The Limited Inflation Bias in Time-Consistent 
Monetary Policy†

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Abstract

This paper studies non-Markovian time-consistent monetary policy in two 
standard Neo-Keynesian models where the monetary authority lacks the ability 
to commit and finds that the so-called “inflation bias” is less severe than previ-ously thought. In the standard price staggering model of Taylor (1980) we 
study two time-consistent inflation paths. Our numerical results show that the 
constant time-consistent inflation rate depends heavily on the degree of market 
power in the model and is quite low when the markup is small. We also find 
in this model that their also generally exist non-constant paths of inflation that 
are time-consistent. Time paths in which the intermediate inflation rate alternates between two values, both lower than the constant time-consistent rate, 
generating final good inflation rates near to zero (above and below, depending 
on the parameterization), are found to be time-consistent. We also study time-consistent policy in a model of Calvo (1983) price stickiness. This model 
presents technical complications compared with the Taylor model as price dis-

cision must now be treated as an endogenous state variable. Here we find 
that the steady states of time-consistent inflation paths are again characterized 
by low levels of inflation and that deflation in initial periods is sometimes time-

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This paper studies time-consistent monetary policy in two standard Neo-Keynesian models. Time-consistent policy is that which the planner announces will be implemented in future periods and subsequently finds optimal to implement once the future arrives. This paper focuses on a within period timing in which policy is announced and implemented before any private decisions are taken. It therefore does not admit reputational enforcement mechanisms or consider the possibility of outcomes with non-rational expectations.

We demonstrate in two standard models that time-consistent policy is not necessarily associated with any significant inflation bias. In a model of Taylor (1980) price staggering, in which time-consistent policy has been previously studied, we find that there exists more than one path of inflation that is time-consistent. While the commonly studied constant time-consistent inflation rate does exhibit a significant inflation under certain parameterizations, we find a second path of time-consistent policy according to which intermediate good inflation alternates between two values implying very small levels of steady state inflation (or deflation, depending on parameter values) in the final good price.

We next study time-consistent policy in a model of Calvo (1983) price stickiness and find that the time-consistent policy in the initial period is characterized by moderate inflation to moderate deflation, depending on the pre-existing level of price dispersion. The steady-state value of inflation is again much lower than the constant time-consistent inflation rate of the Taylor model. Further, this state dependent path of time-consistent inflation is preferable, in welfare terms, to a policy of optimal inflation targeting for some initial conditions.

Kydland and Prescott (1977) and Barro and Gordon (1982, 1983) are early and important contributions to the literature on the time-consistency issues we study here. These papers show that a planner who is unable to commit to policies that had been optimal to announce, but have become sub-optimal to implement, will achieve lower welfare than one who is able to commit. Barro and Gordon (1983) suggest that in a model with nominal frictions that create a Phillips-curve relationship between output and inflation, the cost of discretion manifests in an undesirably high rate of inflation. This has become known as the inflation bias of discretionary policy.

Subsequent research on this topic can be broadly placed into two categories differentiated by the within-period timing of the models studied. Ireland (1997), Albanesi Chari and Christiano (2002, 2003) and Ascher (2005) all study models in which the monetary authority implements policy each period after some private decisions have been taken. This body of research supposes that it is possible for the authority to “surprise” some private agents by implementing unanticipated policy.  

The current research contributes to a second branch of the literature in which all private decisions relevant to the current period’s equilibrium are taken after the policy instrument is publicly observed. For this reason it impossible for the planner to realize superior outcomes by “fooling” some agents. These models

\[^1\] Models of discretionary fiscal policy that have this feature include Chari and Kehoe (1990) and Sargent and Lundqvist (2000).
are made interesting by the fact that the path of policy that maximizes current welfare takes advantage of differences between initial and subsequent periods. Such unconstrained welfare maximizing policies are time inconsistent because the planner always has an incentive to announce that the current period is the initial period. Policies in this environment are deemed time-consistent when the planner has control over the policy instrument in each period yet does not find it optimal to announce policy that had not been previously anticipated.

Wolman (2001), Khan King and Wolman (2001), King and Wolman (2004), and Siu (2005) study this sort of consistency in the Taylor model presented below\(^2\)-\(^3\). Wolman (2001) calculates that the constant time-consistent inflation rate in this model is 15\% per year. We find that the constant time-consistent inflation rate is highly sensitive to the markup charged by intermediate good monopolists and that this inflation rate is around 2\% when the markup is 15\%. We also find that there exists a second non-constant path of time-consistent policy in this model in which there is far more moderate inflation/deflation alternating, for example, between .5\% and -.1\% for the same markup. Such intermediate good inflation rates imply final good inflation rates of around .3\%, a value that, if observed, would hardly be considered evidence of an inflation bias.

This paper finds similar results in the case of time-consistent policy in a model with Calvo stickiness\(^4\). While the solution to this model is made difficult by the fact that price dispersion creates an endogenous state variable that policy effects, it is precisely a result of this feature that the planner has less incentives to inflate. Current inflation increases future price dispersion which lowers the continuation value of future periods. We find that with a steady state inflation rate of below 2.5\% given a 20\% markup, the inflation bias is far smaller than the constant time-consistent policy in the Taylor model. Lastly, we show that time-consistent policy yields higher welfare than does optimal inflation targeting under some initial conditions.

The paper proceeds by presenting the Taylor model and developing the equilibrium conditions in section 1. Section 2 solves for the constant inflation rate time-consistent policy and computes the welfare costs of discretionary policy. Section 3 demonstrates that there are non-constant time-consistent paths of inflation in this model. Section 4 presents the Calvo model and equilibrium conditions and section 5 finds time-consistent Calvo policy and compares these outcomes with optimal inflation targeting outcomes. Section 6 concludes.

SO FAR ONLY SECTIONS 1 AND 2 ARE HERE (STILL TYPING UP THE REST...)

\(^2\)These papers deal with the uniqueness of the equilibrium for a given time-consistent money supply rule and so do not directly address the issue studied here; the inflation bias in time-consistent policy.

\(^3\)See Klein and Rios-Rull (2003) and the references sited therein for a models of fiscal policy considering similar issues of time-consistency.

\(^4\)To our knowledge time-consistent policy in the Calvo model has not been studied with the exception of Woodford (2003) who linearizes the model and thereby avoids any difference between the Calvo and Taylor models.
1 Model

We will study a prefect foresight economy comprised of the planner, a representative household, a representative final good producing firm, and a continuum of mass one of intermediate good firms.

1.1 Households

The representative household’s preferences are represented by

\[ U_t = E_t \sum_{s=0}^{\infty} \beta^s \{ \ln c_{t+s} + \delta \ln (1 - \ell_{t+s}) \}, \]

which is maximized subject to the per-period budget constraint

\[ c_t + q_t b_{t+1} = w_t \ell_t + \phi_t + b_t, \]

where \( c_t \) is per-capita consumption, \( \ell_t \) per-capita labor supply, \( \beta \) the household’s discount factor, \( \delta \) a measure of leisure’s weight in per-period utility, \( b_t \) household asset holdings in the beginning of period \( t \), \( w_t \) the wage rate, \( q_t \) the period \( t \) price of one unit of consumption in period \( t+1 \), and \( \phi_t \) is dividends from the ownership of firms. Taking the sequences \( \{w_t, \phi_t, q_t\}_{t=0}^{\infty} \) as given, the household chooses \( \{c_t, \ell_t, b_t\}_{t=0}^{\infty} \) to maximize utility subject to the per-period budget constraint and the no-ponzi scheme condition that \( \lim_{s \to \infty} \beta^s b_s \geq 0 \).

The following first order necessary conditions characterize the solution to the household’s problem:

\[
\begin{align*}
\delta \frac{c_t}{1 - \ell_t} &= w_t \\
\beta \frac{c_t}{c_{t+1}} &= q_t
\end{align*}
\]

1.2 Producers

The production side of the economy consists of a perfectly competitive, representative final good producer and a continuum of mass one of intermediate good producing monopolists.

1.2.1 Final Goods

Final goods are produced with the production technology

\[ Y_t = \left( \int_0^1 Y(i)_t^{1-\theta} di \right)^{1/\theta}, \]

where \( Y(i)_t \) is the input of intermediate firm \( i \) in period \( t \) and \( \theta < 1 \) is the elasticity of substitution. Let \( P(i)_t \) be the price of intermediate good \( i \) in \( t \) and
$P_t$ be the price of the final good. Profit maximization by final good producers implies that the demand for intermediate good $i$ will be

$$Y(i)_t = P(i)_t^{-\frac{1}{\sigma}} P_t^{\frac{\sigma}{\sigma-1}} Y_t$$  \hspace{1cm} (4)

where the relationship between $P(i)_t$ and $P_t$ is given by

$$P_t = \left( \int_0^1 P(i)_t^{\frac{\sigma-1}{\sigma}} \, di \right)^{-\frac{\sigma}{\sigma-1}}.$$

1.2.2 Intermediate Goods

**Technology** Intermediate goods are produced with labor using the constant returns-to-scale production function

$$Y(i)_t = A \ L(i)_t$$  \hspace{1cm} (5)

where $A$ is a productivity parameter and $L(i)_t$ is the labor input hired by firm $i$ in period $t$.

**Price Stickiness** Intermediate good monopolists face a price-stickiness friction. A firm that was free to adjust its price in the previous period will be able to adjust its price in the current period with probability $1 - \lambda$ and will be stuck charging the previous period’s price with probability $\lambda \leq 1$. All firms that were not free to adjust price in the previous period will be able to adjust its price in the current period. By assuming that in the initial period $\frac{1}{1+\lambda}$ firms are free to adjust price, it will follow that in every period there are $\frac{1}{1+\lambda}$ firms free to adjust and $\frac{\lambda}{1+\lambda}$ firms stuck charging the previous period’s price$^5$. As elsewhere in the Neo-Keynesian literature, output is demand determined for a firm not able to adjust its price, i.e. enough labor must be hired to produce the quantity demanded at the unadjusted price.

**Profit Maximization** Given such price-stickiness, the production technology (5), and the demand function (4), the problem facing intermediate good producer $i$ who is free to adjust price in period $t$ is to find

$$P(i)_t = \arg \max_p \left\{ \left[ p^{1-\frac{1}{\sigma}} P_t^{\frac{\sigma}{\sigma-1}} Y_t - \frac{w_i}{A} p^{-\frac{1}{\sigma}} P_t^{\frac{\sigma}{\sigma-1}} Y_t \right] + \lambda \ q_t \left[ p^{1-\frac{1}{\sigma}} P_t^{\frac{\sigma}{\sigma-1}} Y_{t+1} - \frac{w_{t+1}}{A} p^{-\frac{1}{\sigma}} P_t^{\frac{\sigma}{\sigma-1}} Y_{t+1} \right] \right\},$$  \hspace{1cm} (6)

the price which maximizes the expected discounted value of real profits. The first order necessary condition characterizing the profit maximizing price level is

$$\left[ \frac{\theta - 1}{\theta} P(i)_t^{-\frac{1}{\sigma}} P_t^{-1} + \frac{1}{\theta} \frac{w_i}{A} P(i)_t^{-\frac{1}{\sigma}-1} \right] P_t^{\frac{\sigma}{\sigma-1}} Y_t + \lambda \ q_t \left[ \left( \frac{\theta - 1}{\theta} P(i)_t^{-\frac{1}{\sigma}} P_t^{-1} + \frac{1}{\theta} \frac{w_{t+1}}{A} P(i)_t^{-\frac{1}{\sigma}-1} \right) P_t^{\frac{\sigma}{\sigma-1}} Y_{t+1} \right] = 0,$$

$$\left( \frac{\theta - 1}{\theta} P(i)_t^{-\frac{1}{\sigma}} P_t^{-1} + \frac{1}{\theta} \frac{w_i}{A} P(i)_t^{-\frac{1}{\sigma}-1} \right) P_t^{\frac{\sigma}{\sigma-1}} Y_t + \lambda \ q_t \left[ \left( \frac{\theta - 1}{\theta} P(i)_t^{-\frac{1}{\sigma}} P_t^{-1} + \frac{1}{\theta} \frac{w_{t+1}}{A} P(i)_t^{-\frac{1}{\sigma}-1} \right) P_t^{\frac{\sigma}{\sigma-1}} Y_{t+1} \right] = 0,$$

$^5$This is a generalization of Taylor (1980) which this model encompasses in the case of $\lambda = 1$.  

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or

\[ \frac{P(i)_t}{P_t} = \frac{1}{1 - \theta} \left( \frac{\frac{w_t}{A} + \lambda q_t \frac{Y_{t+1}}{A} \pi_{t+1}}{1 + \lambda q_t \frac{Y_{t+1}}{A} \pi_{t+1}} \right) \]

(8)

where \( \pi_t \equiv \frac{P_{t+1}}{P_t} \) is the rate inflation of the final good price. The intuition for this is clear: the monopolist sets the real price as a markup over a weighted average of current and future real marginal costs, \( \frac{w_t}{A} \) and \( \frac{w_{t+1}}{A} \). The ratio \( \frac{Y_{t+1}}{Y_t} \) determines the relative size of future to current demand and \( \lambda, q_t, \pi_{t+1} \), all increase the impact of the next period’s profits on the current discounted expected value of the firm.

2 Equilibrium

A competitive equilibrium is a set of prices, \( \{w_t, r_t, P_t, P(i)_t\}_{t=0}^\infty \), and an allocation, \( \{L(i)_t, \ell_t, Y_t, Y_i, c_t\}_{t=0}^\infty \), such that households, intermediate good monopolists, and competitive final good producers optimize and such that the labor and good markets clear. Since all intermediate good monopolists adjusting price in period \( t \) face the same problem, the price set by each of these \( \frac{1}{t+1} \) firms will be same. We will denote this as \( P_{0,t} \). The \( \frac{1}{t+1} \) monopolists who do not adjust their price in period \( t \) will all have done so in the previous period and thus be charging \( P_{1,t} = P_{0,t-1} \). Denote by \( \bar{\pi}_t \equiv \frac{P_{0,t}}{P_{0,t-1}} \) the level of intermediate good inflation. Note that firms charging the same price will also face the same demands and thus require the same labor inputs. Let \( Y_{0,t} \) and \( L_{0,t} \), and \( Y_{1,t} \) and \( L_{1,t} \) be the level of output and labor demand of an intermediate good firm that has and has not, respectively, adjusted price in period \( t \). We now list the

\[ ^6 \text{The 0 subscript indicates that the firm has been stuck charging this price for zero previous periods.} \]
complete set of competitive equilibrium conditions.

\[
\begin{align*}
c_t &= Y_t \\
\ell_t &= \frac{1}{1 + \lambda} L_{0,t} + \frac{\lambda}{1 + \lambda} L_{1,t} \\
Y_{0,t} &= \left( \frac{P_{0,t}}{P_t} \right)^{\frac{1}{\lambda}} Y_t \\
Y_{0,t} &= A \frac{L_{0,t}}{L_{1,t}} Y_{1,t} = A \frac{L_{0,t}}{L_{1,t}} Y_t \\
Y_t &= (1 + \lambda)\pi_t^{\frac{1-\theta}{\sigma}} \left( \lambda Y_{0,t}^{1-\theta} + \lambda Y_{1,t}^{1-\theta} \right)^{\frac{1}{1-\sigma}} \\
\frac{P_{0,t}}{P_t} &= (1 + \lambda)\pi_t^{\frac{1-\theta}{\sigma}} \left( 1 + \lambda \frac{\ell_t^{1-\theta}}{\sigma} \right)^{\frac{1}{1-\sigma}} \\
\frac{P_{0,t}}{P_t} &= \frac{1}{1 - \theta} \frac{\ell_t}{\pi_t^{1-\theta}} + \lambda \frac{q_t w_{t+1} Y_{t+1}}{Y_t} Y_{t+1}^{\frac{1}{1-\sigma}} \\
w_t &= \frac{\delta}{1 - \ell_t} c_t \\
q_t &= \frac{\beta c_t}{\ell_t^{1+1}} \\
\pi_t &= \frac{P_t}{P_{t-1}} \pi_t = \frac{P_{0,t}}{P_{t-1}} \\
\end{align*}
\]

Equations (9) and (10) are final good and labor market clearing conditions. Equations (11) follow from optimization of final good producers. Equations (12) and (13) are the technological constraints of final and intermediate good producers. Equation (14) follows from the definition of the price index (it is redundant in the list since equations (11) and (13) together imply (14)). Equation (15) follows from optimization of intermediate good producers. Equations (16) and (17) are the optimality conditions of the households. Equations (18) are the definitions of final and intermediate good price inflation.

Taking the price level \( \{P_t\}_{t=0}^{\infty} \) as the policy instrument, these twelve independent equations characterize the equilibrium values of the twelve endogenous variables \( \{c_t, Y_t, Y_{0,t}, Y_{1,t}, \ell_t, L_{0,t}, L_{1,t}, P_{0,t}, \pi_t, \bar{\pi}_t, q_t, w_t\}_{t=0}^{\infty} \).

**Lemma 1** A sequence of \( \{c_t, \ell_t, w_t, \bar{\pi}_t\}_{t=0}^{\infty} \) satisfying

\[
\begin{align*}
\ell_t &= \frac{w_t}{\delta A f(\bar{\pi}_t) + w_t} \\
c_t &= A \ell_t f(\bar{\pi}_t) \\
w_t \left( 1 + \lambda \pi_t^{1-\theta} \right)^{\frac{1}{1-\theta}} + \lambda \beta \pi_t^{\frac{1}{\sigma}} w_{t+1} \left( 1 + \lambda \pi_{t+1}^{1-\theta} \right)^{\frac{1}{1-\theta}} &= \frac{(1 - \theta) A}{(1 + \lambda) \pi_t^{\frac{1}{\sigma}}} \left[ \left( 1 + \lambda \pi_t^{1-\theta} \right)^{\frac{1}{1-\theta}} - 1 \right] + \lambda \beta \pi_{t+1}^{\frac{1}{\sigma}} \left( 1 + \lambda \pi_{t+1}^{1-\theta} \right)^{\frac{1}{1-\theta}} \\
\end{align*}
\]

for all \( t \geq 0 \), where \( f(\pi) \equiv (1 + \lambda)\pi^{\frac{1-\theta}{\sigma}} \left( \lambda \pi^{1-\theta} \right)^{\frac{1}{1-\theta}} \), also satisfies equations (9) through (18).
This result follows from algebraic manipulation of equations (9) through (18). We will work with this alternative specification of the competitive equilibrium as it is more convenient for handling the monetary authority’s problem. Using equations (19) and (20) to substitute for \(c_t\) and \(\ell_t\) in the household objective function, we see that this specification of the equilibrium reduces the problem of the authority to choosing a welfare maximizing path of \(\tilde{\pi}_t\) and \(w_t\) subject to the constraint in equation (21).

3 Time-Consistent Policy

Each period private agents are able to observe the inflation chosen by the authority before taking any actions. It is convenient to think of the planner as not only implementing a current inflation rate, but also announcing a future time-path of inflation. We now define time-consistency.

**Definition 2** A path of intermediate good inflation \(\{\tilde{\pi}_t\}_{t=0}^\infty\) is **time-consistent** if \(\tilde{\pi}_s\) maximizes the authority’s period \(s\) objective function given \(\{\tilde{\pi}_t\}_{t=s+1}^\infty\) for all \(s \geq 0\).

The period \(s\) objective function of the authority is the period \(s\) discounted lifetime utility of the representative household. Thus, one way to understand this definition is that the authority is constrained by the fact that the current, observed, policy must be optimal given the announcement. If it were not then the announcement would not be credible and private agents will form some alternative beliefs about future policy. Time-consistent policies will be those that the planner can credibly announce and implement. That is, each period’s policy must be optimal given the policies announced for future periods. A policy of price stability, for example, is not time-consistent because zero inflation in the current period is not optimal given the expectations of future price stability.

3.1 The Inconsistency of Price Stability

To see this, consider the problem of the authority choosing the period \(t\) rate of inflation when all agents expect zero inflation in periods \(t+s\) for all \(s \geq 1\). From equation (21) we see that, given \(\tilde{\pi}_{t+s} = 1\) for all \(s \geq 1\), we have \(w_{t+s} = (1 - \theta)A\) for all \(s \geq 1\). Note that price stability implements the flexible price wage rate: because of monopoly power this is inefficiently low. Given these future values of intermediate good inflation and wage rates, all future endogenous variables are pinned down by equations (9) through (18). As a result, when evaluating the optimal decision to be taken in period \(t\), only the impact on period \(t\) variables need be considered.

Returning to equation (21), and given \(\tilde{\pi}_{t+s} = 1\) for all \(s \geq 1\), the period \(t\) wage will be

\[
w_t = w_{p}(\tilde{\pi}_t) = (1 - \theta)A(1 + \lambda)^{\frac{s}{\lambda}} \left(1 + \lambda \frac{\tilde{\pi}_t^{1-s}}{x}\right)^{\frac{\theta}{s}}.
\]  

(22)
Since we are taking outcome after period $t$ is fixed, the problem of the planner is to

$$\max_{\hat{\pi}_t} \{ \log w_{ps}(\hat{\pi}_t) + \log f(\hat{\pi}_t) - (1 + \delta) \log [\delta A f(\hat{\pi}_t) + w_{ps}(\hat{\pi}_t)] \} . \quad (23)$$

Here we have used equations (19) and (20) to substitute for $c_t$ and $\ell_t$ and then combined terms and factored out constants that do not affect the solution to the optimization problem. Since

$$\frac{\partial}{\partial \hat{\pi}_t} \{ \log w_{ps}(1) + \log f(1) - (1 + \delta) \log [\delta A f(1) + w_{ps}(1)] \} > 0, \quad (24)$$

it is clear that zero inflation in the intermediate good is not optimal given future expectations of price stability. Note that condition (24) is true given that $\frac{\partial}{\partial \hat{\pi}_t} w_{ps}(1) > 0$ and $\frac{\partial}{\partial \hat{\pi}_t} f(1) = 0^7$. The first order effect on price dispersion of moving away from price stability is nil while there is a positive first order effect from moving the wage closer to the efficient level. As a result, the planner will not choose $\hat{\pi}_t = 1$ under these circumstances. This demonstrates that a policy of price stability does not meet the requirements of time-consistency: for all $t$, $\hat{\pi}_t = 1$ is not optimal given $\hat{\pi}_{t+s} = 1$ for all $s \geq 1^8$.

We now turn to policies that are time-consistent and show that their exist at least two paths of inflation in this environment that are time consistent. First we focus on the constant time consistent policy and study the sensitivity of this inflation rate to the monopoly power of the intermediate good producers.

### 3.2 Constant Time-Consistent Policy

The simplest time-consistent policy to search for in the Taylor model is one in which the inflation rate is constant. We will denote this as $\hat{\pi}_c$ and require that $\{\hat{\pi}_{t+s}\}_{s=0}^{\infty} = \hat{\pi}_c$ for all $t$ satisfy the conditions that define time-consistency. We do this by taking as given that a certain inflation rate will be chosen in all future periods and then finding the inflation rate that is optimal for the current period. A constant time-consistent inflation rate is any which the monetary authority finds optimal given that this same rate is expected in all future periods.

Let $\{\hat{\pi}_{t+s}\}_{s=1}^{\infty} = \hat{\pi}_f$, i.e. let $\hat{\pi}_f$ be the constant inflation rate expected in all future periods. Returning again to equation (21), the wage in periods $t+s$ for all $s \geq 1$ will be given by

$$w_{t+s} = w_f(\hat{\pi}_f) = (1 - \theta) A(1 + \lambda) \frac{\theta}{\lambda} \left( 1 + \lambda \frac{\hat{\pi}_f}{\hat{\pi}_f} \right) \frac{\lambda \beta}{1 + \lambda \beta} \frac{\hat{\pi}_f^{1-\theta}}{1 + \lambda \beta} . \quad (25)$$

$^7$To be exact, \[ \frac{\partial}{\partial \hat{\pi}_t} \{ \log w_{ps}(1) + \log f(1) - (1 + \delta) \log [\delta A f(1) + w_{ps}(1)] \} = \]

$^8$Of course, the policy that chooses the inflation rate solving the above problem followed by price stability in future periods is not time-consistent either as the above clearly demonstrates that, from the perspective of period $t + 1$, $\hat{\pi}_{t+1} = 1$ is not optimal given $\hat{\pi}_{t+s} = 1$ for all $s \geq 2$.
Again, it is important to note that time $t + 1$ and forward endogenous variables are completely determined by future inflation rates. Thus, considering the effects of the choice of $\tilde{\pi}_t$, the authority need only consider the period $t$ equilibrium. The period $t$ wage rate will be

$$w_t = w_c(\tilde{\pi}_t, \tilde{\pi}_f) = \frac{(1 - \theta)A}{(1 + \lambda)^{\alpha - \beta}} \left[ \left( 1 + \lambda \tilde{\pi}_t^{\frac{1 - \beta}{\alpha - \beta}} \right)^{\frac{\alpha}{\alpha - \beta}} + \frac{\lambda \beta \tilde{\pi}_f^{\frac{1 - \beta}{\alpha - \beta}} (1 - \tilde{\pi}_f)}{(1 + \lambda \tilde{\pi}_f^{\frac{1 - \beta}{\alpha - \beta}})} \right],$$

which follows from using equation (25) together with (21). Following the same procedure as above, we can write the objective of the period $t$ planner as

$$u(\tilde{\pi}_t, \tilde{\pi}_f) = \log w_c(\tilde{\pi}_t, \tilde{\pi}_f) + \log f(\tilde{\pi}_t) - (1 + \delta) \log [\delta Af(\tilde{\pi}_t) + w_c(\tilde{\pi}_t, \tilde{\pi}_f)].$$

(26)

The constant time-consistent inflation rate, \( \tilde{\pi}_c \), is then defined by

$$\tilde{\pi}_c = \arg \max_{\tilde{\pi}_t} \{ u(\tilde{\pi}_t, \tilde{\pi}_c) \}.$$

We solved numerically for $\tilde{\pi}_c$ for a range of $\theta$ values and several $\lambda$ values. See the appendix for the first order conditions characterizing the solution to this problem along with a discussion of the methods used to check that these conditions are necessary and sufficient. We considered $\theta$ values implying zero inflation steady state markups ranging between 10% and 50%9. In the basic Taylor model where firms deterministically alternate between choosing price and not, the expected length of time between price changes is 1.5 years. For the price stickiness parameter we check the results for four values which correspond to expected length of time between price changes of 1.09 years ($\lambda = .1$), 1.29 years ($\lambda = .4$), 1.41 years ($\lambda = .7$), and 1.5 years ($\lambda = 1$). The other parameter values used were $A = 1$, a normalization, $\beta = .96$, for an annual interest rate of roughly 4%, and $\delta = 2$, implying between a quarter and third of the time endowment is spent working. The results of this exercise can be seen in Figure

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9 The zero-inflation steady state gross markup in this economy is $\mu = \frac{1 - \beta}{\alpha - \beta}$. Thus $\theta = \frac{\mu + \lambda - 1}{\mu \lambda}$, and so the values of $\theta$ chosen range from $\frac{1}{\lambda + 1}$ to $\frac{\alpha}{\alpha - \beta}$. 

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1. As does Wolman (2001), we find when prices are stuck for six quarters and the markup is 33%, the constant time-consistent inflation rate\(^1\) is around 15%. While Wolman (2001) does not consider alternative parameterizations of the model, Figure 1 shows that the results change significantly when the markup is lower. With a 10% markup and price adjustments taking place just over every 13 months \((\lambda = .1)\), the constant time-consistent inflation rate is around 1% per year. It is also interesting to note that less stickiness implies lower inflation rates. The intuition for this is that the costs of inflation rise as \(\lambda\) falls\(^2\). When there is inflation, labor is being inefficiently directed from the flexible-price to the stuck-price intermediate good firms. There are more flexible-price firms when prices are less sticky.

Learning that the inflation bias may be much smaller than previously thought leads us to explore the welfare value of commitment. Since we are studying the constant time-consistent inflation rate, a natural benchmark with which to compare is the constant inflation rate welfare maximizing. That is, we will calculate the optimal inflation target. This is the inflation rate that maximizes the household’s objective subject to the constraints in equations (19), (20), and (21), the constraint that intermediate good inflation be constant, and ignoring the constraint imposed by our definition of time-consistency\(^3,4\). We next

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\(^1\)Note that it follows from equation (14) that the final and intermediate good inflation rates are equal when the intermediate good inflation rate is constant.

\(^2\)There is a discontinuity at \(\lambda = 0\).

\(^3\)See Ascher (2005) for a more detailed discussion of the exercise of finding the optimal inflation target in this model.

\(^4\)Two alternative benchmarks would be welfare under price stability and welfare under the full-commitment plan. Given that the optimal inflation target is close to price stability,
compute the permanent percentage increase in consumption a household living with a planner who cannot commit requires to reach the same level of utility as a household living under a planner implementing the optimal inflation target. Figure 2 shows these welfare costs of discretion. We see, for example, that with a price-stickiness parameter of $\lambda = 1$ and a markup of about 33% the representative household living in a discretionary world is indifferent between receiving a permanent .53% increase in consumption and moving to the optimal inflation targeting world.

![Graph](Figure 2)

As figure 2 clearly shows, the welfare costs of discretion vary greatly with the degree of monopoly power and the extent of price-stickiness. Indeed, when the markup is lowered to 10%, holding $\lambda = 1$ constant, the costs are as low as .02%.

This exercise has shown that time-consistent inflation need not be associated with as large a bias as previously thought. Further, we have seen that the welfare gains to commitment may be relatively small. The novelty of this paper, however, will be in identifying non-constant time-consistent policies in the Taylor model and state-contingent time-consistent policies in the Calvo model. We first continue with the Taylor model and find that there do exist additional time-consistent inflation paths to those studied above. These are of interest as time-consistent policy may be qualitatively different in this case to the extent that it may even demonstrate a deflationary bias.

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14This is the parameterization used by Wolman (2001).
3.3 Non-Constant Time-Consistent Policy

Consider again our definition of time-consistency and consider a policy path where inflation alternates between two values, $\pi_e$ in even periods and $\pi_o$ in odd periods. The equilibrium wage rates implied by the optimal pricing condition of firms are

$$w_i = \frac{(1 - \theta)A}{(1 + \lambda)^{1/\nu}} \left( 1 + \lambda \frac{1 - \theta}{\pi_i^{1 - \theta}} \right) \left( \frac{1 - (\lambda \beta)^e \pi_e^{1/\nu}}{1 + \lambda \frac{1 - \theta}{\pi_e^{1 - \theta}} (1 - \pi_j)} + \frac{\lambda \beta \pi_j^{1/\nu}}{1 + \lambda \frac{1 - \theta}{\pi_j^{1 - \theta}}} \right)$$

for $i, j = e, o$ where $i \neq j$