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Abstract

This paper contributes to the literature on business cycles and labor market matching along the following dimensions. First we provide empirical evidence on the dynamics of key aggregate variables in response to identified structural shocks. Using a structural VAR methodology we estimate the impact of technology shocks and of government spending shocks. We show that central labor market indicators, unemployment, vacancies, and labor market tightness, react very sluggishly in response to, in particular, neutral technology shocks. We then construct a fully specified dynamic stochastic general equilibrium model that introduces labor market matching and an endogeneous labor market participation choice as well as other frictions. We estimate key parameters using a limited information approach. We show that the model can successfully account for the dynamics of output and its components following an innovation to the identified structural shocks. However, the model fails very considerably in accounting for the sluggish behavior of the labor market indicators that we document in the data. We argue that this failure is most likely not addressed by introducing wage stickiness but requires real rigidities that lead firms to delay adjustment of employment in response to shocks to the economy.

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1 Introduction

Understanding the cyclical fluctuations in key labor market variables is an important issue in the business cycle literature. Not only are the movements in the labor input of prime importance for understanding the cyclical fluctuations in output, see Kydland (1995), but labor market frictions may also be key to better understanding the propagation mechanism. Furthermore, central labor market indicators often ignored in the business cycle literature, unemployment and job vacancies, display large and persistent fluctuations at the business cycle frequencies. Arguably, ignoring the fluctuations observed in these variables may be too big a shortcut when attempting to better understand business cycle fluctuations and the role of the labor market in propagation shocks over time.

The labor market matching framework proposed by Mortensen and Pissarides (1994) and Pissarides (2000) provides an appealing modeling of the labor market. In this set-up the matching of job vacancies and unemployed workers is frictional and takes both time and resources. In particular, firms need to spend resources on posting job vacancies and matches between firms with vacancies and workers looking for jobs takes time. The matching framework relates the number of new employer-worker matches to the number of job vacancy postings and the number of search active currently unmatched workers. A key variable in this set-up is labor market tightness, the ratio of vacancies to unemployment. When the matching function displays constant returns, this ratio equals the ratio of the probability that a search active worker is matched with a vacancy to the probability that a firm with a job vacancy is matched with a search active worker. This ratio is therefore important for understanding the incentives of firms to post vacancies and the incentive for workers to engage in job search. Moreover, labor market tightness displays large fluctuations over the business cycle with periods of high activity being associated with large increases in labor market tightness and vice versa in recessions.

Recently Hall (2005) and Shimer (2005) have forcefully argued that models that build upon an assumption of flexible wages are unable to account for the volatility of the $vu$-ratio. In the U.S. data, the standard deviation of the $vu$-ratio is more than 10 times higher than the standard deviation of output at the business cycle frequencies. In the standard labor market matching model with flexible wages instead, the $vu$-ratio does not fluctuate much.\(^1\) Shimer discusses this issue in detail and argue that this deficiency is explained by the fact that changes in labor productivity do not give firms much incentive to post vacancies when wages are flexible because employees reap most of the benefits of higher productivity. Shimer (2005) therefore suggest to introduce sticky wages into the labor market matching framework and Hall (2005) analyses such a model.

\(^1\)Neither Merz (1995) nor Andolfatto (1996) report the standard deviation of the $vu$-ratio in their model economies. Adopting Andolfatto’s (1996) calibration however reveals that the $vu$-ratio has a standard deviation that is smaller than output.
Common to these analyses and to those of Andolfatto (1996) and Merz (1995, 1999), who have earlier introduced labor market search and matching into fully specified dynamic stochastic general equilibrium models, is that they study the unconditional moments of the data. We propose instead to use structural VAR methods to derive insights on the conditional moments. We show that such evidence challenges the conclusions drawn by Hall (2005) and Shimer (2005). In particular, the evidence points towards real rigidities as being important for understanding the dynamics of the $vu$–ratio. We study US quarterly data and identify two structural shocks, neutral technology shocks and government spending shocks. These shocks are identified using assumptions that do not rest upon any apriori view on whether wages are flexible or sticky. In particular, we follow Gali (1999), Francis and Ramey (2005), Altig et al (2005) and many others and identify productivity shocks using long-run restrictions on the long-run impact on the level of labor productivity. Government spending shocks are instead identified on the basis of short-run restrictions as in Blanchard and Perotti (2002).

The results reveal substantial inertia in the dynamics of unemployment, vacancies, and of the $vu$–ratio in response to, in particular, the identified technology shock. We find that, in response to an increase in the growth rate of technology, unemployment initially increases but then declines considerable reaching a maximum decline of 2.5 percent with a 2.5 years delay. Similarly, vacancies initially decline very slightly but then increase substantially reaching a peak increase with a similar delay. In combination, the dynamics of unemployment and vacancies imply that the $vu$–ratio displays a marked inverse $U$-shaped dynamics reaching a 5 percent increase 2-2.5 years after an increase in the growth rate of technology that increases long-run output by around 1 percent relative to its trend. This challenges the view of sticky wages explaining the discrepancy between theory and data since sticky wage theories - while generating more volatility in the $vu$–ratio - imply less persistence in the $vu$–ratio and therefore a dynamic pattern that resembles little the inverse $U$–shaped response that we uncover in the data. Similarly, we find that an increase in government spending affects unemployment and vacancies with a prolonged delay although the results are associated with some sampling uncertainty.

We then construct a labor market matching model akin to Andolfatto (1996) and Merz (1995) but augmented along two important dimensions. First, we introduce real frictions in the form of adjustment costs and habit formation. Secondly, we introduce a labor market participation choice. In our set-up agents may either be matched with an employer, unmatched but actively searching for a job, or they may simply choose to be labor market non-participants. Thus, as in the measurement of unemployment in the US statistics, the unemployed agents are characterized by two conditions: (i) they are not currently matched with an employer, and (ii) they are actively looking for a job. We introduce this feature in order better to match the measurement of the data that are of our primary interest, i.e. unemployment.

We model the participation choice as a trade-off between forgoing the possibility
of finding a job and giving up resources needed to participate in labor market search. The introduction of the participation choice, however, leads to a strong tendency for procyclical movements in unemployment. The reason is that any increase in vacancies makes non-participants more likely to engage in labor market search. For that reason we also introduce costs of changing labor market status from non-participation to being search active.

The key parameters of the model are estimated using a limited information approach suggested by Rotemberg and Woodford (1997). As Christiano et al (2005) we estimate a subset of the model parameters on the basis of the impulse response functions to the identified shocks that we uncover in the US data. Our estimates agree broadly with the estimates of Christiano et al (2005) despite the fact that the VAR specifications and the nature of the identified shocks differ. In particular, we find (i) that the growth rate of neutral technology shocks displays significant persistence, and (ii) that there seems to be a significant role for habit formation. Importantly, our results also indicate that labor market status adjustment costs are very large. Quantitatively, the labor market adjustment costs are of a size that a two percentage point change in the labor market participation rate brings about a resource cost of around half a percent of output. It is difficult to see exactly where such large costs may derive from and we thus conclude that there is a labor market participation puzzle in the sense that smaller, and more realistic adjustment costs, bring about the very counterfactual implication that unemployment is procyclical.

On the basis of the estimated structural parameters, we can then examine the extent to which the model brings about dynamics of the key variables that resemble the dynamics of the US data in response to the structural shocks that we identify. We find that the labor market matching model can successfully account for the dynamics of output and its components in response to neutral technology shocks and to changes in government spending. However, the model fails in accounting for the dynamics of unemployment, vacancies, and the \( vu \)-ratio. In particular, despite the presence of real frictions, the model implies little response of the \( vu \)-ratio to the identified neutral technology shock and rather than being \( U \)-shaped, the \( vu \)-ratio initially increases and then reverts fast in a monotonic manner towards its steady-state. Moreover, this feature is robust and becomes even more counterfactual in the absence of labor market status adjustment costs. In the matching model, firms start to hire workers early in response to the boom brought about by a positive technology shock. In the data instead, this adjustment takes a long-time to take effect and is associated with a much larger increase in labor market tightness than predicted by theory. This

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2 The only exception to this concerns the response of consumption to changes in government spending. In the US data, as is currently the focus of a number of papers, private consumption expenditure appears to increase following an increase in government spending. The model that we study implies instead, due to a wealth effect, that private consumption drops. We do not consider this a major problem since we do not introduce features that potentially allow us to address this issue.
indicates that more research is needed to uncover the reasons for the drawn out dynamics of key labor market indicators that we uncover in the US data.

The remainder of the paper is structured as follows. In Section 2 we present the VAR methodology that we apply to identify the structural shocks and we estimate the model using quarterly US data. In Section 3 we present the labor market matching model. Section 4 discusses the statistical procedure that we apply in order to estimate the parameters of the model economy. Section 5 presents the results. Finally, we conclude and summarize in Section 6.

2 Empirical Evidence

In this section we derive evidence on the aggregate effects of technology shocks and of government spending shocks in the United States. We study quarterly data for the sample period 1964-2003. Similarly to Christiano et al (2005) and Altig et al (2005), the identification scheme applies a mixture of long-run and short-run restrictions but the nature of the latter differ due to the nature of the shocks that we identify. The VAR that we consider differs from the majority of previous studies in that we consider also the effects of the identified shocks on unemployment and on job vacancies.

2.1 Identification and Estimation

Consider the following seven-dimensional reduced-form VAR:

\[
\begin{align*}
    x_t &= \alpha + \beta(L) x_{t-1} + e_t \\
    x_t &= \begin{bmatrix}
        \Delta \ln \left( \frac{GDP_t}{hours_t} \right) \\
        \ln \left( \frac{C_t}{GDP_t} \right) \\
        \ln \left( \frac{I_t}{GDP_t} \right) \\
        \ln \left( U_t \right) \\
        \ln \left( V_t \right) \\
        \Delta \ln G_t
    \end{bmatrix} = \begin{bmatrix}
        \Delta a \\
        x_{1t} \\
        x_{5t}
    \end{bmatrix}
\end{align*}
\]

where \( \beta(L) \) is a lag polynomial of order \( P \). \( \Delta \ln \left( \frac{GDP_t}{hours_t} \right) \) is the change in the logarithm of average labor productivity, \( \ln \left( \text{hours}_t \right) \) is the logarithm of average hours worked per adult (the product of employment and hours per worker divided by the adult population), \( \ln \left( \frac{C_t}{GDP_t} \right) \) is the logarithm of private consumption’s share of GDP and \( \ln \left( \frac{I_t}{GDP_t} \right) \) is the logarithm of the investment share. Output is measured as GDP in constant (chain weighted) prices. We measure private consumption expenditure as the sum of private spending on non-durables and on services. Investment expenditure is defined as the sum of fixed investment expenditure and spending on consumer durables. \( \ln \left( U_t \right) \) is the logarithm of the aggregate number of unemployed workers per adult, and \( \ln \left( V_t \right) \) is the logarithm of the number of vacancies per adult.
measured on the basis of the number of job advertisements. Finally, \( \Delta \ln G_{t}^{S} \) denotes the change in the logarithm of real government expenditure per adult. We constructed real government spending as nominal government spending divided by the implicit output deflator.

Equation (1) is the reduced form VAR. Our interest is in identifying the two structural shocks, government spending shocks and technology shocks. We identify the former using a short-run restriction and the latter using a long-run restriction.

To identify government spending shocks, we adopt the approach of Blanchard and Perotti (2002). These authors assume that, in quarterly data, government spending does not react to unexpected movements in any other variable than taxes (net of transfers). Moreover, they find that tax shocks and government spending shocks are almost uncorrelated in US data. Thus, following these suggestions, we identify government spending shocks using the restriction that, at the quarterly frequency, the process for government spending depends on lagged values of government spending and other variables but does not depend on the current realizations of any other structural shocks. Notice that the nature of the short-run restrictions introduced by the identifying assumptions that we make differ from the short-run restrictions customarily made when identifying monetary policy shocks. In particular, the latter are often identified assuming that certain variables are orthogonal to the innovation to monetary policy. Here instead, the restriction assumes that the government spending process is orthogonal to other structural innovations.

Therefore, we can derive government spending shocks from a simple least squares regression:

\[
\Delta g_{t} = \alpha^{g} + \sum_{i=1}^{P} \beta_{g,i}^{g} \Delta g_{t-i} + \sum_{i=1}^{P} \beta_{a,i}^{g} \Delta a_{t-i} + \sum_{i=1}^{P} \beta_{x_{i}}^{g} \Delta x_{1,t-i} + \varepsilon_{t}^{g}
\]

where \( \varepsilon_{t}^{g} \) denotes the identified US government spending shock. This relationship can be estimated with least squares because government spending shocks are assumed not to be affected by any other structural shock.

Technology shocks, instead, are identified using the long-run restriction proposed by Gali (1999). In particular, we assume that neutral technology shocks are the only type of shocks that can affect the long-run level of labor productivity. This restriction holds in the model that we will consider below. Thus, we adopt the estimation strategy of Shapiro and Watson (1988) and identify the technology shock from the following relationship:

\[
\Delta a_{t} = \alpha^{a} + \sum_{i=0}^{P-1} \beta_{g,i}^{a} \Delta^{2} g_{t-i} + \sum_{i=1}^{P} \beta_{a,i}^{a} \Delta a_{t-i} + \sum_{i=0}^{P-1} \beta_{x_{i}}^{a} \Delta x_{1,t-i} + \varepsilon_{t}^{a}
\]

where \( \Delta^{2} \) denotes the double difference operator and \( \varepsilon_{t}^{a} \) is the identified technology shock. This relationship cannot be estimated with least squares because \( x_{1,t} \) may
depend on $\varepsilon_t^y$ and on $\varepsilon_t^g$. We use a two-stage least squares estimator using as instruments a constant, the vector $[x_{1,t-i}, \Delta a_{t-i}, \Delta g_{t-i}]_{i=1}^P$ and $\tilde{\varepsilon}_t^y (\tilde{\varepsilon}_t^g$ denotes the estimated value of $\varepsilon_t^g$). Note that $\tilde{\varepsilon}_t^y$ is a valid instrument because it is assumed orthogonal to $\varepsilon_t^y$.

The system of equations for the components of $x_{1t}$ are estimated using the recursive two-stage least squares technique explained in Altig et al. (2005). The first equation $x_{1t}$ is estimated as:

$$x_{1t}^1 = \alpha^1 + \sum_{i=0}^{P} \beta_{g,i}^1 \Delta g_{t-i} + \sum_{i=0}^{P} \beta_{a,i}^1 \Delta a_{t-i} + \sum_{i=1}^{P} \beta_{x,a,i}^1 x_{1,t-i} + \varepsilon_t^1$$ (4)

using as instruments a constant, $[x_{1,t-i}, \Delta a_{t-i}, \Delta g_{t-i}]_{i=1}^P$ and $[\tilde{\varepsilon}_t^y, \tilde{\varepsilon}_t^A]'$. The second equation is specified as:

$$x_{1t}^2 = \alpha^2 + \sum_{i=0}^{P} \beta_{g,i}^2 \Delta g_{t-i} + \sum_{i=0}^{P} \beta_{a,i}^2 \Delta a_{t-i} + \sum_{i=1}^{P} \beta_{x,a,i}^2 x_{1,t-i} + \beta_{x,1}^2 x_{1t}^1 + \varepsilon_t^2$$

and estimated using two-stage least squares. The instruments, however, now include also the innovation $\varepsilon_t^1$ and thus consist of a constant, $[x_{1,t-i}, \Delta a_{t-i}, \Delta g_{t-i}]_{i=1}^P$ and $[\tilde{\varepsilon}_t^y, \tilde{\varepsilon}_t^A]'$. We continue this procedure recursively for all the variables included in $x_{1t}$. This estimation strategy is applied in order to address the endogeneity aspects of the system of equations formed by $x_{1t}$.

We assume throughout that $P = 4$. The specification in equation (1) assumes that hours worked are stationary, that output cointegrates with consumption and with investment, and that unemployment and vacancies per adult are stationary. The cointegration restrictions are supported by the data. As is well-known, the hours worked series is borderline trend stationary (when one allows for deterministic trends). We will work under the maintained assumption that hours worked are stationary but this does not affect the main conclusions that we draw.

More critical are the assumptions that aggregate unemployment per adult and job vacancies per adult are stationary. Standard unit root tests indicate non-stationarity of these two series, an insight that has been the focus of a large literature. As is well-known, however, it is difficult to reconcile non-stationarity of unemployment with economic theory and the model that we will consider later indeed implies stationarity of unemployment. Moreover, it may also be difficult to distinguish non-stationary time-series from stationary time-series with very high persistence which is a feature that, instead, does indeed characterize unemployment in the model economy that we propose. Thus, in the end, the issue is whether the empirical VAR captures better

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3 We also linearly detrended all the resulting series before estimation.
4 DenHaan (2002) proposes an interesting model in which unemployment may permanently change in response to large shocks. This type of dynamics may be very hard to distinguish from non-stationarity in a time-series sense.
the data when unemployment and vacancies enter in levels or in first differences. We address this issue by examining the sensitivity of the results to the assumption on the trend stationary of unemployment.

The impulse responses of various variables are illustrated in Figure 1A (for technology shocks) and Figure 1B (government spending shocks) for the specification where unemployment and vacancies per adult enter the VAR in levels. Figures 2A and 2B report the results for the VARs where unemployment and vacancies enter in first differences. The pictures show the point estimates together with 95 percent confidence intervals for the key variables of interest to our analysis: Labor productivity, output, consumption, investment, government spending and the labor market indicators, hours worked, unemployment, vacancies, and the $vu$–ratio. We derive the impulse response of the latter by combining the impulse responses of unemployment and vacancies.

### 2.2 The Effects of Technology Shocks

Following a positive productivity shock, we find that aggregate labor productivity rises steadily. Aggregate output also rises up to a new higher level and reaches its maximum level within a forecast horizon of about 2-2.5 years. Aggregate consumption emulates the behavior of output but increases more smoothly than output. Investment instead reacts more elastically and faster than output and consumption. It is also noteworthy that there is a short (but insignificant) reversal of the positive impact of technology shocks on investment within 6 months of the technology shock after which investment rises fast. Government spending rises slowly up to a new higher level over time. These results, with the exception of government spending, are very similar to those reported by Altig et al (2005) and by Christiano et al (2005) despite differences in the specification of the VAR.

Most interesting are the effects on the labor market indicators. In line with the literature which assumes that hours worked is stationary, we find that hours worked increase in response to a technology shock. The response of hours worked is initially quite small and reaches its peak within a two year horizon.

Aggregate unemployment rises on impact in response to a positive technology shock. Moreover, this initial impact is actually marginally significantly positive. The initial positive impact of unemployment, however, is reversed over time and from 3 quarters onwards, unemployment falls rather dramatically in response to the technology shock reaching a maximum decrease of around 2.5 percent 2.5 years after the increase in technology. The impact on vacancies is close to a mirror image of the unemployment response apart from the fact that the response of vacancies is positive from the 2’nd quarter onwards.

Together these results, as shown in the last figure, imply that the $vu$–ratio displays a marked inverse $U$-shaped response to technology shocks. Upon impact, the $vu$–ratio decreases in response to a positive technology shock but this is reversed within the first
year after the increase in technology. From then on and until around 2.5 years after the technology shock, the $vu-$ratio increases significantly reaching a maximum impact above 5 percent. Thereafter, the $vu-$ratio declines gradually over time returning to its original value within the 20 quarter forecast horizon that we consider.

In Figure 2A we assess the sensitivity of the results to the assumptions made on the trend stationarity of unemployment and vacancies. With the exception of government spending, we find very similar - indeed almost identical - impulse responses under the two alternative VAR specifications. Government spending, however, is unaffected by the productivity shock under the assumption that unemployment and vacancies are non-stationary but the estimated standard errors are very large (and the 95 percent confidence interval includes the impulse response function that is generated assuming that unemployment and vacancies are stationary).

We also find approximately identical response of hours worked to the technology shock under the two competing assumptions on the trend stationarity of unemployment and vacancies. The estimated responses of unemployment and vacancies, however, are slightly different. In particular, assuming that unemployment and vacancies are difference stationary leads to lower elasticities of these variables to the technology shock and to slightly faster peak responses. However, we still find an initial increase in unemployment and a delayed peak impact. The maximum decrease in unemployment now occurs after around 2 years and has a size of around 1 percent. This is around 6 months faster and half as big as the impact that we found when assuming trend stationary unemployment and vacancies. Likewise, vacancies reaches a maximum increase of around 1 percent within a 1.5 years horizon as contrasted to a maximum increase of approximately 3 percent with a 2.5 years delay when we assumed trend stationarity. However, we still find an inverse U-shaped response of the $vu-$ratio which initially falls but then rises to a maximum increase of around 2 percent with a significant delay of between 1.5 and 2 years.

The findings of (i) an initial decrease in the $vu$-ratio and (ii) a delayed, but very large increase in the $vu-$ratio is important for current discussions about the specification of labor market matching models. In particular, important contributions of Shimer (2005) and Hall (2005) argue that sticky wages are key to generating sufficient volatility in the $vu-$ratio. The intuition for this is that when wages are sticky, an increase in labor productivity give firms a large incentive to post vacancies as they receive a large share of surplus generated by higher productivity. However, in this case one would expect vacancies and the $vu-$ratio to rise sharply in response to the positive technology shock. Instead we find evidence of a large but delayed increase in vacancies. This evidence therefore seems to seriously question the argument that sticky wages generate more plausible dynamics of the $vu-$ratio.
2.3 The Effects of Government Spending Shocks

Figures 1B and 2B illustrate the dynamic effects of government spending shocks under the two competing assumptions on the trend stationarity of unemployment and vacancies. Based on these results, government spending shocks are persistent increases in government spending.

Consistently with Blanchard and Perotti (2002) and others, we find that an increase in government spending leads to an increase in output but the effect is rather short-lived. We also find that private consumption increases in response to an increase in government spending. Moreover, the increase in private consumption appears to be as large as the increase in output but more persistent. This finding is currently the basis of a growing literature, see e.g. Gali et al (2005) and Ravn et al (2005). As Blanchard and Perotti (2002), we also find that investment drops rather dramatically in response to an increase in government spending. These results are insensitive to the specification of the VAR as regards the assumptions on the trend stationary of unemployment and vacancies. The only notable difference is that government spending shocks appear more persistent when unemployment and vacancies enter the VAR in first differences.

Regardless of the VAR specification, we find that an increase in government spending leads to a short-lived increase in aggregate average hours worked per adult. This effect dies out very quickly after which hours worked are basically unaffected.

The estimated response of unemployment, vacancies, and the $vu$—ratio, instead, depend more on the VAR specification. When we assume that unemployment and vacancies are trend-stationary, we find that unemployment is approximately unaffected by the increase in government spending while vacancies drop (but the responses are insignificantly different from zero, though). The drop in vacancies leads to a 1 percent decrease in the $vu$—ratio one year after the increase in government spending. When unemployment and vacancies are assumed to be difference stationary instead, we find a persistent decrease in unemployment and a persistent increase in vacancies following an increase in government spending. In this case, therefore, the $vu$—ratio increases persistently up to a level of around 3 percent above its initial level. In all cases, the standard errors of the impulse responses are very large indicating that the estimation results are associated with a large amount of uncertainty.

Although the labor market responses to the government spending shock are estimated with a great deal of uncertainty, the results agree with those derived with respect to technology shocks in the sense of a lack of an immediate response of unemployment, vacancies and the $vu$—ratio to the identified structural shock. The results instead indicate that the responses of the labor market variables are sluggish and take effect only with a significant delay in time following the increase in government spending.
3 The Model

In this section we will present a model economy which we will apply to analyze the results derived above. Given that the dynamics of unemployment is at the center of the analysis, we will analyze a model in which frictions prevent instantaneous clearing of the labor market. We adopt the labor market matching set-up of Mortensen-Pissarides (1994) as studied earlier in the business cycle literature by Andolfatto (1996) and by Merz (1995, 1999). In difference to the contributions of these authors, we extend the theoretical set-up and the estimation procedure extensively. First, we allow for multiple shocks and we study the conditional correlation structure rather than unconditional correlations. Secondly, we introduce real rigidities that might be important for the propagation of shocks. In particular, the analysis allows for habit formation and for adjustment costs. Third, and importantly, we introduce a labor market participation choice. This allows us closer to match the model with the data since the definition of unemployment in the model and in data are identical while previous studies implicitly have assumed that all non-employed agents are unemployed. Fourthly, we use formal statistical techniques to estimate key parameters of the model.

3.1 Households, Matching and Technology

There is a continuum of households who maximize expected discounted lifetime utility. We assume that each household consists of a dynasty of agents that pool their idiosyncratic risk. Equivalently, one can assume complete markets. There is also a large number of competitive firms that produce a single homogeneous good.

There are two sources of aggregate risk in the economy. The first is an aggregate productivity shock. Consistently with the empirical analysis in Section 2, we assume that productivity shocks are non-stationary. The second source of aggregate risk consists of stochastic shocks to government spending. Government spending is assumed to grow with productivity over time but is subject to transitory shocks.

The idiosyncratic risk originates in the labor market. We adopt the matching framework of Mortensen and Pissarides (1994) and Pissarides (2000). In this framework, the matching between unemployed workers and firms with vacancies is frictional and takes both time and resources. Existing matches are terminated randomly. New matches occur according to a matching function that links the number of new matches between firms and workers to the number of search active agents and to the number of job vacancy postings.

We assume that job search is associated with a leisure cost. Therefore, not all non-employed agents are necessarily engaged in job search as they face a trade-off between engaging in costly labor market activities and giving up the opportunity to become matched with a job vacancy. Hence, in this model, like in Veracierto (2003) and Ravn (2005), labor market participation is endogenous.
This set-up has two attractive features. First, like Andolfatto (1996), and Merz (1995, 1999), the labor market matching set-up gives rise to frictional (search) unemployment in a fully specified general equilibrium framework and it provides an appealing laboratory in which to analyze the aggregate labor fluctuations. Secondly, unemployment in the model is defined by two conditions: (i) The agent is not employed, and (ii) the agent is actively searching for a job. An agents that fulfills only the former condition is a labor market non-participant. This corresponds directly to the measure of unemployment in the U.S. statistics as applied by the Bureau of Labor Statistics. In contrast the previous literature assumes either that all non-employed agents are search active or that resources are spent only by labor market participants.

A typical household maximizes:

\[ W_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} u \left( C_{t+j} - bC_{t+j-1}, l_{t+j} \right) \] (5)

where \( E_t \) denotes the expectations operator conditional on information available at date \( t \), \( \beta < 1 \) is the subject discount factor, \( C_t \) denotes consumption and \( l_t \) denotes leisure. Depending on the labor market status, a household member’s period utility function is given as:

\[ u \left( C_t - bC_{t-1}, l_t \right) = \ln \left( C_t - bC_{t-1} \right) + \phi \frac{(x_{it})^{1-\eta}}{1-\eta}, \quad i = n, u, l \]

\( b \geq 0 \) is a habit formation parameter. When this parameter is strictly positive, current utility depends both on current and lagged consumption. \( x_{it} \) is the leisure of household members characterized by labor market status \( i \), \( i = n \) indicates the household member is currently matched with an employer, \( i = u \) indicates that the household member is not matched with an employer but is search active (i.e. unemployed), and \( i = l \) indicates that the household member is a labor market non-participant. \( \eta \geq 0 \) determines the curvature of the leisure sub-utility function.

The within period utility of the household is then given as:

\[ u \left( C_t - bC_{t-1}, l_t \right) = \ln \left( C_t - bC_{t-1} \right) \]

\[ + \phi \left[ n_t \left( x_{nt} \right)^{1-\eta} + u_t \left( x_{ut} \right)^{1-\eta} + \left( 1 - n_t - u_t \right) \left( x_{lt} \right)^{1-\eta} \right] \] (6)

where \( n_t \) denotes the share of the household members that are employed, \( u_t \) the share of household members that are not employed but search active, \( (1 - n_t - u_t) \) the share of household members that are labor market non-participants.

We assume that there is a fixed cost \( s \), denoted in units of leisure, of participating in market activities. This fixed cost must be observed by both employed household members and by unemployed (search active) household members. We think of this cost as relating to resources spent on commuting to work, attending job interviews,
etc. Employed household members must work a fixed number of hours, \( f \), but can vary the effort that they exert, \( r_t \). We let \( l_t = f r_t \) denote the effective number of hours supplied for work. Non-participating household members instead enjoy their entire time-endowment as leisure. We normalize the time-endowment to one unit and it therefore follows that \( x_{nt} = 1 - l_t - s, \ x_{ut} = 1 - s, \ x_{lt} = 1 \).

Firms that wish to hire new workers must post a job vacancy. This is associated with resource cost, \( z_t \kappa > 0 \) per vacancy per period (\( z_t \) denotes the level of technology defined below). Firms with vacancies and unemployed (but search active) workers meet randomly in an anonymous matching market. Matches are formed according to the following matching function:

\[
  m_t = M(v_t, u_t) \leq \min(v_t, u_t)
\]

where \( m_t \) is the number of new matches between unemployed workers and vacant jobs in period \( t \) and \( v_t \) denotes the number of vacancies posted in period \( t \). The function \( M \) is assumed to be increasing and concave in each of its arguments, and to be homogeneous of degree one in vacancies and unemployment jointly. The restriction that \( m_t \leq \min(v_t, u_t) \) is a consistency requirement which simply bounds the number of matches from above.

Given the constant returns assumption, we can express the matching function as:

\[
  m_t = u_t \varphi(\theta_t)
\]

where \( \theta_t = v_t / u_t \) is the ratio of vacancies to unemployment, and \( \varphi(\theta_t) \equiv M(\theta_t, 1) \).

Thus, the probability that a search active worker finds a job vacancy, \( m_t / u_t = \varphi(\theta_t) \), is an increasing function of \( \theta_t \) while the probability that a job vacancy is matched with an unemployed worker, \( m_t / v_t = \varphi(\theta_t) / \theta_t \), is a decreasing function of \( \theta_t \). Hence, it is clear that the \( vu \)-ratio, labor market tightness, is a key variable since it determines the matching market prospects of firms and workers.

Each period firms and employed households face an exogenously given probability that their match is terminated. This probability is given by \( \sigma \in [0; 1] \). Thus, the transition equation for employment is given as:

\[
  n_{t+1} = (1 - \sigma) n_t + u_t \varphi(\theta_t)
\]

Output is produced using inputs of effective labor (the product of employment and total effort), \( n_t l_t \), capital, \( K_t \), and is subject to stochastic productivity shocks, \( z_t \). The production function is specified by:

\[
  Y_t = f(K_t, n_t l_t, z_t)
\]

which we assume satisfies the Inada conditions, is increasing and strictly concave in \( K_t \) and in \( n_t l_t \), and homogeneous of degree one in \( (K_t, n_t l_t) \).

We note that, since employment is predetermined in this economy, this set-up is similar to the labor hoarding models of Burnside et al (1993) and Burnside and
Eichenbaum (1996). In the present set-up however, there is an extra friction in the labor market due to the Mortensen-Pissarides set-up which makes the matching process costly and non-instantaneous.\footnote{For this reason, labor effort fluctuates more persistently in our set-up than in Burnside and Eichenbaum (1996) since the matching framework implies that it takes more than one period to adjust employment.}

The capital stock evolves over time according to:

\[ K_{t+1} = (1 - \delta) K_t + I_t \]  \hspace{1cm} (10)

where \( \delta \in (0, 1) \) denotes the depreciation rate, and \( I_t \) is investment. Total investment expenditure need to cover also adjustment costs. We assume that total investment expenditure is given as:

\[ I_t^T = I_t + D(\Delta K_t/\Delta z_t) \]  \hspace{1cm} (11)

where \( D(\Delta K_t/\Delta z_t) \) is an increasing and convex function. We assume that there exist some \( \widetilde{i} \) for which \( D(\widetilde{i}) = D'(\widetilde{i}) = 0 \).

For reasons that shall become clear, we assume that there are costs of entering the labor force. We think of these costs as referring to costs of acquiring the skills to look for job vacancies etc. We model these costs as:

\[ H_t = z_t h(p_t - p) \]

where \( p_t \) denotes the participation rate. We assume that \( h \) is increasing and convex, and that \( h(0) = h'(0) = 0 \).

The resource constraint of the economy is then given by:

\[ Y_t \geq C_t + I_t + \kappa z_t v_t + H_t + G_t \]  \hspace{1cm} (12)

The stochastic processes for productivity shocks and for government spending shocks are given as:

\[ \ln(z_{t+1}/z_t) = (1 - \rho_z) \ln \sigma + \rho_z \ln(z_t/z_{t-1}) + \varepsilon_{t+1} \]  \hspace{1cm} (13)

\[ G_t/z_{t-1} = \exp(g_t) \]  \hspace{1cm} (14)

\[ g_{t+1} = (1 - \rho_g) \sigma + \rho_g g_t + \varepsilon_{t+1}^g \]  \hspace{1cm} (15)

where \( \rho_z \in (-1; 1) \), \( \sigma \) denotes the mean growth rate of technology, and \( \varepsilon_{t+1} \) is assumed to be normally and independently distributed over time with mean 0 and variance \( \nu_e \). Thus, technology shocks are assumed to be non-stationary with possibly persistent growth rates.

We assume that \( \rho_g \in (-1; 1) \) and \( \sigma \) is a constant. The specification in (14) is consistent with the identifying assumption made in Section 2 that government spending is orthogonal to the current realization of the innovation to the productivity shock process. However, it introduces more assumptions than those made in the
empirical section since we do not allow for feedback dynamics on the fiscal policy rule. This short-cut, however, is made only for simplicity and does not affect the results.\textsuperscript{6}

This economy is non-stationary since technological progress brings about growth. We can, however, easily transform it into a stationary economy by “detrending” the growing variables by the level of technology. We define the following variables: $c_t = C_t/z_t$, $i_t = I_t/z_t$, $k_t = K_t/z_{t-1}$ and $g_t = G_t/z_{t-1}$. We can then compute the competitive allocation on the basis of the social planner’s problem for the stationary representation of the economy.

### 3.2 The Role of Labor Market Participation Adjustment Cost

Before continuing, however, let us return to the potential importance of the costs of entering the labor force. Assume for simplicity, but without loss of generality, that there are no habits in consumption. As discussed in Ravn (2005), when $h(p_t - \bar{p}) = 0$, the model gives rise to a consumption - labor market participation volatility puzzle. In particular, in the equilibrium, the following condition holds:

$$\Theta(\theta_t) = \omega c_t$$

where $\omega$ is a constant and $\Theta(\theta_t) = \frac{\phi'(\theta_t) - \theta_t \phi''(\theta_t)}{\phi''(\theta_t)}$ denotes the ratio of the marginal impact of unemployment on matches to the marginal impact of vacancies on matches. When the matching function is specified by a Cobb-Douglas function, as we will assume later, this condition can be expressed as:

$$\theta_t = \tilde{\omega} c_t$$

where $\tilde{\omega}$ is another constant.

This condition has two counterfactual implications. First, the volatility of the $\nu u$–ratio should be equal to the volatility of (detrended) consumption. In the data instead, labor market tightness has a standard deviation that is around 20 times higher than that of consumption at the business cycle frequencies. Secondly, unemployment has a strong tendency of becoming procyclical in this set-up.

Consider a situation with strong productivity growth. In this case, the planner’s optimal policy will involve increasing employment. Due to the matching set-up, an increase employment can be generated by increasing vacancies and/or unemployment. Thus, the planner will wish to increase vacancies and, if possible, unemployment. In standard models with no participation choice, the latter possibility is not an option but in the present set-up unemployment can be increased even when employment is increasing by lowering the share of non-participants. Alternatively, the mechanism can be understood by considering the choice problem of a non-participant. For such

\textsuperscript{6}Moreover, it turns out that the impulse response to the identified government spending shock are approximately identical to those that derive assuming instead that government spending is exogenous.
a person, an increase in vacancies increases the prospects of becoming matched with a firm with a vacancy. Therefore, increases in vacancies make it more attractive to be search active rather than not participating.

The costs of entering the labor force included through \( H \) are introduced in order to improve the model’s performance in terms of potentially allowing for countercyclical unemployment and substantial volatility of the ictu—ratio. In the presence of these costs, non-participants are less likely to enter the labor force. Therefore, an increase in productivity growth may be associated with lower unemployment once adjustment costs are allowed for. Clearly, if large costs are needed in order to account for key features, this calls for a more profound analysis of the type of costs that prevent labor market non-participants from being search active in periods of high pay-offs from labor market participation.

4 Estimation

We follow the approach of Rotemberg and Woodford (1997), Christiano et al (2005), Altig et al (2005) and estimate the parameters of the model using a limited information approach. In particular, on the basis of the impulse response functions to the identified structural shocks that were discussed in Section 2, we estimate a subset of the parameters that enter the model proposed in the previous section.

As a first step we specify the functional forms for the production function, the matching function, and the two adjustment cost functions. We assume that:

\[
\begin{align*}
 f (K_t, n_t l_t, z_t^\gamma) &= \chi_y K_t^{\theta} (z_t n_t l_t)^{1-\theta} \\
 M (v_t, u_t) &= \chi_m v_t^\alpha u_t^{1-\alpha} \\
 h (p_t - p) &= \gamma_p (p_t - p)^2 \\
 D (\Delta K_{t+1}/z_t) &= \gamma_k \left( \Delta K_{t+1}/z_t - \bar{z} \right)^2
\end{align*}
\]  

The first of these specifies the production function by a Cobb-Douglas technology. We follow the great majority of the labor market matching literature and specify the matching technology by a Cobb-Douglas production function as well. Both adjustment cost functions are given by quadratic functions.

We divide the “deep” parameters into two subsets, \( \Upsilon \) and \( \Psi \). The set of parameters in \( \Psi \) will be estimated while those in \( \Upsilon \) will be calibrated. We let \( \Upsilon \) be given by \((\beta, \delta, \gamma, \sigma, f, \eta, \bar{z}, \bar{g})\). We discuss the calibration of these parameters below. The vector of variables that we estimate consist of \( \Psi = (b, \gamma_p, \gamma_k, \alpha, \theta, \rho_z, \sigma_z, \rho_g, \sigma_g) \). These variables are key for the results and are therefore interesting objects to estimate. The habit formation parameter, \( b \), is important for the propagation of shocks over time. As discussed earlier, the participation adjustment costs are important for the determination of the cyclical variation in unemployment and in labor market tightness. The matching function parameter, \( \alpha \), determines the elasticity of the matching
technology with respect to vacancies and with respect to unemployment. This is an important determinant of the cyclical variations of vacancies and of unemployment.

In order to estimate the parameters that enter $\Psi$ we employ the following procedure. Let the estimated impulse response function be given by the vector $\lambda = \left[\hat{\lambda}_{11}, \hat{\lambda}_{21}, ..., \hat{\lambda}_{NT} \right]$ where $\hat{\lambda}_{it}$ denotes the estimated response of variable $i$ at forecast horizon $\tau$ to an innovation of size $\sigma_z$ or $\sigma_g$, depending on the source of the shock, at date 1. Next, let $\Pi (\Psi_s; \Upsilon)$ be the equivalent impulse responses generated by the model when $\Psi = \Psi_s$. We then estimate the set of parameters in $\Psi$ by minimizing the following quadratic form:

$$
\tilde{\Psi} = \arg\min_{\Psi} Q (\Psi; \lambda, \Upsilon) = \left( \lambda - \Pi (\Psi; \Upsilon) \right)' V_\lambda^{-1} \left( \lambda - \Pi (\Psi; \Upsilon) \right) \quad (20)
$$

where $V_\lambda$ is a weighting matrix. We follow Christiano, Eichenbaum and Evans (2005) and specify $V_\lambda$ as a diagonal matrix with the sample variances of $\lambda$ along the diagonal. Technically, we solve this minimization problem using a quasi Newton-Raphson algorithm.\footnote{Technically, we solve this minimization problem the following way. Given $\Upsilon$ and $\Psi$, we solve the model using a quadratic approximation to the return function around the steady-state (of the “detrended” version of the economy) using a log-transformation as in Christiano, 1988. The resulting model (with a quadratic return function and linear transition equations) is solved using a Ricatti equation approach. We then compute the impulse responses of the model equivalents of $\hat{\lambda}$ on the basis of the impulse responses of the “detrended” variables converted into levels by multiplying with their steady-state levels and multiplying the growing variables with $Z_t \left( Z_{t-1} \right. \left. \text{in the case of the capital stock and government spending} \right)$. We then use a quasi Newton-Raphson algorithm to solve the minimization problem in (20). For the first few iterations we do not update the step-size since the algorithm becomes very slow for arbitrary initial values.} The standard errors of the resulting estimate of $\Psi$, $\tilde{\Psi}$, are computed using a delta method.

We use a forecast horizon of 20 quarters. We select the following impulse response functions to be minimized in the estimation algorithm: The responses of output, consumption, investment, unemployment and vacancies to an identified technology shock; and the responses of output, consumption, investment, government spending, unemployment and vacancies to an identified technology shock. We do not include the response of hours worked among the moments that we match. The reason for this is the underlying assumption that the econometrician does not observe the fluctuations in labor effort. Thus, the “true” labor input does not match measured labor input. Moreover, given that we do not introduce feedback dynamics in the government spending rule, we do not include the response of government spending to technology shocks among the moments to be matched.

### 4.1 Calibration of $\Upsilon$

We set the subjective discount factor equal to 0.99. The depreciation rate is calibrated so that it corresponds to an annual rate of depreciation just around 10 percent. We
set the rate of productivity growth (in the absence of technology shocks) equal to 1.004 which implies a steady-state growth rate of around 1.6 percent.

We assume that, in the steady-state, the participation rate is equal to 63.9 percent. This number corresponds to the average ratio of labor market participants to the civilian population above 16 years of age in the US over the sample 1964-2004. Over the same period, the average aggregate unemployment rate was 5.9 percent which corresponds to 3.79 percent of the civilian population. Thus, we assume that \( \bar{\pi} = 3.79 \) percent and that \( \pi = 60.11 \) percent.

Care must be taken in calibrating the job-separation rate, \( \sigma \). Along the balanced growth path, equation (8) implies that

\[
\sigma \bar{\pi} = \bar{M}
\]

Andolfatto (1996) assumes that \( \sigma = 15 \) percent per quarter which implies that the quarterly (measure) number of matches is around 9 percent. This calibration obviously cannot be adopted here since it implies that \( \bar{M} > \pi \).\(^8\) Merz (1999) calibrates \( \sigma \) on the basis of average quarterly flow to and from unemployment as a percentage of the labor force. This number is approximately 7.5 percent in the US for the period 1977-1996. It follows therefore that \( \sigma = (\bar{M}/(\bar{\pi} + \pi)) (\bar{\pi} + \pi) / \bar{\pi} = 7.97 \) percent per quarter. Although this is almost 50 percent lower that the job-turnover rate used by Andolfatto (1996), it is still problematic since it implies that \( \bar{M} = 4.79 > \pi \).\(^9\)

These problems can be avoided by taking temporal aggregation issues into account. It follows from Merz (1999) that the monthly flow from unemployment to employment corresponds to 2.5 percent of the labor force. Therefore, \( \sigma_{\text{month}} = 2.66 \) percent. Next, according to the Bureau of Labor Statistics, over the period 1964-2005, the average duration of unemployment in the US was around 13.9 weeks (or \( \text{dur}_{\text{month}} = 3.2 \) months). We can then compute the quarterly job-turnover rate as follows. Along the steady-state path, in the first month of the quarter, \( \sigma_{\text{month}} \bar{\pi} \) employed worker lose their jobs. Next month another measure \( \sigma_{\text{month}} \bar{\pi} \) of the employed workers lose their jobs but only the fraction \((1 - 1/\text{dur}_{\text{month}})\) of those that lost their jobs the previous month will still be unmatched. In the third month another share \( \sigma_{\text{month}} \bar{\pi} \) of the employed workers lose their jobs but only the fractions \((1 - 1/\text{dur}_{\text{month}})^2\) of those that lost their jobs in the first month and \((1 - 1/\text{dur}_{\text{month}})^2\) of those that lost their jobs in the second month are still unmatched. Hence, the quarterly net job-separation rate is given as:

\[
\sigma = \sigma_{\text{month}} \left(1 + (1 - 1/\text{dur}_{\text{month}}) + (1 - 1/\text{dur}_{\text{month}})^2\right) = 5.74 \text{ percent}
\]

\(^8\)Andolfatto (1996) assumes that all non-employed are search active, i.e. that \( \pi = 1 - \bar{\pi} \). Therefore, although he assumes a very high value of \( \sigma \) (15 percent per quarter), his calibration is still consistent with \( \bar{M} < \pi \) along the balanced growth path. When unemployment is measured on the basis of the search active non-employed, as in our model, \( \sigma = 0.15 \) instead leads to a violation of the requirement that \( \bar{M} > \pi \).

\(^9\)This inconsistency does not arise in Merz’ model because temporary layoffs are allowed for.
which is almost 30 percent lower than the estimate in Merz (1999) and less than 40 percent of the value used by Andolfatto (1996).\textsuperscript{10} This value for \( \sigma \) implies that \( \bar{M} = 3.45 \) percent which is consistent with \( \bar{M} < \bar{\mu} \).

In order to calibrate the number of vacancies we adopt Andolfatto’s (1996) calibration of \( \bar{q} = \frac{\bar{M}}{\bar{\mu}} \), the probability of filling a vacancy within a month. Citing empirical estimates of van Ours and Ridder (1992), Andolfatto (1996) sets \( \bar{q} = 0.90 \). From this it follows that \( \bar{\mu} = 3.83 \) percent.

We normalize \( r \), the steady-state effort level, to unity and we set \( f \), the number of hours worked equal to \( \frac{1}{3} \) of the time-endowment. We assume that \( \eta \), the curvature of the leisure sub-utility function, is equal to 0.95 which implies that the utility function displays slightly more curvature in consumption than in leisure. The investment adjustment cost function constant, \( \tilde{i} \), is calibrated so that it equals \( \Delta \bar{K}_{t+1}/z_t \) along the balanced growth path, i.e. \( \tilde{i} = (1 - 1/\gamma) k \). Over the sample period considered in Section 2, the sample mean of the output share of government spending equals 20.4 percent and we use this to calibrate \( \bar{q} \). Finally, we calibrate \( y \) so that it equals unity along the balanced growth path.

A number of other parameters are implicitly estimated as functions of \( \Upsilon \) and \( \Psi \). First, given the estimate of \( \theta \), the steady-state capital-output ratio and the constant \( \chi_y \) are given as:

\[
\frac{k}{y} = \frac{\theta}{(1/\beta - (1 - \delta) / \gamma)}
\]

\[
\chi_y = \frac{\gamma^\theta}{(k/y)^\theta (nl)^{1-\theta}}
\]

Next, the fixed cost of participating in labor market activities, \( s \), can be determined given an estimate of \( \alpha \) from the condition that, along the balanced growth path, a non-employed agent must be indifferent between searching for a job (which potentially produces a match with a vacancy) and being a labor market nonparticipant. This gives rise to the following non-linear condition for \( s \):

\[
(1 - l - s)^{1 - \eta} + (1 - \eta) l (1 - l - s)^{-\eta} = 1 + (1 - (1 - s)^{1 - \eta}) \left( \frac{1}{\beta} - (1 - \sigma) \frac{\bar{\pi}}{\bar{m}(1 - \alpha)} \right)
\]

which we solve numerically.

Given the estimates of \( s, \theta \) and \( \alpha \) we can also derive the vacancy posting cost parameter \( \kappa \) and the matching function constant, \( \chi_m \), as:

\[
\kappa = \frac{1}{(1 - \eta) (1 - \alpha) \bar{m} \bar{\pi} (1 - l - s)^{\eta} (1 - (1 - s)^{1 - \eta})}
\]

\[
\chi_m = \frac{\sigma \bar{m}}{\bar{\pi} \bar{m}^{\gamma - 1 - \alpha}}
\]

\textsuperscript{10}Repeating this computation for weekly or daily frequencies implies \( \sigma = 5.29 \) percent or \( \sigma = 5.18 \) percent, respectively. The continuous time limit is \( \sigma = 5.17 \) percent.
Finally, given these estimates, we can derive consumption’s share of output and the utility weight \( \phi \) as:

\[
\frac{c}{y} = 1 - \left(1 - \frac{1 - \delta}{\gamma}\right) \frac{k}{y} - \frac{q}{y} \frac{\kappa \bar{v}}{y},
\]

\[
\phi = \frac{1 - \beta b / \gamma y (1 - \theta)}{1 - b / \gamma c \cdot \bar{\eta}} (1 - l - s)^\eta.
\]

5 Estimation Results

Table 1 reports the estimation results. The first two columns contain the parameter estimates and their standard errors when we use the impulse responses (and their sample covariance matrix) estimated from the VAR specification in which unemployment and vacancies enter in levels. The third and fourth columns instead report the results when we apply the results from the VAR where unemployment and vacancies enter in first differences. We concentrate on the former of these and use the latter as a check on the robustness of the results.

5.1 Parameter Estimates

We find a point estimate of the habit persistence parameter, \( b \), of 0.479 with relatively small standard error. This estimate is smaller than the estimate of Altig et al (2005) but still relatively large. The estimate of \( \alpha \), the matching function elasticity to vacancies, is equal to 0.409 with a standard error of half a percent. This estimate is extremely similar to Blanchard and Diamond (1989) who estimate \( \alpha \) to be close to 40 percent. The estimate of the elasticity of output with respect to capital, \( \theta \), is 0.318. This is similar to the estimates typically applied in the business cycle literature. That literature, however, typically calibrates this parameter on the basis of the capital share of income which is approximately 30-35 percent. In the matching framework adopted in the present analysis, \( \theta \) no longer corresponds to the capital share of income due to the labor market search distortion. Nevertheless, the implied labor share in the present model (70.3 percent) is in the range of values assumed in standard calibration exercises.\[^{11}\]

These values imply a capital output-ratio of 8.16 which in the range of values usually considered reasonable. The implied estimate of the vacancy posting cost as a share of steady-state output is 0.5 percent, half the 1 percent estimate in Andolfatto (1996). The parameter \( s \), the fixed cost of participating in labor market activities,

\[^{11}\text{If we assume that wages are determined by a Nash-Bargain and that workers bargaining weight equals } (1 - \alpha) \text{ (the so-called Hosios condition), it is straightforward to show that the steady-state labor income share is given as } (1 - \alpha)(1 - \theta) + \alpha \bar{\pi} (c (1 - b)/(1 - \beta b)) [\zeta_1 + \zeta_2] \text{ where } \zeta_1 = \phi / (1 - \eta) \left(1 - (1 - l - s)^{1-\eta}\right) \text{ and } \zeta_2 = (1 - \alpha)/\alpha(v/u)\kappa (1 - \beta b)/ (c (1 - b)) \text{. Inserting the parameter estimates imply a labor share of 70.3 percent.} \]
is estimated to be 9.9 percent of the time endowment. Assuming that the time available for market activities corresponds to 108 hours per week (ie. 6 hours of sleep per night and one day off), this implies that the estimate of \( \delta \), the fixed time cost of participating in the labor market, is approximately 10 hours per week. Burnside and Eichenbaum (1996) instead assume that the fixed cost of participating in the labor market corresponds to 5 percent of the time endowment.

The estimates of the persistence of the two structural shocks are both relatively high. We find a point estimate of \( \rho_g \) of 0.916 so that half the initial shock to government spending persists within the first 2 years after the innovation. The standard error of this estimate is, however, relatively high. The point estimate of \( \rho_z \) equals 0.573 which implies that changes in the growth rate of technology are quite persistent. Although high, this estimate is nevertheless significantly below the estimate of Altig et al (2005) who derive an estimate of \( \rho_z \) of around 0.90. It is interesting to note that our estimate as well as that of Altig et al (2005) imply that the productivity shock process is significantly different from the random walk specifications adopted in many previous business cycle analyses.

The estimates of the adjustment cost parameters, \( \gamma_p \) and \( \gamma_k \), are important for understanding the dynamics of the model economy. We find a point estimate of \( \gamma_p \) of 12.5. This implies that an increase in the participation rate from 64 percent (its steady-state value) to 66 percent gives rise to a cost that corresponds to approximately 0.5 percent of steady-state output. We consider this a large cost and it is much larger than the capital adjustment costs that we instead find to be very moderate.

### 5.2 Implied Dynamics

Figure 3 illustrates the implied impulse responses of the model economy parametrized on the basis of \( \Upsilon \) and \( \hat{\Psi} \) together with the sample estimates of the impulse response functions that we have used for the estimation of \( \hat{\Psi} \).

\( \Pi(\hat{\Psi}; \Upsilon) \) provides a good fit of \( \lambda \) as regards the response of output components to the identified technology shock. As in the empirical impulse responses, the theoretical impulse responses imply that output and consumption rise gradually over time in response to an increase in the growth rate of technology. The model also implies a gradual increase in investment over time but is unable to reproduce the temporary setback in investment indicated by the empirical impulse responses. The gradual increase in investment is due to the high persistence of the technology shock and the presence of adjustment costs. Models that assume low persistence of the growth rate of technology investment displays an “overshooting” type dynamics initially increasing beyond its new steady-state level and then declining gradually over time. This type of implied dynamics of investment in response to random walk technology shocks has been stressed eg. by Rotemberg and Woodford’s (1996) in their criticism of RBC models’ inability to “fit” VAR evidence. However, when the growth rate of technology is persistent and in the presence of adjustment costs, as our estimates
indicate, investment rises gradually over time.

Similarly, the model fits relatively well the impulse responses of output and its components in response to the government spending shock. As the figure indicates, the government spending process is extremely well approximated by an autoregressive process with high persistence. The model predicts, consistently with the data, that an increase in government spending gives rise to a temporary rise in output and a drop in investment. While the model does not generate the U-shaped response of investment, the implied dynamics are still reasonable similar to the data. However, in the data, an increase in government spending appears to be associated with an increase in private consumer’s expenditure. The model instead predicts that private consumption decreases (due to a wealth effect). We do not consider this a major problem since the present model excludes features that would allow us to address this feature of the data.

The major puzzle that derives from Figure 3 is concerned with the labor market dynamics. As is clear from Figure 3, in the matching model, unemployment and vacancies remain approximately constant in response to the higher growth rate of technology. This contrasts sharply with the results in Section 2 that indicate that following a positive technology shock, there is a delayed but large decrease in unemployment and a large, but again, delayed increase in vacancies. Evidently, the matching model implies far too little volatility in unemployment and vacancies.

Moreover, in the data the implied a large elasticity of the $vv$-ratio but with a significant delay of around 2-2.5 years. Given $\Psi$, the labor market matching model instead implies an initial very small increase in the $vv$-ratio that dies out monotonically within the first year following the positive technology shock. We illustrate this in Figure 4 where we plot together the empirical and model based impulse response of the $vv$-ratio in response to a technology shock. This plot makes the deficiencies of the model very clear: In the data, the $vv$-ratio follows an inverse U-shaped response to technology with a peak increase of 5 percent which occurs 9 quarters after the improvement in technology. The labor market matching model instead implies a monotonically decreasing dynamic pattern which consists of half a percentage point initial increase and a smooth a fast convergence to the steady-state.

Evidently, in the data the technology shock leads to a large incentive for firms to hire new workers but only with a substantial delay. The matching model instead gives the employer an incentive to start the hiring process early in anticipation of the future technology improvements signalled by the current technology shock. Hall (2005) and Shimer (2005) have recently proposed to introduce sticky wages into the matching framework in order to generate higher volatility of the $vv$-ratio. The evidence here, however, suggest that this may not be appropriate since this will generate an even larger increase in vacancies upon impact therefore not address the inverse U-shaped dynamics observed in the data.

Thus, in conclusion, while the matching model can successfully account for the dynamics of output and its components in response to identified technology shocks
and government spending shocks, the model cannot account for the dynamics of unemployment, vacancies, and the $vu$–ratio. We consider this an important insight since the main reason for introducing the labor market search and matching features into the business cycle model was to provide an improved framework in which to understand the movements in key aggregate labor market indicators.

### 5.3 Sensitivity Analysis

In rows 3–8 of Table 1 we report the estimation results for alternative specifications. Rows 3–4 consider the case in which we base the estimation of the structural parameters on the impulse responses of the VAR specification that includes unemployment and vacancies in first-differences. In rows 5–8 we constrain the value of the participation rate adjustment costs. Rows 5 and 6 report the results when we assume that $\gamma_p = 1$ so that it is almost costless to enter and exit the labor force. In rows 7–8 instead we set $\gamma_p = 200$ which basically implies that the participation rate is constant.

When we consider the alternative empirical impulse response function, the estimates of $\alpha$, $\theta$ and the parameters of the technology shock process are basically identical to those that we report in rows 1 and 2 but the standard errors are somewhat higher. The point estimate of the habit persistence parameter, $b$, instead is considerably higher under the alternative VAR specification and very close to 0.9. Similarly, we find much higher adjustment costs but, in all cases, the sampling uncertainty of these parameters is much higher as well. As discussed in Section 2, the government spending process appears much more consistent when we assume that unemployment and vacancies are difference stationary. For that reason, we find a higher estimate of $\rho$ which, in fact, is very close to unity. These estimates imply output dynamics that are similar to those reported in Figure 3 except that consumption, due to the very high habit parameter, displays very smooth dynamics. This implies that this model actually leads to countercyclical vacancies subject to technology shocks as firms respond to the higher growth rate of technology by lowering employment temporarily. Given the high size of the labor force adjustment cost, this leads to procyclical movements in unemployment. While this allows the model to fit the temporary drop in vacancies and the temporary increase in unemployment that we estimated in Section 2, it obviously also implies that the model does a poor job at fitting the key dynamics of the US labor market indicators.

When we set $\gamma_p = 1$, the point estimates of the unconstrained parameters are approximately unchanged apart from a higher value of the investment adjustment cost parameter $\gamma_k$. We show in Figure 5 the dynamics of unemployment, vacancies and the $vu$–ratio following a technology shock (Panel A) and following an increase in government spending using these parameter estimates. A positive technology shock, leads to a short-lived drop in unemployment after which it increases above its steady-state. Vacancies, in contrast, fall persistently, and, in combination, this leads to a
persistent drop in the $vu$-ratio. These impulse responses are almost the opposite of what is observed in the data. The intuition for the matching model’s predictions are to be found in two aspects of the parameter estimates. First, technology shocks are persistent. Therefore, a current positive technology shock signals high future technology growth as well. Secondly, the habit persistence parameter is high and therefore agents prefer very smooth consumption profiles. Together this implies that the optimal outcome is to decrease employment slightly. At the same time, resources for current consumption and investment can be freed up by lowering vacancy posting and increasing unemployment rates (which counters the negative effect of fewer vacancy postings on the number of new job matches). An increase in government spending instead leads to a relatively large increase in both unemployment and vacancies. The reason is that households are unwilling to cut their consumption expenditure and wish to increase employment which is accomplished by boosting both unemployment and vacancy postings. This has the counterfactual implication that a large positive correlation is induced between unemployment and vacancies.

We get better results when we basically close down the participation choice. The parameter estimates of the unconstrained parameters are very similar to those reported in the general case apart from the smaller capital adjustment costs. Figures 6A and 6B show the implied dynamics of the key labor market variables in the face of the two structural shocks. A positive technology shock here leads to an increase in vacancies. Since this increases employment, and since the participation rate is constant, unemployment drops persistently before returning to its steady-state value. Therefore, the $vu$ – ratio increases in response to the technology shock and then returns monotonically to the steady-state. The model therefore implies some volatility of the $vu$–ratio, but is inconsistent with the inverse $U$–shaped dynamics that we discussed in Section 2. Similarly, an increase in government spending now implies higher volatility of the $vu$–ratio since the increase in vacancies increases employment gradually over time.

In sum, none of the alternative scenarios that we consider enables the matching model to capture the sluggish adjustment of the key labor market indicators that we documented exist in the US data.

6 Summary and Conclusions

This paper has made the following contributions to the literature on business cycles and labor market dynamics. First, we provided evidence on the conditional dynamics of key US labor market indicators in response to identified structural shocks. The empirical results are interesting for we document that unemployment, vacancies, and the $vu$–ratio change in sluggish manner in response to the two structural shocks that we consider. Perhaps most interesting, we showed that neutral technology shocks bring about lower unemployment, higher vacancies, and an increase in labor market tightness but only with a long delay of around 2-2.5 years after the technological
improvement. This delayed response may be seen as a challenge to theories, such as models with sticky wages, meant to generate higher volatility in the $vu$–ratio by increasing the short-term elasticity of vacancy postings to economic shocks. Secondly, we introduced a labor market matching model into a fully specified general equilibrium framework that incorporates real rigidities as well as a novelty, an endogenous participation choice. Thirdly, we estimated key parameters of the theoretical model by using a limited information strategy. The estimates of standard parameters are broadly in line with calibrations typically applied in the business cycle literature and with parameter estimates of other recent papers using similar estimation strategies. We also found that there must be large costs of entering and exiting the labor force over the business cycle frequencies. In the absence of such costs, the matching model implies very counterfactual dynamics of the labor market indicators (such as procyclical unemployment). We believe that it might be important to consider the nature of such costs in order better to understand the lack of large procyclicality of the participation rate. However, even when these costs are large, we showed that the labor market matching model is incapable of generating the sluggish adjustment of unemployment, vacancies and labor market tightness that we documented in the data.

Thus, the conclusion of this paper is that much more research is needed to explain the behavior of the labor market. While existing models can successfully account for much of the dynamics of output and its components in response to technological shocks, monetary shocks and to changes in government spending, they cannot account for key features of the dynamics of central labor market indicators even when combined with frictional labor market theories.

References


[18]


## Tables and Figures

### Table 1. Estimates of Structural Parameters

<table>
<thead>
<tr>
<th>Specification</th>
<th>U and V in levels</th>
<th>U and V differenced</th>
<th>low $\gamma_p$</th>
<th>high $\gamma_p$</th>
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<tr>
<td>$\hat{\Psi}$</td>
<td>$\hat{s}(\hat{\Psi})$</td>
<td>$\hat{\Psi}$</td>
<td>$\hat{s}(\hat{\Psi})$</td>
<td>$\hat{\Psi}$</td>
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<tr>
<td>$b$</td>
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<td>0.898</td>
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<td>12.2</td>
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Figure 1.A The Effects of Technology Shocks

Note: The VAR includes unemployment and vacancies in levels
Figure 1.A The Effects of Technology Shocks, cont’d

Note: The VAR includes unemployment and vacancies in levels
Figure 1.B The Effects of Government Spending Shocks

Note: The VAR includes unemployment and vacancies in levels
Figure 1.B The Effects of Government Spending Shocks, cont’d

Note: The VAR includes unemployment and vacancies in levels
Figure 2.A The Effects of Technology Shocks

Note: The VAR includes unemployment and vacancies in first differences
Figure 2.A The Effects of Technology Shocks, cont’d

Note: The VAR includes unemployment and vacancies in first differences
Figure 2.B The Effects of Government Spending Shocks

Note: The VAR includes unemployment and vacancies in first differences
Figure 2.B The Effects of Government Spending Shocks, cont’d

Note: The VAR includes unemployment and vacancies in first differences
Figure 3A: Empirical and Theoretical Impulse Responses: Technology Shocks

Note: Full lines show the empirical impulse responses. The dashed lines are the theoretical impulse responses.
Figure 3B: Empirical and Theoretical Impulse Responses: Gov’t Sp. Shocks
Figure 4: The Dynamics of the $VU$–ratio: Technology Shock

Note: The full drawn line illustrates the empirical impulse response of the $vu$–ratio in response to an identified technology shock. The dashed line shows the impulse response of the $vu$–ratio in the matching model.
Figure 5A: Labor Market Dynamics when $\gamma_p$ is Low: Technology shocks

Note: The full drawn line illustrates the response of unemployment, the dashed line the dynamics of vacancies, and the dotted line the dynamics of the vu-ratio.
Figure 5B: Labor Market Dynamics when $\gamma_p$ is Low: Government Spending Shocks

Note: The full drawn line illustrates the response of unemployment, the dashed line the dynamics of vacancies, and the dotted line the dynamics of the vu-ratio.
Figure 6A: Labor Market Dynamics when $\gamma_p$ is High: Technology shocks

Note: The full drawn line illustrates the response of unemployment, the dashed line the dynamics of vacancies, and the dotted line the dynamics of the vu-ratio.
Figure 6B: Labor Market Dynamics when $\gamma_p$ is High:
Government Spending Shocks

Note: The full drawn line illustrates the response of unemployment, the dashed line the dynamics of vacancies, and the dotted line the dynamics of the vu-ratio.