 1) \end{aligned} \tag{B3}$$

Note that (B3) essentially converts a project division problem into the one analyzed in Section 2 by summarizing the effect of division only on the final value. Now, we are ready to prove the conclusion of Proposition 7.

Suppose that $s(tk) \leq ts(k)$ for $t > 1$ and that the number of suppliers is unaffected by the division of the project. Take any $k \in \{1, 2, \dots\}$. We want to show that the buyer prefers dividing the project into k subprojects over $k+1$, i.e., $B_{k+1}(\rho^{k+1}) < B_k(\rho^k)$. According to (B3), $\tilde{B}_{k+1}(\rho^{k+1})$ is buyer's expected surplus from a project with $(k+1)$ subprojects whose final value is $s(k+1)V$, or equivalently $\tilde{B}_{k+1}(\rho^{k+1})$ is the expected surplus from a project with k subprojects whose final value is $B_{k+1}(\rho^1)$. With this interpretation in mind, if we can compare $B_{k+1}(\rho^1)$ and $s(k)V$, then we reach the conclusion by Lemma B1. Now, recall that, from Proposition 1, the total surplus increases at an increasing rate as the project moves forward. Thus, $\tilde{W}_{k+1}(\rho^1) < [k/(k+1)]s(k+1)V$. Letting $t = (k+1)/k$, $s(tk) \leq ts(k)$ implies that $[k/(k+1)]s(k+1) \leq s(k)$. This means $B_{k+1}(\rho^1) < \tilde{W}_{k+1}(\rho^1) < s(k)V$. Lemma B1, then, implies that $\tilde{B}_{k+1}(\rho^{k+1}) < B_k(\rho^k)$, which in turn implies $s(k+1)B_{k+1}(\rho^{k+1}) < s(k)B_k(\rho^k)$. Because $s(k+1) \geq s(k)$, we have $B_{k+1}(\rho^{k+1}) < B_k(\rho^k)$. \square

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