Policy Implications of the New Keynesian Phillips Curve

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The theoretical framework within which optimal monetary policy was studied before the arrival of the new Keynesian Phillips curve (NKPC) but after economists had become comfortable using dynamic, optimizing, general equilibrium models and a welfare-maximizing criterion for policy analysis was one in which the central source of nominal non-neutrality was a demand for money. At center stage in this literature was the role of money as a medium of exchange (as in cash-in-advance models, money-in-the-utility-function models, or shopping-time models) or as store of value (as in overlapping generations models).

Notably absent in this strand of the literature were nominal rigidities stemming from the sluggish adjustment of product or factor prices. With the arrival of the new Keynesian Phillips curve these features became fundamental for understanding the monetary transmission mechanism. The new Keynesian paradigm represents a profound shift in monetary economics from viewing the role of money primarily as a medium of exchange to viewing money—sometimes exclusively—as a unit of account. In the early 1990s, Christopher Sims and Michael Woodford demonstrated that the mere assumption that product prices are quoted in units of fiat money, even if such money is physically inexistent, gives rise to a theory of price level determination. Combining the assumption of nominal price stickiness with the assumption of money as unit of account delivers a trade off between output and inflation that in optimizing models came to be known as the new Keynesian Phillips curve.

The inessential role that money plays in the neo Keynesian literature along with the actual

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conduct of monetary policy in the United States and elsewhere over the past thirty years naturally shifted theoretical interest away from money growth rate rules and toward interest-rate rules: In the work of academic monetary economists, Milton Friedman's celebrated k-percent growth path for the money supply gave way to Taylor's equally influential interest-rate feedback rule.

In this article, we survey recent advancements in the theory of optimal monetary policy in models with a new Keynesian Phillips curve. To provide perspective, however, we begin with a brief account of the state of the literature on optimal monetary policy before the advent of the new Keynesian revolution.

1 Optimal Monetary Policy Pre NKPC

Within the pre NKPC framework, under quite general conditions, optimal monetary policy calls for a zero opportunity cost of holding money, a result known as the Friedman rule. In fiat money economies in which assets used for transactions purposes do not earn interest, the opportunity cost of holding money equals the nominal interest rate. Therefore, in the class of models commonly used for policy analysis before the emergence of the new Keynesian Phillips curve, the optimal monetary policy prescribed that the risk-less nominal interest rate—the return on Federal funds, say—be set at zero at all times.

In the early literature to which we are alluding, a demand for money is motivated in a variety of ways, including a cash-in-advance constraint (Lucas, 1982), money in the utility function (Sidrauski, 1967), a shopping-time technology (Kimbrough, 1986), or a transactions-cost technology (Feenstra, 1986). Regardless of how a demand for money is introduced, the intuition for why the Friedman rule is optimal in this class of model is straightforward: a zero nominal interest rate maximizes holdings of a good—real money balances—that has a negligible production cost. At the same time, a positive interest rate can distort the efficient allocation of resources. For instance, in the cash-in-advance model with credit and cash goods, a positive interest rate distorts the allocation of private spending across these two types of goods. In models in which money ameliorates transaction costs or shortens shopping time, a positive interest rate introduces a wedge in the consumption-leisure choice.

To illustrate the optimality of the Friedman rule, consider augmenting a neoclassical model with a transaction technology that is decreasing in real money holdings and increasing in consumption spending. Specifically, consider an economy populated by a large number of identical households. Each household has preferences defined over processes of consumption and leisure and described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{1}$$

where c_t denotes consumption, h_t denotes labor effort, $\beta \in (0,1)$ denotes the subjective discount factor, and E_0 denotes the mathematical expectation operator conditional on information available in period 0. The single period utility function U is assumed to be increasing in consumption, decreasing in effort, strictly concave, and twice continuously differentiable.

Final goods are produced using a production function $z_t F(h_t)$ that takes labor, h_t as the only factor input and is subject to an exogenous productivity shock, z_t.

A demand for real balances is introduced into the model by assuming that money holdings, denoted M_t, facilitate consumption purchases. Specifically, consumption purchases are subject to a proportional transaction cost $s(v_t)$ that is decreasing in the household's money-to-consumption ratio, or consumption-based money velocity,

$$v_t = \frac{P_t c_t}{M_t}, \tag{2}$$

where P_t denotes the nominal price of the consumption good in period t. The transaction cost function, $s(v)$, satisfies the following assumptions: (a) $s(v)$ is nonnegative and twice continuously differentiable; (b) There exists a level of velocity $\underline{v} > 0$, to which we refer as the satiation level of money, such that $s(\underline{v}) = s'(\underline{v}) = 0$; (c) $(v - \underline{v})s'(v) > 0$ for $v \neq \underline{v}$;

and (d) $2s'(v) + vs''(v) > 0$ for all $v \geq \underline{v}$. Assumption (b) ensures that the Friedman rule, i.e., a zero nominal interest rate, need not be associated with an infinite demand for money. It also implies that both the transaction cost and the distortion it introduces vanish when the nominal interest rate is zero. Assumption (c) guarantees that in equilibrium money velocity is always greater than or equal to the satiation level. Assumption (d) ensures that the demand for money is decreasing in the nominal interest rate.

Households are assumed have access to risk-free pure discount bonds, denoted B_t. These bonds are assumed to carry a gross nominal interest rate of R_t when held from period t to period $t+1$.

The flow budget constraint of the household in period t is then given by:

$$P_t c_t[1 + s(v_t)] + P_t \tau_t^L + M_t + \frac{B_t}{R_t} = M_{t-1} + B_{t-1} + P_t z_t F(h_t), \tag{3}$$

where τ_t^L denotes real lump sum taxes. In addition, it is assumed that the household is subject to a borrowing limit that prevents it from engaging in Ponzi-type schemes.

The government is assumed to follow a fiscal policy whereby it rebates any seignorage income it receives from the creation of money in a lump sum fashion to households.

A stationary competitive equilibrium can be shown to be a set of plans $\{c_t,\ h_t,\ v_t\}$, satisfying the following three conditions:

$$v_t^2 s'(v_t) = \frac{R_t - 1}{R_t} \tag{4}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{z_t F'(h_t)}{1 + s(v_t) + v_t s'(v_t)} \tag{5}$$

$$[1 + s(v_t)]c_t = z_t F(h_t), \tag{6}$$

given monetary policy $\{R_t\}$, with $R_t \geq 1$, and the exogenous process $\{z_t\}$. The first equilibrium condition can be interpreted as a demand for money or liquidity preference function. Given our maintained assumptions about the transactions technology $s(v_t)$, the implied money demand function is decreasing in the gross nominal interest rate R_t. Further, our assumptions imply that as the interest rate vanishes, or R_t approaches unity, the demand for money reaches a finite maximum level given by $c_t/\underline{v}$. At this level of money demand, households are able to perform transactions costlessly, as the transactions cost, $s(v_t)$, becomes nil. The second equilibrium condition shows that a level of money velocity above the satiation level $\underline{v}$, or, equivalently, an interest rate greater than zero, introduces a wedge between the marginal rate of substitution of consumption for leisure and the marginal product of labor. This wedge, given by $1+s(v_t)+v_t s'(v_t)$, induces households to move away from consumption and toward leisure. It is positive as long as velocity is above the satiation level $\underline{v}$, or as long as the nominal interest rate is positive. Moreover, the wedge is increasing in the nominal interest rate, implying that the larger is the nominal interest rate, the more distorted is the consumption-leisure choice. The final equilibrium condition states that a positive interest rate entails a resource loss in the amount of $s(v_t)c_t$. This resource loss is increasing in the interest rate and vanishes only when the nominal interest rate equals zero.

We wish to characterize optimal monetary policy in the context of this model. A monetary policy is said to be optimal if it yields the highest level of welfare among all feasible monetary policies. Inspecting the three equilibrium conditions above, it is clear that if the policy maker sets the monetary policy instrument, which we take to be the nominal interest rate, such that velocity is at the satiation level, ($v_t = \underline{v}$), then the equilibrium conditions become identical to an economy without the money demand friction. Moreover, the above equilibrium conditions would be identical to the equilibrium conditions of a Pareto optimal real economy. From equation (4), it follows that setting the nominal interest rate to zero ($R_t = 1$), ensures that $v_t = \underline{v}$ and hence that the real allocation and the level of welfare are the same as in the Pareto optimal economy without a demand for money. For this reason, optimal monetary policy takes the form of a zero nominal interest rate at all times.

Under the optimal monetary policy, the rate of change in the aggregate price level varies over time. Because, to a first approximation, the nominal interest rate equals the sum of the real interest rate and the expected rate of inflation, and because under the optimal monetary policy the nominal interest rate is held constant, the degree to which the inflation rate fluctuates depends on the equilibrium variations in the real rate of interest. In general, optimal monetary policy in a model in which a role for monetary policy arises solely from the presence of money demand, is not characterized by inflation stabilization.

A second important consequence of optimal monetary policy in the context of the present model is that inflation is on average negative. This is because with a zero nominal interest rate, the inflation rate equals on average the negative of the real rate of interest.

As we will see below, the arrival of the new Keynesian Phillips curve turned the optimality of the Friedman rule on its head. At a practical level, monetary policy makers never actually warmed up to the policy prescriptions coming out of the type of model with a demand for money as the sole source of nominal frictions. On the contrary, zero or near zero nominal interest rates were for the most part regarded not as optimal but rather as detrimental to the economic wellbeing of a society. Instead in the past thirty years most central banks adopted criteria for optimal policy that sought to stabilize some combination of output and inflation fluctuations. The emergence of the new Keynesian Phillips curve in the mid 1990s provided theoretical grounds for the reluctance of policy makers to embrace the Friedman rule.

2 Implications of the NKPC for Optimal Monetary Policy

The new Keynesian Phillips curve can be briefly defined as the dynamic output-inflation tradeoff that arises in a dynamic general equilibrium model populated by utility-maximizing households and profit-maximizing firms—like the one laid out in the previous section—and augmented with some kind of rigidity in the adjustment of nominal product prices. The foundations of the new Keynesian Phillips curve were laid by Calvo (1983) and Rotemberg (1982). Woodford (1996) and Yun (1996) completed the development of the New Keynesian

Phillips curve by introducing optimizing behavior on the part of firms facing Calvo-type dynamic nominal rigidities.

The most important policy implication of models featuring a new Keynesian Phillips curve is the optimality of price stability. (See Goodfriend and King (1997) for an early discussion of this result). We will discuss the price-stability result in a variety of theoretical models, including ones with a realistic set of real and nominal rigidities, policy instruments and policy constraints, and sources of aggregate fluctuations. We start, however, with the simplest structure within which the price-stability result can be obtained. To this end, we strip the model presented in the previous section from its money demand friction and instead introduce costs of adjusting nominal product prices. In the resulting model, sticky prices represent the sole source of nominal friction.

To introduce sticky prices into the model of the previous section assume that the consumption good c_t is a composite good made of a continuum of intermediate differentiated goods. The aggregator function is of the Dixit-Stiglitz type. Each household/firm unit is the monopolistic producer of one variety of intermediate goods. In turn, intermediate goods are produced using a technology like the one given in the previous section. The household/firm unit hires labor, $\tilde{h}_t$, from a perfectly competitive market.

The demand faced by the household/firm unit for the intermediate input that it produces is of the form $Y_t d(\tilde{P}_t/P_t)$, where Y_t denotes the level of aggregate demand, which is taken as exogenous by the household/firm unit, $\tilde{P}_t$ denotes the nominal price of the intermediate good produced by the household/firm unit, and P_t is the price of the composite consumption good. The demand function $d(\cdot)$ is assumed to be decreasing in the relative price $\tilde{P}_t/P_t$ and to satisfy $d(1) = 1$ and $-d'(1) \equiv \eta > 1$, where η denotes the price elasticity of demand for each individual variety of intermediate goods that prevails in a symmetric equilibrium. The restrictions on $d(1)$ and $d'(1)$ are necessary for the existence of a symmetric equilibrium. The monopolist sets the price of the good it supplies taking the level of aggregate demand as given, and is constrained to satisfy demand at that price, that is, $z_t F(\tilde{h}_t) \geq Y_t d(\tilde{P}_t/P_t)$.

Price adjustment is assumed to be sluggish à la Rotemberg (1982). Specifically, the household/firm unit faces a resource cost of changing prices that is quadratic in the inflation rate of the good it produces.

$$\text{Price adjustment cost } = \frac{\theta}{2}\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1\right)^2. \tag{7}$$

The parameter θ measures the degree of price stickiness. The higher is θ the more sluggish is the adjustment of nominal prices. When θ equals zero, prices are fully flexible.

The flow budget constraint of the household/firm unit in period t is then given by:

$$c_t + \tau_t^L \leq (1 - \tau_t^D) w_t h_t + \left[\frac{\tilde{P}_t}{P_t} Y_t d\left(\frac{\tilde{P}_t}{P_t}\right) - w_t \tilde{h}_t - \frac{\theta}{2}\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1\right)^2\right],$$

where τ_t^D denotes an income tax rate. We introduce this tax as a way to offset the distortions arising from the presence of monopolistic competition.

We restrict attention to a stationary symmetric equilibrium in which all household/firm units charge the same price for the intermediate good they produce. Letting $\pi_t \equiv P_t/P_{t-1}$ denote the gross rate of inflation, the complete set of equilibrium conditions is then given by

$$\pi_t(\pi_t - 1) = \frac{\eta c_t}{\theta}\left[\frac{w_t}{z_t F'(h_t)} - \frac{\eta - 1}{\eta}\right] + \beta E_t \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \pi_{t+1}(\pi_{t+1} - 1), \tag{8}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = (1 - \tau_t^D) w_t, \tag{9}$$

and

$$z_t F(h_t) - \frac{\theta}{2}(\pi_t - 1)^2 = c_t. \tag{10}$$

The above three equations provide solutions for the equilibrium processes of consumption, c_t, hours, h_t, and the real wage, w_t, given processes for the rate of inflation, π_t, and for the tax rate, τ_t^D, which we interpret to be outcomes of the monetary and fiscal policies in place, respectively.

The first equilibrium condition is the new Keynesian Phillips curve, to which the current volume is devoted. It describes an equilibrium relationship between current inflation, π_t, the current deviation of marginal cost, $w_t/z_tF'(h_t)$, from marginal revenue, $(\eta-1)/\eta$, and expected future inflation. Under full price flexibility, the firm would always set marginal revenue equal to marginal cost. However, in the presence of price adjustment costs this practice is costly. To smooth out price changes over time, firms set prices to equate an average of current and future expected marginal costs to an average and future expected future marginal revenues. This optimal price-setting behavior gives rise to a relation whereby, given expected inflation, current inflation is an increasing function of marginal costs. Intuitively, this relation is steeper the more flexible are prices (i.e., the lower is θ), the more competitive are product markets (i.e., the higher is η), and the higher is the current level of demand (i.e., the larger is c_t). At the same time, given marginal cost, current inflation is increasing in expected future inflation. This is because, with quadratic costs of changing nominal prices, a firm expecting higher inflation in the future would like to smooth out the necessary price adjustments over time by beginning raising prices already in the current period.

We have derived the new Keynesian Phillips curve in the context of the Rotemberg (1982) model of price stickiness. However, a similar relationship emerges under other models of nominal rigidity. This is the case, for instance, for the Calvo-Woodford-Yun model. In that model, price stickiness arises because firms are assume to receive an idiosyncratic random signal each period indicating whether they are allowed to reoptimize their posted prices or not. A difference between the Rotemberg and the Calvo-Woodford-Yun models is that the latter displays equilibrium price dispersion across firms even in the absence of aggregate uncertainty. Up to first order, the new Keynesian Phillips curves implied by the Rotemberg and Calvo-Woodford-Yun models are identical.

The second equilibrium condition presented above states that the marginal rate of substitution of consumption for leisure is equated to the after-tax real wage rate. And the third equilibrium condition is a resource constraint requiring that aggregate output net of price adjustment costs equal private consumption.

It is straightforward to establish that in this economy, the optimal monetary policy, that is, the policy that maximizes the welfare of the representative household, is one in which the inflation rate is nil at all times. Formally, the optimal monetary policy must be consistent with an equilibrium in which

$$\pi_t = 1$$

for all $t \geq 0$. This result holds exactly provided the fiscal authority subsidizes labor income to a point that fully offsets the distortion arising from the existence of imperfect competition among intermediate goods produces. Specifically, the income tax rate τ_t^D must be set at a constant and negative level given by

$$\tau_t^D = \frac{1}{1-\eta},$$

for all $t \geq 0$.

To see that the proposed policy regime is optimal, we demonstrate that it implies a set of equilibrium conditions which coincides with the one that arises in an economy with fully flexible prices ($\theta = 0$) and perfect competition in product markets ($\eta = \infty$). In effect, when $\pi_t = 1$ for all t, equilibrium condition (10) collapses to $c_t = z_t F(h_t)$. In addition, under zero inflation the new Keynesian Phillips curve, equation (8), reduces to $w_t = z_t F'(h_t)(\eta - 1)/\eta$. Using this expression along with the proposed optimal level for the income tax rate in the equilibrium labor supply, equation (9), yields the efficiency conditions $-U_h(c_t, h_t)/U_c(c_t, h_t) = z_t F'(h_t)$, which, together with the resource constraint $c_t = z_t F(h_t)$, constitute the equilibrium conditions of a flexible-price perfectly competitive economy.

3 Optimal Monetary Policy with Money Demand and Sticky Prices

At this point, it is of interest to summarize and compare the results obtained in this and the previous sections. We have shown that when prices are fully flexible and the only nominal friction is a demand for money, then optimal monetary policy takes the form of complete stabilization of the interest rate at a value of zero ($R_t = 1$ for all t). We have also established that in a cashless economy in which the only source of nominal friction is given by product price stickiness, optimal monetary policy calls for full stabilization of the rate of inflation at a value of zero ($\pi_t = 1$ for all t). Under optimal policy, in the monetary, flexible-price economy inflation is time varying and equal to the negative of the real interest rate on average, whereas in the cashless, sticky-price economy inflation is constant and equal to zero at all times. Also, in the flexible-price monetary economy optimal policy calls for a constant nominal interest rate equal to zero at all times, whereas in the cashless sticky-price economy it calls for a time-varying nominal interest rate equal to the real interest rate on average.

These results raise the question of what the characteristics of optimal monetary policy are in a more realistic economic environment in which both a demand for money and price stickiness coexist. In particular, in such an environment a policy tradeoff emerges between the benefits of targeting zero inflation—i.e., minimizing price-adjustment costs—and the benefits of deflating at the real rate of interest—i.e., minimizing the opportunity cost of holding money. In the canonical economies with only one nominal friction studied in this and the previous sections, the characterization of the optimal rate of inflation is relatively straightforward. As soon as both nominal frictions are incorporated jointly, it becomes impossible to analytically determine the optimal rate of inflation. One is therefore forced to resort to numerical methods.

The resolution of the Friedman-rule-versus-price-stability tradeoff was studied by, among others, Khan, King, and Wolman (2003) and Schmitt-Grohé and Uribe (2004, 2007). As one would expect, when both the money demand and sticky-price frictions are present, the optimal rate of inflation falls between zero and the one called for by the Friedman rule. The question of interest, however, is where exactly in this interval the optimal inflation rate lies. Khan, King, and Wolman find, in the context of a stylized model calibrated to match aspects of money demand and price dynamics in the postwar United States, that the optimal rate of inflation is -0.76 percent per year. By comparison, in their model the Friedman rule is associated with a deflation rate of 2.93 percent per year. Thus, in the study of Khan, King, and Wolman, the optimal policy is closer to price stability than to the Friedman rule. Taking these numbers at face value, one might conclude that price stickiness is the dominant friction in shaping optimal monetary policy. However, Khan, King, and Wolman (2003) and Schmitt-Grohé and Uribe (2004, 2007) show that the resolution of the tradeoff is quite sensitive to plausible changes in the values taken by the structural parameters of the model.

In Schmitt-Grohé and Uribe (2007), we find that the most striking characteristic of the optimal monetary regime is the high sensitivity of the welfare-maximizing rate of inflation with respect to the parameter governing the degree of price stickiness for the range of values of this parameter that is empirically relevant. The model underlying the analysis of Schmitt-Grohé and Uribe (2007) is a medium-scale model of the US economy featuring, in addition to money demand by households and sticky product prices, a number of real and nominal rigidities including wage stickiness, a demand for money by firms, habit formation, capital accumulation, variable capacity utilization, and investment adjustment costs. The structural parameters of the model are assigned values that are consistent with full- as well as limited-information approaches to estimation of this particular model.

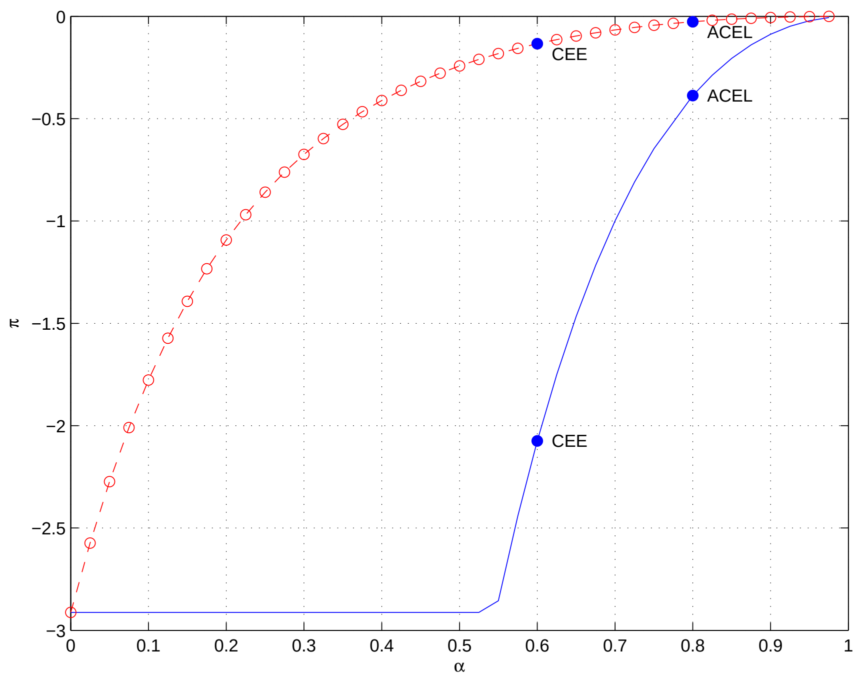
In the Schmitt-Grohé and Uribe (2007) model, the degree of price stickiness is captured by a parameter denoted α, measuring the probability that a firm is not able to optimally set the price it charges in a particular quarter. So that $1/(1-\alpha)$ represents the average time elapsed between two consecutive optimal price adjustments. Available empirical estimates of the degree of price rigidity using macroeconomic data vary from 2 to 5 quarters, or

$\alpha \in [0.5, 0.8]$. For example, Christiano, Eichenbaum, and Evans (2005) estimate α to be 0.6. By contrast, Altig, Christiano, Eichenbaum, and Lindé (2005) estimate an marginal-cost-gap coefficient in the Phillips curve that is consistent with a value of α of around 0.8. Both Christiano, Eichenbaum and Evans and Altig, Christiano, Eichenbaum, and Lindé use an impulse-response matching technique to estimate the price-stickiness parameter α. Bayesian estimates of this parameter include Del Negro, Schorfheide, Smets, and Wouters (2004), Levin, Onatski, Williams, and Williams (2005), and Smets and Wouters (2007) who report posterior means of 0.67, 0.83, and 0.66, respectively, and 90-percent posterior probability intervals of (0.51,0.83), (0.81,0.86), and (0.56,0.74), respectively. On the other hand, evidence on price stickiness based on microeconomic data suggest a much higher frequency of price changes than the evidence based on macro data. The findings reported in Bils and Klenow (2004) and Golosov and Lucas (2003), for example, suggest values of α of around 1/3, or a degree of price stickiness of about 1.5 quarters.

Figure 1 displays with a solid line the relationship between the degree of price stickiness, α, and the optimal rate of inflation in percent per year, π, implied by the model studied in Schmitt-Grohé and Uribe (2007). When α equals 0.5, the lower range of the available empirical evidence using macro data, the optimal rate of inflation is -2.9 percent, which is the level called for by the Friedman rule. For a value of α of 0.8, which is near the upper range of the available empirical evidence using macro data, the optimal level of inflation rises to -0.4 percent, which is close to price stability. Also evident from figure 1 is the fact that values of α based on microeconomic evidence, around 1/3, imply that the Friedman rule is optimal in the long-run.

The above analysis suggests the importance of refining the available estimates of the degree of price stickiness. This research should aim not only at narrowing the range of values that stem from macro evidence but also at reconciling the apparent disconnect between estimates emerging from macro and micro data. Significant progress along this last dimension has been made by Carvalho (2006), who shows that in the presence of heterogeneity in the

Figure 1: Price Stickiness, Fiscal Policy, and Optimal Inflation



__________ Lump-Sum Taxes -o-o- Optimal Distortionary Taxes

Note: CEE and ACEL indicate, respectively, the values for the parameter α estimated by Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Lindé (2005).

degree of price stickiness, there is an upward aggregation bias in econometric estimates of the degree of price stickiness using aggregate data and the standard new Keynesian Phillips curve. Specifically, Carvalho (2006) shows that to explain observed inflation dynamics, an economy with an homogeneous degree of price stickiness across firms requires a frequency of price changes ($1/(1-\alpha)$) that is about three times lower than the average of an economy with a degree of heterogeneity in price stickiness consistent with microeconomic evidence on price changes from the United States. As reported above, micro evidence suggests that firms change prices about every 1.5 quarters, whereas macro evidence is consistent with price changes taking place on average every 3.5 quarters. The ratio of the macro-based price change frequency to its micro-based counterpart is thus 3.5/1.5, which is close to the ratio suggested by Carvalho's theoretical analysis.[1]

There is also empirical research devoted to reconciling the micro and macro evidence on price stickiness. For example, Klenow and Kryvtsov (2005) and Nakamura and Steinsson (2007b) find that excluding sales from price data underlying the consumer price index significantly reduces the frequency of price changes. Specifically, Klenow and Kryvtsov report that regular (nonsale) prices change on average every 7.2 months, whereas regular prices including sales change on average every 4.3 months. Nakamura and Steinsson, using a longer sample period and a somewhat different methodology, find even longer price spells of 8 to 11 months when excluding sales.

Besides the uncertainty surrounding the estimation of the degree of price stickiness, a second aspect of the apparent difficulty in establishing reliably the optimal long-run level of inflation has to do with the shape of the relationship linking the degree of price stickiness to the optimal level of inflation. The problem resides in the fact that, as is evident from figure 1, this relationship becomes significantly steep precisely for that range of values of α that is empirically most compelling. The problem would not arise if the steep portion of the relationship would take place at values of α below 1/3 or above 0.8, say. It turns

[1] Nakamura and Steinsson (2007a) arrive at similar results.

out that an important factor determining the shape of the function relating the optimal level of inflation to the degree of price stickiness is the underlying fiscal policy regime. We address the interaction between optimal monetary and fiscal policies in the context of a model featuring a new Keynesian Phillips curve in the next section.

Before turning to the interaction between fiscal and monetary policy, we wish to draw attention to the fact that, quite independently of the precise degree of price stickiness, the optimal inflation target is below zero. In light of this robust result, it is puzzling that all countries that self-classify as inflation targeters set inflation targets that are positive. In effect, in the developed world inflation targets range between 2 and 4 percent per year. Somewhat higher targets are observed across developing countries. An argument often raised in defense of positive inflation targets is that negative inflation targets imply nominal interest rates that are dangerously close to the zero lower bound on nominal interest rates and hence may impair the central bank's ability to conduct stabilization policy. In Schmitt-Grohé and Uribe (2007) we find, however, that this argument is of no relevance in the context of the medium-scale estimated model within which we conduct policy evaluation. The reason is that under the optimal policy regime, the mean of the nominal interest rate is about 4.5 percent per year with a standard deviation of only 0.4 percent. This means that for the zero lower bound to pose an obstacle to monetary stabilization policy, the economy must suffer from an adverse shock that forces the interest rate to be more than 10 standard deviations below target. The likelihood of such an event is practically nil.

4 Implications of the NKPC for Optimal Monetary and Fiscal Policy

Thus far we have ignored fiscal considerations by implicitly or explicitly assuming the existence of lump-sum taxes that not only balance the government budget at all times but also allow the government to subsidize output to a degree that the level of production is as high in the model with monopolistic competition as in a perfectly competitive economy. This assumption is clearly unrealistic —however, it is an assumption commonly maintained in the literature on optimal monetary policy in models featuring a new Keynesian Phillips curve. Here we wish to argue that taking explicitly into account the fiscal side of an optimal policy problem has crucial consequences for the optimal long-run level and cyclical properties of inflation.

4.1 The Optimal Level of Inflation

Fiscal considerations fundamentally change the long-run tradeoff between price stability and the Friedman rule. To see this, we now consider an economy where lump-sum taxes are unavailable ($\tau^L = 0$). Instead, the fiscal authority must finance government purchases by means of proportional capital and labor income taxes. The social planner sets jointly monetary and fiscal policy in a Ramsey-optimal (i.e., welfare maximizing) fashion. The details of this environment are contained in Schmitt-Grohé and Uribe (2006). Figure 1 displays the relationship between the degree of price stickiness, α, and the optimal rate of inflation, π. The solid line corresponds to the case discussed earlier featuring lump-sum taxes. The dash-circled line corresponds to the economy with optimally chosen distortionary income taxes analyzed in Schmitt-Grohé and Uribe (2006). In stark contrast to what happens under lump-sum taxation, under optimal distortionary taxation the function linking π and α is flat and very close to zero for the entire range of macro-data-based empirically plausible values of α, namely 0.5 to 0.8. In other words, when taxes are distortionary and optimally determined, price stability emerges as a prediction that is robust to the existing uncertainty about the exact degree of price stickiness. Even if one focuses on the evidence of price stickiness stemming from micro data, the model with distortionary Ramsey taxation predicts an optimal long-run level of inflation that is much closer to zero than to the level called for by the Friedman rule.

Our intuition for why price stability arises as a robust policy recommendation in the economy with optimally set distortionary taxation runs as follows. Consider the economy with lump-sum taxation. Deviating from the Friedman rule (by raising the inflation rate) has the benefit of reducing price adjustment costs. Consider next the economy with optimally chosen income taxation and no lump-sum taxes. In this economy, deviating from the Friedman rule still provides the benefit of reducing price adjustment costs. However, in this economy increasing inflation has the additional benefit of increasing seignorage revenue thereby allowing the social planner to lower distortionary income tax rates. Therefore, the Friedman-rule versus price-stability tradeoff is tilted in favor of price stability.

It follows from this intuition that what is essential in inducing the optimality of price stability is that on the margin the fiscal authority trades off the inflation tax for regular taxation. Indeed, it can be shown that if distortionary tax rates are fixed, even if they are fixed at the level that is optimal in a world without lump-sum taxes, and the fiscal authority has access to lump-sum taxes on the margin, the optimal rate of inflation is much closer to the Friedman rule than to zero. In this case, increasing inflation no longer has the benefit of reducing distortionary taxes. As a result, the Ramsey planner has less incentives to inflate.

4.2 The Optimal Volatility of Inflation

Two distinct branches of the existing literature on optimal monetary policy deliver diametrically opposed policy recommendations concerning the cyclical behavior of prices and interest rates. One branch follows the theoretical framework laid out in Lucas and Stokey (1983). It studies the joint determination of optimal fiscal and monetary policy in flexible-price environments with perfect competition in product and factor markets. In this strand of the literature, the government's problem consists in financing an exogenous stream of public spending by choosing the least disruptive combination of inflation and distortionary income taxes. The criterion under which policies are evaluated is the welfare of the representative private agent.

Calvo and Guidotti (1990, 1993) and Chari, Christiano, and Kehoe (1991) characterize optimal monetary and fiscal policy in stochastic environments with nominal non-state-contingent government liabilities. A key result of these papers is that it is optimal for the government to make the inflation rate highly volatile and serially uncorrelated. For instance, Schmitt-Grohé and Uribe (2004) show, in the context of a flexible-price model calibrated to the U.S. economy, that under the optimal policy the inflation rate has a standard deviation of 6.8 percent per year and a serial correlation of -0.04. The intuition for this result is that, under flexible prices, highly volatile and unforecastable inflation is nondistorting and at the same time carries the fiscal benefit of acting as a lump-sum tax on private holdings of government-issued nominal assets. The government is able to use surprise inflation as a nondistorting tax to the extend that it has nominal, non-state-contingent liabilities outstanding. Thus, price changes play the role of a shock absorber of unexpected innovations in the fiscal deficit. This 'front-loading' of government revenues via inflationary shocks allows the fiscal authority to keep income tax rates remarkably stable over the business cycle.

On the other hand, a more recent literature focuses on characterizing optimal monetary policy in environments with nominal rigidities and imperfect competition. (See, for example, the authoritative treatment in Woodford, 2003.) Besides its emphasis on the role of price rigidities and market power, this literature differs from the earlier one described above in two important ways. First, it assumes, either explicitly or implicitly, that the government has access to (endogenous) lump-sum taxes to finance its budget. An important implication of this assumption is that there is no need to use unanticipated inflation as a lump-sum tax; regular lump-sum taxes take up this role. Second, as discussed in section 2, the government is assumed to be able to implement a production (or employment) subsidy to eliminate the distortion introduced by the presence of monopoly power in product and factor markets.

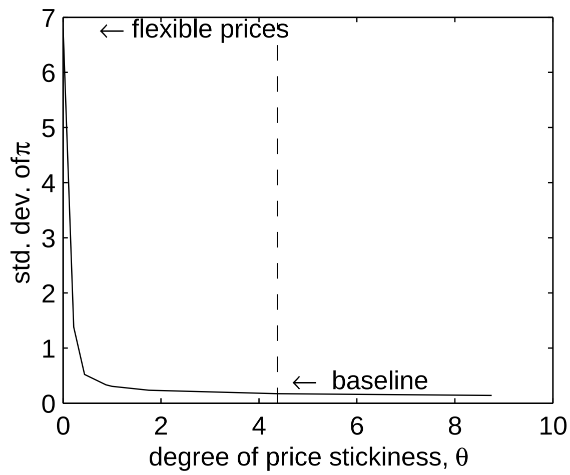
A key result of this literature, which we presented in sections 2 and 3, is that the optimal monetary policy features an inflation rate that is zero or close to zero at all times. The reason why price stability turns out to be optimal in environments of the type described there is that it minimizes (or completely eliminates) the costs introduced by inflation under nominal rigidities.

Taken together, these two strands of research on optimal monetary policy leave the monetary authority without a clear policy recommendation. Should the central bank pursue policies that imply high or low inflation volatility? In Schmitt-Grohé and Uribe (2004), we analyze the resolution of this policy dilemma by incorporating in a unified framework the essential elements of the two approaches to optimal policy described above. Specifically, we build a model that shares two elements with the earlier literature: (a) The only source of regular taxation available to the government is distortionary income taxes. As a result, the government cannot implement production subsidies to undo distortions created by the presence of imperfect competition; and (b) The government issues only nominal, one-period, non-state-contingent bonds. At the same time, the setup shares two important assumptions with the more recent body of work on optimal monetary policy: (a) Product markets are imperfectly competitive. And (b) Product prices are assumed to be sticky and hence the model features a new Keynesian Phillips curve. Schmitt-Grohé and Uribe (2004) introduce price stickiness as in the previous section by assuming that firms face a convex cost of price adjustment (Rotemberg, 1982). An assumption maintained here is that the government has the ability to fully commit to the implementation of announced fiscal and monetary policies.

In this environment, the government faces a tradeoff in choosing the path of inflation. On the one hand, the government would like to use unexpected inflation as a non-distorting tax on nominal wealth. In this way the fiscal authority could minimize variations in distortionary income taxes over the business cycle. On the other hand, changes in the rate of inflation come at a cost, for firms face nominal rigidities.

When price changes are brought about at a cost, it is natural to expect that a benevolent government will try to implement policies consistent with a more stable behavior of prices than when price changes are costless. However, the quantitative effect of an empirically plausible degree of price rigidity on optimal inflation volatility is not clear a priori. In Schmitt-Grohé and Uribe (2004), we show that for the degree of price stickiness that has

Figure 2: Degree of Price Stickiness and Optimal Inflation Volatility



Note. The parameter θ is governs the cost of adjusting nominal prices, as defined in equation (7). Its baseline value is 4.4, in line with available empirical estimates. The standard deviation of inflation is measured in percent per year.

been estimated for the U.S. economy, this tradeoff is overwhelmingly resolved in favor of price stability. The Ramsey allocation features a dramatic drop in the standard deviation of inflation from about 7 percent per year under flexible prices to a mere 0.17 percent per year when prices adjust sluggishly. Siu (2004) obtains similar results in a cash-credit economy where nominal rigidities are introduced by assuming that a fraction of firms must set their price one period in advance and the only source of uncertainty are government purchases shocks.

Indeed, the impact of price stickiness on the optimal degree of inflation volatility turns out to be much stronger than suggested by the numerical results reported in the previous paragraph. Figure 2, taken from Schmitt-Grohé and Uribe (2004), shows that a minimum amount of price stickiness suffices to make price stability the central goal of optimal policy. Specifically, when the degree of price stickiness, embodied in the parameter θ (see equation (7)), is assumed to be 10 times smaller than the estimated value for the U.S. economy, the optimal volatility of inflation is below 0.52 percent per year, 13 times smaller than under full price flexibility.

A natural question elicited by figure 2 is why even a modest degree of price stickiness can turn undesirable the use of a seemingly powerful fiscal instrument, such as large re- or devaluations of private real financial wealth through surprise inflation. Our conjecture is that in the flexible-price economy, the welfare gains of surprise inflations or deflations are very small. Our intuition is as follows. Under flexible prices, it is optimal for the central bank to keep the nominal interest rate constant over the business cycle. This means that large surprise inflations must be as likely as large deflations, as variations in real interest rates are small. In other words, inflation must have a near-i.i.d. behavior. As a result, high inflation volatility cannot be used by the Ramsey planner to reduce the average amount of resources to be collected via distortionary income taxes, which would be a first-order effect. The volatility of inflation serves primarily the purpose of smoothing the process of income tax distortions—a second-order source of welfare losses—without affecting their average level.

An additional way to gain intuition for the dramatic decline in optimal inflation volatility that takes place even at very modest levels of price stickiness is to interpret price volatility as a way for the government to introduce real state-contingent public debt. Under flexible prices the government uses state-contingent changes in the price level as a non-distorting tax or transfer on private holdings of government assets. In this way, non-state contingent nominal public debt becomes state-contingent in real terms. So, for example, in response to an unexpected increase in government spending the Ramsey planner does not need to increase tax rates by much because by inflating away part of the public debt he can ensure intertemporal budget balance. It is therefore clear that introducing costly price adjustment is as if the government was limited in its ability to issue real state-contingent debt. It follows that the larger is the welfare gain associated with the ability to issue real state-contingent public debt—as opposed to non-state contingent debt—the larger is the amount of price stickiness required to reduce the optimal degree of inflation volatility. Aiyagari, Marcet, Sargent, and Seppala (2002) show that indeed the level of welfare under the Ramsey policy in an economy without real state-contingent public debt is virtually the same as in an economy with state-contingent debt. Our finding that a small amount of price stickiness is all it takes to bring the optimal volatility of inflation from a very high level to near zero is thus perfectly in line with the finding of Aiyagari, Marcet, Sargent, and Seppala (2002).

If this intuition is correct, then the behavior of tax rates and public debt under sticky prices should resemble that implied by the Ramsey allocation in economies without real state-contingent debt. Indeed, in financing the budget, the Ramsey planner replaces front-loading with standard debt and tax instruments. For example, in response to an unexpected increase in government spending the planner does not generate a surprise increase in the price level. Instead, he chooses to finance the increase in government purchases partly through an increase in income tax rates and partly through an increase in public debt. The planner minimizes the tax distortion by spreading the required tax increase over many periods. This tax-smoothing behavior induces near-random walk dynamics into the tax rate and public debt. By contrast, under full price flexibility (i.e., when the government can create real-state contingent debt) tax rates and public debt inherit the stochastic process of the underlying shocks.

An important conclusion of this analysis is thus that the Aiyagari, Marcet, Sargent, and Seppala (2002) result, namely, that optimal policy imposes a near random walk behavior on taxes and debt, does not require the unrealistic assumption that the government can issue only non-state-contingent real debt. This result emerges naturally in economies with nominally non-state contingent debt—clearly the case of greatest empirical relevance—and a minimum amount of price rigidity.

It is of interest to relate the near random walk in taxes and debt that emerges as the optimal policy outcome in a model featuring a new Keynesian Phillips curve with the celebrated tax smoothing result of Barro (1979). In Barro's formulation the objective function of the government is the expected present discounted value of squared deviations of tax rates from a target or desired level. The government minimizes this objective function subject to a sequential budget constraint, which is linear in debt and tax rates. The resulting solution resembles the random walk model of consumption with taxes taking the place of consumption and public debt taking the place of private debt. The analysis in Schmitt-Grohé and Uribe (2004) departs from Barro's ad-hoc loss function and replaces it by the utility function of the representative optimizing household inhabiting a fully-articulated, dynamic, stochastic, general-equilibrium economy. In this environment, the random-walk result obtains form a more subtle channel, namely, the introduction of a miniscule amount of nominal rigidity in product prices.

We note that an important assumption driving the result that significantly less inflation volatility is desirable in the presence of sticky prices is that government debt is state-noncontingent. This assumption is empirically appealing because the majority of of outstanding public debt in the United States and other industrialized countries is nominally risk free, and therefore nominally non-state contingent. However, if government debt is assumend to be state contingent, the presence of sticky prices may introduce no difference in the Ramsey real allocation (see Correia, Nicolini, and Teles, 2008). The reason for this result is that, as shown in Lucas and Stokey (1983), if government debt is state contingent and prices are fully flexible, the Ramsey allocation does not pin down the price level uniquely. In this case there is an infinite number of price level processes (and thus of money supply processes) that can be supported as Ramsey outcomes. Loosely speaking, the introduction of price stickiness simply 'uses this degree of freedom' without altering other aspects of the Ramsey solution. This is not possible under state-noncontingent debt. For in this case the price level is uniquely determined in the flexible-price economy. Thus, the presence of nominal rigidities modifies the optimal real allocation in fundamental ways.

5 Implementation of Optimal Policy

We established that in the simple new Keynesian model of section 2, the optimal (i.e., welfare-maximizing) policy consists in setting the inflation rate equal to zero at all times and imposing a constant output subsidy. The optimal inflation rate fully eliminates price-adjustment costs, and the income subsidy fully eliminates the distortion arising from imperfect competiiton. The question we pursue in this section is how to implement this optimal policy.

Because central banks in the United States and elsewhere use the short-term nominal interest rate as the monetary policy instrument, it is of empirical interest to search for interest-rate rules that implement the optimal allocation. One might be tempted to believe that implementation of optimal policy is trivial once the optimal allocation (or Ramsey equilibrium) has been solved for. Specifically, in the Ramsey equilibrium, the nominal interest rate can be expressed as a function of the current state of the economy. Then, the prescription would be to simply use this function as a policy rule in setting the nominal interest rate at all dates and under all circumstances. It turns out that conducting policy in this fashion would in general not deliver the intended results. The reason is that although such a policy would be consistent with the optimal equilibrium it would at the same time open the door to other (suboptimal) equilibria. It follows that the solution to the optimal policy promblem (i.e., the Ramsey equilibrium) is mute with respect to the issue of implementation of such policy. To see this, it is convenient to consider as an example a log-linear approximation to the equilibrium conditions associated with the cashless, sticky-price model presented in section 2. It can be shown that the resulting linear system is given by[2]

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{c}_t - \gamma \hat{z}_t,$$

$$-\sigma \hat{c}_t = \hat{R}_t - \sigma E_t \hat{c}_{t+1} - E_t \hat{\pi}_{t+1}$$

where $\sigma, \kappa, \gamma > 0$ are parameters. All hatted variables denote percent deviaitons of the corresponding nonhatted variables from their respective values in the deterministic steady

[2] The log-linearization is performed around the nonstochastic steady state of the Ramsey equilibrium. In performing the linearization, we assume that the period utility function is separable in consumption and hours and that the production function is linear in labor.

state of the Ramsey equilibrium. The first results from combining equations (8), (9), and (10) and is typically referred to as the new Keynesian Phillips curve. The second equation is a linearized version of an Euler equation that prices nominally risk-free bonds, where R_t denotes the gross nominal risk-free interest rate between periods t and $t+1$. This equation is typically refferred to as the intertemporal IS equation. This expression does not appear in the definition of equilibrium given in section 2. We introduce it here because be are interested in analyzing the equilibrium consequences of alternative interest-rate feedback rules. In deriving the above system of equations we set the tax rate τ_t^D equal to its constant optimal level. We have established earlier that under the welfare-maximizing allocation $\hat{\pi}_t = 0$ at all times. Substituting this value into the new Keynesian Phillips curve, we have that the optimal soluiton for consumption satisfies $\hat{c}_t = \gamma/\kappa\hat{z}_t$. The goal of this section is to identify interest-rate feedback rules that can suppor this outcome as the unique competitive equilibrium.

The above system is in two equatons and three unknowns, $\hat{c}_t$, $\hat{\pi}_t$, and $\hat{R}_t$. To close the model, one must specify monetary policy. Suppose, following the argument given above, we were to pick as a policy rule the Ramsey solution for the nominal interest rate. Recall that under the optimal solution, $\hat{\pi}_t$ equals zero at all times. It then follows from the intertemporal IS curve that the nominal interest rate is given by $\hat{R}_t = \sigma E_t(\hat{c}_{t+1} - \hat{c}_t)$, which states that under the optimal policy the nominal and real rates of interest coincide. Suppose for a moment that the central bank adopted this expression as a policy feedback rule. The question we wish to entertain is whether this feedback rule would uniquely implement the optimal (or Ramsey) competitive equilibrium.

Let us split this question into two parts by first considering whether the proposed policy rule can support the optimal equilibrium. Substituting the proposed interest-rate rule into the intertemporal IS equaiton, we obtain $E_t\hat{\pi}_{t+1} = 0$, or that expected inflaiton must be nil at all times. Using this result in the new Keynesian Phillips curve, we obtain $\hat{\pi}_t = \kappa\hat{c}_t - \gamma\hat{z}_t$. The optimal paths of inflation and consumption, namely $\hat{\pi}_t = 0$ and $\hat{c}_t = \gamma/\kappa\hat{z}_t$ solve this equation. It follows that the proposed policy rule is consistent with the optimal equilibrium.

The second part of the question we wish to address is whether the proposed rule $\hat{R}_t = \sigma E_t(\hat{c}_{t+1} - \hat{c}_t)$ implements the optimal allocation uniquely. The answer to this question is no. To see this, consider a solution of the form $\hat{\pi}_t = \epsilon_t$, where ϵ_t is any white noise process. Then, we have that $E_t\hat{\pi}_{t+1} = E_t\epsilon_{t+1} = 0$, as required by the combination of the proposed interest-rate feedback rule and the intertemporal IS equation. In addition, the solution for consumpiton can be read off the new Keynessian Phillips curve as being $\hat{c}_t = \gamma/\kappa\hat{z}_t + (1/\kappa)\epsilon_t$. Clearly, the resulting competitive equilibrium is different from the welfare-maximizing competitive equilibrium.

We have thus demonstrated that using the Ramsey solution for the interest rate as a policy rule opens the door to an infinite number of suboptimal cmpetitive equilibria. In each of these equilibria, inflation varies over time, causing unnecessary price adjustment costs. It follows that the use of the Ramsey solution for the nominal interest rate as a policy rule fails to implement the desired competitive equilibrium.

5.1 Can the Taylor Rule Implement Optimal Policy?

A Taylor-type rule is an interest-rate feedback rule whereby the nominal interest rate is set as an increasing linear function of inflaiton and deviations of real output from trend, with an inflation coefficient greater than unity and an output coefficient greater than zero. Formally, a Taylor-type interest-rate rule can be written as $\hat{R}_t = \alpha_\pi\hat{\pi}_t + \alpha_y\hat{y}_t$, where $\alpha_\pi > 1$ and $\alpha_y > 0$ are parameters and $\hat{y}_t$ represents the percent deviation of real output from trend. Taylor's rule has been widely studied in monetary economics since the publication of Taylor's (1993) seminal article. It has been advocated as a desirable policy specification and is considered an adequate approximation to actual monetary policy in the United States and many other developed countries. For this reason, we now consider the question of whether a Taylor rule can implement the optimal allocation. The answer is that in general an interest-rate feedback rule of the type proposed by Taylor is unable to support the Ramsey optimal equilibrium.

This issue was first analyzed by Woodford (2001).

To establish whether the Taylor rule presented above can implement the optimal allocation, set the inflation rate at its optimal value of zero ($\hat{\pi}_t = 0$) and combine the intertemporal IS equation with the Taylor rule. This yields $\alpha_y \hat{c}_t = \sigma(E_t \hat{c}_{t+1} - \hat{c}_t)$. Now replacing $\hat{c}_t$ with its optimal value of $\gamma/\kappa \hat{z}_t$, we obtain $E_t \hat{z}_{t+1} = (1 + \alpha_y/\sigma)\hat{z}_t$. This expression represents a contradiction because the productivity shock $\hat{z}_t$ is assumed to be an exogenous stationary process whose first-order approximation need not have a first-order autoregressive representation. Further, even if $\hat{z}_t$ did have an $AR(1)$ representation, stationarity would require that the $AR(1)$ coefficient be less than one in absolute value. In turn, this requirement would imply that α_y is negative, which is contrary to the definition of a Taylor-type rule given above.

We have established that the proposed Taylor rule fails to implement the optimal equilibrium. This result also obtains when the nominal interest rate is assumed to respond to deviations of output from its natural level, which is the level of output associated with the optimal equilibrium. Letting $y_t^n \equiv \gamma/\kappa \hat{z}_t$ denote the natural rate of output, such a rule can be written as $\hat{R}_t = \alpha_\pi \hat{\pi}_t + \alpha_y(\hat{y}_t - \hat{y}_t^n)$.

The optimal allocation can indeed be implemented by a modified Taylor rule of the form $\hat{R}_t = \hat{r}_t^n + \alpha_\pi \hat{\pi}_t$ as long as $\alpha_\pi > 1$. In this rule $\hat{r}_t^n \equiv \sigma\gamma/\kappa(E_t \hat{z}_{t+1} - \hat{z}_t)$ represents the risk-free real and nominal interest rate that prevails in the Ramsey optimal equilibrium. The first term in the modified Taylor rule makes the Ramsey allocation feasible as an equilibrium outcome. The second term makes it unique. The key difference between the standard and modified Taylor rules is that the latter features a time-varying intercept that allows the nominal interest rate to accommodate movements in the real interest rate one for one without requiring changes in the price level. More generally, the optimal competitive equilibrium can be implemented via rules of the form $\hat{R}_t = \hat{r}_t^n + \alpha_\pi \hat{\pi}_t + \alpha_y(\hat{y}_t - \hat{y}_t^n)$, with policy parameters α_π and α_y satisfying the restrictions imposed by the definition of a Taylor-type rule given above.

Applying this type of rule can be quite impractical. For it would require knowledge on the part of the cantral bank of current and expected future values taken by all of the shocks that affect the real interest rate as well as of the function mappint such values to the natural rate of interest. This difficulty raises the quesiton of how close to implementing the optimal equilibrium a less sophisticated interest-rate rule would get. We turn to this issue next.

6 Optimal, Simple, and Implementable Rules

We wish to ascertain the ability of simple, implementable interest-rate rules to approximate the outcome of optimal policy. We draw from our previous work (Schmitt-Grohé and Uribe, 2007), where we perform policy evaluation in the context of a calibrated model of the U.S. business cycle featuring monopolistic competition and sticky prices in product markets and capital accumulation.[3] In the model, business cycles are driven by stochastic variations in the level of total factor productivity and government consumption. We impose two requirements for an interest-rate rule to be implementable. First, the rule must deliver a unique rational expectations equilibrium. And second, it must induce nonnegative equilibrium dynamics for the nominal interest rate. At the same time, for an interest-rule to be simple we require that the interest rate be set as a function of a small number of easily observable macroeconomic indicators. Specifically, we study interest-rate feedback rules that respond to measures of inflation, output and lagged values of the nominal interest rate. The family of rules we consider is of the form

$$\ln(R_t/R^*) = \alpha_R \ln(R_{t-1}/R^*) + \alpha_\pi E_t \ln(\pi_{t-i}/\pi^*) + \alpha_y E_t \ln(y_{t-i}/y^*); \quad i = -1, 0, 1, \tag{11}$$

where y^* denotes the nonstochastic Ramsey steady-state level of aggregate demand, and R^*, π^*, α_R, α_π,, and α_y are parameters. The index i can take three values 1, 0, and -1. When $i =$ 1, we refer to the interest rate rule as backward looking, when $i = 0$ as contemporaneous, and

[3] See also Rotemberg and Woodford (1997).

Table 1: Evaluating Interest-Rate Rules

	α_π	α_y	α_R	Welfare Cost	σ_π	σ_R
Ramsey Policy	–	–	–	0	0.01	0.27
Optimized Rules						
Contemporaneous ($i = 0$)						
Smoothing	3	0.01	0.84	0.000	0.04	0.29
No Smoothing	3	0.00	–	0.001	0.14	0.42
Backward ($i = 1$)	3	0.03	1.71	0.001	0.10	0.23
Forward ($i = -1$)	3	0.07	1.58	0.003	0.19	0.27
Non-Optimized Rules						
Taylor Rule ($i = 0$)	1.5	0.5	–	0.522	3.19	3.08
Simple Taylor Rule	1.5	–	–	0.019	0.58	0.87
Inflation Targeting	–	–	–	0.000	0	0.27

Notes: (1) The interest-rate rule is given by $\ln(R_t/R^*) = \alpha_R \ln(R_{t-1}/R^*) + \alpha_\pi E_t \ln(\pi_{t-i}/\pi^*) + \alpha_y E_t \ln(y_{t-i}/y^*)$; $i = -1, 0, 1$. (2) In the optimized rules, the policy parameters α_π, α_y, and α_R are restricted to lie in the interval $[0, 3]$. (3) The welfare cost is defined as the percentage decrease in the Ramsey optimal consumption process necessary to make the level of welfare under the Ramsey policy identical to that under the evaluated policy. Thus, a positive figure indicates that welfare is higher under the Ramsey policy than under the alternative policy. (4) The standard deviation of inflation and the nominal interest rate is measured in percent per year.

when $i = -1$ as forward looking. The optimal simple and implementable rule is the simple and implementable rule that maximizes welfare of the representative agent. Specifically, we characterize values of α_π, α_y, and α_R that are associated with the highest level of welfare of the representative agent within the family of smple and implementable interest-rate feedback rules defined by equation (11). As a point of comparison for policy evaluation, we also compute the real allocation associated with the Ramsey optimal policy.

The first row of table 1 shows that under the Ramsey policy inflation is virtually equal to zero at all times.[4] One may wonder why in an economy featuring sticky prices as the single nominal friction, the volatility of inflation is not exactly equal to zero at all times under the Ramsey policy. The reason is that we do not follow the standard practice of subsidizing

[4]In the deterministic steady state of the Ramsey economy, the inflation rate is zero.

factor inputs to eliminate the distortion introduced by monopolistic competition in product markets. Introducing such a subsidy would result in a constant Ramsey-optimal rate of inflation equal to zero.

The remaining rows of table 1 report policy evaluations. The welfare associated with each interest-rate feedback rule is compared to the level of welfare associated with the Ramsey optimal policy. Specifically, the welfare cost is defined as the fraction of the consumption stream an agent living in the Ramsey economy would be willing to give up to be as well off as in an economy in which monetary policy takes the form of the respective interest-rate feedback rule shown in the table.

We consider seven different monetary policies: Four constrained optimal interest-rate feedback rules and three non-optimized rules. In the constrained optimal rule labeled no-smoothing, we search over the policy coefficients α_π and α_y keeping α_R fixed at zero. The second constrained-optimal rule, labeled smoothing in the table, allows for interest-rate inertia by setting optimally all three coefficients, α_π, α_y and α_R.

We find that the best no-smoothing interest-rate rule calls for an aggressive response to inflation and a mute response to output. The inflation coefficient of the optimized rule takes the largest value allowed in our search, namely 3.[5] The optimized rule is quite effective as it delivers welfare levels remarkably close to those achieved under the Ramsey policy. At the same time, the rule induces a stable rate of inflation, a feature that also characterizes the Ramsey policy.

We next study a case in which the central bank can smooth interest rates over time. Our numerical search yields that the optimal policy coefficients are $\alpha_\pi = 3$, $\alpha_y = 0.01$, and $\alpha_R = 0.84$. The fact that the optimized rule features substantial interest-rate inertia means that the monetary authority reacts to inflation much more aggressively in the long run than in the short run. The fact that the interest rule is not superinertial (i.e., α_R does not exceed

[5]Removing the upper bound on policy parameters optimal policy calls for a much larger inflation coefficient, a zero output coefficient and yields a negligible improvement in welfare. The unconstrained policy-rule coefficients are $\alpha_\pi = 332$ and $\alpha_y = 0$. The associated welfare gain is about one thousandth of one percent of consumption.

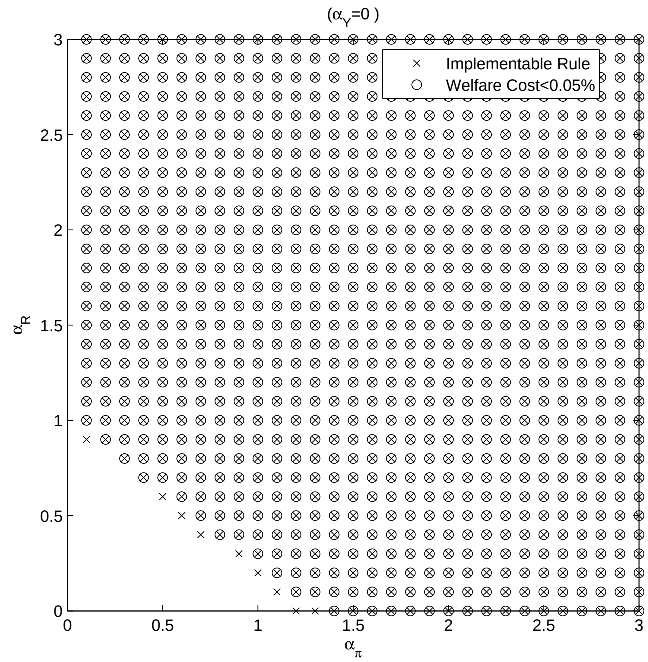
unity) means that the monetary authority is backward looking. So, again, as in the case without smoothing optimal policy calls for a large response to inflation deviations in order to stabilize the inflation rate and for no response to deviations of output from the steady state. The welfare gain of allowing for interest-rate smoothing is insignificant. Taking the difference between the welfare costs associated with the optimized rules with and without interest-rate smoothing reveals that agents would be willing to give up less than 0.001 percent, that is, less than 1 one-thousandth of one percent, of their consumption stream under the optimized rule with smoothing to be as well off as under the optimized policy without smoothing.

The finding that allowing for optimal smoothing yields only negligible welfare gains spurs us to investigate whether rules featuring suboptimal degrees of inertia or responsiveness to inflation can produce nonnegligible welfare losses at all. Panel (a) of figure 3 shows that provided the central bank does not respond to output, $\alpha_y = 0$, varying α_π and α_R between 0 and 3 typically leads to economically negligible welfare losses of less than five one-hundredth of one percent of consumption. The graph shows with crosses combinations of α_π and α_R that are implementable and with circles combinations that are implementable and that yield welfare costs less than 0.05 percent of consumption relative to the Ramsey policy.

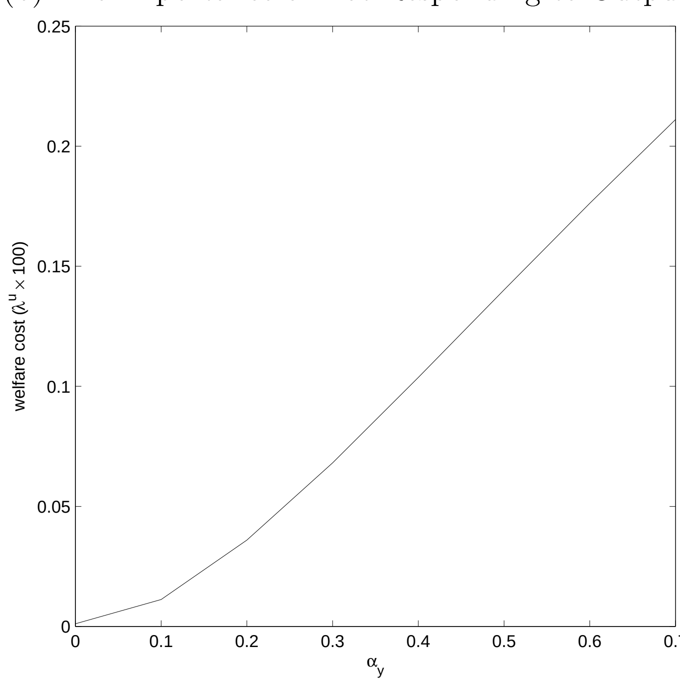
The blank area in the figure identifies α_π and α_R combinations that are not implementable either because the equilibrium fails to be locally unique or because the implied volatility of interest rates is too high. This is the case for values of α_π and α_R such that the policy stance is passive in the long run, that is $\frac{\alpha_\pi}{1-\alpha_R} < 1$. For these parameter combinations the equilibrium is not locally unique. This finding is a generalization of the result, that when the inflation coefficient is less than unity ($\alpha_\pi < 1$) the equilibrium is indeterminate, which obtains in the absence of interest-rate smoothing ($\alpha_R = 0$). We also note that the result that passive interest-rate rules renders the equilibrium indeterminate is typically derived in the context of models that abstract from capital accumulation. It is therefore reassuring that this particular abstraction appears to be of no consequence for the finding that (long-run) passive policy is inconsistent with local uniqueness of the rational expectations equilibrium.

Figure 3: The Cashless Economy

(a) Implementability and Welfare



(b) The Importance of Not Responding to Output



Similarly, we find that determinacy obtains for policies that are active in the long run, $\frac{\alpha_\pi}{1-\alpha_R} > 1$.

More importantly, panel (a) of figure 3 shows that virtually all parameterizations of the interest-rate feedback rule that are implementable yield about the same level of welfare as the Ramsey equilibrium. This finding suggests a simple policy prescription, namely, that any policy parameter combination that is irresponsive to output and active in the long run is equally desirable from a welfare point of view.

One possible reaction to the finding that implementability-preserving variations in α_π and α_R have little welfare consequences may be that in the class of models we consider welfare is flat in a large neighborhood around the optimum parameter configuration, so that it does not really matter what the government does. This turns out not to be the case in the economy studied here. Recall that in the welfare calculations underlying panel (a) of figure 3 the response coefficient on output, α_y, was kept constant and equal to zero. Indeed, as we show in the next subsection, interest-rate rules that lean against the wind by raising the nominal interest rate when output is above trend can be associated with sizable welfare costs.

Panel (b) of figure 3 illustrates the consequences of introducing a cyclical component to the interest-rate rule. It shows that the welfare costs of varying α_y can be large, thereby underlining the importance of not responding to output. The figure shows the welfare cost of deviating from the optimal output coefficient ($\alpha_y \approx 0$) while keeping the inflation coefficient of the interest-rate rule at its optimal value ($\alpha_\pi = 3$) and not allowing for interest-rate smoothing ($\alpha_R = 0$). Welfare costs are monotonically increasing in α_y. When $\alpha_y = 0.7$, the welfare cost is over two tenths of one percent of the consumption stream associated with the Ramsey policy. This is a significant figure in the realm of policy evaluation at business-cycle frequency.[6] This finding suggest that bad policy can have significant welfare costs in our model and that policy mistakes are committed when policy makers are unable to resist the

[6] A similar result obtains if one allows for interest-rate smoothing with α_R taking its optimized value of 0.84.

temptation to respond to output fluctuations.

It follows that sound monetary policy calls for sticking to the basics of responding to inflation alone.[7] This point is conveyed with remarkable simplicity by comparing the welfare consequences of a simple interest-rate rule that responds only to inflation with a coefficient of 1.5 to those of a standard Taylor rule that responds to inflation as well as output with coefficients 1.5 and 0.5, respectively. Table 1 shows that the Taylor rule that responds to output is significantly welfare inferior to the simple interest-rate rule that responds solely to inflation. Specifically, the welfare cost of responding to output is about half a percentage point of consumption.[8]

The Ramsey-optimal monetary policy implies near complete inflation stabilization (see table 1). It is reasonable to conjecture, therefore, that inflation targeting, interpreted to be any monetary policy capable of bringing about zero inflation at all times ($\pi_t = 1$ for all t), would induce business cycles virtually identical to those associated with the Ramsey policy. We confirm this conjecture by computing the welfare cost associated with inflation targeting. The welfare cost of targeting inflation relative to the Ramsey policy is virtually nil.

An important issue in monetary policy is what measures of inflation and aggregate activity the central bank should respond to. In particular, a question that has received considerable attention among academic economists and policymakers is whether the monetary authority should respond to past, current, or expected future values of output and inflation. Here we address this question by computing optimal backward- and forward-looking interest-rate rules. That is, in equation (11) we let i take the values -1 and $+1$. Table 1 shows that there are no welfare gains from targeting expected future values of inflation and output as opposed to current or lagged values of these macroeconomic indicators. Also a muted response to output continues to be optimal under backward- or forward-looking rules.

Under a forward-looking rule without smoothing ($\alpha_R = 0$), the rational expectations

[7] Other authors have also argued that countercyclical interest-rate policy may be undesirable (e.g., Ireland, 1997; and Rotemberg and Woodford, 1997).

[8] The simple interest-rate rule that responds solely to inflation is implementable, whereas the standard Taylor rule is not, because it implies too high a volatility of nominal interest rates.

equilibrium is indeterminate for all values of the inflation and output coefficients in the interval [0,3]. This result is in line with that obtained by Carlstrom and Fuerst (2005). These authors consider an environment similar to ours and characterize determinacy of equilibrium for interest-rate rules that depend only on the rate of inflation. Our results extend the findings of Carlstrom and Fuerst to the case in which output enters in the feedback rule.

We close this section by noting that most of the results presented here, extend to a model economy with a much richer battery of nominal and real rigidities. In Schmitt-Grohé and Uribe (2007), we consider an economy featuring four real rigidities: habit formation, variable capacity utilization, investment adjustment costs, and monopolistic competition in product and labor markets. The economy in that study also includes four nominal frictions. Namely, sticky prices, sticky wages, money demand by households, and money demand by firms. Finally, the model features a more realistic shock structure that includes permanent stochastic variations in total factor productivity, permanent stochastic variations in the relative price of investment, and stationary stochastic variations in government spending. The values assigned to the structural parameters are based on existing econometric estimations of the model. These studies, in turn, argue that the model explains satisfactorily observed short-term fluctuations in the postwar United States. We find that the Ramsey policy calls for stabilizing price inflation. More importantly, a simple interest-rate rule that responds only to inflation (with mute responses to wage inflation or output) attains a level of welfare remarkably close to that associated with the Ramsey optimal equilibrium.

References

Aiyagari, R., Albert Marcet, Thomas J. Sargent, and Juha Seppälä, "Optimal Taxation Without State-Contingent Debt," *Journal of Political Economy 110,* 2002, 1220-1254.

Altig, David, Lawrence J. Christiano, Martin Eichenbaum, and Jesper Lindé, "Firm-Specific Capital, Nominal Rigidities, and the Business Cycle," NBER WP 11034, January 2005.

Barro, Robert J., "On the Determination of Public Debt," *Journal of Political Economy 87,* 1979, 940-971.

Bils, Mark and Pete Klenow, "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy 112,* October 2004, 947-985.

Calvo, Guillermo, "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics 12,* September 1983, 383-98.

Calvo, Guillermo A. and P. E. Guidotti, "Indexation and Maturity of Government Bonds: An Exploratory Model," in R. Dornbusch and M. Draghi, Eds., *Public Debt Management: Theory and History,* Cambridge University Press, Cambridge, 1990, 52-82.

Calvo, Guillermo A. and P. E. Guidotti, "On the Flexibility of Monetary Policy: The Case of the Optimal Inflation Tax," *Review of Economic Studies 60,* July 1993, 667-87.

Carlstrom, Charles T. and Timothy S. Fuerst, "Investment and interest rate policy: a discrete time analysis," *Journal of Economic Theory 123,* July 2005, 4-20.

Carvalho, Carlos, "Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks," *The BE Journal of Macroeconomics,* 2 (Frontiers), 2006.

Chari, V. V., Christiano, Lawrence, and Kehoe, Patrick, "Optimal Fiscal and Monetary Policy: some recent results," *Journal of Money Credit and Banking 23,* 1991, 519-539.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy 113,* February 2005, 1-45.

Correia, Isabel, Juan Pablo Nicolini, and Pedro Teles, "Optimal Fiscal and Monetary Policy: Equivalence Results," *Journal of Political Economy 168,* 2008, 141-170.

Del Negro, Marco, Frank Schorfheide, Frank Smets, and Raf Wouters, "On the Fit and Forecasting Performance of New-Keynesian Models," manuscript, November 2004.

Feenstra, Robert C., "Functional Equivalence Between Liquidity Costs and the Utility of Money," *Journal of Monetary Economics 17,* 1986, 271-291.

Golosov, Mikhail and Robert E. Lucas, Jr, "Menu Costs and Phillips Curves," mimeo, MIT, August 2006.

Goodfriend, Marvin and Robert G. King, "The New Neoclassical Synthesis and the Role of Monetary Policy," in Ben Bernanke and Julio J. Rotemberg, eds., *NBER Macroeconomics Annual 1997,* Cambridge MA: MIT Press, 1997, 231-283.

Ireland, Peter, "A Small, Structural, Quarterly Model for Monetary Policy Evaluation," *Carnegie-Rochester Conference Series on Public Policy 47,* December 1997, 83-108.

Khan, Aubhik, Robert G. King, and Alexander Wolman, "Optimal Monetary Policy," *Review of Economic Studies 70,* October 2003, 825-860.

Kimbrough, Kent P., "The Optimum Quantity of Money Rule in the Theory of Public Finance," *Journal of Monetary Economics 18,* 1986, 277-284.

Klenow, Pete and Oleksiy Kryvtsov, "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?," mimeo, Stanford University, July 2005.

Levin, Andrew, Alexei Onatski, John Williams and Noah Williams, "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models," in Gertler Mark and Kenneth Rogoff, eds., *NBER Macroeconomics Annual 2005,* Volume 20, Cambridge: MIT Press, 2006.

Lucas, Robert E. Jr., "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics 10,* 1982, 335-360.

Lucas, Robert E. Jr., and Nancy L. Stokey, "Optimal Fiscal and Monetary Policy in an Economy Without Capital," *Journal Monetary Economics 12,* 1983, 55-93.

Nakamura, Emi and Jón Steinsson, "Monetary Non-Neutrality in a Multi-Sector Menu Cost Model," mimeo, Harvard University, May 2007a.

Nakamura, Emi and Jón Steinsson, "Five Facts About Prices: A Reevaluation of Menu Cost Models," mimeo, Harvard University, May 2007b.

Rotemberg, Julio J, "Sticky Prices in the United States," *Journal of Political Economy 90,* December 1982, 1187-1211.

Rotemberg, Julio J., and Michael Woodford, "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," in Bernanke, Ben S. and Julio J. Rotemberg, eds., *NBER macroeconomics annual 1997,* Cambridge and London: MIT Press, 1997, 297-346.

Schmitt-Grohé, Stephanie, and Martín Uribe, "Optimal Fiscal and Monetary Policy under Sticky Prices," *Journal of Economic Theory 114,* February 2004, 198-230.

Schmitt-Grohé, Stephanie and Martín Uribe, "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model," in Gertler, Mark and Kenneth Rogoff, eds., *NBER Macroeconomics Annual 2005,* Cambridge and London: MIT Press, 2006, 383-425.

Schmitt-Grohé, Stephanie and Martín Uribe, "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model," in Klaus Schmidt-Hebbel and Rick Mishkin, eds., *Monetary Policy Under Inflation Targeting,* Central Bank of Chile, Santiago, Chile, 2007, 125-186.

Sidrauski, Miguel, "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review, Papers and Proceedings 57,* 1967, 534-544.

Siu, Henry E., "Optimal Fiscal and Monetary Policy with Sticky Prices," *Journal of Monetary Economics 51,* April 2004, 575607.

Smets, Frank and Raf Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review 97,* June 2007, 586-606.

Taylor, John B., "Discretion versus Policy Rules in Practice," *Carnegie Rochester Conference Series on Public Policy 39,* December 1993, 195-214.

Woodford, Michael, "Control of the Public Debt: A Requirement for Price Stability?," NBER WP 5684, July 1996.

Woodford, Michael, "The Taylor Rule and Optimal Monetary Policy," *American Economic Review Papers and Proceedings 91,* May 2001, 232-237.

Woodford, Michael, *Interest and Prices,* Princeton University Press: Princeton, New Jersey, 2003.

Yun, Tack, "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics 37,* July 1996, 345-70.