Notes on
A Life Cycle Model of Fertility Control and Women’s Allocation Decisions

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We outline a simple theory of life cycle fertility and women’s time allocation decision-making which incorporates two key behavioral assumptions about childbearing and rearing behavior: (i) births are stochastic events that are only partially controlled by parental contraceptive actions and (ii) the care of young children is relatively intensive in the mother’s time when they are young but becomes relatively (market-) goods intensive as they grow up. We explore the model’s implications for the relationship between early childbearing and a woman’s subsequent fertility and time allocation decisions and what can be learned about the causal effects of the early childbearing from alternative sources of variation in the timing of first births.

Consider women making optimal fertility, and time and resource allocation decisions over their finite lifetimes. For now, we assume that women’s husbands or partners, if present, play no role in these decisions, other than as sexual partners and providers of income. Let \( i \) index a woman and \( t \) her age. For our purposes, the childbearing phase of a woman’s life begins at age \( t_0 \), the age at which she becomes fecund, and ends at age \( t_m \), when she reaches menopause; we assume that she lives until age \( T \). At each age, her choices are constrained by limitations on her own time, her financial resources, her ability to control her fertility and the obligations she has for existing children. Let the vector \( b_{it} \) denote the \( i \)th woman’s birth history when she is age \( t \),
where,

\[ b_{it} = (b_{i,t-1}, \ldots, b_{i,t-r}, \ldots, b_{it}) , \]

\( b_{i,t-r} \) denotes whether there exists an \( r \)-year old child when the mother is age \( t \) (i.e., \( b_{i,t-r} = 1 \) if such a child exists and \( = 0 \) otherwise) and \( n_{it} = \sum_{r=1}^{b_{i,t-r}} b_{i,t-r} \) denote the number of children the woman has borne as of age \( t \).

We assume that parental lifetime utility depends on the satisfaction they receive from children and from the own consumption. More precisely, lifetime utility is given by

\[ \sum_{t=0}^{T} \beta^t u(n_{it}, z_{it}, \ell_{it}; \chi_i), \]  

where \( \beta \) is a subjective discount factor (\( \beta \in (0,1) \)) and \( u(\cdot) \) is a concave function of the number of existing children, \( n_{its} \), the consumption of the parents, \( z_{its} \), and \( \ell_{its} \), the woman’s leisure time—that time she devotes to activities other than labor market work and child care—and \( \chi_i \) is an age-invariant, preference parameter, which varies across women in the population and indexes differences in their marginal utility of own consumption, i.e., \( \partial u / \partial \chi \geq 0 \).

A key feature of the model is the birth, or reproduction, process. Following the demographic literature (Sheps and Menken, 1968), births are viewed as a stochastic process, but as in earlier economic models of fertility (Rosenzweig and Schultz, 1985) this process may be controlled, in part, by a couple’s contraceptive actions. Consider the following reproduction function,

\[ b_{it} = p_i(\mu_i, e_i) + \zeta_{it} \]
\[ = \mu_i - \kappa e_{it} + \zeta_{it} . \]

The probability of a birth at age \( t \), \( p_i(\mu_i, e_i) \), is assumed to be a linear function of \( \mu_i \), a woman-

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\(^1\) As in the equivalence scales literature, \( z \) correspond to exclusively adult goods.
specific fertility component indexing her *fecundity* parameter and \( e_{it} \), the contraceptive action the couple takes at \( t \). Births can also be affected by idiosyncratic random variation, \( \zeta_{it} \), which are assumed to be distributed independently across individuals and over time. Let \( e_{it} \in [\underline{e}, \bar{e}] \), where, for a given \( \mu_i \), \( \bar{e} \) is that contraceptive strategy which *minimizes* the likelihood of a birth while \( \underline{e} \) *maximizes* it.\(^2\)

Once born, children must be reared. We assume that it takes both maternal time and market goods and services to rear a child and that the process for the production of the care of children is of the following form. Let \( c_r \) denote the mother’s time and \( a_r \) the market inputs into the care of an \( r \) year-old child. To capture the feature that rearing young children is time intensive compared to older ones,\(^3\) we use the following simple specification for how \( c_r \) and \( a_r \) change with the age of the child,

\[
\begin{align*}
  c_r &= \gamma \delta^{r-1}, \quad \gamma > 0, \quad 0 < \delta < 1 \\
  a_r &= \psi, \quad \psi > 0
\end{align*}
\]

for all \( r \), i.e., the ratio of inputs,

\[
\frac{a_r}{c_r} = \frac{\psi}{\gamma \delta^{r-1}}.
\]

*increases* in a child’s age. Thus, the total amounts of maternal time and market inputs devoted to the \( i^{th} \) woman’s children when she is age \( t \) is

\[
\begin{align*}
  c_{it} &= \sum_{r=1}^{t-t_0} c_r b_{i,t-r} = \gamma \sum_{r=1}^{t-t_0} \delta^{r-1} b_{i,t-r} \\
  a_{it} &= \sum_{r=1}^{t-t_0} a_r b_{i,t-r} = \psi \sum_{r=1}^{t-t_0} b_{i,t-r} = \psi n_{it}.
\end{align*}
\]

Furthermore, parents can influence the child services they receive, and their associated costs, over their lifetimes by influencing the timing and spacing of births.

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\(^2\) If induced abortions are available, then it is feasible to avoid any births with probability one. The psychic and monetary costs of such abortions may make it unlikely that the risks of unwanted births are completely eliminated.

\(^3\) The time intensity of young children was first incorporated into a static model of fertility and women’s time allocation by Willis (1972) and into a life cycle model by Moffitt (1984) and Hotz and Miller (1986).
Parental choices are subject to two sets of additional constraints: per-period maternal time and budget constraints. The mother’s time at any age is constrained to be $L$ so that

$$L = c_i + \ell_i + h_i, \quad (6)$$

where $h_i$ are the number of hours she devotes to the labor market. Assuming that parents cannot borrow or lend, the budget constraint they face is

$$z_i + a_i = y_i + w_i h_i = y_i + w_i(L - c_i - \ell_i), \quad (7)$$

where the prices of all market goods are normalized to one, $y_i$ is either the husband’s income, if present, or any non-work income the mother may receive—the latter may include returns from assets and transfers from relatives or the government—and $w_i$ is the wage rate the mother could receive in the labor market.\footnote{Note that the costs associated with the existing children is $a_i + w_i c_i$.} While some sources of income, such as public assistance, may depend on parental choices, i.e., presence of children and maternal labor supply, we assume that $y_i$ is exogenous for sake of simplicity. For now, we assume that the wage rate facing the woman is also exogenous, although it may vary over the woman’s life cycle.

\subsection{1.1 Effects of Early Births when Contraception is Totally Ineffective ($\kappa = 0$)}

Our interest is in analyzing the causal effects of early childbearing, i.e., those changes in subsequent behavior which can be attributed solely to having a birth at an early age or, equivalently, a woman not delaying her childbearing until she is older. To assess the predictions of the model concerning such effects, it is convenient to first analyze a special case of the above model. We first consider an admittedly unrealistic case in which women have no effective control over their fertility at any age, i.e., $\kappa = 0$ for all $t$. Under this assumption, a woman has no incentive to take contraceptive actions which are costly and, in effect, she only exercises choice, at each age,
with respect to her own consumption and hours of work. Substituting the budget and time con-
straints into \( u(n_{it}, z_{it}, \ell_{it}, \chi) \), it follows that

\[
\max_{h_{it}} u(n_{it}, [y_{it} + w_{it}h_{it} - a_{it}], [L - c_{it} - h_{it}]; \chi),
\]

(8)

and the first order condition is

\[
u_2 w_{it} - u_3 \leq 0
\]
or

\[
\eta(0; \Omega_t) \equiv \eta(n_{it}, [y_{it} + w_{it} \cdot 0 - a_{it}], [L - c_{it} - 0]; \chi) \geq w_{it},
\]

(9)

where \( \eta(h; \Omega_{it}) \equiv u_3/u_2 \) is the shadow price of the woman’s time, evaluated at \( h_{it} = h \), and \( \Omega_{it} \) de-
notes the woman’s information set at the beginning of age \( t \); \( \Omega_{it} \) contains \( b_{it}, y_{its}, w_{it}, \chi_t \), and \( \mu_t \).

For women who participate in the labor force, the optimal labor supply choice at age \( t, \) \( h_{it} = h^*_i \), is the solution (1)

\[
\eta(h^*_i; \Omega_{it}) = w_{it}.
\]

Under the conditions of exogenous and stochastic fertility, teen births, as are all births,
are purely random events, i.e., such births are “accidental.” To examine the effect of an acciden-
tal teen birth on subsequent fertility and labor supply decisions for two identical women (i and j,
say), where one has an exogenous birth at \( t_0 \), i.e., a positive draw from \( \zeta_{it} \) at \( t_0 \), the other does not
and all of their remaining realizations on \( b_{it}, t > t_0 \) are identical. Given that fertility over the life
cycle is assumed to be exogenous—i.e., women are assumed not to be able to control it via their
choices of \( e_{it} \)—it follows immediately that the teen mother (woman i) will have one more child at
each age \( t, t > t_0 \), than the non-teen mother (woman j). That is,

\[
\frac{\partial n}{\partial \zeta_{it_0}} = n_{it} - n_{jt} = 1, \text{ for all } t \geq t_0. \tag{10}
\]

However, because fertility is exogenous, it follows that the teen birth, itself, has no effect on a
woman’s subsequent childbearing behavior, i.e.,

$$\frac{\partial p_i(t)}{\partial \zeta_{t_0}} = 0, \text{ for all } t > t_0. \quad (11)$$

With respect to labor force participation and labor supply decisions, consider first the effects of changes in $c_{it}$ and $n_{it}$ on the woman’s shadow price of time:

$$\frac{\partial \eta(0;\Omega_{it})}{\partial c_{it}} = \frac{u_i u_{23} - u_i u_{33}}{u_i^2} \geq 0, \quad (12)$$

where the sign holds so long as $u_{23} \geq 0$ and

$$\frac{\partial \eta(0;\Omega_{it})}{\partial u_{it}} = \frac{u_i u_{22} - u_i u_{32}}{u_i^2} \leq 0, \quad (13)$$

where the sign holds so long as $u_{32} \geq 0$. Thus, the effect of an exogenous birth at age $t_0$ on the shadow price of a woman’s time at age $t$ is:

$$\frac{\partial \eta(0;\Omega_{it})}{\partial \zeta_{t_0}} = \frac{\partial \eta(0;\Omega_{it})}{\partial c_{it}} \frac{\partial c_{it}}{\partial \zeta_{t_0}} + \frac{\partial \eta(0;\Omega_{it})}{\partial u_{it}} \frac{\partial u_{it}}{\partial \zeta_{t_0}} = \frac{\partial \eta(0;\Omega_{it})}{\partial c_{it}} \gamma^{S-t_0-1} + \frac{\partial \eta(0;\Omega_{it})}{\partial u_{it}} \psi. \quad (14)$$

It follows from (12) and (13) that the sign of (14) is ambiguous. Immediately after a teen birth, whether the probability that the woman works decreases (increases) if the positive effect that the maternal time “costs” of the first born child on the woman’s shadow price at age $t$ are newborn is greater (less) than the negative effect that the cost of market inputs for its care have on the shadow price. Causal observation would suggest that the $\partial \eta(0;\Omega_{it})/\partial c_{it}$ term is larger in absolute magnitude and, thus, the shadow price of the $i^{th}$ woman’s time rises relative to that of the $j^{th}$ woman. (Recall that the probability that a woman works is decreasing in $\eta(0;\Omega_{it})$.) But note that because the time costs of the first child born to the $i^{th}$ woman declines with age, it follows that over time (14) is increasingly likely to be negative, i.e., as the teen birth grows older, the $i^{th}$
woman is *more* likely to participate in the labor force compared to her $j$th counterpart. That is,

$$\frac{\partial^2 \eta(0; \Omega_{it})}{\partial \zeta_{t_0} \partial t} \leq 0.$$ 

For women who are working ($\eta(0; \Omega_{it}) < \omega_t$), the effect of a teenage birth on the number of hours they work is ambiguous, but the model above implies that it will tend to rise with the mother’s (and first-child’s) age. To see this, totally differentiate (9) w.r.t. $h_{it}$ and the “pre-determined” variables, $c_{it}$ and $n_{it}$

$$\left[u_{22}w^2_{it} + u_{33} - 2u_{23}w_{it}\right]dh_{it} - \left[u_{22}w_{it} - u_{33}\right]dc_{it} - \left[u_{22}w_{it} - u_{32}\right]da_{it} = 0$$

so that it follows that:

$$\frac{\partial h_{it}}{\partial c_{it}} = \frac{u_{23}w_{it} - u_{33}}{u_{22}w^2_{it} + u_{33} - 2u_{23}w_{it}} \leq 0,$$

and

$$\frac{\partial h_{it}}{\partial a_{it}} = \frac{u_{22}w_{it} - u_{32}}{u_{22}w^2_{it} + u_{33} - 2u_{23}w_{it}} \geq 0$$

where the signs of these effects follow by assuming that $u_{23} \geq 0$ and, from the second order conditions for a maximum of (8), that $(u_{22}w^2_{it} + u_{33} - 2u_{23}w_{it}) \leq 0$. Then it follows that:

$$\frac{\partial h_{it}}{\partial \zeta_{t_0}} = \frac{\partial h_{it}}{\partial \zeta_{t_0}} \frac{\partial \zeta_{t_0}}{\partial c_{it}} + \frac{\partial h_{it}}{\partial c_{it}} \frac{\partial c_{it}}{\partial \zeta_{t_0}}$$

$$= \frac{\partial h_{it}}{\partial c_{it}} \gamma \delta^{\psi_{t_0} - 1} + \frac{\partial h_{it}}{\partial a_{it}} \psi,$$

and, by reasoning similar to that for the effects on a woman’s labor force participation decision,

$$\frac{\partial^2 h_{it}}{\partial \zeta_{t_0} \partial t} \geq 0.$$ 

Thus, in the case where life cycle fertility is governed by an exogenous stochastic process, the simple model developed above implies that having an (exogenously-generated) birth as a teen has no behavioral effect on subsequent fertility, although it does result in a larger (by one) com-
pleted family size, but it does affect women’s subsequent labor market behavior, causing teen mothers to work less early in their lifetimes but to work more at older ages.

1.2 Effects of Early Births when Contraception is Effective (κ ≠ 0)

To the extent that available contraceptive methods are effective in changing p, i.e., κ ≠ 0, births are the result, at least in part, of contraceptive decisions which are jointly determined with a woman’s life cycle time allocation and depend upon her preferences, fecundity, wage and income possibilities, available contraceptive technology and past endogenous choices. As such, we now consider the consequences of allowing for fertility control (eit) to be jointly determined over her life cycle with a woman’s time allocation decisions. This optimization problem can be can solved in two stages: (1) at each age, choose time and market inputs to maximize the parents’ own consumption (zit) subject to the time and budget constraints and Ωit and (2) the parents’ make contraceptive choices, at ages t = t0,…,tm, conditional on optimal choices of zit, Ωit, and the expectations about future birth realizations. The first stage corresponds to the solution in the case treated in the previous section, where life cycle fertility was stochastic and exogenous. The second stage of the solution determines a woman’s optimal contraceptive strategy, and, thus, the likelihood that she bears a child, at each age up to tm. Let e(Ωit) denote that method, which is made according to the following decision rule:

\[ e(\Omega_{it}) = \begin{cases} e & \text{if and only if } v(\Omega_{it}) \geq 0 \\ \tilde{e} & \text{if and only if } v(\Omega_{it}) < 0, \end{cases} \]  

(19)

where \( v(\Omega_{it}) \equiv V(b_{it}=1;\Omega_{it}) - V(b_{it}=0;\Omega_{it}) \) is the difference between woman’s valuation of having and not having a birth at age t, and these valuations are defined by:

\[ V(b_{it}=k;\Omega_{it}) \equiv \max_{\{e_{j}\}_{j=t+1}^{T} \mid h_{j}} \sum_{t=0}^{T} e^{t} \beta^{t} \left[ n_{t}^{+} + \sum_{j=t+1}^{T} b_{j} \left( y_{j} + w_{j} (h_{j} - a_{j}) \right) - c_{j} - h_{j} \right] \mathbb{E}_{t} \]  

(20)

and where \( E_{t} \) is the expectations operator condition on the information set \( \Omega_{it} \) augmented by \( b_{it} = \)
Consider again the comparison of two women who have the same exogenous characteristics, including $\mu_i$, $\chi_i$, $w_{it}$ and $y_{it}$, over their life cycles and have the same realizations on $\zeta_{it}$, for $t > t_0$, but differ with respect to $\zeta_{t_0}$, i.e., the $i^{th}$ woman has a teen births and the $j^{th}$ woman does not. Now, however, these women’s subsequent fertility, and thus their subsequent time allocation decisions, may differ because they find it in their interests to choose different contraceptive strategies, according to (19) in response to their prior fertility realizations. Unfortunately, obtaining clear-cut comparative dynamic predictions for the sorts of life cycle model outlined above, in general, are not feasible.\(^5\) However, one can get gain some insight into the likely ways in which early childbearing will affect a woman’s life cycle behavior when she is able to exercise some control over her subsequent fertility and time allocation immediately after having a first birth as a teen and immediately before she reaches menopause, $t_m$.

Consider first the ages immediately following $t_0$, for the $i^{th}$ woman, who is a teen mother, and the $j^{th}$ woman, who is not. Because of her child born at $t_0$ requires maternal time, $c_{it}$ and market inputs, $a_{it}$, as would a birth born at any other age, the $i^{th}$ woman has an incentive to take contraceptive actions which reduce her risk of a subsequent birth, i.e., $e(\Omega_{i,t_0+1}) = \bar{e}$. In contrast, the $j^{th}$ woman’s contraceptive strategy depends on whether she wants to have or avoid a birth at $t_0+1$, which depends on her information set, $\Omega_{j,t_0+1}$, which is the same as that of the $i^{th}$ woman but for their childbearing histories. If, for example, woman $j$, and, as such, woman $i$, wanted to have a birth in period $t_0$, but only the latter realized one, then it is likely that woman $j$’s optimal contraceptive strategy at age $t_0+1$ is $e(\Omega_{j,t_0+1}) = \bar{e}$. Alternatively, either neither woman wanted a birth at

\(^5\) This is because one cannot, in general, establish that the valuation function is concave in the two key fertility variables, namely the number of children born, $n_x$, and the maternal time required to care for existing children, $c_x$. 
to or the jth woman’s exogenous circumstances changed and she wants to avoid a birth at age $t_0 + 1$ in which case $e(\Omega_{j,t_0+1}) = e$. In either case, it follows that the costliness of rearing a newborn child implied by the above model, implies that a teen mother is more likely to contracept immediately after that birth than is her identical counterpart who did not experience a birth as a teen.

More formally, allowing for (partially) effective fertility control implies that the probability of a birth at any age $t$ can be expressed as a function of $e(\Omega_{it})$, and, thus, $v(\Omega_{it})$:

$$p(\mu_t, e(\Omega_{it})) = p \cdot 1\{v(\Omega_{it}) < 0\} + \bar{p} \cdot 1\{v(\Omega_{it}) \geq 0\}$$

$$= p + (\bar{p} - p) 1\{v(\Omega_{it}) \geq 0\},$$

(21)

where $1\{\cdot\}$ is the indicator function. Because children are especially “costly” to care for immediately after a birth, one can conjecture that $v(\Omega_{it})$ is decreasing in both $c_{it}$ and $a_{it}$ (and, thus, $n_{it}$) immediately after an exogenously induced first birth, i.e., $\partial v(\Omega_{it}) / \partial c_{it} \leq 0$ and $\partial v(\Omega_{it}) / \partial n_{it} \leq 0$, at ages immediately after $t_0$. If that conjecture holds, then the derivative of (21) with respect to $\zeta_{t_0}$ is proportional to

$$\frac{\partial p(\mu_t, e(\Omega_{it}))}{\partial \zeta_{t_0}} = (\bar{p} - p) \left[ \frac{\partial v(\Omega_{it})}{\partial c_{it}} \frac{\partial c_{it}}{\partial \zeta_{t_0}} + \frac{\partial v(\Omega_{it})}{\partial n_{it}} \frac{\partial n_{it}}{\partial \zeta_{t_0}} \right] = (\bar{p} - p) \left[ \frac{\partial v(\Omega_{it})}{\partial c_{it}} \gamma_0 \delta^{-t_0 - 1} + \frac{\partial v(\Omega_{it})}{\partial n_{it}} \frac{\partial n_{it}}{\partial \zeta_{t_0}} \right] \leq 0,$$

(22)

for ages immediately after $t_0$. That is, the effect of an exogenously-generated first birth early in life on the probability of having a second birth is likely to be negative initially, due to the fact that the child born at $t_0$ will subsequently require the mother’s time, $c$, and market inputs, $a$, to be reared and because there are diminishing returns to additional children ($u_{11} < 0$). However, it is not the case, ceteris paribus, that birth probabilities after a birth will attain the level of a woman’s first birth probability since, following this conjecture, $v(\Omega_{it})$ is declining in $n$ due to diminishing returns to children and the permanent per-period costs of market goods needed in the care of a
child. Thus, it is unclear from this model whether a woman’s expected number of births over her lifetime will necessarily be higher as a result of having an early first birth as the model, i.e., it is unclear whether \( n_{t_0} \) is an increasing or decreasing function of \( \zeta_{t_0} \).

By a similar line of argument, the above conjecture would suggest that the labor force participation and labor supply responses of a teen mother (woman \( i \)) compared to her “equivalent” non-teen mother (woman \( j \)) will correspond to the predictions in (14) and (18), at least at ages immediately after a first birth. That is, the teen mother’s probability of working and hours of work, if she works, will decline in response to an exogenously determined teen birth but that both will tend to rise as the first-born child ages.

As noted above, one cannot formally demonstrate that the above conjecture holds because one cannot establish that \( \nu(\Omega_{it}) \) is concave in \( c_{it} \) and \( n_{it} \) at any age but \( t_m \). At that age, a woman faces no possibility of having any births in the future and, as such, does not need to consider the ramifications of her current contraceptive choices in light of what births she might experience in the future. At that age, the conditions for optimal choices for both \( e_{itm} \) and \( h_{itm} \) can be shown to imply that \( \nu(\Omega_{itm}) \) is concave in \( c_{itm} \) and \( a_{ itm} \). (A proof of this proposition is provided in the Appendix.) Thus the validity of the above conjectures depends on the extent that this concavity property holds at earlier ages.

In summary, the above model suggests that the causal response of women who begin their childbearing as a teenager\(^7\) is to contracept, at least temporarily to try to avoid having another child while her newborn requires a great deal of her time in child care. Moreover, to the ex-

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\(^6\) Hotz and Miller (1986) establish that \( \nu(\Omega_{it}) \) will be approximately concave in \( c_{it} \) and \( a_{it} \) as the difference, \((\overline{p} - p)\), goes to zero.

\(^7\) Note that the causal effect of teenage childbearing could be discerned from an exogenous postponement of childbearing to a later age.
tent that children entail persistent monetary costs to feed, cloth and shelter them to the extent that there are diminishing utility returns to children, women who begin their childbearing early are likely to attempt to control their subsequent fertility. Finally, the causal effect of early childbearing on a woman’s labor supply is likely to vary over the woman’s life cycle. Her initial response in labor supply is likely to be driven by the time-intensity of rearing a newborn child, but, at the child ages, women who start the childbearing early are increasingly likely to work in order to help meet the on-going financial obligations of a child. Thus their labor supply will tend to be higher at earlier ages than if they had postponed their childbearing.

1.3 Heterogeneous Tastes and Fecundity and Potential Biases in Measuring Causal Effects

An concern in obtaining unbiased estimates of the causal effects of an early birth is how to measure the sorts of causal effects considered in the previous two sections. In particular, the above analysis considered the effects of an exogenously generated birth at an early age \( t_0 \) and held constant women’s tastes for own-consumption and her fecundity. But the sorts of comparisons which underly the statistical associations noted in the Introduction are based on comparisons of outcomes for teen mothers and women who postpone their childbearing until adulthood. It is likely that such comparisons produce biased estimates of causal effects, since these two groups of women are likely to differ in either their tastes or levels of fecundity. In the remainder of this section, we briefly the likely biases that when variations in \( b_{t_0} \) reflect not only exogenous and transitory variations in early fertility (i.e., \( \zeta_{t_0} \)) but also differences in either fecundity (\( \mu \)) or a woman’s tastes for her own-consumption (\( \chi \)).

To focus on essential ideas, consider the association between teenage childbearing and a woman’s subsequent fertility when tastes and fecundity are allowed to vary. That is
\[ \frac{dp_i(\mu_i)}{db_{t_0}} = \frac{\partial p_i(\mu_i)}{\partial \xi_{t_0}} + \frac{dp_i(\mu_i)}{d\mu} \geq 0 \]

and

\[ \frac{dn_i}{db_{t_0}} = \frac{\partial p_i(\mu_i)}{\partial \xi_{t_0}} + \left\{ \frac{dp_i(\mu_i)}{d\mu} + \sum_{t=t_0+1}^{t-1} \frac{dp_i(\mu_i)}{d\mu} \right\} \geq 1, \]

for all \( t \geq t_0 \), since \( dp_i(\mu_i)/d\mu_i \) is positive as women with higher levels of fecundity have more children, on average, at each age. With respect to the apparent effect of a teen birth on a woman’s subsequent labor supply behavior, failure to control for differences in \( \mu \) or \( \chi \) implies that

\[ \frac{d\eta(0;\Omega_u)}{db_{t_0}} = \frac{\partial \eta(0;\Omega_u)}{\partial \xi_{t_0}} + \frac{d\eta(0;\Omega_u)}{d\mu} + \frac{d\eta(0;\Omega_u)}{d\chi}, \]

where

\[ \frac{d\eta(0;\Omega_u)}{d\mu} = \frac{\partial \eta(0;\Omega_u)}{\partial \xi_{t_0}} d\mu_i + \frac{\partial \eta(0;\Omega_u)}{\partial \chi_{t_0}} d\mu_i + \sum_{t=t_0+1}^{t-1} \left( \frac{\partial \eta(0;\Omega_u)}{\partial \chi_{it}} d\mu_i \right) \frac{dp_i(\mu_i)}{d\mu} \]

and

\[ \frac{dh_{it}}{db_{t_0}} = \frac{\partial h_{it}}{\partial \xi_{t_0}} + \frac{dh_{it}}{d\mu} + \frac{dh_{it}}{d\chi}, \]

where

\[ \frac{dh_{it}}{d\mu_i} = \frac{\partial h_{it}}{\partial \xi_{t_0}} \frac{d\mu_i}{d\mu_i} + \frac{\partial h_{it}}{\partial \chi_{t_0}} \frac{d\mu_i}{d\mu_i} + \sum_{t=t_0+1}^{t-1} \left( \frac{\partial h_{it}}{\partial \chi_{it}} \frac{d\mu_i}{d\mu_i} \right) \frac{dp_i(\mu_i)}{d\mu} \]

Suppose that tastes for children are held constant across women, but fecundity is not. While more fecund women have more children at each age, including \( t_0 \), than those less fecund
(dp_i(A)/d\mu_i \geq 0), on average, fecund women have more young children at each age \( \tau \geq t_0 \). Accordingly, fecund women who cannot control their fertility are less likely to work at each age than a woman who had a teen birth but was less fecund and, if they work, are likely to work fewer hours. That is

\[
\frac{\partial \eta(0; \Omega_i)}{\partial \zeta_{t_0}} \leq \frac{d\eta(0; \Omega_i)}{d\mu_i} \quad \text{and} \quad \frac{\partial h_i}{\partial \zeta_{t_0}} \geq \frac{dh_i}{d\mu_i}.
\]

Thus, to the extent that teen mothers are more fecund than non-teen mothers, the above model implies that comparing teen mothers with non-teen mothers will provide a negatively biased estimate of the effect of teenage childbearing on a woman’s subsequent rate of labor force participation and hours of work.

All else equal, women with high tastes for children are less likely to likely to contracept and more likely to have children early in life compared to those with lower levels of \( \chi \). But women with high values of \( \chi \) are also less likely to contracept, and more likely to have births, at all subsequent ages, everything else equal. As such, comparing the subsequent outcomes of teen mothers and women who began their childbearing at later ages—the sort of comparisons which have fueled the recent concerns about the teenage childbearing problem in the U.S. noted in the Introduction—reflect the variations in many factors.

### 1.4 Generalizations

The model has imposed a number of important simplifications that, if relaxed, could affect the predictions it yields concerning the effects of exogenous variations in early childbearing. We briefly consider the likely implications of two potential extensions: allowing for human capital acquisition and for more flexible technologies for the rearing of children, i.e., the production
With respect to human capital, allowing the woman to devote time to the acquisition of human capital, either via schooling or on the job, introduces another possible use of her time and changes her intertemporal time allocation motives by allowing devotion of time to acquiring human capital early in life to increase her wages, and thus time spent in the market, later in life.\footnote{See Moffitt (1984) for a discussion of this model feature within a life cycle model of fertility and labor supply.} Models of human capital acquisition generally depend on a woman’s ability to finance the monetary costs, if any, of investing in human capital, the efficiency with which human capital can be acquired per unit of her time (i.e., her ability) and the rate at which her human capital depreciates over time or with non-use. To the extent that a woman can finance things like schooling and is efficient at learning and acquires capital which has “durability,” i.e., does not depreciate rapidly, the option of investing in skill acquisition is likely to increase the costs associated with early childbearing and to limit the number of children a woman bears over her life cycle. As such, such women are likely more likely to postpone childbearing and, if they accidentally become pregnant, i.e., experience a contraceptive failure, are more likely to have an induced abortion, than are women who do not have these traits. Thus, the presence of a human capital accumulation motive is likely to account for an important source of heterogeneity across women who are observed to have early births and those observed to delay their fertility, providing further concerns about using a comparison of the outcomes of these two types of women to measure the impacts of early childbearing on subsequent behavior.

In the model presented above, we do not allow women to substitute market inputs for her

\footnote{Also note that the model presented above assumes that capital markets are perfectly imperfect, i.e., there is no borrowing or lending possible. Thus, another possible extension would be to allow for more flexible intertemporal budget constraints that have been considered in some of the existing life cycle fertility literature. See Hotz, Klerman and Willis (1997) for a discussion of this model feature as well as for the human capital and child quality production technologies discussed in the text.}
time in the care of children at any age and impose, as an assumption, that young children are relatively time intensive. Furthermore, we do not allow parents to choose the level of quality, or care, they produce for their children. The above model could also be generalized to allow for greater substitution possibilities in the production of child quality and could allow parents to choose different levels of quality so as to alter the flow of services they receive from children. Greater substitution possibilities in inputs could mitigate or overturn the predictions noted above for how a woman’s time allocation and subsequent fertility is likely to change as her children age (and she ages).