A Search for a Structural Phillips Curve

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Objective

- Re-examine the Calvo model of inflation dynamics

Question

- Is it a *structural* model of the dynamics of inflation?
  - Are pricing parameters invariant to changes in policy regime?
  - How does allowing for different policy regimes (different levels of trend inflation) alter the relative importance of *forward* versus *backward* components in the model?
  - How do our estimates accord with microeconomic evidence on the frequency of price changes?
Approach

• Generalize the Calvo model to allow for shifts in trend inflation

• Study a reduced form representation of the system of variables, with parameter drifts

• Estimate Calvo parameters to satisfy the cross-equation restrictions implied by the model
  – innovation: time-varying parameter representation for the reduced form
Outline

- Motivation
- The generalized model of inflation dynamics
- Empirical methodology
- Evidence of VAR parameter drifts
- Estimation of Calvo parameters
- Conclusion and further research
Motivation

- Empirical evidence shows instability in U.S. economy
  - More muted responses to policy shocks in the post-‘82 period (Boivin-Giannoni 2002)
  - Reduced volatility in economic time series (Stock-Watson 2002)
  - Instability in reduced form parameters for inflation/unemployment (Cogley-Sargent 2001)

- Is NKPC a structural model of inflation dynamics?
  - Standard NKPCs are derived as log-linear approximations around ss with zero inflation
  - If long-run level of inflation changes, can we identify stable ‘Calvo’ parameters?

- Our strategy: use drifts in reduced-form parameters to identify structural model parameters.
Generalized Calvo model

- Includes indexation to previous inflation rate
- Includes source of strategic complementarities
- Allows for trend inflation
  - Implies a restriction across mean values of inflation and marginal cost
    * equilibrium condition at steady state
  - Implies a restriction across second moments of inflation and marginal costs
    * approximate equilibrium conditions (obtained from log-linearization around a non-zero steady state inflation)
Calvo equation

\[ \hat{\pi}_t = \hat{\rho}\hat{\pi}_{t-1} + \zeta s_t + b_1 E_t \hat{\pi}_{t+1} + b_2 \sum_{j=2}^{\infty} \gamma_1^{j-1} E_t \hat{\pi}_{t+j} + \]

\[ + \chi (\gamma_2 - \gamma_1) \sum_{j=0}^{\infty} \gamma_1^j E_t \hat{R}_{t+j,t+j+1} \]

\[ + \chi (\gamma_2 - \gamma_1) E_t \sum_{j=0}^{\infty} \gamma_1^j \hat{\gamma}_{y,t+j+1} \]

- Role of trend inflation
  - Impacts on coefficients of past inflation and marginal cost
  - Makes further forward-looking terms matter
\( \zeta (\psi) = \frac{1}{\Delta} \left( \frac{1 - \alpha \bar{\pi}(\theta - 1)(1 - \varrho)}{\alpha \bar{\pi}(\theta - 1)(1 - \varrho)} \right) \left( \frac{1 - \gamma_2}{1 + \theta \omega} \right) \)

where

\( \Delta = 1 + \varrho \gamma_2 - \frac{1 - \alpha \bar{\pi}(\theta - 1)(1 - \varrho)}{\alpha \bar{\pi}(\theta - 1)(1 - \varrho)} \phi_0 \)

\( \phi_0 = \frac{\varrho \theta (\gamma_1 - (1 + \omega) \gamma_2) - \varrho \gamma_1}{1 + \theta \omega} \)

\( \gamma_1 = \alpha \bar{\beta} \bar{\pi}(\theta - 1)(1 - \varrho) \)

\( \gamma_2 = \alpha \bar{\beta} \bar{\pi} \theta(1 - \varrho)(1 + \omega) \)

\( - \) Coefficient of past inflation

\( \tilde{\varrho} (\psi) = \frac{\varrho}{\Delta} \)
Implication

• Changes in the level of trend inflation imply instability of the coefficients
  – particularly affect estimated inertia

Our objective

• To estimate the parameters that characterize the model
  – frequency of price changes $\alpha$
  – demand elasticity $\theta$
  – indexation $\varrho$

• To ask whether they are stable, even if the Calvo coefficients are not.
Empirical Methodology

• Exploit restrictions that the model imposes on a time series representation of the data

• General idea:
  – give the model a (restricted) autoregressive representation
  – give the variables an (unrestricted) reduced form autoregressive representation
  – estimate ‘deep parameters’ by satisfying the restriction imposed by the model on the coefficients of the reduced form
Implementation: 2-step estimation strategy

- Start from a joint representation of the time series of interest
  \[ x_t = \left( \pi_t, s_t, R_t, \gamma_{yt} \right)' \sim VAR(p) \]

- Define a vector \( z_t = (x_t, x_{t-1}, \ldots, x_{t-p+1})' \)
  \[ z_t = \mu + Az_{t-1} + \varepsilon_{zt} \]

- Take conditional expectation of inflation from the VAR
  \[ E(\hat{\pi}_t/\hat{z}_{t-1}) = e'_\pi A\hat{z}_{t-1} \]

- Take conditional expectations of inflation from the model
  \[ E(\hat{\pi}_t/\hat{z}_{t-1}) = \bar{\pi}e'_\pi \hat{z}_{t-1} + \zeta e'_s A\hat{z}_{t-1} + b_1 e'_\pi A^2\hat{z}_{t-1} \]
  \[ + b_2 e'_\pi \gamma_1 (I - \gamma_1 A)^{-1} A^3 \hat{z}_{t-1} + \]
  \[ + \chi (\gamma_2 - \gamma_1) e'_R (I - \gamma_1 A)^{-1} A\hat{z}_{t-1} \]
  \[ + \chi (\gamma_2 - \gamma_1) e'_y (I - \gamma_1 A)^{-1} A^2 \hat{z}_{t-1} \]
  \[ \equiv g(A, \psi)\hat{z}_{t-1} \]
• Equate right hand sides of above $\rightarrow$ equality must hold for any value of $\hat{z}_{t-1}$

• Define a distance function (between the “data” and the “model”)

$$F_1(\mu, A, \psi) = e'_\pi A - g(A, \psi).$$

• Add restriction between the means of inflation and marginal cost (from ss relation)

$$F_2(\mu, A, \psi) = \left(1 - \alpha \pi (\theta - 1)(1 - \phi)\right)^{1+\theta \omega} \left(1 - \alpha \overline{R}_y \pi^{1+\theta(1-\phi)(1+\omega)}\right)$$

$$\cdot \frac{1 - \alpha \overline{R}_y \pi^{\theta - \phi(\theta - 1)}}{1 - \alpha \overline{R}_y \pi^{\theta - \phi(\theta - 1)}} - (1 - \alpha)^{1+\theta \omega} \left(\frac{\theta}{\theta - 1}\right) \overline{s},$$

• Let $F(\mu, A, \psi) = (F'_1 F'_2)'$: If the model is true, there exists a $\psi$ that satisfies $F(\cdot, \psi) = 0$. 
– Estimate $\psi$ by solving an equally weighted GMM problem

$$\min_{\psi} F(\cdot, \psi)'F(\cdot, \psi).$$
Innovation here

- Process $z_t$ is modeled as a time-varying VAR

$$z_t = \mu_t + A_t z_{t-1} + \varepsilon_{zt}$$

  - Moment conditions defined for $\mu_t$, $A_t$, estimated for a number of dates $t$.

  - Objective function is $\mathcal{F}(\psi)'^T \mathcal{F}(\psi)$

    * $\mathcal{F}(.) = [F'_{t1}, F'_{t2}, \ldots F'_{tn}]$

    * $F_{t_i}(.)$ represents restrictions $F(\mu_{t_i}, A_{t_i}, \psi)$, for representative date $t_i$

- We estimate two versions of the model:

  - holding $\psi$ constant

    $$F_t(\cdot) = F(\mu_t, A_t, \psi)$$

  - allowing $\psi$ to differ across dates

    $$F_t(\cdot) = F(\mu_t, A_t, \psi_t)$$
VAR with drifting parameters

- **Objective**: to provide empirical evidence about the evolving joint process of inflation, marginal costs, output growth and discount rate

- **Methodology**: Bayesian vector autoregression with drifting coefficients and stochastic volatilities (as in Cogley and Sargent (2004))

- **Specification**: we fit a $VAR(2)$ representation

- **Data**
  - quarterly data from 1960:1 through 2003:4 - [Data from 1954:1-1959:4 used to initialize the prior].
  - time series on GDP, implicit GDP deflator, nominal compensations and hours for the NFB sector.
Bayesian VAR - summary of methodology

• Vector of $VAR$ coefficients as $\vartheta_t$ evolve as driftless random walks

$$\vartheta_t = \vartheta_{t-1} + v_t$$

• Innovation $v_t$ is normally distributed, with mean 0 and variance $Q$.

• The driftless random walk component is represented by a joint prior

$$f (\vartheta^T, Q) = f (\vartheta^T | Q) f (Q) = f (Q) \prod_{s=0}^{T-1} f (\vartheta_{s+1} | \vartheta_s, Q)$$

• We allow for stochastic volatility:

  - $VAR$ innovations $\varepsilon_{xt}$ is

$$\varepsilon_{xt} = V_t^{1/2} \xi_t$$
– $\xi_t$ is a standard normal vector, $E(\xi_t, \nu_s) = 0$,
– $V_t$ is

$$V_t = B^{-1}H_tB^{-1}'$$

* Elements of $H_t$ (stochastic volatilities) evolve as driftless geometric random walks

$$\ln h_{it} = \ln h_{it-1} + \sigma_i \eta_{it}$$

* specification for $h_{it}$ chosen to represent permanent shifts in innovation variance

- We use Markov Chain Monte Carlo methods to simulate the posterior density

$$p (\vartheta^T, Q, \sigma, b, H^T | X^T)$$
Evidence on parameter drifts

- Structure of innovation variance of $\theta_t$, matrix $Q$

Principal components of $Q$

<table>
<thead>
<tr>
<th></th>
<th>Variance</th>
<th>Cumulative Proportion of $tr(Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC 1</td>
<td>0.0554</td>
<td>0.637</td>
</tr>
<tr>
<td>PC 2</td>
<td>0.0132</td>
<td>0.789</td>
</tr>
<tr>
<td>PC 3</td>
<td>0.0065</td>
<td>0.864</td>
</tr>
<tr>
<td>PC 4</td>
<td>0.0057</td>
<td>0.930</td>
</tr>
<tr>
<td>PC 5</td>
<td>0.0016</td>
<td>0.947</td>
</tr>
<tr>
<td>PC 6</td>
<td>0.0011</td>
<td>0.961</td>
</tr>
</tbody>
</table>

- $Q$ (36 $\times$ 36) - only 4 or 5 significant principal components
  - many linear combinations of $\theta$ are approximately time invariant.
Trend inflation is $\ln \tilde{\pi}_t = e_\pi^I (I - A_t|T)^{-1} \mu_t|T$. 
Implications

- Trend inflation varies in our sample
  - 2.3% (early ‘60)
  - 4.75% (‘70s)
  - 1.65% (late ‘90s)

- Trend-based estimates of inflation gap are less persistent than mean-based measures
  - decrease in gap persistence after the Volcker disinflation
  - the first-order autocorrelation for the trend-based gap: 0.75 prior to 1985 and 0.34 thereafter.
Changes in inflation persistence
Estimates of deep parameters

• Coefficients of Calvo eq’n \((b_1, b_2, \gamma_1, \gamma_2, \bar{\rho}, \chi, \zeta)\) are functions of parameters \(\psi = [\alpha, \tilde{\beta}, \theta, \rho, \omega, \pi_t]^{\prime}\).
  
  – When trend inflation varies, the coefficients are necessarily unstable

  – We want to assess whether the parameters in \(\psi\) are stable.

• Free parameters: only \(\alpha, \rho,\) and \(\theta\)
  
  – \(\omega = a/(1-a), \quad 1-a\) is the CD labor elasticity, set to 0.7

  – \(\tilde{\beta}_t = \gamma y_t R_t \pi_t\): estimated in the VAR estimation of stage 1
• We select five representative dates:
  – 1961.Q3 - low and stable inflation
  – 1978.Q3 - height of the great inflation,
  – 1983.Q3- end of Volker disinflation

• We stack the restrictions at each date in a function

\[
\mathcal{F}(\cdot) = \begin{bmatrix}
F_{1961}(\cdot) \\
F_{1978}(\cdot) \\
F_{1983}(\cdot) \\
F_{1995}(\cdot) \\
F_{2003}(\cdot)
\end{bmatrix},
\]

• we estimate \(\alpha, \varrho, \theta\) to \(\min \mathcal{F}(\cdot)' \mathcal{F}(\cdot)\).
• Moment restrictions

– To assess the effect of first stage uncertainty, we estimate best-fitting values of \( \{\alpha, \varrho, \theta\} \) for every draw in the posterior sample \( \mu_{t|T}(i) \) and \( A_{t|T}(i) \)

– Moment restrictions, for each date, are

\[
F_t \left( \alpha_i, \varrho_i, \theta_i, \mu_{t|T}(i), A_{t|T}(i) \right) = (F'_{1t}, F'_{2t})'
\]

– We report median estimates of the distribution of \( \{\alpha_i, \varrho_i, \theta_i\} \)

\[
\{\alpha_i, \varrho_i, \theta_i\} = \arg \min \mathcal{F}'(\cdot) \mathcal{F}(\cdot).
\]

• We also report best fitting value from the posterior VAR means \( \mu_{t|T} \) and \( A_{t|T} \)

• We estimate two versions

– \( \{\alpha, \varrho, \theta\} \) held constant across dates

– \( \{\alpha, \varrho, \theta\} \) free to differ across dates
Estimates imposing constant $\alpha, \varrho, \theta$

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\rho$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>@VAR mean</td>
<td>0.603</td>
<td>0</td>
<td>10.55</td>
</tr>
<tr>
<td>median</td>
<td>0.602</td>
<td>0</td>
<td>9.97</td>
</tr>
<tr>
<td>mad</td>
<td>0.048</td>
<td>0</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Interpretation

- $\varrho = 0$
  - inflation gap computed on a trend-varying is less persistent;
  - $\varrho > 0$ may arise from omitted variable problem

- implied inertia
  - median duration of prices: 4.1 months

- steady-state mark-up: 11%
Fit of the model

- Test of cross-equation restrictions

\[ J = \hat{F}' Var(\hat{F})^{-1} \hat{F} \]

\[ \hat{F} = F(\hat{\alpha}, \hat{\theta}, \hat{\phi}, \mu_{t|T}, A_{t|T}) \]

- \( \hat{\alpha}, \hat{\phi}, \hat{\theta} \) are best fitting values corresponding to \( \mu_{t|T} \) and \( A_{t|T} \),

- \( Var(\hat{F}) \) is the sample variance of the moment conditions in the cross section.

- \( J = 22.2 \) (constant-parameter model)

  - fail to reject model’s over-identifying restrictions.

  - Caution: chi-square approximation may not be valid

  - non-normality of \( \alpha_i, \phi_i, \theta_i \), and \( F(\alpha_i, \phi_i, \theta_i, \cdot) \)
Inflation Dynamics

Actual and Predicted Inflation Gaps

Correlation = 0.90
Estimates letting $\alpha$, $\varrho$, $\theta$ vary at each date

<table>
<thead>
<tr>
<th>Year</th>
<th>$\alpha$ mean</th>
<th>$\alpha$ median</th>
<th>$\alpha$ mad</th>
<th>$\rho$</th>
<th>$\theta$ mean</th>
<th>$\theta$ median</th>
<th>$\theta$ mad</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961</td>
<td>0.611</td>
<td>0.590</td>
<td>0.093</td>
<td>0</td>
<td>11.31</td>
<td>11.32</td>
<td>1.47</td>
</tr>
<tr>
<td></td>
<td>@VAR mean</td>
<td>@VAR median</td>
<td>@VAR mad</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1978</td>
<td>0.566</td>
<td>0.567</td>
<td>0.053</td>
<td>0</td>
<td>9.60</td>
<td>10.30</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>@VAR mean</td>
<td>@VAR median</td>
<td>@VAR mad</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1983</td>
<td>0.625</td>
<td>0.591</td>
<td>0.065</td>
<td>0</td>
<td>12.96</td>
<td>11.56</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>@VAR mean</td>
<td>@VAR median</td>
<td>@VAR mad</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>0.724</td>
<td>0.672</td>
<td>0.117</td>
<td>0</td>
<td>10.47</td>
<td>10.71</td>
<td>1.87</td>
</tr>
<tr>
<td></td>
<td>@VAR mean</td>
<td>@VAR median</td>
<td>@VAR mad</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>0.734</td>
<td>0.682</td>
<td>0.124</td>
<td>0</td>
<td>11.05</td>
<td>10.90</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td>@VAR mean</td>
<td>@VAR median</td>
<td>@VAR mad</td>
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</tbody>
</table>
- Histograms report estimates for each draw of the VAR parameters (5000 estimates at each date)

Figure 5 - Histograms for Calvo Parameters
Discussion

- Values for $\rho$ pile up at zero
  - little support for an important indexing or backward-looking component
  - fraction that pile up at zero highest after Volker disinflation

- Estimates of $\rho$ and $\theta$ are stable across dates $\rightarrow$ only little variation in their histograms

- There is some evidence of variation in $\alpha$
  - the histograms shift as New Keynesian theory predicts (BMR, ‘88)
    * prices were most flexible in ‘78 when inflation highest and most variable
    * prices least flexible during Greenspan era
Conclusion

- methodological contribution
  - extend the 2-step distance estimator used in previous work with constant parameter VAR to drifting parameters VAR
  - use the time-variation of reduced form as a test for structural stability

- implication of results
  - inflation deviations from drifting trend are less persistent
  - a purely forward looking model fits well once shifts in inflation trend are taken into account