Monetary Policy and Subgame Perfect Equilibrium Outcomes in the Absence of Commitment

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Abstract

This paper studies implementable monetary policy in a standard dynamic perfect foresight general equilibrium staggered pricing model in which the monetary authority is unable to commit. We consider Subgame Perfect Equilibria (SPE) and allow the private sector to use non-Markovian trigger strategies. We compare these implementable outcomes with those that would be chosen by a monetary authority under three alternative specifications of the authority’s power to commit. Specifically, we compare implementable SPE outcomes with those that obtain when 1) the planner can implement the “optimal from the timeless perspective policy” 2) the planner can implement an inflation target and 3) the planner can implement the full-commitment solution. We find that the outcome that obtains in each case is also an outcome that is implementable as a SPE outcome. We show that the optimal from the timeless perspective outcome of price stability obtains as a Nash Equilibrium (NE), and thus is implementable even when non-Markovian trigger strategies are not allowed. We find that the optimal inflation target, which yields higher welfare than does price stability is small but positive. The inflation target is a SPE outcome when the private sector punishes monetary authority deviation by expecting price stability in future periods. Lastly, we show that the full-commitment solution involves non-monotonic convergence of the inflation rate to a steady state of price stability and find that this path of inflation is a SPE outcome when private agents use a “stick-and-carrot” strategy that punishes the authority more severely than does reversion to the zero inflation NE outcome.

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There is an ongoing debate among economists regarding the welfare costs of discretionary policy. This paper will study discretionary monetary policies in a standard general equilibrium staggered pricing model and compare these with several policies that have been suggested by researchers who have assumed that the monetary authority is served by an exogenous commitment technology. We depart from much of the related literature\textsuperscript{1} by allowing the expectations of private agents to respond to past values of the policy instrument. We find that such private sector expectations create a powerful deterrent which endogenizes the authority’s ability to achieve commitment outcomes. Our finding is unambiguous: all relevant commitment outcomes are attainable by a discretionary monetary authority.

In the study of discretionary policy one often finds that the inflation rate the monetary authority wants private agents to anticipate differs from that which the authority will in fact find optimal to implement. This occurs when private agents whose optimal choices depend on anticipated policy make decisions before learning the true inflation rate. In many frameworks, the policy maker then has an incentive to deviate from the policy that was anticipated. A Nash Equilibrium (NE) outcome in this setting obtains when the action that is optimal for the policy maker given the preceding choices of private agents coincides with the policy that had been expected. In infinite horizon economies this type of timing is conducive to constructing equilibria in which private actions taken before policy implementation depend not only on expectations of the future but also on past policy actions. The resulting non-Markovian Subgame Perfect Equilibrium (SPE) outcomes are of interest because they typically allow the policy maker to achieve higher welfare than does infinite repetition of any given NE.

Optimal monetary policy research has considered various assumptions regarding the nature of the exogenous commitment technology. We will compare SPE outcomes with policies that are optimal for three different, and common, commitment regimes. The weakest form of commitment we will consider is that studied by Woodford (2003); policy that is optimal from the timeless perspective. The timeless perspective policy is that which the planner would currently implement had the full commitment plan been announced many periods in the past. In the model studied in this paper, as in Woodford (2003), the optimal from the timeless perspective policy is one of price stability. We will find that this is also the unique Markovian NE discretionary policy. When private actions depend only on anticipated policy, we will see that the monetary authority will always choose to minimize price dispersion. This is achieved through a policy of price stability.

Given this uniqueness result, it is clear that if more desirable policies exist, SPE will be needed to attain the associated levels of welfare. The second commitment outcome we will consider is that which obtains under the optimal inflation targeting rule. Wolman (2001) and Woodford (2004) study inflation targets within this modeling framework and find that a small but positive target

\textsuperscript{1}See below.
is optimal. An inflation targeting rule is of interest both because it allows for higher welfare than does price stability and because such rules evidently govern the behavior of several central banks including, notably, the Bank of England. Our finding is that the welfare maximizing level of constant inflation does obtain in a simple SPE. When private agents punish the policy maker by expecting price stability in all periods following a deviation, it is optimal for the monetary authority to validate private expectations of inflation targeting policy.

Interestingly, however, we do not find that such simple SPE allow the discretionary authority to achieve all commitment outcomes. The third such outcome we consider is that which implements the full commitment, welfare maximizing, or Ramsey, path of inflation. We find that this path is characterized by non-monotonic convergence to price stability\(^2\). This follows from the fact that current and future inflation rates have a qualitatively different impact on the current equilibrium. We find that when deviation results in an expectation by private agents of price stability in future periods the authority will choose to deviate. Following Abrue (1988), we next construct a “Stick and Carrot” (S&C) SPE with a value low enough to serve as a deterrent sufficiently strong so as to support the Ramsey time path of inflation as a SPE outcome. The S&C SPE features one period of inflation high enough to cause monopolists who cannot adjust price to shutdown, an outcome commonly ignored in the staggered pricing literature, followed by many periods of price stability.

Important early contributions to the literature of discretionary policy are found in Kydland and Prescott (1977) and Barro and Gordon (1983a, 1983b). This research suggests that a lack of commitment leads to an inflationary bias in monetary policy that is welfare reducing\(^3\). Barro and Gordon (1983b) suggest that this problem may be alleviated when private sector expectations depend on past policy choices. Subsequently, models of both fiscal and monetary policy have been studied focusing on the difficulties created for the discretionary planner by the within period timing of the model. In the realm of monetary policy Ireland (1997) and Albanesi Char and Christiano (2002, 2003) are closest in spirit to the current research\(^4\). One important difference is that the models therein studied have the feature that the full commitment solution is a policy of constant deflation at the rate of subjective discount. Albanesi Char and Christiano do not consider non-Markovian SPE\(^5\). While Ireland does, his SPE rely on another undesirable feature of his model; the single period NE is one of

\(^2\)This result to our knowledge, is absent in the literature, though Wolman (2001) does allude to it in a footnote.

\(^3\)This result was also confirmed more recently in the review of Clarida Gali and Gertler (1999).

\(^4\)Char and Kehoe (1990) and Sargent and Lundquist (2000) address fiscal policy.

\(^5\)Other papers studying discretionary policy that do incorporate non-Markovian SPE include Wolman (2001), Kahn King and Wolman (2001), King and Wolman (2005), Ascher (2005), and Siu (2005). This research studies an alternative within period timing in which policy is implemented before private agents act. Discretion in this setting raises issues of time inconsistency in which the planner wishes to revise the announced plan in future periods. For the literature on time inconsistency in fiscal policy, see Klein Krussel and Rios-Rull (2004) and the references therein.
unbounded money growth and zero production.

The paper will proceed by describing preferences and technology in section 1. Section 2 will then characterize a competitive equilibrium in the economy for an arbitrary monetary policy. Section 3 will study outcomes under commitment and section 4 will compare these with SPE outcomes. Section 5 concludes.

1 Model

We will study a prefect foresight economy comprised of the planner, a representative household, a representative final good producing firm, and a continuum of mass one of intermediate good firms.

1.1 Households

The representative household’s preferences are represented by

\[ U_t = E_t \sum_{s=0}^{\infty} \beta^s \{ \ln c_{t+s} + \delta \ln (1 - \ell_{t+s}) \}, \]

which is maximized subject to the per-period budget constraint

\[ c_t + q_t b_{t+1} = w_t \ell_t + \phi_t + b_t, \]

where \( c_t \) is per-capita consumption, \( \ell_t \) per-capita labor supply, \( \beta \) the household’s discount factor, \( \delta \) a measure of leisure’s weight in per-period utility, \( b_t \) household asset holdings in the beginning of period \( t \), \( w_t \) the wage rate, \( q_t \) the period \( t \) price of one unit of consumption in period \( t+1 \), and \( \phi_t \) is dividends from the ownership of firms. Taking the sequences \( \{ w_t, \phi_t, q_t \}_{t=0}^{\infty} \) as given, the household chooses \( \{ c_t, \ell_t, b_t \}_{t=0}^{\infty} \) to maximize utility subject to the per-period budget constraint and the no-ponzi scheme condition that \( \lim_{s \to \infty} \beta^s b_s \geq 0 \).

The following first order necessary conditions characterize the solution to the household’s problem:

\[ \delta \frac{c_t}{1 - \ell_t} = w_t \quad (1) \]

\[ \beta \frac{c_t}{c_{t+1}} = q_t \quad (2) \]

1.2 Producers

The production side of the economy consists of a perfectly competitive, representative final good producer and a continuum of mass one of intermediate good producing monopolists.
1.2.1 Final Goods

Final goods are produced with the production technology

\[ Y_t = \left( \int_0^1 Y(i)_t^{1-\theta} \, di \right)^{-\frac{\theta}{\sigma}}, \]

where \( Y(i)_t \) is the input of intermediate firm \( i \) in period \( t \) and \( \theta < 1 \) is the elasticity of substitution. Let \( P(i)_t \) be the price of intermediate good \( i \) in \( t \) and \( P_t \) be the price of the final good. Profit maximization by final good producers implies that the demand for intermediate good \( i \) will be

\[ Y(i)_t = P(i)_t^{-\frac{\theta}{\sigma}} P_t^\theta Y_t \]

where the relationship between \( P(i)_t \) and \( P_t \) is given by

\[ P_t = \left( \int_0^1 P(i)_t^{\frac{\sigma}{\theta-\sigma}} \, di \right)^{-\frac{\theta}{\sigma}}. \]

1.2.2 Intermediate Goods

Technology Intermediate goods are produced with labor using the constant returns-to-scale production function

\[ Y(i)_t = A \cdot L(i)_t \]

where \( A \) is a productivity parameter and \( L(i)_t \) is the labor input hired by firm \( i \) in period \( t \).

Price Stickiness Intermediate good monopolists face a price-stickiness friction. A firm that was free to adjust its price in the previous period will be able to adjust its price in the current period with probability \( 1 - \lambda \) and will be stuck charging the previous period’s price with probability \( \lambda \leq 1 \). All firms that were not free to adjust price in the previous period will be able to adjust its price in the current period. By assuming that in the initial period \( \frac{1}{1+\lambda} \) firms are free to adjust price, it will follow that in every period there are \( \frac{1}{1+\lambda} \) firms free to adjust and \( \frac{\lambda}{1+\lambda} \) firms stuck charging the previous period’s price.\(^6\) As elsewhere in the Neo-Keynesian literature, output is demand determined for a

\(^6\) To our knowledge, this exact specification of price stickiness has not been used in the literature. It offers several advantages, both technical and theoretical, over the most commonly used specifications, those introduced by Taylor (1980), where firms deterministically alternate between periods in which they are and are not able to adjust price, Calvo (1977), where every firm has some constant probability, regardless of past experience, of not being able to adjust price, and Rotemberg (1985), where there are explicit menu costs. As with Taylor and Calvo, but unlike Rotemberg, this specification features a cost to inflation that is an equilibrium result, following from the cost of price dispersion, rather than an assumption. Unlike Calvo, there is no probability that a firm is not able to adjust for many periods, which has the un-
firm not able to adjust its price, i.e. enough labor must be hired to produce the quantity demanded at the unadjusted price. We will however allow firms who face a loss to temporarily shut down. As we will see below, there is a threshold level of inflation beyond which firms who are not able to adjust their price will earn negative profits.

**Profit Maximization** Given such price-stickiness, the production technology (5) and the demand function (4), the problem facing intermediate good producer $i$ who is free to adjust price in period $t$ is to find

$$P(i)_t = \arg \max_P \left\{ p^{1-\frac{\theta}{n}} P_t^\frac{\theta}{n} Y_t - \frac{w_t}{A} p^{-\frac{\theta}{n}} P_t^\frac{\theta}{n} Y_t \right\} + \lambda_q t \left[ p^{1-\frac{\theta}{n}} P_{t+1}^\frac{\theta}{n} Y_{t+1} - \frac{w_{t+1}}{A} p^{-\frac{\theta}{n}} P_{t+1}^\frac{\theta}{n} Y_{t+1} \right],$$

(6)

the price which maximizes the expected discounted value of real profits. The first order necessary condition characterizing the profit maximizing price level is

$$\left[ \frac{\theta - 1}{\theta} P(i)_t^{\frac{\theta}{n}} P_t^{-1} + \frac{1}{\theta} \frac{w_t}{A} P(i)_t^{\frac{\theta}{n}-1} \right] P_t^\frac{\theta}{n} Y_t + \lambda_q t \left[ \left( \frac{\theta - 1}{\theta} P(i)_t^{\frac{\theta}{n}} P_t^{-1} + \frac{1}{\theta} \frac{w_{t+1}}{A} P(i)_t^{\frac{\theta}{n}-1} \right) P_{t+1}^\frac{\theta}{n} Y_{t+1} \right] = 0,$$

or

$$\frac{P(i)_t}{P_t} = \frac{1}{1 - \frac{w_t}{A} + \frac{w_{t+1}}{A} \frac{Y_{t+1}}{Y_t} \frac{\theta}{n} \pi_{t+1}}$$

(7)

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the rate inflation of the final good price. The intuition for this is clear: the monopolist sets the real price as a markup over a weighted average of current and future real marginal costs, $\frac{w_t}{A}$ and $\frac{w_{t+1}}{A}$. The ratio $\frac{Y_{t+1}}{Y_t}$ determines the relative size of future to current demand and $\lambda, q_t, \pi_{t+1}$ all increase the impact of the next period’s profits on the current discounted expected value of the firm.

2 Equilibrium

A competitive equilibrium is a set of prices, $\{w_t, r_t, P_t, P(i)_t\}_{t=0}^\infty$, and an allocation, $\{L(i)_t, \ell_t, Y_{it}, Y_t, c_t\}_{t=0}^\infty$, such that households, intermediate good monopolists, and competitive final good producers optimize and such that the labor

desirable implication that firms with high levels of production can operate indefinitely with negative profits. A technical advantage over Calvo is that inherited price dispersion must be treated as a state variable under Calvo’s model, upon which policy must be conditioned whereas, given the specification we will apply, there is only one price inherited from the past, which can further be treated as the numeraire. Finally, unlike Taylor price staggering, price stickiness is not deterministic and, further, price-stickiness can be parameterized separately from period length. In fact, note that $\lambda = 1$ is exactly price staggering à la Taylor.

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and good markets clear. Since all intermediate good monopolists adjusting price in period $t$ face the same problem, the price set by each of these $\frac{1}{1+\lambda}$ firms will be same. We will denote this as $P_{0, t}$. The $\frac{1}{1+\lambda}$ monopolists who do not adjust their price in period $t$ will all have done so in the previous period and thus be charging $P_{1, t} = P_{0, t-1}$. Denote by $\bar{\pi}_t \equiv \frac{P_{0, t}}{P_{0, t-1}}$ the level of intermediate good inflation. Note that firms charging the same price will also face the same demands and thus require the same labor inputs. Let $Y_{0, t}$ and $L_{0, t}$, and $Y_{1, t}$ and $L_{1, t}$ be the level of output and labor demand of an intermediate good firm that has and has not, respectively, adjusted price in period $t$. We now list the complete set of competitive equilibrium conditions$^8$.

\[ c_t = Y_t \]
\[ \ell_t = \frac{1}{1+\lambda} L_{0, t} + \frac{\lambda}{1+\lambda} L_{1, t} \]
\[ Y_{0, t} = \left( \frac{P_{0, t}}{P_t} \right)^{\frac{1}{\theta}} Y_t \quad Y_{1, t} = \left( \frac{P_{0, t-1}}{P_t} \right)^{\frac{1}{\theta}} Y_t \]
\[ Y_t = (1 + \lambda)^{\frac{1}{\theta}} \left( \frac{Y_{0, t}^{1-\theta} + \lambda Y_{1, t}^{1-\theta}}{1+\lambda} \right) \frac{1}{\theta} \]
\[ P_{0, t} = (1 + \lambda)^{\frac{1}{\theta}} \left( 1 + \lambda \bar{\pi}_t^{\frac{1}{\theta}} \right) \frac{1}{\theta} \]
\[ P_{0, t} = 1 + \theta q_t \frac{Y_{0, t}}{1+\lambda} \pi_t^{\frac{1}{\theta}} \]
\[ w_t = \frac{\delta_c}{1-\ell_t} \]
\[ q_t = \frac{\beta c_t}{c_t+1} \]
\[ \pi_t = \frac{P_t}{P_{t-1}} \quad \bar{\pi}_t = \frac{P_{0, t}}{P_{0, t-1}} \]

Equations (9) and (10) are final good and labor market clearing conditions. Equations (11) follow from optimization of final good producers. Equations (12) and (13) are the technological constraints of final and intermediate good producers. Equation (14) follows from the definition of the price index (it is redundant in the list since equations (11) and (13) together imply (14)). Equation (15) follows from optimization of intermediate good producers. Equations (16) and (17) are the optimality conditions of the households. Equations (18) are the definitions of final and intermediate good price inflation.

Taking the price level $\{P_t\}_{t=0}^\infty$ as the policy instrument, these twelve independent equations characterize the equilibrium values of the twelve endogenous variables.

$^7$The 0 subscript indicates that the firm has been stuck charging this price for zero previous periods.

$^8$Here we are ignoring the possibility of firm shutdown. We will return to it later.
\{c_t, Y_t, Y_{0,t}, Y_{1,t}, \ell_t, L_{0,t}, L_{1,t}, P_{0,t}, \pi_t, q_t, w_t\}_{t=0}^{\infty}.

**Lemma 1** A sequence of \{c_t, \ell_t, w_t, \pi_t\}_{t=0}^{\infty} satisfying

\begin{align}
\ell_t &= \frac{w_t}{\delta A f(\pi_t) + w_t} \\
c_t &= A \ell_t f(\pi_t)
\end{align}

\begin{align}
w_t \left(1 + \lambda \pi_t^{1 - \phi} \right)^{1 - \phi} + \lambda \beta \pi_{t+1}^{1 - \phi} = \frac{(1 - \theta)A}{(1 + \lambda)\pi_t^{1 - \phi}} \left[1 + \lambda \pi_t^{1 - \phi} \right]^{-1} + \lambda \beta \pi_{t+1}^{1 - \phi} \left[1 + \lambda \pi_{t+1}^{1 - \phi} \right]^{-1}
\end{align}

for all \(t \geq 0\), where \(f(\pi) \equiv (1 + \lambda)\pi^{1 - \phi} \left[1 + \lambda \pi^{1 - \phi} \right]^{-1}\), also satisfies equations (9) through (18).

This result follows from algebraic manipulation of equations (9) through (18). We will work with this alternative specification of the competitive equilibrium as it is more convenient for handling the monetary authority’s problem. Using equations (19) and (20) to substitute for \(c_t\) and \(\ell_t\) in the household objective function, we see that this specification of the equilibrium reduces the problem of the authority to choosing a welfare maximizing path of \(\pi_t\) and \(w_t\) subject to the constraint in equation (21).

### 3 Optimal Monetary Policy with Commitment

In this section we consider outcomes available to policy makers under three alternative assumptions regarding commitment. We have shown above that the equilibrium time path of endogenous variables is determined by the time path of inflation in the intermediate goods. We do not focus on the mechanism used to implement policy and assume that the planner is able to directly choose a path for intermediate good inflation.

#### 3.1 Optimal From the Timeless Perspective Policy

Woodford (2003), recognizing that policy in initial periods enters the planner’s problem in a qualitatively different way than does future policy, and taking as given the inconsistency of the policy that solves the full commitment problem of the Ramsey Planner, focuses on the steady state policy that obtains under the Ramsey solution. Such policy is called Optimal from the Timeless Perspective (OTP). We may also think of OTP policy as the policy which would be followed had the full commitment Ramsey solution been implemented beginning many periods in the past.

Following Woodford (2003), we will find the OTP policy by taking the time limit of policy under the full commitment solution. Given the equilibrium
conditions in equations (19), (20), and (21), the problem of the Ramsey Planner can be written as

$$
\max_{(w_t, \bar{p}_t)_{t \geq 0}} \left\{ \sum_{t=0}^{\infty} \beta^t \left( \log w_t + \log f(\bar{p}_t) - (1 + \delta) \log [\delta A f(\bar{p}_t) + w_t] \right) \right\}
$$

subject to the sequence of constraints in equation (21). Here we have used equations (19) and (20) to substitute for \( c_t \) and \( \xi_t \) and then combined terms and factored out constants that do not affect the solution to the optimization problem.

The first order conditions characterizing the solution to the full commitment solution are

$$\frac{1}{w_0} - \frac{1 + \delta}{\delta A f(\bar{p}_0) + w_0} + \mu_0 (1 + \lambda \bar{p}_0)^{\frac{1-\theta}{\theta}} = 0, \quad (22)$$

$$\frac{f'(\bar{p}_0)}{f(\bar{p}_0)} - \frac{(1 + \delta) \delta A f(\bar{p}_0)}{\delta A f(\bar{p}_0) + w_0} - \frac{\mu_0 w_0 \lambda}{(1 + \lambda \bar{p}_0)^{\frac{1-\theta}{\theta}}} \frac{\lambda A(1-\theta)^2 (1 + \lambda \bar{p}_0)^{\frac{1-\theta}{\theta}}}{(1 + \lambda \bar{p}_0)^{\frac{1-\theta}{\theta}}} = 0, \quad (23)$$

$$\frac{1}{w_t} - \frac{1 + \delta}{\delta A f(\bar{p}_t) + w_t} + \mu_t (1 + \lambda \bar{p}_t)^{\frac{1-\theta}{\theta}} + \mu_{t-1} \lambda \bar{p}_t^{\frac{1-\theta}{\theta}} (1 + \lambda \bar{p}_t)^{\frac{1-\theta}{\theta}} = 0 \quad \forall \ t \geq 0, \quad (24)$$

$$\frac{f'(\bar{p}_t)}{f(\bar{p}_t)} - \frac{(1 + \delta) \delta A f(\bar{p}_t)}{\delta A f(\bar{p}_t) + w_t} - \mu_t w_t \lambda \frac{\lambda A(1-\theta)^2 (1 + \lambda \bar{p}_t)^{\frac{1-\theta}{\theta}}}{(1 + \lambda \bar{p}_t)^{\frac{1-\theta}{\theta}}} = 0 \quad \forall \ t \geq 0,$$

where \( \beta \mu_t \) is the Lagrange multiplier associate with the period \( t \) version of the constraint in equation (20).

Since the OTP policy is the limit of the Ramsey solution we focus on the steady state of equations (24) and (22). The following result follows.

**Proposition 2** \( \bar{p}^{\text{otp}} = 1 \) is the policy rule that is Optimal from the Timeless Perspective

**Proof.** in the appendix. \( \blacksquare \)

The proof shows that there is a steady state of the above system of equations in which \( \bar{p} = 1 \). Thus, the OTP policy rule is one of price stability.
3.2 Optimal Inflation Target

Within this deterministic setting, an inflation targeting rule is simply a single inflation rate that the policy maker commits to. The OTP policy of zero inflation in all periods is, therefore, one such target. There is no reason to believe, however, that this target maximizes welfare within the set of all constant inflation rate rules. The Optimal Inflation Target (OT) will be the constant inflation rate that maximizes welfare subject to the equilibrium conditions in equations (19), (20), and (21). Given that inflation will be constant, equation (21) can be rewritten as

\[ w(\hat{\pi}) = (1 - \theta)A(1 + \lambda)^{\theta/\tau} \left( 1 + \lambda \beta^{1 - \theta/\tau} \right) \frac{\tau^\theta}{1 + \lambda \beta^{1 - \theta/\tau}} \]  

(25)

and the problem of the monetary authority will be to maximize

\[ \max_{w, \hat{\pi}} \{ \log w + \log f(\hat{\pi}) - (1 + \delta) \log [\delta A f(\hat{\pi}) + w] \} \]

subject to (25)

Proposition 3 Assuming the planners problem above is everywhere concave, the optimal inflation target is positive, i.e. \( \hat{\pi}^{opt} > 0 \).

The proof is in the appendix.

We numerically consider how the optimal inflation target changes with \( \theta \), the elasticity of substitution in the in the final good production function. In the absence of pricing frictions the markup over marginal cost that would be charged is \( markup = \frac{1}{1 - \delta} \). Figure 1 plots the optimal inflation target as a
function of the markup when $\beta = 1$, $\delta = 2$, $\lambda = 1$, and $A = 1^{9,10}$.

\[ w(1) = (1 - \theta)A \]
\[ \frac{\partial f}{\partial \pi} \bigg|_{\hat{\pi} = 1} = 0 \quad \frac{\partial w}{\partial \pi} \bigg|_{\hat{\pi} = 1} = (1 - \theta)A \frac{\lambda}{1 + \lambda}(1 - \beta) > 0, \quad (26) \]

that is, by increasing inflation above zero the policy maker can increase the wage rate. At zero inflation, the wage reflects the full monopoly mark-up of $\frac{1}{1-\theta}$, i.e. the wage is only $(1 - \theta)$ percent of the first-best rate. All else equal, welfare is increased as the wage increases to $A$. In general, all else is not equal since there is a cost of inflation; lower Total Factor Productivity (TFP, which in this model is $A f(\hat{\pi})$) is a result of price dispersion. However at the margin, beginning from the point of price stability the effect of inflation on TFP is of second order importance as $\hat{\pi} = 1$ maximizes final output per unit of labor. Thus, the wage effect dominates in terms of welfare gains and so the optimal rule calls for a positive level of inflation.

Digging a bit deeper, the problem of the price setting monopolist facing an inflationary environment is to weigh the costs of charging sub-optimal markups across the current and future period. All monopolists place a weight of $\lambda \times \beta$ on

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9. $\beta = .96$ implies a real interest rate of around 4%. $\delta = 2$ was chosen so that when $\lambda = 1$ one third of the time endowment is supplied as labor in the price-frictionless first best equilibrium and 28% of the endowment is supplied in the price-frictionless with monopoly power equilibrium.

10. $\hat{\pi}^{opt}$ is extremely insensitive to $\lambda$ and the values shown in figure 1 are approximately the same for all values of $\lambda$ between .1 and 1.
the future period which is the probability of being stuck with the price currently chosen times the discount factor. Since $\lambda$ percent of the firms will actually be stuck in each period, the individual firms underweight the effect of not being able to adjust relative to the aggregate. This follows from the fact, in equilibrium, that all firms discount by $\beta$ the state in which they are not able to adjust. In fact,

**Lemma 4** when there is no discounting the optimal inflation target is one of price stability.

This demonstrates the importance of discounting in the result that the optimal inflation target is positive. It follows immediately from the fact that in equation (26) $\frac{\partial T}{\partial x} |_{x=1} = 0$ when $\beta = 1$.

### 3.3 The Full Commitment Policy

The problem facing the planner with full commitment power and first order conditions characterizing the solution to the problem are presented above. We can numerically solve these equations for a given parameterization. For researchers facing similar problems, the method for doing so is described briefly.

Equations (22) and (23) are combined to eliminate the multiplier. We next solve for the values of $w_0$ that satisfy these equations as a function of $\tilde{x}_0$ and then use these in conjunction with equation (23) to find the associated value of $\mu_0$. Equations (21), (24), (??) are three equations in $\pi_{t-1}$, $\pi_t$, $w_{t-1}$, $w_t$, $\mu_{t-1}$, and $\mu_t$ for $t \geq 1$. We proceed by taking a candidate triple of $\pi_0$, $w_0$, and $\mu_0$ satisfying and equations (22) and (23) iterate over equations (21), (24), (??) to create a sequence of $\{\pi_t, w_t, \mu_t\}_{t=0}^\infty$. For each parameterization considered we found a single triple of $\pi_0$, $w_0$, and $\mu_0$ that did not lead to a divergent path. Further these paths satisfied the condition that $\lim_{t \to \infty} \{\pi_t, w_t\} = \{1, (1-\theta)A\}$, which we have seen above is a steady state of the Ramsey solution.

For the parameterization above$^{11}$ and a markup of 20%, we find that the time path of intermediate inflation in figure 2 to be the full commitment time

$^{11}$ $\beta = 1$, $\delta = 2$, $\lambda = 1$, and $A = 1$
The basic intuition is that the planner can get the period zero wage closest to the marginal product of labor with positive intermediate inflation in period 0 and negative intermediate good inflation in period 1. This relationship can be seen in equation (21). When there is deflation in period 1, firms setting price in period 0 are concerned about being stuck with too high a price in the following period. The result is that they are willing to reduce their real price in the current period to be closer to the optimal real price in period 1. Given that both current inflation and future deflation reduce the current markup and that current inflation reduces current TFP, the planner chooses at least a small amount of deflation in period 1. Note also that the period 0 wage is decreasing in $w_1$. This helps to explain why the planner chooses a $\hat{\pi}_2 > \hat{\pi}_1$. The period 1 wage is lower when period 2 inflation is above that is period 1.

4 Optimal Monetary Policy without Commitment

If the government cannot commit to policy then the within period timing of the model becomes relevant. Chari and Kehoe (1996), Ireland (1997), Sargent and Lundquist (2001) and Albanesi Chari and Christiano (2003) also study issues of time-inconsistency by exploring the within-period order of events.

We assume that households make some decision before the monetary authority. Specifically, we will assume that first the household makes either its consumption or labor supply decision. After this the government announces the current inflation rate and final good and intermediate good firms set their
prices optimally. The household value not chosen in the begining of the pe-
riod is then determined by market clearing conditions. Either the household
makes a consumption decision at the begining of the period and then supplies
whatever amount of labor is required to produce the recourses consistent with
that level of consumption or makes a labor supply decision at the begining of
the period and then consumes whatever amount of goods this labor supply is
consistent with making available. We will show shortly that the equilibrium
outcomes are independent of which decision the household makes first.

4.1 Nash Equilibrium

Now consider the problem of the monetary authority once the household decision
has been made. Suppose that the household has chosen a labor supply, \( \ell_s = \bar{\ell} \).
The period \( s \) problem of the monetary authority that cannot commit is to choose

\[
\pi_s^{br}(\bar{\ell}) = \arg \max \left\{ \log [A \bar{\ell} f(\pi_s)] + \delta \log \left[ 1 - \frac{\bar{\ell}}{\bar{\ell} f(\pi_s)} \right] + \sum_{t=s}^{\infty} \beta^t \left( \log w_{t+s} + \log f(\pi_{t+s}) - (1 + \delta) \log [\delta A f(\pi_{t+s}) + w_{t+s}] \right) \right\}
\]

subject to equation (21) for all \( t \geq s \).

Suppose instead that the household has first chosen a level of consumption
\( c_s = \bar{c} \). The period \( s \) problem of the monetary authority that cannot commit
is to choose

\[
\pi_s^{br}(\bar{c}) = \arg \max \left\{ \log \bar{c} + \delta \log \left[ 1 - \frac{\bar{c}}{A f(\pi_s)} \right] + \sum_{t=s}^{\infty} \beta^t \left( \log w_{t+s} + \log f(\pi_{t+s}) - (1 + \delta) \log [\delta A f(\pi_{t+s}) + w_{t+s}] \right) \right\}
\]

subject to equation (21) for all \( t \geq s \).

Lemma 5 Regardless of weather the household first chooses a labor supply or
consumption level, the optimal response of the monetary authority will be to
choose zero inflation.

Proof. Since \( \arg \min_{\pi} f(\pi) = 1 \), \( \pi_s^{br} = 1 \) is the solution to both of the above
problems.

Inflation has the effect of reducing monopoly power and creating price dispersion. Once the household has already committed to a consumption or labor supply decision, the monopoly power of intermediate firms is irrelevant. The goal of the monetary authority is then to reduce dispersion which can be achieved with zero intermediate good inflation.

Proposition 6 The Nash Equilibrium outcome for any period \( t \) is \( \pi_t^{ne} = 1 \),
\( \ell_t^{ne} = \frac{1 - \theta}{1 - \theta + \pi} \), \( c_t^{ne} = \Delta \ell_t^{ne} 

Proof. This follows immediately from the above.
Thus the NE outcome for any given period coincides with the OTP policy.
Proposition 7 The Nash Equilibrium outcome repeated in all periods is a SPE

This is a standard result. SPE requires that the strategies used induce a NE on every Subgame. If the single period NE obtains in each period then this must be true. To see this consider any period $t$ in which private agents and monetary authority expect price stability in all future periods. We have seen that regardless of the choice of private agents the best response of the monetary authority is $\tilde{\pi}_t^{br} = 1$. Given this, private agents should rationally expect $\tilde{\pi}_{t} = 1$.

4.2 SPE Implementing the Optimal Inflation Target

We have shown that the NE outcome for any given period is for the monetary authority to choose zero inflation and for households and firms to respond accordingly. We will now consider SPE outcomes in which households consider past outcomes when making the labor supply choice\footnote{From here on we will assume that the household chooses its labor supply before learning the current inflation. Making the alternative assumption that consumption decisions are made first doesn’t qualitatively change any of the following results.}. When the monetary authority chooses unexpected inflation rates, households respond by adjusting expectations of future government behavior. Such updating of household expectations alters the optimal household choice and therefore the optimal response of the monetary authority.

We are interested in studying such equilibria in order to characterize outcomes other than price stability that are attainable in the absence of commitment on the part of the policy maker. The mechanism supporting these equilibria is quite natural: households are willing to believe that the monetary authority intends to implement a desirable outcome as long as the authority’s actions are consistent with such an outcome. The consequences of lost faith in the monetary authority serves as a deterrent that prevents the authority from surprising households in order to realize short term gains. Here we will show that the optimal inflation target can be supported as a SPE when deviation is punished by reversion to the NE outcome above.

Consider the equilibrium allocation when the monetary authority implements the optimal inflation target $\tilde{\pi}^{ot} > 0$ identified in section (3.2). The wage rate will be given by $w^{ot} = (1-\theta)A(1+\lambda)\frac{\theta^\theta}{\psi} \left( 1 + \lambda \left( \frac{\tilde{\pi}^{ot}}{\tilde{\pi}^{ot} + \psi} \right) \right)^{\frac{\psi}{1+\lambda(\tilde{\pi}^{ot} + \psi)}},$
the labor supply of households by $\ell^{ot} = \frac{\tilde{\pi}^{ot}}{\tilde{\psi}(\tilde{\pi}^{ot} + \psi)}$, and the level of consumption by $c^{ot} = A \ell^{ot} f(\tilde{\pi}^{ot})$.

Households Denote by $\zeta^{t-1} = \{\pi_s\}_{s=0}^{t-1}$ the history of policy observable by households at date $t$ and consider the following strategy for households

$$\ell_t(\zeta^{t-1}) = \begin{cases} \ell^{ot} & \text{if } \zeta^{t-1} = \{\tilde{\pi}^{ot}\}_{s=0}^{t-1} \\
\ell^{ne} = \frac{1-\theta}{1-\theta + \tilde{\pi}^{ot}} & \text{otherwise} \end{cases}.$$
Clearly, such behavior is consistent with a perfect foresight, rational expectations equilibrium if \( \hat{\pi}_t = \hat{\pi}^{ol} \) for all \( t \). In fact, in this case the equilibrium allocation is identical to inflation targeting commitment equilibrium of section (3.2).

**Monetary Authority** To confirm that \( \hat{\pi}^{ol} \) is in fact a SPE outcome we construct such an equilibrium. To do so we must specify the monetary authority’s choice of intermediate good inflation following every history and then confirm that the choices are optimal. There are two relevant histories to consider, those in which \( \zeta^{t-1} = \{ \hat{\pi}^{ol} \}_{s=0}^{t-1} \) (inclusive of \( t = 0 \)) and those where \( \zeta^{t-1} \neq \{ \hat{\pi}^{ol} \}_{s=0}^{t-1} \). We begin with the second case where the authority has already deviated from \( \hat{\pi}^{ol} \) in at least one previous period. According to the household’s strategy \( \ell^{ne} = \frac{1-\theta}{1-\theta+\delta} \) will now be chosen in each period. The problem of the authority in period \( t \) is to choose \( \hat{\pi}_t \) to solve

\[
\max_{\hat{\pi}_t} \left\{ \log \left[ A \frac{1-\theta}{1-\theta+\delta} f(\hat{\pi}_t) \right] + \delta \log \left[ \frac{\delta}{1-\theta+\delta} \right] + \sum_{s=1}^{\infty} \beta^s \left[ \log \left[ A \frac{1-\theta}{1-\theta+\delta} f(\hat{\pi}_{t+s}) \right] + \delta \log \left[ \frac{\delta}{1-\theta+\delta} \right] \right] \right\},
\]

the solution to which is clearly \( \hat{\pi}_t = 1 \). Note that by virtue of the household’s decision rule, the policy maker’s choice has no impact on future outcomes. Also note that from this point forward the equilibrium allocation is identical to that which obtains when the policy maker implements a policy of price stability.

Now consider the authority’s choice in periods in which \( \zeta^{t-1} = \{ \hat{\pi}^{ol} \}_{s=0}^{t-1} \). Since the household labor supply is fixed at \( \ell^{ol} \) for the current period, it is clear that the authority can maximize the current period’s utility by choosing \( \hat{\pi}_t = 1 \) which will yield a return of \( \log \left[ A \ell^{ol} \right] + \delta \log \left[ 1 - \ell^{ol} \right] \). This payoff must be weighed against the cost of choosing an action such that \( \zeta^{t} \neq \{ \hat{\pi}^{ol} \}_{s=0}^{t} \). As we have just shown, this will result in a payoff of \( \log \left[ A \frac{1-\theta}{1-\theta+\delta} \right] + \delta \log \left[ \frac{\delta}{1-\theta+\delta} \right] \) in every future period. The alternative for the authority is to choose the \( \hat{\pi}_t \) that is consistent with households’ forecasts. This is \( \hat{\pi}^{ol} \). This will be optimal if

\[
\frac{1}{1-\beta} \left\{ \log \left[ A \ell^{ol} f(\hat{\pi}^{ol}) \right] + \delta \log \left[ 1 - \ell^{ol} \right] \right\} \geq \log \left[ A \ell^{ol} \right] + \delta \log \left[ 1 - \ell^{ol} \right] \tag{29}
\]

\[
\frac{\beta}{1-\beta} V^{ne}
\]

where \( V^{ne} = \log \left[ A \frac{1-\theta}{1-\theta+\delta} \right] + \delta \log \left[ \frac{\delta}{1-\theta+\delta} \right] \), the per-period value of a implementing price stability in all periods. If \( \beta \geq \beta^* \) where \( \beta^* = \frac{\log \left[ \log \left[ \frac{1-\theta+\delta}{1-\theta} \right] + \delta \log \left[ 1 - \ell^{ol} \right] \right] + \log f(\hat{\pi}^{ol})}{\log \left[ A \ell^{ol} \right] + \delta \log \left[ 1 - \ell^{ol} \right]} \), then condition (29) is true. Since \( \hat{\pi}^{ol} \) (and therefore \( \ell^{ol} \)) depend on \( \beta \) the right hand side of this condition is not independent of \( \beta \). Nonetheless, this condition appears to be satisfied across a large set of parameterization of the model. In particular we have verified numerically that the condition in equation (29) holds for all combinations of values of \( \beta \) between .8 and .99, values of \( \theta \) between .05 and .4, and values of \( \lambda \) between .1 and 1 when \( A = 1 \) and \( \delta = 2 \).
Thus the optimal inflation target is a SPE outcome. The punishment of reverting to the price stability NE outcome is sufficient to deter the planner from surprising households.

### 4.3 SPE Implementing the Full Commitment Outcome

Now we turn to the full commitment inflation path and show that it can be supported as a SPE outcome. First, however, we will show that the reversion to the NE is not sufficient to prevent deviation from the Ramsey Plan. Following this we construct a SPE with a value lower than the NE. This will be a “stick and carrot” (S&C) SPE with one period of high inflation followed by the NE. Finally we show that the stick and carrot punishment is sufficient to prevent deviation from the Ramsey plan.

#### 4.3.1 Punishing Deviation with Price Stability

We have seen that the unique Markovian SPE coincides with the optimal from the timeless perspective policy of price stability. We are now interested in whether the full commitment path of inflation can be supported as a SPE outcome when deviation punished by the indefinite future expectation of price stability. Denote by \( \pi^*_t \) and \( \ell^*_t \) the period \( t \) values of intermediate good inflation and equilibrium labor supply along the optimal full commitment path. We are considering whether the household behavior

\[
\ell_t(\xi^{t-1}) = \begin{cases} 
\ell^*_t & \text{if } \xi^{t-1} = \{\pi^*_s\}^t_{s=0} \\
\ell^{ne} & \frac{1-\theta}{1-\theta+\delta} & \text{otherwise}
\end{cases},
\]

which is clearly a rational expectations equilibrium path of labor supply if \( \pi_t = \pi^*_t \) for all \( t \), is a SPE strategy. As above we must check whether it is optimal for the monetary authority to choose \( \pi^*_t \) in all periods. As with above, the deviation that maximizes the current payoff given the household’s fixed labor supply is zero intermediate good inflation and since the future following deviation depends only on the fact that there was a deviation, it is clear that the authority will choose \( \pi_t = 1 \) in any \( t \) in which it does choose to surprise households.

The condition we check to determine whether the planner will deviate is

\[
\sum_{s=0}^{\infty} \beta^s \left\{ \log \left[ A \ell^s_{t+s} f(\pi^*_{t+s}) \right] + \delta \log \left[ 1 - \ell^s_{t+s} \right] \right\} \geq \log \left[ A \ell^*_t \right] + \delta \log \left[ 1 - \ell^*_t \right] + \frac{\beta}{1-\beta} V^{ne}.
\]

The LHS has the payoff from period \( t \) forward of implementing the full commitment plan while the RHS has the payoff to deviating in period \( t \). If this satisfied for all \( t \) then the strategy in (30) induces a SPE outcome. Figure 3 shows the LHS and RHS of condition (31) for the first 15 periods given the parameterization
used above.

![Graph](image)

Figure 3

Evidently, the full commitment outcome is not a SPE outcome when expectations of price stability follow a deviation. In particular, note that the planner has an incentive to deviate to price stability in the first period. Apparently we must find a SPE associated with lower welfare than is price stability if we hope to support the full commitment outcome as a SPE. In the spirit of the “stick and carrot” (S&C) punishment suggested by Abrue (1991), we now study whether a SPE in which one period of high inflation is followed by price stability and show that the welfare associated with this path of inflation is sufficiently low to support the full commitment outcome.

4.4 The Stick

We are searching for a SPE with a value sufficiently low so as to deter the monetary authority from deviating from the full commitment path of inflation. The SPE we study is one in which there is a high level of intermediate good inflation in the first period followed by price stability in subsequent periods. In determining the value of such a path of inflation, we rely on the ability of intermediate good firms who are making negative profits to temporarily shut down. This Lemma proves useful in evaluating this SPE.

**Lemma 8** When $\hat{\pi}_{t+s} = 1$ for all $s \geq 1$, there exists a $\hat{\pi}_1 = \bar{\pi} > 1$ at which firms not able to adjust price will shut down.

The proof is in the appendix. The intuition for this result is simple: the real price of intermediate good firms not adjusting price decreases with intermediate good inflation. For the parameter values we have been considering $\bar{\pi}$ is exactly
20%. Given the previous results we were apparently justified in ignoring \( \bar{\pi} \) earlier in the paper.

When these firms shut down, it follows that \( L_{0,t} = \ell_t \). The second implies that

\[
c_t = (1 + \lambda)^{\bar{\pi}_t} A \ell_t,
\]

which follows from final good market clearing and the fact that there is no production by intermediate producers who have not changed price. This is decreasing in \( \lambda \) as higher values imply that a greater mass of intermediate firms have shut down. Since final good producers are buying intermediate goods that all have the same price, the price level is given by \( P_t = P_{0,t} \). If inflation in future periods is zero then it must be that \( P_{0,t+1} = P_{0,t} \) implying that \( \pi_{t+1} = \bar{\pi}_{t+1} = 1 \).

By virtue of the future price stability, we know that \( w_{t+s} = (1 - \theta)A \). The current wage rate, \( w_t \), must then also be \( w_t = (1 - \theta)A \) in order to satisfy the condition characterizing the participating firms’ optimal pricing decision, equation (8). It then follows from equations (16) and (32) that

\[
\ell_{stick} = \frac{(1 - \theta)A (1 + \lambda) \bar{\pi}_t}{\delta (1 + \lambda)^{\bar{\pi}_t} + (1 - \theta)}
\]

\[
1 - \ell_{stick} = \frac{\delta (1 + \lambda)^{\bar{\pi}_t}}{\delta (1 + \lambda)^{\bar{\pi}_t} + (1 - \theta)}
\]

while in future periods \( (s > 0) \) when there is no inflation it will be the case that \( c_{t+s} = c_{carrot} = c_{ne} = \frac{A (1 - \theta)}{1 - \theta + \delta} \) and \( \ell_{t+s} = \ell_{carrot} = \ell_{ne} = \frac{1 - \theta}{1 - \theta + \bar{\pi}} \).

We will confirm that such a path of inflation can be supported as an SPE by demonstrating that it is self-sustaining. A self-sustaining SPE is one in which the punishment used to prevent deviation has the same value as does the SPE under consideration itself. In other words, we will check whether this is an SPE outcome when the punishment to deviating is to restart the SPE from the beginning. The strategy we are considering by households is thus

\[
\ell_t(\zeta_t^{t-1}) = \begin{cases} 
\ell_{stick} & \text{if } t = 0 \text{ or if } \zeta_t^{t-1} \neq \{ \bar{\pi}, 1, 1, \ldots \} \\
\ell_{ne} & \text{otherwise}
\end{cases}
\]

Since all periods but the first feature the NE outcome, it is clear that deviation is not optimal as long as the first period payoff is lower than the NE payoff. It is simple to show that this is true as long as \( (1 + \delta) \log \left( \frac{\delta + (1 - \theta)(1 + \lambda)^{\bar{\pi}}}{\delta + (1 - \theta)} \right) < 0 \), which is evidently always true. Deviation in the first period will be for the planner to set \( \bar{\pi}_1 = 1 \) and thereby eliminate distortions and maximize consumption given the predetermined labor input. This will yield

\[
c^{*d.} = \frac{(1 - \theta)A}{\delta (1 + \lambda)^{\bar{\pi}_1} + (1 - \theta)}
\]

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(where the superscript s.d. is for stick deviation) units of consumption. This follows from the fact that when there is zero intermediate good inflation all firms will participate sharing the labor input efficiently. Given these results, it is simple to prove the following:

**Proposition 9** The “stick and carrot” path of inflation of $\bar{\pi}_t = \bar{\pi}$ and $\pi_{t+s} = 1$ for all $s \geq 1$ is a self-sustaining SPE if $\beta \geq \beta^* = \frac{\frac{1}{1-\delta} \log(1+\lambda)}{(1+\delta) \log \left( \frac{\lambda^{1-\delta} (1+\lambda)^{1-r/\delta}}{\bar{\pi}^{1-\delta} (1+\delta)^{r}} \right)}$.

Since we have already shown that the authority does not want to deviate in subsequent periods, proving the above reduces to proving that it is not optimal for the planner to deviate in the initial period. When the future is valued enough, the punishment of having to wait an additional period to return to the price stability NE is severe enough to deter deviation. For the parameterization we have been considering in this paper, $\beta^* = .8698$.

With this result in hand we now return to the question of whether the full commitment outcome obtains in any SPE. The household strategy in such an equilibrium will be

$$
\ell_t(\zeta^{t-1}) = \begin{cases} 
\ell^ne & \text{if } \zeta^{t-1} = \{\bar{\pi}^t\}_{s=0}^{t-1} \text{ or if } t = 0 \\
\ell^{stick} & \text{otherwise}
\end{cases}
$$

In studying the S&C SPE, we have seen that the authority will choose $\bar{\pi}$ in period immediately after deviating and $\bar{\pi}_t = 1$ in any $t$ after having deviated deviated in the past. To determine whether the above strategy results in a rational expectations SPE outcome then reduces to determining if the authority has any incentive to choose $\tilde{\pi}_t \neq \bar{\pi}^t$ in any $t$. If not then the authority will optimally implement the full commitment path of inflation and the private sector will rationally expect this. The analogous condition to (31) is

$$
\sum_{s=0}^{\infty} \beta^s \left\{ \log \left[ A \ell_{t+s}^c f(\tilde{\pi}_t^{t+s}) \right] + \delta \log \left[ 1 - \ell_{t+s}^c \right] \right\} \geq \log \left[ A \ell_t^c \right] + \delta \log \left[ 1 - \ell_t^c \right] + \beta \left\{ \log \ell^{stick} + \delta \log(1 - \ell^{stick}) \right\} + \frac{\beta^2}{1-\beta} V^{ne} \quad (33)
$$

where $\log \ell^{stick} + \delta \log(1 - \ell^{stick}) + \frac{\beta}{1-\beta} V^{ne} < \frac{1}{1-\beta} V^{ne}$ is the payoff from the S&C SPE described above. Figure 4 shows the LHS and RHS of condition (31)
for the first 15 periods given the parameterization used above.

![Graph showing Ramsey Welfare and "Stick and Carrot" Deviation Welfare over time.]

Thus, the full commitment outcome is a SPE outcome when expectations inducing the S&C SPE follow a deviation. In steady state the planner clearly does not have an incentive to deviate since their will be no gain to deviation in the first period and following this the stick punishment will lead to lower welfare. We see from figure 4 that neither will there be deviation in the initial periods. Thus, the authority will optimally implement the full commitment path of inflation and the private sector will rationally expect this.

5 Conclusion

We have shown that commitment outcomes, including the optimal full commitment policy, are sustainable as SPE outcomes.
6 Appendix

Proposition 1: \( \tilde{\pi}^{opt} = 0 \) is the policy rule that is Optimal from the Timeless Perspective

Consider the system of equations (24) and (37) and let \( \tilde{\pi}^s = \lim_{t \to -\infty} \tilde{\pi}_t, \ w^s = \lim_{t \to -\infty} w_t, \ \mu^s = \lim_{t \to -\infty} \mu_t \) be the steady state values of the intermediate inflation rate, wage, and lagrange multiplier when the Ramsey plan is implemented. In steady state equation (37) is

\[
f'(\tilde{\pi}^s) \left[ f(\tilde{\pi}^s)^{-1} - \frac{\delta(1+\delta)A}{\delta A f(\tilde{\pi}^s) + w^s} \right] = \mu_s w^s \frac{\lambda}{\theta} \left( 1 + \lambda \left( \tilde{\pi}^s \right)^{\frac{1}{1-\phi}} \right)^{\frac{1}{1-\phi}} \left( \tilde{\pi}^s \right)^{\frac{1-\phi}{\phi}} (1 - \tilde{\pi}^s).
\]

Given that \( f'(1) = 0 \), it is clear that \( \tilde{\pi}^s = 1 \) satisfies this condition.

Proposition 2: Assuming the planners problem above is everywhere concave, the optimal inflation target is positive, i.e. \( \tilde{\pi}^{opt} > 0 \)

Lemma 2: When there is no discounting the optimal inflation target is price stability

When \( \beta = 1 \) the wage rate as a function of the constant rate of inflation \( \tilde{\pi} \) is

\[
(1 - \theta)A(1 + \lambda) \frac{\phi}{\sigma - 1} \left( 1 + \lambda \tilde{\pi}^{\frac{1}{1-\phi}} \right)^{\frac{1}{1-\phi}}
\]

which is maximized at \( \tilde{\pi} = 1 \). Thus, since \( \tilde{\pi} = 1 \) both maximizes the real wage rate and minimizes price dispersion it is simple to show that it is the welfare maximizing inflation target.

Lemma 3: When \( \tilde{\pi}_{t+s} = 1 \) for all \( s \geq 1 \), there exists a \( \tilde{\pi}_{t} = \tilde{\pi} > 1 \) at which firms not able to adjust price will shut down.

Equation (14) can be used to find the real price of the intermediate good producers who are not able to adjust price as a function of intermediate good inflation. This is

\[
\frac{P_{t+s}}{P_t} = (1 + \lambda) \frac{\phi}{\sigma - 1} \left( \lambda + \tilde{\pi}_{t+s}^{\frac{1}{1-\phi}} \right)^{\frac{\phi}{1-\phi}}.
\]

We have already found the wage that obtains when inflation is constant at zero. When \( \tilde{\pi}_{t+s} = 1 \) for all \( s \geq 1 \) the first order condition characterizing the optimal pricing plan tells us that \( w_{t+s} = (1 - \theta)A \) for \( s \geq 1 \). Thus, when \( \tilde{\pi}_{t+s} = 1 \) for all \( s \geq 1 \) the period \( t \) pricing condition tells us that the period \( t \) wage rate is a function of the intermediate good inflation rate which can be written as

\[
w_t = (1 - \theta)A(1 + \lambda) \frac{\phi}{\sigma - 1} \left( 1 + \lambda \tilde{\pi}_t^{\frac{1}{1-\phi}} \right)^{\frac{\phi}{1-\phi}}.
\]
Intermediate good firms will shut down when their per-period profits are negative. This will happen when the real price of producers who cannot adjust their price falls below the real marginal cost. This happens when

\[
\frac{P_{t+1}}{P_t} < \frac{w_t}{A}
\]
or

\[
(1 + \lambda) \frac{\pi_t}{\pi_{t+1}} \left(1 - \theta \right) (1 + \lambda) \frac{\pi_t}{\pi_{t+1}} \left(1 + \lambda \frac{\pi_t}{\pi_{t+1}} \right)^{\frac{\theta}{1 + \theta}} < (1 - \theta)(1 + \lambda) \frac{\pi_t}{\pi_{t+1}} \left(1 + \lambda \frac{\pi_t}{\pi_{t+1}} \right)^{\frac{\theta}{1 + \theta}}
\]

which can be simplified to

\[
\frac{\pi_t}{\pi_{t+1}} - (1 - \theta) \left(1 + \lambda \frac{\pi_t}{\pi_{t+1}} \right)^{\frac{\theta}{1 + \theta}} < (1 - \theta) \left(1 + \lambda \frac{\pi_t}{\pi_{t+1}} \right)^{\frac{\theta}{1 + \theta}} - \lambda.
\]

Let \(\bar{\pi}\) be the value of \(\bar{\pi}_t\) at which the above holds with equality. While the RHS of the above is a constant, the LHS of this equation is a strictly decreasing function of \(\bar{\pi}_t\) that becomes infinitely negative as \(\bar{\pi}_t\) becomes arbitrarily large. Since \(\bar{\pi}_t = 1\) does not satisfy this condition, the \(\bar{\pi}\) that does is strictly greater than 1.

\textbf{Proposition 3:} The “stick and carrot” path of inflation of \(\bar{\pi}_t = \bar{\pi}\) and \(\pi_{t+s} = 1\) for all \(s \geq 1\) is a self-sustaining SPE if \(\beta \geq \beta^* = \frac{1}{(1 + \delta) \log \left( \frac{\beta (1 - \theta) A + \delta \log \delta}{\beta (1 - \theta) A + \delta \log \delta} \right)}.
\]

Here we write the condition that must be satisfied for the authority to choose to not deviate in the first period of this SPE and verify that it is equivalent to the one in the proposition. Let \(V^s\) be the value from the first period of the “stick and carrot” punishment. It is given by

\[
V^s = \log c^{stick} + \delta \log (1 - \ell^{stick}) + \frac{\beta}{1 - \beta} \left[ \log c^{new} + \delta \log (1 - \ell^{new}) \right]
\]

\[
= \frac{1}{1 - \beta} \left[ \log((1 - \theta)A) + \delta \log \delta \right]
\]

\[
- (1 + \delta) \log \left( \delta (1 + \lambda \frac{\pi_t}{\pi_{t+1}} + (1 - \theta) \right) + \frac{\beta}{1 - \beta} (1 + \delta) \log (\delta + (1 - \theta))
\]

Since we are evaluating whether the proposed inflation obtains in a self-sustaining SPE, the planner will choose not to deviate if

\[
\log c^{s,d.} + \delta \log (1 - \ell^{stick}) + \beta V^s \leq V^s.
\]

This expression can be easily reduced to that appearing in the proposition.
References


