Time-Consistent Majority Rules with Interdependent Valuations

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Abstract

This paper studies collective decision-making with interdependent valuations, where, unlike the extant literature, the committee cannot commit to a majority rule, necessitating it to be ex post optimal or "time-consistent". We find that (1) a majority rule is time-consistent if and only if the expected number of affirmative votes it generates is approximately the rule itself; (2) the preference interdependence exacerbates the commitment problem by creating incentives for strategic (pivotal) voting and in turn expanding the set of time-consistent rules; in particular, any majority rule, including the unanimity, becomes time-consistent under pure common values; and (3) as the committee size grows without bound, the commitment problem disappears owing to the dilution of incentives for strategic voting.

JEL Classifications: C7, D7

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1 Introduction

Decision-making by committees is commonplace in modern organizations. Perhaps, the greatest advantage of a committee is its ability to draw upon diverse opinions of its members. To the extent that these opinions are successfully shared, the committee will make a better decision. There are, however, two potential difficulties: First, information leading to one's opinion is likely to be private; and second, each member is likely to favor his own

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information. One source of such a "bias" could be representing a different constituency as would be the case for members of an inter-disciplinary university committee or a Food and Drug Administration (FDA) advisory panel composed of scientists and various industry and consumer advocates. Another source of the bias could be individuals' commonly known cultural, psychological, and demographic backgrounds that distort their processing of others' information.[1] Yet a third source of the bias could be one's field of speciality as noted by Moldovanu and Shi (2012). These authors cogently argue that the growing complexity of issues in the modern world often requires decision-making by a committee of "specialists". Indeed, a multidisciplinary team approach is being increasingly recommended and adopted in many areas of health care (Porter and Olmsted-Teisberg, 2007; Schuetz et al., 2010).[2]

While communication in real committees is probably more complicated, in this paper, we follow the extensive literature on committees[3] and assume that opinions are expressed by binary votes.[4] We depart from the literature by considering situations in which the committee cannot commit to a decision rule; instead, given the votes, the committee takes the best action that maximizes its members' joint payoff. For instance, it is hard to imagine that: a team of physicians would carry out a radical surgery when their opinions suggest a serious residual risk for the patient's health;[5]a university will start a new interdisciplinary program when the votes still cast doubt on its value; or the FDA will approve a new drug when the advisory panel's votes do not fully eliminate the safety and efficacy concerns.[6]

Our model consists of a group of agents who vote whether or not to implement a "project". Each agent receives an independent private signal about the project, but his valuation of the project is a weighted average of all the signals, placing a (weakly) greater weight on his own.

[1]A similar explanation is often used for why agents may "agree to disagree"; e.g. Morris (1995).

[2]Aside from the complexity of the cases, such an approach is also believed to alleviate the specialist bias. For instance, Jang et. al (2010) report that one-third of prostate cancer patients who had only seen a urologist underwent prostate surgery while radiation therapy was the most common treatment for those patients who visited both a urologist and a radiation oncologist.

[3]See Gerling et al. (2005), and Li and Suen (2009) for reviews.

[4]Li et al. (2001) show that a general voting or scoring procedure can be the optimal mechanism to pool information in committees since absent side payments, an optimal decision rule mitigates members' incentives to manipulate private (continuous) information by "garbling" it almost everywhere. They also show that the garbling categories depend on the degree of preference heterogeneity – the greater the heterogeneity is, the less fine the categories are. In this sense, our restriction to binary or two-category voting is not without loss of generality, but it is often used in practice and consistent with the literature.

[5]In fact, evidence suggests that the fear of malpractice liability drives physicians to adopt "defensive medicine" (Kessler and McClellan, 1996).

[6]The FDA is not required to follow the panel's advice, but it often does. According to one study, the FDA overruled only 26% of such recommendations between 2007 and 2010 (*Forbes*, 10/12/2010). In this paper, if the decision-maker is different from the committee, we will assume that her objective coincides with the committee's.

In the language of auction theory, agents have (linearly) *interdependent* valuations, ranging from pure private values (with no regard for others' signals) to pure common values (with equal weight given to them).[7] Agents simultaneously submit their votes to a utilitarian social planner who values each signal equally, i.e., the committee as whole is less biased than each of its members. The planner accepts the project if its expected payoff exceeds that of the status quo, which amounts to choosing the majority rule conditional on the votes. In effect, agents play a simultaneous game with each other as well as with the planner. Not surprisingly, only the ex post optimal, or *time-consistent*, majority rules can emerge in equilibrium. Identifying these rules is important because it relates to the commitment problem and affects information aggregation in the committee.

We make three key observations. First, a majority rule k is time-consistent if and only if the expected number of affirmative votes it generates is approximately k. Otherwise, if the expected number were, say, much higher, then this would be indicative of too lenient a standard for an affirmative vote and lead to the rejection –rather than the acceptance– of the project upon receiving exactly k affirmative votes.

Second, as the degree of preference interdependence increases, the set of time-consistent majority rules expands. This is due to an increased level of "strategic" voting: agents with stronger interdependencies place a greater weight on the information gleaned from being pivotal, making their votes more sensitive to the majority rule. This implies that the commitment problem is more severe, the stronger the interdependencies are. For instance, the planner would face no commitment problem if agents possessed pure private values since they would always vote "sincerely", i.e., based only on their own signals. In fact, we show that ignoring integer problems, the unique time-consistent rule under pure private values is the ex ante optimal one whereas *any* majority rule, including the unanimity, becomes time-consistent under pure common values.

Third, as the committee size grows without bound, the only "percentage" rule that is time-consistent is the one for pure private values. This makes sense: in a large group, each agent has a sharper prediction of the average of others' signals, which lowers the amount of information that can be inferred from the pivotal event and in turn weakens the incentives for strategic voting.

Related Literature. Building on the Condorcet Jury Theorem, there is an early literature on voting as a means of information aggregation in committees, which is ably summarized

[7]See Krishna (2009) for a review.

by Gerling et al. (2005), and Li and Suen (2009). Austen-Smith and Banks (1996) pointed out that sincere voting assumed in this literature is unlikely to occur in equilibrium "even when individuals have [such] a common preference."[8] Our analysis reveals that individuals with a common preference may actually vote the most strategically. Feddersen and Pesendorfer (1997, 1998) have investigated the consequences of strategic voting on information aggregation in large common value elections; and in particular, they have demonstrated that the unanimity rule performs poorly in this regard. Duggan and Martinelli (2001) extend their results to a continuous signal space – a model we discuss in Section 5. Our investigation complements these papers by also uncovering that without integer problems, the unanimity rule is never ex ante optimal, but it is always time-consistent (or ex post optimal) under common preferences.[9] That is, if jurors were not told a voting rule, they could well form rational beliefs about the unanimity being the rule and vote accordingly.

Our paper also relates to several recent studies on optimal decision rules in committees with common values. Among them, Gerardi and Yariv (2008), Gersbach (1995), Gershkov and Szentes (2009), Li (2001), and Persico (2004) show that the ex ante optimal rule is, in general, ex post inefficient. This means that the social planner would face an ex post commitment problem but only to incentivize agents to acquire costly information, which is not a feature of our setting.

Finally, as in ours, Gruner and Kiel (2004), and Moldovanu and Shi (2012) also represent committee members' biases via interdependent valuations. These papers, however, fix decision rules; so their optimality or time-consistency are not at stake.[10]

The remainder of the paper is organized as follows. In the next section, we lay out the model. In Section 3, we characterize equilibrium voting for a fixed majority rule. In Section 4, we study time-consistent majority rules. We discuss the time-consistency within a Condorcet-type model in Section 5, followed by a discussion of nonlinear interdependencies in Section 6. Section 7 concludes. Proofs that do not appear in the text are relegated to an appendix.

[8] Ali et al. (2008) offer some experimental evidence in favor of strategic voting in a Condorcet-type model.

[9] See also Costinot and Kartik (2007) for a complementary explanation as to the ex ante suboptimality of the unanimity rule under common preferences.

[10] The time-consistency problem also arises in dynamic models of collective decision-making due to shocks to the economy. See, e.g., Dal Bo (2006) and Riboni (2010) who show that supermajority rules can strike the right balance between commitment and flexibility. These papers, however, assume commitment to the rule within each period whereas our model is static and focuses on the commitment problem within the period.

2 The Model

For concreteness, there is a committee that consists of $n \geq 2$ "specialist" members who decide whether to implement a "project"; e.g., undertaking an invasive surgery, hiring a job candidate, or approving a new drug. Based on his specialization or constituency, each member i can evaluate only one attribute of the project, which yields a private signal θ_i independently drawn from a common distribution F, with support $[\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}$ where $\underline{\theta} < 0 < \overline{\theta}$. F is assumed twice differentiable, with a positive density $f = F'$ in its interior, and whose mean, $E[\theta_i]$, is normalized to 0. Member i is biased toward his specialization but he also cares about other attributes of the project. We capture this "conflict" by the following interdependent payoffs, which is often exploited in the auction theory literature (see Krishna (2009) for a review):[11]

$$v_i = (1 - \alpha + \frac{\alpha}{n})\theta_i + \frac{\alpha}{n} \sum\nolimits_{j \neq i} \theta_j, \tag{1}$$

where $\alpha \in [0, 1]$. The parameter α measures the degree of interdependence in the committee; in particular, $\alpha = 0$ refers to pure private values whereas $\alpha = 1$ refers to pure common values. The reservation payoff from dismissing the project is 0. Given that $E[v_i] = 0$, this implies no status quo bias.[12] For technical convenience, we will restrict attention to $\alpha \in (0, 1)$, though one can always take the limits of α.

Based on their private signals, the committee members simultaneously submit "yes" or "no" votes to a social planner. The planner accepts the project if and only if there are k or more "yes" votes. The planner is a utilitarian agent who maximizes the committee's welfare:

$$w \equiv \frac{\sum_i v_i}{n} = \frac{\sum_i \theta_i}{n}. \tag{2}$$

Note that w is independent of α. That is, the degree of interdependence does not have a direct effect on the committee's welfare; but it will have an indirect effect through voting.

Remark 1 *The social planner can be viewed as a fictitious player who weighs each attribute equally. That is, committee members collectively agree that each piece of information should be treated equally in the final decision even though they are personally biased toward their own except for the special case of pure common values. The planner can also be viewed as a "real" player who delegates the decision to the committee or shares its objective.*

[11] We briefly discuss nonlinear payoffs in Section 6.

[12] Introducing a status quo bias, i.e., $E[\theta_i] \neq 0$, does not change our qualitative results, as is clear from the proof of Lemma A1, for instance.

We begin the analysis by characterizing voting behavior for a fixed majority rule, and then proceed to endogenizing the rule.

3 Equilibrium Characterization

Let the majority rule, k, be fixed and publicly known before votes are cast. Since signals are independent and v_i is strictly increasing in θ_i, it is readily verified that member i follows a cutoff strategy such that he votes "yes" if $\theta_i > \widehat{\theta}_i$, and "no" if $\theta_i < \widehat{\theta}_i$ for some signal $\widehat{\theta}_i$.[13] As is customary in the literature, we focus on symmetric (Bayesian-Nash) equilibrium in undominated strategies throughout.[14]

Suppose that all members but i adopt a cutoff $\widehat{\theta}$. In determining his, member i needs to consider only the pivotal event in which there are exactly $k-1$ "yes" and $n-k$ "no" votes except for his. Conditional on this event, member i's expected payoff from (1) is:

$$v(\theta_i;\widehat{\theta},k,n,\alpha) \equiv (1-\alpha+\frac{\alpha}{n})\theta_i + \frac{\alpha}{n}\left((k-1)E^y[\widehat{\theta}] + (n-k)E^n[\widehat{\theta}]\right),$$

where $E^y[\widehat{\theta}] \equiv E[\theta|\theta > \widehat{\theta}]$ and $E^n[\widehat{\theta}] \equiv E[\theta|\theta < \widehat{\theta}]$. The symmetric cutoff $\widehat{\theta}$ constitutes an equilibrium if and only if it satisfies the following indifference condition:

$$v(\widehat{\theta};\widehat{\theta},k,n,\alpha) = 0. \tag{3}$$

Lemma 1 *For any feasible k, n, and α, there exists a unique symmetric equilibrium, and it is interior, i.e., $\underline{\theta} < \widehat{\theta}(k,n,\alpha) < \overline{\theta}$.*

Proof. Let $V(x;k,n,\alpha) \equiv v(x;x,k,n,\alpha)$. Since $E^y[x] > E^n[x]$ for any $x \in [\underline{\theta},\overline{\theta}]$, $V(x;k,n,\alpha)$ is strictly increasing in k. Together with $E[\theta]=0$, it follows that $V(\underline{\theta};k,n,\alpha) \leq V(\underline{\theta};n,n,\alpha) = [1-(n-1)\frac{\alpha}{n}]\underline{\theta} < 0$, and $V(\overline{\theta};k,n,\alpha) \geq V(\overline{\theta};1,n,\alpha) = [1-(n-1)\frac{\alpha}{n}]\overline{\theta} > 0$. In addition, since (with appropriate limit arguments for $x = \underline{\theta}$ and $\overline{\theta}$)

$$E^{y\prime}[x] = \frac{f(x)}{1-F(x)}(E^y[x]-x) > 0 \text{ and } E^{n\prime}[x] = \frac{f(x)}{F(x)}(x-E^n[x]) > 0, \tag{4}$$

it also follows that $V_x(x;k,n,\alpha) > 0$, where subscripts of functions denote partial derivatives throughout. From these three facts, there exists a unique and interior solution, $\widehat{\theta}(k,n,\alpha)$, to $V(x;k,n,\alpha) = 0$. ■

[13] His decision when indifferent is immaterial as it is a zero probability event.

[14] It is well-known in the literature that under nonunanimity voting rules, there are also "unresponsive" symmetric equilibria, where agents adopt extremal cutoffs with probability one, irrespective of their private information. These equilibria, however, involve weakly dominated strategies.

The equilibrium existence is trivial. Its uniqueness follows from the fact that both truncated means, E^y and E^n, are strictly increasing in the cutoff.

To perform comparative statics on the voting strategy, we first introduce the notion of "sincere" or nonstrategic voting in our model. Agent i is said to vote sincerely if he conditions his vote only on his private information (Austen-Smith and Banks, 1996). Absent any status quo bias, sincere voting corresponds to adopting a cutoff of 0 in our model. Equilibrium voting will, however, be strategic in general except (perhaps) for one specific majority rule. Let $k = k^s(n)$ be this rule. Setting $\widehat{\theta} = 0$ in (3) and solving for k, we find:

$$k^s(n) \equiv F(0) + (1 - F(0)) \times n, \tag{5}$$

where we use the fact that

$$F(x) \times E^n[x] + (1 - F(x)) \times E^y[x] = E[\theta] = 0. \tag{6}$$

$k^s(n)$ is generically noninteger, which means that equilibrium voting is generically strategic. Proposition 1 shows the direction of this strategic behavior.

Proposition 1 *In equilibrium,*

(a) $\widehat{\theta}(k+1, n, \alpha) < \widehat{\theta}(k, n, \alpha)$

(b) $\widehat{\theta}_\alpha(k, n, \alpha) =^{sign} \widehat{\theta}(k, n, \alpha) =^{sign} k^s(n) - k$

(c) $E^n[\widehat{\theta}(.)] < 0 < E^y[\widehat{\theta}(.)]$.

Proof. From the proof of Lemma 1, recall that $V(x; k, n, \alpha)$ is strictly increasing in x and in k. Moreover, in equilibrium $V(\widehat{\theta}(k, n, \alpha); k, n, \alpha) = 0$. Thus, $\widehat{\theta}(k, n, \alpha)$ must be strictly decreasing in k, i.e., $\widehat{\theta}(k+1, n, \alpha) < \widehat{\theta}(k, n, \alpha)$. To prove part (b), we differentiate both sides of eq.(3) with respect to α: $\widehat{\theta}_\alpha(.) = -\frac{V_\alpha(\widehat{\theta}(.),.)}{V_x(.)}$. Next, we observe that $V_\alpha(\widehat{\theta}(.), .) = -\frac{\widehat{\theta}(.)}{\alpha}$. Since $V_x(.) > 0$ and $\alpha > 0$, we have $\widehat{\theta}_\alpha(k, n, \alpha) =^{sign} \widehat{\theta}(k, n, \alpha)$. Now, note that $\widehat{\theta}(1, .) = \frac{n-1}{n}\alpha\left(\widehat{\theta}(1, .) - E^n[\widehat{\theta}(1, .)]\right) > 0$ and $\widehat{\theta}(n, .) = \frac{n-1}{n}\alpha\left(\widehat{\theta}(n, .) - E^y[\widehat{\theta}(n, .)]\right) < 0$. By part (a), $\widehat{\theta}(k, n, \alpha)$ is strictly decreasing in k, and thus the unique real number $k^s(n)$ in (5) solves: $\widehat{\theta}(k, n, \alpha) = 0$. This means that $\widehat{\theta}(k, n, \alpha) > 0$ for $k < k^s(n)$, and $\widehat{\theta}(k, n, \alpha) < 0$ for $k > k^s(n)$. To prove the last part, suppose, to the contrary, that $E^y[\widehat{\theta}(.)] \leq 0$. Given that $E^n[x] < E^y[x]$, we have $E^n[\widehat{\theta}(.)] < 0$. Moreover, from (3), $\widehat{\theta}(k, n, \alpha) > 0$. But, this means $E^y[\widehat{\theta}(.)] > 0$ – a contradiction. Hence, $E^y[\widehat{\theta}(.)] > 0$. A similar argument shows that $E^n[\widehat{\theta}(.)] < 0$. ∎

Consistent with the strategic voting literature (e.g., Austen-Smith and Banks, 1996; Duggan and Martinelli, 2001; Feddersen and Pesendorfer, 1997, 1998),[15] part (a) says that as the decision rule requires more affirmative votes for the project's approval, individuals relax their own standards due to holding a more positive view of others' information in the event of being pivotal.

Part (b) has two implications: if the majority rule is less (resp. more) stringent than the one that induces sincere voting, then members vote strategically by approving the project less (resp. more) often than sincere voting would dictate; and in either case, a stronger interdependence leads to more strategic voting. The first implication follows from part (a) that voting strategy is strictly decreasing in the majority requirement. The second implication follows because with stronger interdependence, each member puts more weight on the information deduced from the pivotal event and in turn adjusts his strategy further away from the sincere one.

Armed with the equilibrium characterization for a fixed majority rule, k, we are ready to investigate the optimal rules.

4 Time-Consistent Majority Rules

Note that for an arbitrary cutoff x the probability that there are exactly m "yes" and $n-m$ "no" votes is the binomial coefficient, $b(x;m,n) = \binom{n}{m}[1-F(x)]^m[F(x)]^{n-m}$, and with this vote profile, the ex post committee welfare from implementing the project is

$$w(x;m,n) \equiv \frac{mE^y[x]+(n-m)E^n[x]}{n}, \tag{7}$$

and the ex ante committee welfare is

$$\overline{w}(x;k,n) \equiv \textstyle\sum_{m=k}^{n} b(x;m,n)w(x;m,n). \tag{8}$$

If the social planner can commit to the voting rule, then she will pick the one that maximizes (8) while taking its impact on subsequent voting. Formally, the ex ante optimal rule solves:

$$k^o = \arg\max_k \overline{w}(\widehat{\theta}(k,n,\alpha);k,n).$$

Lemma 2 $k^o \in \{\lfloor k^s(n)\rfloor, \lceil k^s(n)\rceil\}$.

[15]See Section 5 for a brief overview of the Duggan-Martinelli model.

Proof. Lemma A1 in the appendix establishes that

$$\overline{w}_x(x;k,n) =^{sign} -(x+(k-1)E^y[x]+(n-k)E^n[x]).$$

Since the expression, $x+(k-1)E^y[x]+(n-k)E^n[x]$, is strictly increasing in x by (4); strictly negative at $x=\underline{\theta}$; and strictly positive at $x=\overline{\theta}$, it follows that $\overline{w}(x;k,n)$ is strictly quasi-concave in x, with an interior maximum. Given the (equilibrium) constraint, $x=\widehat{\theta}(k,n,\alpha)$, this maximum occurs when $\overline{w}_x(\widehat{\theta}(k,n,\alpha),k,n)=0$, or equivalently when $\widehat{\theta}(k,.)+((k-1)E^y[\widehat{\theta}(k,.)]+(n-k)E^n[\widehat{\theta}(k,.)])=0$. Together with (3) and $\alpha\in(0,1)$, the optimal cut-off must be $x^o=\widehat{\theta}(k,.)=0$. This means that k^o satisfies: $0+(k^o-1)E^y[0]+(n-k^o)E^n[0]=0$, whose unique solution is $k^o=k^s(n)$, as defined in (5). Since $k^s(n)$ may be noninteger, $k^o\in\{\lfloor k^s(n)\rfloor,\lceil k^s(n)\rceil\}$.
■

Lemma 2 is best understood when $k^s(n)$ is integer so that sincere voting is feasible in equilibrium. Recall from (2) that the planner cares only about individuals' signals – *not* the degree of interdependence among them – and signals are most informative about the project under sincere voting. Therefore, it is welfare-maximizing to set $k^o=k^s(n)$. When $k^s(n)$ is noninteger, the planner chooses the rule that is close to engendering sincere voting.

In the absence of ex ante commitment, the planner can only set the majority rule that best responds to individuals' voting strategies; and in anticipation, individuals best respond to this rule as well as to each other's voting strategy. Let (k^*,θ^*) be an equilibrium of this game, which, by definition, lies at the intersection of the players' best responses:

$$k^*=\arg\max_k \overline{w}(\theta^*;k,n), \text{ and } \theta^*=\widehat{\theta}(k^*,n,\alpha). \tag{9}$$

Given θ^*, note that the ex post welfare $w(\theta^*;m,n)$ is strictly increasing in the number of "yes" votes, m, since $E^y(\theta^*)>E^n(\theta^*)$, and that $b(\theta^*;m,n)>0$ since $\theta^*\in(\underline{\theta},\overline{\theta})$. Thus, the ex ante welfare $\overline{w}(\theta^*;k,n)$ is maximized with respect to k by including all the terms $w(\theta^*;m,n)$, which are nonnegative. Formally, $w(\theta^*;k^*,n)\geq 0$ and $w(\theta^*;k^*-1,n)<0$.[16] In words, without the commitment power, k^* must be ex post optimal or time-consistent. Lemma 3 offers a necessary and sufficient condition for time-consistent rules.

Lemma 3 *The majority rule k is time-consistent if and only if $0\leq\Delta(k,n,\alpha)<1$, where $\Delta(k,n,\alpha)\equiv k-[1-F(\widehat{\theta}(k,n,\alpha))]\times n$.*

[16]Though nongeneric and unimportant for the analysis, we assume that the project is accepted whenever the planner is indifferent.

Proof. Recall that k is time-consistent if and only if $w(\theta^*; k, n) \geq 0$ and $w(\theta^*; k-1, n) < 0$. Using (7), $w(\theta^*; k, n) \geq 0$ if and only if $k \geq \frac{-E^n(\theta^*)}{E^y(\theta^*) - E^n(\theta^*)} n$. By (6), it then follows that $w(\theta^*; k, n) \geq 0$ if and only if $k \geq [1 - F(\theta^*)] \times n$; or equivalently $\Delta(k, n, \alpha) \geq 0$ given that $\theta^* = \widehat{\theta}(k, n, \alpha)$ in equilibrium. An exact argument reveals that $w(\theta^*; k-1, n) < 0$ if and only if $k - 1 < [1 - F(\theta^*)] \times n$, or equivalently $\Delta(k, n, \alpha) < 1$. Combining the two inequalities, we reach the desired conclusion. ■

According to Lemma 3, a majority rule is time-consistent as long as the expected number of approval votes it generates is not too different from itself. If the expected number of approval votes were much higher, then it would mean that members have adopted too lenient a standard for approval and the rule needs to be more stringent to rectify this. If, on the other hand, the expected number of approval votes were much lower, then the rule should be relaxed. Time-consistency curbs both incentives.

Remark 2 *By re-writing the condition in Lemma 3, we also find bounds on the equilibrium probability of an approval vote:* $\frac{k^*}{n} - \frac{1}{n} \leq 1 - F(\theta^*) < \frac{k^*}{n}$. *In particular, for a relatively large committee, the probability of an approval vote is approximately equal to the percentage majority rule itself.*

In order to sharpen our characterization of time-consistent rules, we ensure that $\Delta(k, n, \alpha)$ is increasing in k by imposing a monotonicity condition on the truncated means.

Condition M. $(E^y[x] - x)$ *is decreasing in* x, *and* $(x - E^n[x])$ *is increasing in* x.

Condition M is mild. Bagnoli and Bergstrom (2005) record that if signal distribution has an increasing hazard rate, i.e., $\frac{d}{d\theta}\left[\frac{f(\theta)}{1-F(\theta)}\right] \geq 0$, or equivalently if $1 - F$ is log-concave, then $E^y[x] - x$ is decreasing in x. On the other hand, if $\frac{d}{d\theta}\left[\frac{f(\theta)}{F(\theta)}\right] \leq 0$, or equivalently if F is log-concave, then $x - E^n[x]$ is increasing in x.[17] Both hazard rate conditions are satisfied if the signal density, f, is log-concave; and many well-known probability densities such as the uniform and Normal are log-concave, as listed by Bagnoli and Bergstrom.

Condition M also has a natural interpretation. Given the cutoff x, $E^y[x] - x > 0$ is the net informational value of an approve vote. It is reasonable that such a vote will contain less information as the cutoff rises. Similarly, $x - E^n[x] > 0$ is the net informational value of a disapprove vote, and such a vote will convey less information as the cutoff diminishes. Under Condition M, we present the main result of this paper.

[17] Bagnoli and Bergstrom note that in Industrial Engineering, $(E^y[x] - x)$ corresponds to the mean-residual-lifetime function, and $(x - E^n[x])$ corresponds to the mean-advantage-over-inferiors.

Proposition 2 *Suppose that Condition M holds. Then, given α and n, there exist unique (real) cutoffs $\underline{k}(\alpha, n) < \overline{k}(\alpha, n)$ such that k is time-consistent if and only if $\underline{k}(\alpha, n) \leq k < \overline{k}(\alpha, n)$. In addition, $\underline{k}(\alpha, n) < k^s(n) < \overline{k}(\alpha, n)$; and*

- $\underline{k}(\alpha, n)$ *is strictly decreasing in α; and* $\lim_{\alpha \to 0} \underline{k}(\alpha, n) = [1 - F(0)] \times n$ *and* $\lim_{\alpha \to 1} \underline{k}(\alpha, n) = 0$.

- $\overline{k}(\alpha, n)$ *is strictly increasing in α; and* $\lim_{\alpha \to 0} \overline{k}(\alpha, n) = 1 + [1 - F(0)] \times n$ *and* $\lim_{\alpha \to 1} \overline{k}(\alpha, n) = 1 + n$.

Proof. Suppose that Condition M holds. Then, by Lemma A2 in the appendix, $\Delta(k, n, \alpha)$ is strictly increasing in k. Let $\underline{k}$ and $\overline{k}$ be the unique (real) solutions to $\Delta(k, n, \alpha) = 0$ and $\Delta(k, n, \alpha) = 1$, respectively. By Lemma 1, k is time-consistent if and only if $\underline{k} \leq k < \overline{k}$. Moreover, since, by definition, $\widehat{\theta}(k^s, n, \alpha) = 0$ and $k^s = F(0) + (1 - F(0)) \times n$, we have $\Delta(k^s, n, \alpha) = F(0) \in (0, 1)$, which implies that $\underline{k} < k^s < \overline{k}$.

To prove the comparative statics, we first observe that $\Delta_\alpha(.) = n \times f(\widehat{\theta}(.)) \times \widehat{\theta}_\alpha(.) =^{sign} k^s - k$ by Proposition 1. Given $\underline{k} < k^s$, $\Delta_\alpha(.) > 0$ and therefore $\underline{k}$ must be strictly decreasing in α to satisfy $\Delta(\underline{k}, n, \alpha) = 0$. As $\alpha \to 0$, $\widehat{\theta}(.) \to 0$ and $\underline{k} \to [1 - F(0)] \times n$. As $\alpha \to 1$, eq.(3) and $\Delta(\underline{k}, n, \alpha) = 0$ require that $E^y[\widehat{\theta}(\underline{k}, n, 1)] = \widehat{\theta}(\underline{k}, n, 1)$, which in turn requires that $\widehat{\theta}(\underline{k}, n, 1) = \overline{\theta}$. Hence, $\underline{k} \to 0$ as $\alpha \to 1$. As for $\overline{k}$, it must be strictly increasing in α to satisfy $\Delta(\overline{k}, n, \alpha) = 1$ because $k^s < \overline{k}$ and thus $\Delta_\alpha(.) < 0$. Then, similar arguments to $\underline{k}$ confirm the limits for $\overline{k}$. ■

Two corollaries help understand Proposition 2.

Corollary 1 *Suppose that $k^s(n)$ is an integer. Then, the ex ante optimal rule k^o is time-consistent for all α. Moreover, for some $\underline{\alpha} \in (0, 1)$, k^o is the unique time-consistent rule if and only if $\alpha < \underline{\alpha}$.*

Proof. The first part is immediate from Proposition 2. For the second part, note that $1 < k^s(n) < n$ since $n \geq 2$ and $k^s(n)$ is integer. Let α^1 and α^2 be the (unique) solutions to $\underline{k}(\alpha, n) = k^s(n) - 1$ and $\overline{k}(\alpha, n) = k^s(n) + 1$, respectively. Evidently, $\alpha^1 \in (0, 1)$ and $\alpha^2 \in (0, 1)$. Defining $\underline{\alpha} = \min\{\alpha^1, \alpha^2\}$, the result obtains. ■

Corollary 1 is explained by the intimate relationship between the ex post welfare and the pivotal voting incentive. The ex ante rule k^o dictates that the project be rejected in the marginal event of $k^o - 1$ "yes" and $n - k^o + 1$ "no" votes; and this is what the pivotal voting incentive implies. Recall that $k^o = k^s(n)$ induces sincere voting, which requires that, in the pivotal event, each individual has a zero expectation of $n - 1$ signals with $k^o - 1$ "yes" and $n - k^o$ "no" votes. Thus, with an additional "no" vote, the ex post welfare must be strictly negative and lead to the project's rejection. This reasoning holds for any degree of interdependence, α. If individuals possess pure private values, i.e. $\alpha = 0$, so that sincere voting becomes a dominant strategy, the ex ante rule k^o emerges as the unique time-consistent rule. Corollary 1 indicates that by continuity of voting strategies in α, this uniqueness remains true for sufficiently weak interdependencies.

For a sufficiently strong interdependency, $\alpha \geq \underline{\alpha}$, there are multiple time-consistent rules. In fact, since $\underline{k}(\alpha, n)$ is strictly decreasing and $\overline{k}(\alpha, n)$ is strictly increasing in α, Proposition 2 reveals that the set of time-consistent voting rules grows with the degree of interdependence, covering all feasible rules as payoffs approach pure common values.

Corollary 2 *If k is time-consistent under α^1, then it is also time-consistent under $\alpha^2 > \alpha^1$. In the limit, as $\alpha \to 1$, any $k \in \{1, ..., n\}$ is time-consistent.*

Proof. The limit result follows because $\underline{k}(\alpha, n) \to 0$ and $\overline{k}(\alpha, n) \to 1 + n$, as $\alpha \to 1$. ■

The intuition for Corollary 2 follows from Proposition 1: agents with stronger interdependencies are more likely to tailor their strategies to the majority rule; and as a result, they can hold rational beliefs for a wider range of majority rules. This suggests that the commitment problem is more severe in a committee that exhibits stronger interdependencies. Somewhat ironically it is the most severe when members have pure common values. We illustrate Proposition 2 by an example.

Example 1 *Consider a committee of 5 members, who each independently draw a signal from a uniform distribution on $[-1, 1]$. Trivial algebra reveals that $\underline{k} = \frac{25(1-\alpha)}{10-9\alpha}$ and $\overline{k} = \frac{35-29\alpha}{10-9\alpha}$. The ex ante optimal rule is $k^o = 3$, which remains the unique time-consistent rule for $\alpha < \frac{5}{7}$. For $\frac{5}{7} \leq \alpha < \frac{15}{16}$, only $k = 2, 3, 4$, and for $\alpha \geq \frac{15}{16}$, all $k = 1, ..., 5$ are time-consistent.*

Remark 3 *If F is symmetric about $\theta = 0$, then clearly $k^o = \left\lfloor \frac{n+1}{2} \right\rfloor, \left\lceil \frac{n+1}{2} \right\rceil$, and it is time-consistent for all α. If F is not symmetric, then due to rounding issues, I could not prove or disprove that the time-consistency of k^o is generic. Nonetheless, since $\underline{k}(\alpha, n) < k^s(n) < \overline{k}(\alpha, n)$, Corollary 2 implies that for sufficiently large α's, k^o must be time-consistent.*

- **Large Committees**

Our investigation can also inform us how time-consistent rules change with committee size, n. To distill the scale effect, however, we look at the percentage rule, $\frac{k^*(\alpha,n)}{n}$, and let $\theta^*(\alpha, n) = \widehat{\theta}(k^*(\alpha, n), n, \alpha)$.

Proposition 3 *As $n \to \infty$, we have $\theta^*(\alpha, n) \to 0$ and $\frac{k^*(\alpha,n)}{n} \to 1 - F(0)$.*

Proof. From Lemma 3, note that $0 \leq \frac{k^*(\alpha,n)}{n} - [1 - F(\theta^*(\alpha, n))] < \frac{1}{n}$. Let $l(\alpha) \equiv \lim_{n\to\infty} \frac{k^*(\alpha,n)}{n}$. The Sandwich Theorem implies that $l(\alpha) = 1 - F(\theta^*(\alpha, \infty))$. To determine $\theta^*(\alpha, \infty)$, we evaluate eq.(3) as $n \to \infty$:

$$(1-\alpha)\theta^*(\alpha,\infty) + \alpha\{l(\alpha) \times E^y[\theta^*(\alpha,\infty)] + [1 - l(\alpha)]E^n[\theta^*(\alpha,\infty)]\} = 0. \tag{10}$$

Inserting $l(\alpha) = 1 - F(\theta^*(\alpha, \infty))$, the term inside the curly bracket reduces to $E[\theta] = 0$, revealing that $\theta^*(\alpha, \infty) = 0$. Hence, $\lim_{n\to\infty} \frac{k^*(\alpha,n)}{n} = 1 - F(0)$. ■

Proposition 3 says that as committee size grows without bound, the strategic voting incentive vanishes completely and the ex ante optimal voting rule becomes the only time-consistent rule. The reason is that the correlation between agents' valuations gets weaker in a larger committee[18] because, by the logic of the law of large numbers, each agent has a sharper estimate of the average signal of others, which diminishes the need to infer it in equilibrium. It is worth remarking that if the voting rule did not adjust in equilibrium, sincere voting need not obtain in a large committee. For instance, if we exogenously set $\frac{k(n)}{n} = \mu \neq 1 - F(0)$, it is clear from (10) that $\theta^* \nrightarrow 0$. Note also that because it is a limit result, Proposition 3 does not rely on Condition M.

5 Robustness

In this section, we discuss the robustness of our key findings under a traditional Condorcet Jury model and under nonlinear interdependencies.

5.1 The Condorcet Jury

As alluded to in the Introduction, following the Condorcet's jury theorem, there is a voluminous literature on group decision-making in which voters share a common interest but they receive differential information about an unknown state of the world. While the standard Condorcet setup is not directly captured by our model, we demonstrate here that our key results continue to hold; in particular, as with pure common values, any majority rule k becomes time-consistent for the Condorcet jury, too.

To make our point, we adopt the jury model of Duggan and Martinelli (2001), hereafter DM, because they allow for continuous signals as we do. Specifically, the jury consists of

[18] It is readily verified that $Cov(v_i, v_j) = \frac{\alpha(2-\alpha)}{n}$.

$n \geq 2$ members who each vote whether to convict, C, or acquit, A, a defendant. The defendant's unknown state of innocence, I, or guilt, G, occurs with probabilities $P(I)$ and $P(G)$. Conditional on the state, each juror i receives an independent private signal, $s_i \in (\underline{S}, \overline{S})$, from distributions, $F(s|I)$ and $F(s|G)$, with respective densities $f(s|I)$ and $f(s|G)$. The jurors simultaneously vote to convict or acquit. The defendant is convicted if the number of convict votes is k or more, and acquitted otherwise. Jurors' preferences satisfy: $u(C|G) = u(A|I) = 0$ while $u(C|I) < 0$ and $u(A|G) < 0$. As in DM, let the relative ex ante cost of acquittal be

$$\rho = \frac{u(A|G)}{u(C|I)} \frac{P(G)}{P(I)}.$$

We keep DM's regularity assumptions A1-A4 about signal distributions; in particular, the likelihood ratio, $\frac{f(s|I)}{f(s|G)}$ is strictly decreasing in $s \in (\underline{S}, \overline{S})$. That is, higher signals are more indicative of guilt than innocence. DM prove that voting strategies are of cutoff types: convict if $s > \overline{s}_i$, and acquit if $s < \overline{s}_i$. Focusing on symmetric (and responsive) equilibrium, DM show that the unique equilibrium cutoff, $s^* = \widehat{s}(k, n, \rho)$, solves

$$\left(\frac{1 - F(s^*|I)}{1 - F(s^*|G)} \right)^{k-1} \left(\frac{F(s^*|I)}{F(s^*|G)} \right)^{n-k} \frac{f(s^*|I)}{f(s^*|G)} - \rho = 0. \tag{11}$$

Since $F(s|G) \leq F(s|I)$ by the monotone likelihood ratio, it is readily verified that $\widehat{s}(.)$ is decreasing in k, which is in line with our Lemma 1.

Time-Consistency. With homogenous jurors, the social planner's objective coincides with a representative juror's. As before, we say that a majority rule k is time-consistent if, given the voting strategy, $s^* = \widehat{s}(k, n, \rho)$, (a) the planner convicts the defendant after receiving $m \geq k$ convict votes, and (b) she acquits him otherwise. Formally, letting $b^l(s; m, n) = \binom{n}{m}(1 - F(s|l))^m (F(s|l))^{n-m}$, $l = I, G$, the requirement (a) amounts to:

$$P(I) \times u(C|I) \times b^I(s^*; m, n) \geq P(G) \times u(A|G) \times b^G(s^*; m, n),$$

which, given that $u(C|I) < 0$, reduces to:

$$\left(\frac{1 - F(s^*|I)}{1 - F(s^*|G)} \right)^{m} \left(\frac{F(s^*|I)}{F(s^*|G)} \right)^{n-m} \leq \rho \text{ for } m \geq k. \tag{12}$$

Since $F(s|G) \leq F(s|I)$, (12) is satisfied if and only if

$$\left(\frac{1 - F(s^*|I)}{1 - F(s^*|G)} \right)^{k} \left(\frac{F(s^*|I)}{F(s^*|G)} \right)^{n-k} \leq \rho. \tag{13}$$

Similarly, the requirement (b) can be written as

$$P(I) \times u(C|I) \times b^I(s^*; k-1, n) < P(G) \times u(A|G) \times b^G(s^*; k-1, n),$$

which is equivalent to

$$\rho < \left(\frac{1-F(s^*|I)}{1-F(s^*|G)}\right)^{k-1} \left(\frac{F(s^*|I)}{F(s^*|G)}\right)^{n-k+1}. \tag{14}$$

Using (11), the inequalities (13) and (14) reveal that k is time-consistent if and only if

$$\frac{1-F(s^*|I)}{1-F(s^*|G)} \leq \frac{f(s^*|I)}{f(s^*|G)} < \frac{F(s^*|I)}{F(s^*|G)}. \tag{15}$$

As also recorded in DM's Lemma 0, the monotone likelihood ratio implies (15), which leads us to

Proposition 4 *Any majority rule k is time-consistent in the Jury model, as examined by Duggan and Martinelli (2001).*

Similar to Corollary 1, Proposition 4 follows from the direct connection between the pivotal voting incentive and the planner's marginal decision ex post.

5.2 Nonlinear Interdependence

We now discuss to what extent our results generalize to a nonlinear payoff environment. For consistency, we keep the same signal structure. Let agent i's payoff be given by:

$$v_i = u(\theta_i, \boldsymbol{\theta}_{-i}),$$

where u is twice continuously differentiable and symmetric in its last $n-1$ components. To ensure positive interdependence, we assume that $\frac{\partial}{\partial \theta_j} u(.) > 0$ for all j. Moreover, we set $E_{\boldsymbol{\theta}}[u(\boldsymbol{\theta})] = 0$ to eliminate the status quo bias, and set $u(\mathbf{0}) = 0$ to guarantee that sincere voting means choosing a cutoff of 0 as before. Note that the case of pure common values refers to $u(\boldsymbol{\theta})$ that is symmetric in $\boldsymbol{\theta}$.

Proposition 5 *In the nonlinear environment described so far, there exists a unique symmetric equilibrium, and it is interior, i.e.,* $\underline{\theta} < \widehat{\theta}(k,n) < \overline{\theta}$. *In addition, (1)* $\widehat{\theta}(k+1,n,\alpha) < \widehat{\theta}(k,n,\alpha)$*; and (2) any majority rule k is time-consistent under pure common values.*

Proof. The symmetric cutoff $\widehat{\theta}(k,n)$ constitutes an equilibrium if and only if it solves

$$\overline{u}(x,k-1,n) \equiv u(x,\underbrace{E^y[x],...,E^y[x]}_{k-1 \text{ times}},\underbrace{E^n[x],...,E^n[x]}_{n-k \text{ times}}) = 0. \tag{16}$$

Since $u(\mathbf{0})=0$, $E^y[\underline{\theta}]=0$ and $E^n[\underline{\theta}]<0$, we have $\overline{u}(\underline{\theta},k-1,n)<0$. With a similar argument, $\overline{u}(\overline{\theta},k-1,n)>0$. By continuity of u, these imply that there is an interior solution to (16). Moreover, since $\overline{u}_x(x,k-1,n)>0$, the solution is unique. Next, note that since $E^y[x] > E^n[x]$, $\overline{u}(x,k-1,n)$ is strictly increasing in k. Thus, to satisfy (16), $\widehat{\theta}(k,n)$ needs to be strictly decreasing in k, as proposed. Finally, under pure common values, majority rule k is time-consistent if and only if

$$u(\underbrace{E^y[\theta^*],...,E^y[\theta^*]}_{k \text{ times}},\underbrace{E^n[\theta^*],...,E^n[\theta^*]}_{n-k \text{ times}}) \geq 0 \text{ and } u(\underbrace{E^y[\theta^*],...,E^y[\theta^*]}_{k-1 \text{ times}},\underbrace{E^n[\theta^*],...,E^n[\theta^*]}_{n-k+1 \text{ times}}) < 0,$$

where $\theta^* = \widehat{\theta}(k,n)$. But, given the facts that $E^y[\theta^*] > \theta^* > E^n[\theta^*]$ and the monotonicity of u, these inequalities immediately follow from (16). ■

Proposition 5 generalizes previous observations from Lemma 1 and Proposition 1. Interestingly, it does not require that agent i be biased toward his own information, i.e., $\frac{\partial}{\partial\theta_i}u(\theta_i,\boldsymbol{\theta}_{-i}) \geq \frac{\partial}{\partial\theta_j}u(\theta_i,\boldsymbol{\theta}_{-i})$ for $j \neq i$. But this is the familiar "single-crossing" condition in mechanism design, which implies that the ex post valuations of different agents will be ordered as their signals (Krishna, 2009; p. 102) and that a truth-telling equilibrium is possible. This condition plays an important role in determining time-consistent rules: if it did not hold, then every majority rule could be time-consistent without requiring pure common values.[19] The intuition parallels Proposition 2: agents who are biased toward their own information vote less strategically which in turn shrinks the set of time-consistent rules.

6 Concluding Remarks

Our analysis suggests that preference interdependence worsens the committee's ability to commit to a majority rule by increasing incentives for strategic voting. This is precisely because agents with stronger interdependencies care more about others' information, making

[19] To see this, suppose, to the contrary, that $\frac{\partial}{\partial\theta_i}u(\theta_i,\boldsymbol{\theta}_{-i}) \leq \frac{\partial}{\partial\theta_j}u(\theta_i,\boldsymbol{\theta}_{-i})$ for $j \neq i$. Note that a majority rule k is time-consistent if and only if $w(\theta^*;k,n) \equiv \frac{k\overline{u}(E^y[\theta^*],k-1,n)+(n-k)\overline{u}(E^n[\theta^*],k,n)}{n} \geq 0$ and $w(\theta^*;k-1,n) < 0$ where $\theta^* = \widehat{\theta}(k,n)$ and $\overline{u}(\theta^*,k-1,n)=0$. Since $E^y[\theta^*] > \theta^* > E^n[\theta^*]$, we have $\overline{u}(E^n[\theta^*],k,n) \geq \overline{u}(E^y[\theta^*],k-1,n) > 0$ and thus $w(\theta^*;k,n) \geq 0$. In addition, we have $\overline{u}(E^y[\theta^*],k-2,n) \leq \overline{u}(E^n[\theta^*],k-1,n) < 0$, which implies that $w(\theta^*;k-1,n) < 0$.

their votes more sensitive to the rule. Understanding the extent of the commitment problem is valuable as it relates to the set of ex ante suboptimal aggregation rules that can emerge in equilibrium. Perhaps ironically, the commitment problem is most pronounced for committees with pure common values and indeed, any majority rule is possible in equilibrium in such committees. Our analysis also suggests that given the preferences, the commitment problem can be alleviated by increasing the committee size.

In closing, we note two issues that were not addressed here. The first one is pre-voting communication. Although many committee voting models assume away such communication, recent papers have pointed out its potential importance on voting outcomes (Coughlan 2000; Gerardi and Yariv 2007). While some communication between voters can and does occur in reality, like Persico (2004), we believe that certain institutional and physical barriers may still render it to be imperfect. Nonetheless, it would interesting to enrich the present model with this dimension and see how it affects the commitment problem. Second, we did not allow for "asymmetric" interdependence among members. It is, however, conceivable that members may have heterogenous levels of biases or they value others' opinions differently. In fact, it could even be the planner's choice how to best form the committee when such biases are commonly known. These issues await future research.

7 Appendix

Lemma A1 *The ex ante welfare stated in (8) satisfies*

$$\overline{w}_x(x;k,n) = -b(x;k-1,n-1) \times f(x) \times ((k-1)E^y[x] + (n-k)E^n[x] + x),$$

where $b(x;m,n) = \binom{n}{m}[1-F(x)]^m[F(x)]^{n-m}$.

Proof. We do not assume here $E[\theta] = 0$ to demonstrate that it is inessential. Letting $p \equiv 1 - F(x)$, we write

$$\begin{aligned} \overline{w}(x;k,n) &\equiv \phi(p,x,k,n) \equiv \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i} \left(\frac{iE^y[x] + (n-i)E^n[x]}{n} \right) \\ &= \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i} \left(\frac{i}{n} \frac{\int_x^{\overline{\theta}} \theta dF(\theta)}{p} + \frac{n-i}{n} \frac{\int_{\underline{\theta}}^{x} \theta dF(\theta)}{1-p} \right). \end{aligned}$$

Since $\frac{i}{n}\binom{n}{i} = \binom{n-1}{i-1}$, $\frac{n-i}{n}\binom{n}{i} = \binom{n-1}{i}$, and

$$\sum_{i=k}^{n} \binom{n-1}{i-1} p^{i-1}(1-p)^{n-i} = \sum_{i=k-1}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i},$$

we have

$$\begin{aligned} \phi(p,x,k,n) &= \int_x^{\overline{\theta}} \theta dF(\theta) \left[\sum_{i=k-1}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} \right] + \int_{\underline{\theta}}^{x} \theta dF(\theta) \left[\sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} \right] \\ &= E[\theta] \sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} + \int_x^{\overline{\theta}} \theta dF(\theta) \left[\binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \right]. \end{aligned}$$

Next, observe that

$$\frac{\partial}{\partial p} \sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} = (n-1) \binom{n-2}{k-1} p^{k-1} (1-p)^{n-1-k}.$$

Thus,

$$\begin{aligned} \overline{w}_x(x;k,n) &= \phi_p(.) \times \frac{\partial p}{\partial x} + \phi_x(.) \\ &= -f(x) \Bigg[E[\theta](n-1) \binom{n-2}{k-1} p^{k-1}(1-p)^{n-1-k} \\ &\quad + \binom{n-1}{k-1} \left((k-1)p^{k-2}(1-p)^{n-k} - (n-k)p^{k-1}(1-p)^{n-1-k} \right) \int_x^{\overline{\theta}} \theta dF(\theta) \Bigg] \\ &\quad + \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} (-xf(x)). \end{aligned}$$

Inserting the facts: $\int_x^{\overline{\theta}} \theta dF(\theta) = pE^y[x]$ and $pE^y[x] + (1-p)E^n[x] = E[\theta]$, it follows that

$$\begin{aligned} \overline{w}_x(x;k,n) &= -f(x)\left\{E[\theta](n-1)\binom{n-2}{k-1}p^{k-1}(1-p)^{n-1-k}\right. \\ &\quad +\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}\left[(k-1)E^y[x]+(n-k)E^n[x]\right] \\ &\quad -\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}(n-k)\frac{E[\theta]}{1-p} \\ &\quad \left.+\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}x\right\}. \end{aligned}$$

Note that the first and the third terms inside the curly brackets on the r.h.s. cancel out, leaving

$$\begin{aligned} \overline{w}_x(x;k,n) &= -f(x)\left\{\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}\left[(k-1)E^y[x]+(n-k)E^n[x]\right]\right. \\ &\quad \left.+\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}x\right\} \\ &= -f(x)\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}\left\{(k-1)E^y[x]+(n-k)E^n[x]+x\right\}. \end{aligned}$$

Substituting back for $p \equiv 1-F(x)$, the desired result for $\overline{w}_x(x;k,n)$ is then obtained. ■

Lemma A2 *Suppose that Condition M holds. If* $0 \leq \Delta(k,n,\alpha) < 1$ *and* $0 \leq \Delta(k+1,n,\alpha) < 1$, *then* $\Delta(k+1,n,\alpha) > \Delta(k,n,\alpha)$, *where* $\Delta(k,n,\alpha) \equiv k - [1-F(\widehat{\theta}(k,n,\alpha))] \times n$.

Proof. Suppose that Condition M holds so that $(E^y[x]-x)$ is decreasing and $(x-E^n[x])$ is increasing in x. Let $\theta_0 = \widehat{\theta}(k,n,\alpha)$ and $\theta_1 = \widehat{\theta}(k+1,n,\alpha)$. From Proposition 1, $\theta_1 < \theta_0$. Since $0 \leq \Delta(k,n,\alpha) < 1$, Lemma 2 implies that $w(\theta_0;k,n) \geq 0$ and $w(\theta_0;k-1,n) < 0$. Similarly, $w(\theta_1;k+1,n) \geq 0$ and $w(\theta_1;k,n) < 0$. Next, as noted in the proof of Lemma 2, since $[1-F(\widehat{\theta}(k,n,\alpha))] \times n = \frac{-E^n[\widehat{\theta}(.)]}{E^y[\widehat{\theta}(.)]-E^n[\widehat{\theta}(.)]}n$, $\Delta(k,n,\alpha)$ can be written: $\Delta(k,n,\alpha) \equiv k + \frac{E^n[\widehat{\theta}(.)]}{E^y[\widehat{\theta}(.)]-E^n[\widehat{\theta}(.)]}n$; and therefore

$$\begin{aligned} \Delta(k+1,n,\alpha)-\Delta(k,n,\alpha) &= k+1+\frac{E^n[\theta_1]}{E^y[\theta_1]-E^n[\theta_1]}n-\left(k+\frac{E^n[\theta_0]}{E^y[\theta_0]-E^n[\theta_0]}n\right) \\ &= 1+\frac{w(\theta_1;k,n)}{E^y[\theta_1]-E^n[\theta_1]}-\frac{w(\theta_0;k,n)}{E^y[\theta_0]-E^n[\theta_0]}. \end{aligned}$$

Adding and subtracting θ_1 and θ_0 in denominators, we find:

$$\begin{aligned} \Delta(k+1,n,\alpha)-\Delta(k,n,\alpha) &= 1+\frac{w(\theta_1;k,n)}{E^y[\theta_1]-\theta_1+\theta_1-E^n[\theta_1]}-\frac{w(\theta_0;k,n)}{E^y[\theta_0]-\theta_0+\theta_0-E^n[\theta_0]} \\ &\geq 1+\frac{w(\theta_1;k,n)}{E^y[\theta_0]-\theta_0+\theta_1-E^n[\theta_1]}-\frac{w(\theta_0;k,n)}{E^y[\theta_0]-\theta_0+\theta_1-E^n[\theta_1]} \quad \text{(A-1)} \\ &= 1+\frac{1}{E^y[\theta_0]-\theta_0+\theta_1-E^n[\theta_1]}[w(\theta_1;k,n)-w(\theta_0;k,n)], \quad \text{(A-2)} \end{aligned}$$

where (A-1) follows because $E^y[\theta_1] - \theta_1 \geq E^y[\theta_0] - \theta_0$ and $w(\theta_1; k, n) < 0$; and because $\theta_1 - E^n[\theta_1] \leq \theta_0 - E^n[\theta_0]$ and $w(\theta_0; k, n) \geq 0$.

To sign (A-2), we employ (3), which reveals that

$$\begin{aligned} (1-\alpha)\theta_1 + \frac{\alpha}{n}\left(\theta_1 - E^n[\theta_1]\right) + \alpha w(\theta_1; k, n) &= 0, \\ (1-\alpha)\theta_0 + \frac{\alpha}{n}\left(\theta_0 - E^y[\theta_0]\right) + \alpha w(\theta_0; k, n) &= 0. \end{aligned}$$

Then, $w(\theta_1; k, n) - w(\theta_0; k, n) = \frac{1-\alpha}{\alpha}(\theta_0 - \theta_1) - \frac{1}{n}\left(E^y[\theta_0] - \theta_0 + \theta_1 - E^n[\theta_1]\right)$. Inserting this fact into (A-2), we conclude:

$$\Delta(k+1, n, \alpha) - \Delta(k, n, \alpha) \geq 1 + \frac{\frac{1-\alpha}{\alpha}(\theta_0 - \theta_1)}{E^y[\theta_0] - \theta_0 + \theta_1 - E^n[\theta_1]} - \frac{1}{n} > 0.$$

■

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