Macroeconomics and Asset Markets: some Mutual Implications.

Harald Uhlig

Fachbereich Wirtschaftswissenschaften
Humboldt Universität zu Berlin
Deutsche Bundesbank, CentER and CEPR
uhlig@wiwi.hu-berlin.de
The Issue

- Economic Risks: unemployment, stock returns, business cycles. How much do they matter?
- Macroeconomic Risks: not diversifiable.
- Price of Risks: Asset Markets.
Asset Markets vs Macro

- Asset pricing literature: take economic choices as given, determine prices from preferences or vice versa.

- Macro literature: take preferences as given, solve for economic choices.

- Each imposes discipline on the other.
Some literature


- \[ E[u'(c_i)] \neq u'(E[c_i]) \]: Constantinides, Storesletten et al. Taxes: McGrattan-Prescott.

Goal and Results

• Goal: deepen the understanding of the mutual cross-discipline.
  • “Generic” real business cycle model.
  • Nonseparability: consumption vs leisure. Underexplored so far, showcase for this methodology, new results.
  • Exogenous law for wage movements.
  • Two-agent economy (Guvenen, 2003).

• Results:
  • Asset price implications for nonseparability between consumption and leisure ...
  • ... and their macro consequences.
  • Elusive quest! Labor market is key.
Overview

1. **Facts and a generic model**
2. Asset pricing ●
3. Macroeconomic consequences ●
4. A two agent model. ●
5. Conclusions. ●
Asset Market Facts.

• Campbell (2004)

• Equity premium: 7.2% annually. Volatility: 15.5%. Sharpe Ratio: 0.46.

• Excess returns are forecastable, in particular at longer horizons.

• Lettau-Ludvigson. $cay_t$: consumption, assets and income are cointegrated. Deviations predict corrections in asset prices, not changes in consumption.

• The safe rate is not very volatile: 1.7% annually.
Macroeconomic Facts.

- Cooley and Prescott (1999)
- Labor, labor productivity, consumption are all procyclical.
- Consumption fluctuates less than output, hours nearly as much, and investment much more.
A generic model

\[ \max E \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right] \]

\[ C_t + X_t = Y_t = Z_t F(K_{t-1}, N_t) \]

\[ K_t = (1 - \delta) K_{t-1} + G \left( \frac{X_t}{K_{t-1}} \right) K_{t-1} \]

\[ 1 = N_t + L_t \]

\( C_t \): consumption. \( L_t \): leisure. \( X_t \): investment. \( K_t \): capital. \( Y_t \): output. \( N_t \): labor. \( Z_t \): TFP. \( U \): utility function. \( F \): production function for output, const. ret. to scale. \( G \): adjustment cost function for capital.
Utility function

\[
\eta_{cc} = - \frac{U_{CC}(\bar{C}, \bar{L})\bar{C}}{U_C(\bar{C}, \bar{L})}
\]
\[
\eta_{cl,c} = \frac{U_{CL}(\bar{C}, \bar{L})\bar{C}}{U_L(\bar{C}, \bar{L})}
\]
\[
\eta_{cl,l} = \frac{U_{CL}(\bar{C}, \bar{L})\bar{L}}{U_C(\bar{C}, \bar{L})}
\]
\[
\eta_{ll} = - \frac{U_{LL}(\bar{C}, \bar{L})\bar{L}}{U_L(\bar{C}, \bar{L})} \geq \left( \frac{\eta_{cl,c}}{\eta_{cl,l}} \right) \left( \frac{\eta_{cl,l}^2}{\eta_{cc}} \right)
\]
Production function

\[ \theta = \frac{F_K(\bar{K}, \bar{N})\bar{K}}{F(\bar{K}, \bar{N})} \]

\[ \phi_{kk} = -\frac{F_{KK}(\bar{K}, \bar{N})\bar{K}}{F_K(\bar{K}, \bar{N})} \]

\[ \phi_{nn} = -\frac{F_{NN}(\bar{K}, \bar{N})\bar{N}}{F_N(\bar{K}, \bar{N})} \]
Thus,

\[
\phi_{kk} = \frac{F_{KN}(\bar{K}, \bar{N})\bar{N}}{F_K(\bar{K}, \bar{N})}
\]

\[
\phi_{nn} = \frac{F_{KN}(\bar{K}, \bar{N})\bar{K}}{F_N(\bar{K}, \bar{N})}
\]

Cobb-Douglas: \( \phi_{kk} = 1 - \theta \) and \( \phi_{nn} = \theta \)
Adjustment cost function

- $G(\delta) = \delta$
- $G''(\delta) = 1$
- $\xi = -\frac{1}{G'''(\delta)\delta} > 0$
- $\xi = \infty$: no adj. cost. $\xi = 0.23$. 
### Loglinearization

<table>
<thead>
<tr>
<th>Category</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>feasib.</td>
<td>$y_t = \bar{X}_Y x_t + (1 - \bar{X}_Y) c_t$</td>
</tr>
<tr>
<td>goods prod.:</td>
<td>$y_t = \theta k_{t-1} + (1 - \theta) n_t$</td>
</tr>
<tr>
<td>cap. prod.:</td>
<td>$k_t = (1 - \delta) k_{t-1} + \delta x_t$</td>
</tr>
<tr>
<td>wages:</td>
<td>$w_t = z_t + \phi_{nn}(k_{t-1} - n_t)$</td>
</tr>
<tr>
<td>dividends:</td>
<td>$d_t = z_t - \phi_{kk}(k_{t-1} - n_t)$</td>
</tr>
<tr>
<td>time:</td>
<td>$l_t = -\frac{1-L}{L} n_t$</td>
</tr>
<tr>
<td>shadow v. of c:</td>
<td>$\lambda_t = -\eta_{cc} c_t + \eta_{cl} l_t$</td>
</tr>
<tr>
<td>shadow v. of l:</td>
<td>$\lambda_t + w_t = \eta_{cl} c_t - \eta_{ll} l_t$</td>
</tr>
<tr>
<td>adj. cost:</td>
<td>$\psi_t = \frac{1}{\xi} (x_t - k_{t-1})$</td>
</tr>
<tr>
<td>ret. on cap.:</td>
<td>$r_t = \frac{R-1+\delta}{R} d_t - \psi_{t-1} + \frac{1}{R} \psi_t$</td>
</tr>
<tr>
<td>asset pric.:</td>
<td>$0 = E_t [\lambda_{t+1} - \lambda_t + r_{t+1}]$</td>
</tr>
</tbody>
</table>
## Free Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>econ.</th>
<th>calibr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>cap. share</td>
<td>0.36</td>
</tr>
<tr>
<td>$\delta$</td>
<td>deprec.</td>
<td>0.025</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>gross cap. ret.</td>
<td>1.01</td>
</tr>
<tr>
<td>$\phi_{nn}$</td>
<td>elast. of w.</td>
<td>$\theta$ (Cobb-Douglas)</td>
</tr>
<tr>
<td>$\phi_{kk}$</td>
<td>elast. of d.</td>
<td>$1 - \theta$ (Cobb-Douglas)</td>
</tr>
<tr>
<td>$\xi \geq 0$</td>
<td>adj. cost</td>
<td>0.23 or $\infty$</td>
</tr>
<tr>
<td>$\bar{L}$</td>
<td>leis. share</td>
<td>2/3</td>
</tr>
<tr>
<td>$\eta_{cc}$</td>
<td>cons. risk. av.</td>
<td>$[1, \infty)$</td>
</tr>
<tr>
<td>$\eta_{cl,l}$</td>
<td>cross der.</td>
<td>$(-\infty, \infty)$</td>
</tr>
</tbody>
</table>
## Parameter Restrictions

<table>
<thead>
<tr>
<th>parameter</th>
<th>Restrictions</th>
<th>theory</th>
<th>econ.</th>
<th>calibr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\bar{X}}{\bar{Y}})</td>
<td>(\kappa = \frac{\eta_{cl,c}}{\eta_{cl,l}})</td>
<td>\begin{align*} \bar{X} &amp;= \frac{\delta \theta}{R - 1 + \delta} \ \eta_{ll} \geq \frac{\kappa \eta_{cl,l}^2}{\eta_{cc}} \end{align*}</td>
<td>inv. share</td>
<td>25.7%</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>(\eta_{cl,c} \leq \eta_{cl,l})</td>
<td></td>
<td>rel. exp. sh.</td>
<td>0.58</td>
</tr>
<tr>
<td>(\eta_{ll})</td>
<td></td>
<td></td>
<td>leis. risk. av.</td>
<td>([0, \infty))</td>
</tr>
</tbody>
</table>
Overview

1. Facts and a generic model
2. Asset pricing
3. Macroeconomic consequences
4. A two agent model
5. Conclusions
Overview: asset pricing.

1. Facts and a generic model

2. Asset pricing
   (a) Theory
   (b) Data
   (c) Preference implications

3. Macroeconomic consequences

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5. Conclusions.
Asset Pricing: theory

\[ 1 = E_t[\beta \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1}] \] (1)

- Notation. \( \tilde{\lambda}_{t+1} = \log \Lambda_{t+1} \), etc. No tilde: log-deviation.

\[ 0 = \log \beta + \log \left( E_t \left[ \exp \left( \Delta \tilde{\lambda}_{t+1} + \tilde{r}_{t+1} \right) \right] \right) \] (2)

- Assume joint log-normality
Asset Pricing: theory 2

- Risk-free rate:

\[
rf_t = - \log \beta - E_t[\Delta \tilde{\lambda}_{t+1}] - \frac{1}{2} \sigma^2_{\lambda,t} \tag{3}
\]

- Define the **Sharpe Ratio**

\[
SR_t = \frac{\log E_t[R_{t+1}] - rf_t}{\sigma_{r,t}}
\]

- Result:

\[
SR_t = - \rho_{\lambda,r,t} \sigma_{\lambda,t} \tag{4}
\]

\[
SR_t^{\max} = \sigma_{\lambda,t} \tag{5}
\]
Asset pricing: theory 3

\[ \lambda_t = -\eta_{cc} c_t + \eta_{cl,l} l_t \]

• For risk free rate:

\[ E_t [\Delta \tilde{\lambda}_{t+1}] = -\eta_{cc} E_t [\Delta \tilde{c}_{t+1}] + \eta_{cl,l} E_t [\Delta \tilde{l}_{t+1}] \]

• Sharpe ratio with nonseparable utility:

\[ \mathcal{SR}_t = \eta_{cc} \rho_{c,r,t} \sigma_{c,t} - \eta_{cl,l} \rho_{l,r,t} \sigma_{l,t} \]

(6)

• The Sharpe ratio also depends on the cross-derivative term \( \eta_{cl,l} \). Therefore, asset pricing facts can be explained with low cons. risk aversion, if \( \eta_{cl,l} \) etc. are chosen appropriately.
Overview: asset pricing.

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5. Conclusions.
Asset pricing: data

- Holding period: $k$ quarters. Volatilities: of $\Delta_k c_t$ etc.. Ignores predictable part except unconditional mean.

- Calculate correlations at various horizons.

- Data: log excess return of S&P 500 (with dividends reinvested) versus 1-year T-bill.
### Asset pricing: data 2

<table>
<thead>
<tr>
<th>$k$</th>
<th>std.dev. of $r_{t+1}$</th>
<th>Sharpe ratio</th>
<th>Ann. Sharpe ratio, $\mathcal{R} \sqrt{4/j}$</th>
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<tbody>
<tr>
<td>1</td>
<td>6.87</td>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td>10.37</td>
<td>0.21</td>
<td>0.29</td>
</tr>
<tr>
<td>3</td>
<td>13.18</td>
<td>0.24</td>
<td>0.28</td>
</tr>
<tr>
<td>4</td>
<td>15.40</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>8</td>
<td>22.21</td>
<td>0.36</td>
<td>0.26</td>
</tr>
<tr>
<td>12</td>
<td>26.75</td>
<td>0.47</td>
<td>0.27</td>
</tr>
<tr>
<td>16</td>
<td>29.47</td>
<td>0.63</td>
<td>0.31</td>
</tr>
<tr>
<td>20</td>
<td>31.41</td>
<td>0.82</td>
<td>0.37</td>
</tr>
</tbody>
</table>
### Asset pricing: data 3

<table>
<thead>
<tr>
<th>$k$ (Quart.)</th>
<th>$\sigma_l$ (leis.)</th>
<th>$\sigma_c$ (cons.)</th>
<th>$\rho(c, l)$</th>
<th>$\rho(l, r)$</th>
<th>$\rho(c, r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.45</td>
<td>0.67</td>
<td>-0.33</td>
<td>-0.07</td>
<td>0.27</td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
<td>1.04</td>
<td>-0.42</td>
<td>-0.08</td>
<td>0.34</td>
</tr>
<tr>
<td>3</td>
<td>1.11</td>
<td>1.33</td>
<td>-0.51</td>
<td>-0.15</td>
<td>0.37</td>
</tr>
<tr>
<td>4</td>
<td>1.36</td>
<td>1.64</td>
<td>-0.55</td>
<td>-0.21</td>
<td>0.39</td>
</tr>
<tr>
<td>8</td>
<td>2.10</td>
<td>2.42</td>
<td>-0.62</td>
<td>-0.39</td>
<td>0.42</td>
</tr>
<tr>
<td>12</td>
<td>2.46</td>
<td>2.73</td>
<td>-0.62</td>
<td>-0.50</td>
<td>0.34</td>
</tr>
<tr>
<td>16</td>
<td>2.49</td>
<td>3.01</td>
<td>-0.57</td>
<td>-0.58</td>
<td>0.39</td>
</tr>
<tr>
<td>20</td>
<td>2.39</td>
<td>3.11</td>
<td>-0.48</td>
<td>-0.59</td>
<td>0.41</td>
</tr>
</tbody>
</table>
Asset pricing: data 4

• Time variation in volatility: GARCH,

\[ \sigma^2_{l,t} = (1 - \phi)\sigma^2_{l,t-1} + \phi(l_t - l_{t-1} - E[l_t - l_{t-1}])^2 \]

etc.

• Assuming constant correlations,

\[ \Delta SR_{t+1} = \eta_{cc}\rho_{c,r}\Delta \sigma_{c,t+1} - \eta_{cl,l}\rho_{l,r}\Delta \sigma_{l,t+1} \quad (7) \]

• Interpretation: surprise increases in macroeconomic volatility (i.e. rising business cycle uncertainty) should lead to surprise falls in stock prices. Data: they do for leisure.
Macroeconomic Volatility

![Graph showing the volatility of consumer and leisure spending over time. The x-axis represents dates from 1970 to 2005, and the y-axis represents percent values ranging from 0 to 1.4. The graph features two lines: one in red labeled "cons. std. dev." and one in blue labeled "leisure std. dev." The lines show fluctuations over time.]
## Asset pricing: data 5

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\sigma$ of $\sigma_{l,t}$</th>
<th>$\sigma$ of $\sigma_{c,t}$</th>
<th>$\rho(\sigma_c, \sigma_l)$</th>
<th>$\rho(\sigma_1, \mathbf{r})$</th>
<th>$\rho(\sigma_c, \mathbf{r})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.01</td>
<td>0.18</td>
<td>0.06</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>0.02</td>
<td>0.22</td>
<td>-0.01</td>
<td>-0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.02</td>
<td>0.02</td>
<td>0.24</td>
<td>-0.13</td>
<td>-0.01</td>
</tr>
<tr>
<td>4</td>
<td>0.02</td>
<td>0.03</td>
<td>0.21</td>
<td>-0.23</td>
<td>-0.00</td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
<td>0.07</td>
<td>0.17</td>
<td>-0.46</td>
<td>0.02</td>
</tr>
<tr>
<td>12</td>
<td>0.04</td>
<td>0.10</td>
<td>0.24</td>
<td>-0.53</td>
<td>-0.07</td>
</tr>
<tr>
<td>16</td>
<td>0.05</td>
<td>0.11</td>
<td>0.38</td>
<td>-0.52</td>
<td>-0.11</td>
</tr>
<tr>
<td>20</td>
<td>0.05</td>
<td>0.10</td>
<td>0.43</td>
<td>-0.52</td>
<td>-0.09</td>
</tr>
</tbody>
</table>
Leisure vol. and returns, $k = 8$. 
Overview: asset pricing.

1. Facts and a generic model

2. Asset pricing
   (a) Theory
   (b) Data
   (c) Preference implications

3. Macroeconomic consequences

4. A two agent model.

5. Conclusions.
Preference implications

• Benchmark case: $\eta_{cl,l} = 0$. Campbell (2004).

$$\eta_{cc} = \frac{SR}{\rho_{c,r} \sigma_c} = \frac{0.27}{1.64\% \times 0.39} = 42 \text{ for } k = 4$$ (8)

• With $\eta_{cl,l} \neq 0$:

$$\eta_{cl,l} = \frac{SR - \eta_{cc} \rho_{c,r} \sigma_c}{-\rho_{l,r} \sigma_l}$$ (9)

• Desirable: $\eta_{ll}$ as low as possible. Thus,

$$\eta_{ll} = \frac{\kappa \eta_{cl,l}^2}{\eta_{cc}}$$
### Preference implications 2

<table>
<thead>
<tr>
<th>$\eta_{cc}$</th>
<th>$\eta_{cl_l}$</th>
<th>$\eta_{ll}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k=4$</td>
<td>$k=8$</td>
</tr>
<tr>
<td>3.0</td>
<td>84.7</td>
<td>41.1</td>
</tr>
<tr>
<td>5.0</td>
<td>80.4</td>
<td>38.7</td>
</tr>
<tr>
<td>10.0</td>
<td>69.5</td>
<td>32.5</td>
</tr>
<tr>
<td>15.0</td>
<td>58.5</td>
<td>26.3</td>
</tr>
<tr>
<td>20.0</td>
<td>47.6</td>
<td>20.1</td>
</tr>
<tr>
<td>30.0</td>
<td>25.8</td>
<td>7.7</td>
</tr>
<tr>
<td>40.0</td>
<td>4.0</td>
<td>-4.7</td>
</tr>
<tr>
<td>50.0</td>
<td>-17.9</td>
<td>-17.1</td>
</tr>
</tbody>
</table>
The cross-derivative term $\eta_{cl,l}$. 

![Graph showing the relationship between $\eta_{cl,l}$ and $\eta_{cc}$ for two different values of $k$.](image-url)
Leisure risk aversion $\eta_l$.
Preference implications 3

- Alternative: targeting only a quarter of the observed Sharpe ratio.

<table>
<thead>
<tr>
<th>$\eta_{cc}$</th>
<th>$\eta_{cl}$</th>
<th>$\eta_{ll}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 k=4</td>
<td>20.6</td>
<td>247.2</td>
</tr>
<tr>
<td>2.0 k=8</td>
<td>10.0</td>
<td>57.8</td>
</tr>
<tr>
<td>3.0 k=4</td>
<td>16.3</td>
<td>51.2</td>
</tr>
<tr>
<td>4.0 k=8</td>
<td>7.5</td>
<td>10.9</td>
</tr>
<tr>
<td>5.0 k=4</td>
<td>11.9</td>
<td>16.5</td>
</tr>
<tr>
<td>6.0 k=8</td>
<td>5.0</td>
<td>2.9</td>
</tr>
<tr>
<td>7.0 k=4</td>
<td>7.5</td>
<td>4.7</td>
</tr>
<tr>
<td>8.0 k=8</td>
<td>2.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Overview

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Macroeconomic Implications

- Exogenous process for technology:

\[ z_t = 0.95 z_{t-1} + \epsilon_t \]

- Adjust \( \sigma_\epsilon \) so that the HP-filtered output has a standard deviation of 2%. Compare to the benchmark \( \sigma_\epsilon = 0.712 \).

- Try \( \xi = \infty \) and \( \xi = 0.23 \). Vary \( \eta_{cc} \). Calculate the other preference parameters via
  - Target 1: the observed Sharpe ratio
  - Target 2: the observed Sharpe ratio / 4

Holding period: \( k = 8 \) quarters.
Target: Sharpe ratio. Table A.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Labor</th>
<th>Cons.</th>
<th>Inv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{cc}$</td>
<td>$\sigma_\epsilon$</td>
<td>$\sigma_{H P}^{n}$</td>
<td>$\rho_{n,y}^{H P}$</td>
</tr>
<tr>
<td>$\xi = 0.23$:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.85</td>
<td>0.64</td>
<td>-1</td>
</tr>
<tr>
<td>10</td>
<td>2.27</td>
<td>1.48</td>
<td>-1</td>
</tr>
<tr>
<td>15</td>
<td>2.84</td>
<td>2.64</td>
<td>-1</td>
</tr>
<tr>
<td>20</td>
<td>3.67</td>
<td>4.33</td>
<td>-1</td>
</tr>
<tr>
<td>$\xi = \infty$:</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>1.07</td>
<td>1.04</td>
<td>0.66</td>
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<tr>
<td>10</td>
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<td>15</td>
<td>1.16</td>
<td>0.99</td>
<td>0.73</td>
</tr>
<tr>
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<td>1.23</td>
<td>0.91</td>
<td>0.69</td>
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Target: Sharpe ratio/4. Table A.

<table>
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<tr>
<th>Param.</th>
<th>Labor</th>
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<tr>
<td>$\eta_{cc}$</td>
<td>$\sigma_\epsilon$</td>
<td>$\sigma_{n}^{HP}$</td>
<td>$\rho_{n,y}^{HP}$</td>
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<td>$\xi = 0.23$:</td>
<td></td>
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<tr>
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### Target: Sharpe ratio. Table B.

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<th>Sᵣᵩₙₙ.</th>
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<tr>
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<td>ξ = 0.23:</td>
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<td>ξ = ∞:</td>
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<td>7.37</td>
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</table>
**Target: Sharpe ratio/4. Table B.**

<table>
<thead>
<tr>
<th>$\eta_{cc}$</th>
<th>$\sigma_{\Delta g_c}$</th>
<th>$\sigma_{\Delta g_l}$</th>
<th>$\mathcal{S} \mathcal{R}_{ann.}$</th>
<th>$\sigma_{r,f}$</th>
<th>$\sigma_{r}$</th>
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<tbody>
<tr>
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<td></td>
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<tr>
<td>1</td>
<td>5.16</td>
<td>0.44</td>
<td>0.01</td>
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<tr>
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<td>0.02</td>
<td>0.15</td>
<td>0.16</td>
</tr>
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</table>
Target: Sharpe ratio. Figures.

\[ \eta_{cc} = 5, \xi = 0.23 \]

\[ \eta_{cc} = 20, \xi = 0.23 \]

\[ \eta_{cc} = 5, \xi = \infty \]

\[ \eta_{cc} = 20, \xi = \infty \]

\[ \eta_{cc} = 1, \xi = .23 \]

Impulse responses to a shock in technology

\[ \eta_{cc} = 7, \xi = .23 \]

Impulse responses to a shock in technology

\[ \eta_{cc} = 1, \xi = \infty \]

Impulse responses to a shock in technology

\[ \eta_{cc} = 7, \xi = \infty \]

Impulse responses to a shock in technology
An exogenous law for wages

- Replace FOC wrt leisure with

\[ w_t = \gamma w_{t-1} + \alpha n_{t-1} \]

- Correlations higher at longer lags.
Wage-Hours Correlations

Correlation of $w(t)$ with $n(t-j)$

Correlation

Lag for labor

0  5  10  15  20
An exogenous law for wages

- Estimate first-order VAR in quadratically detrended log-hours and log-wages using the 12-th lag instead.

\[
\begin{bmatrix}
    w_t \\
    n_t
\end{bmatrix} = B^{12} \begin{bmatrix}
    w_{t-12} \\
    n_{t-12}
\end{bmatrix}. \quad B = \begin{bmatrix}
    1.04 & 0.15 \\
    -0.28 & 0.72
\end{bmatrix}
\]

- Thus, law for wages is
  \[ w_t = 1.04 w_{t-1} + 0.15 n_{t-1} \]

- Interesting Dynamics
Wage-Hours Dynamics

Dynamics of wages and hours

- Wages
- Hours

Years after shock

Percent

-2 -1 0 1 2 3 4 5 6 7 8 9 10

Macroeconomics and Asset Markets: some Mutual Implications. Harald Uhlig, Humboldt Universität zu Berlin. uhlig@wiwi.hu-berlin.de – p. 47/85
Variations in $\eta_{cc}$ and $\xi$

- Vary $\eta_{cc}$ and $\xi$. Calculate $\eta_{cl,l}$, $\eta_{cl,c}$ and $\eta_{ll}$ from targeting Sharpe ratio.
Sharpe ratio results
Risk-free rate vol.
Consumption volatility.
Consumption correlation.
Labor correlation
Tradeoff: criterion

\[ \chi = 1000(SR - 0.27)^2 + (\sigma_{r^f} - 1.7)^2 \\
+ (\sigma_{n,HP} - 1.79)^2 \\
+ (\max(\sigma_{c,HP} - 1.30/2.13, 0) \\
\quad + \min(\sigma_{c,HP} - 0.82/1.74, 0))^2 \\
+ (\max(\sigma_{x,HP} - 6.87/1.74, 0) \\
\quad + \min(\sigma_{x,HP} - 8.07/2.13, 0))^2 \\
+ (\text{corr}_{c,y} - 0.80)^2 \\
+ (\text{corr}_{n,y} - 0.86)^2 \\
+ (\text{corr}_{x,y} - 0.83)^2 \]
Tradeoff: criterion
“Optimal” tradeoff

- $\eta_{cc} = 40$, $\xi = 0.55$

- Moments:
  
  $\sigma_{y,HP} = 2$, need: $\sigma_\epsilon = 0.65$
  
  $\sigma_{n,HP} = 2.09$, $\sigma_{c,HP} = 0.55$, $\sigma_{x,HP} = 6.36$,
  
  $\rho_{n,y} = 0.93$, $\rho_{c,y} = 1$, $\rho_{x,y} = 1$,

- Asset Pricing:
  
  $\mathcal{S}_R = 0.22$, $\sigma_{r^f} = 3.95$, $\sigma_{r_k} = 11.12$
“Optimal” tradeoff

Impulse responses to a shock in technology
Macro-Implications: Summary

- With endogenous choices, agents react by smoothing those variables, which are subject to high risk prices.

- Adjustment costs help in generating higher Sharpe ratios (no consumption smoothing via investment), but worsens corr(n,y), corr(c,y).

- Labor and consumption move in opposite directions, in contrast to the data.

- Thus key: alternative labor market paradigm. Exogenous law for wage movement does the trick!
Overview

1. Facts and a generic model
2. Asset pricing
3. Macroeconomic consequences
4. A two agent model.
5. Conclusions.
A two-agent economy.

- Campbell-Cochrane (1999): highly nonlinear, external habit explain asset pricing observations.

- Ljungqvist-Uhlig (2003): endogenizing consumption choices with CC preferences has unusual consequences. E.g., an agent is significantly better off by periodically destroying parts of a constant stream of endowment. Reason: utility has local and global nonconcavities.

A two-agent economy.

- “Capitalist”: owns capital, does not work, trades in the riskless bond. Risk aversion = 2.


- Guvenen (2003): emphasizes nonlinearities, etc. Here: extend benchmark model and study the loglinearized dynamics. Easier to understand.

- Guvnenen (2003) keeps labor fixed: $\eta_l = \infty$. Thus, no explanation of employment fluctuations, a key feature of business cycles.

- We consider $\eta_l = 0$, $\eta_u = \infty$ and exog. wages.
The capitalist.

$$\max E \left[ \sum_{t=0}^{\infty} \beta^t U^{(C)}(C_t^{(C)}) \right]$$

$$C_t^{(C)} + B_t + X_t = D_t K_{t-1} + R_{t-1}^f B_{t-1}$$

$$K_t = (1 - \delta) K_{t-1} + G \left( \frac{X_t}{K_{t-1}} \right) K_{t-1}$$
The worker.

\[
\max E \left[ \sum_{t=0}^{\infty} \beta^t U(W)(C_t(W), L_t) \right]
\]

\[
C_t(W) - B_t = W_t N_t - R_{t-1}^f B_{t-1}
\]

\[1 = N_t + L_t\]

Three Possibilities:

- \(\eta_{ll}^{(W)} = \infty\): Guvenen.
- \(\eta_{ll}^{(W)} = 0\).
- Exogenous law of motion for wages.
Loglinearization: the changes.

\[
y_t = \frac{\bar{X}}{Y} x_t + \frac{\bar{C}^{(C)}}{Y} c_t^{(C)} + \frac{\bar{C}^{(W)}}{Y} c_t^{(W)}
\]

\[
\lambda_t^{(W)} = -\eta_{cc}^{(W)} c_t^{(W)} + \eta_{cl,l}^{(W)} l_t
\]

\[
\lambda_t^{(W)} + w_t = \eta_{cl,c}^{(W)} c_t - \eta_{ll}^{(W)} l_t
\]

\[
\lambda_t^{(C)} = -\eta_{cc}^{(C)} c_t^{(C)}
\]

\[
\frac{\bar{C}^{(W)}}{Y} c_t^{(W)} - b_t = (1 - \theta)(w_t + n_t) - \bar{R} \bar{B} r_{t-1}^f - \bar{R} b_{t-1}
\]

\[
0 = E_t \left[ \lambda_{t+1}^{(C)} - \lambda_t^{(C)} + r_{t+1} \right]
\]

\[
0 = E_t \left[ \lambda_{t+1}^{(C)} - \lambda_t^{(C)} + r_f^t \right]
\]

\[
0 = E_t \left[ \lambda_{t+1}^{(W)} - \lambda_t^{(W)} + r_f^t \right]
\]
### Free parameters

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<th>econ.</th>
<th>calibr.</th>
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<td>$\phi_{nn}$</td>
<td>elast. of wages</td>
<td>$\theta$</td>
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<td>$\bar{L}$</td>
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<td>$\eta_{cl,l}$</td>
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## Parameter restrictions

<table>
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<tr>
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<th>Restrictions</th>
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</thead>
</table>
| $\frac{\bar{X}}{\bar{Y}}$, $\frac{\bar{C}(W)}{\bar{Y}}$, $\frac{\bar{C}(C)}{\bar{Y}}$, $\kappa = \frac{\eta_{cl,c}^{(W)}}{\eta_{cl,l}}$, $\eta_{ll}^{(W)}$ | \[ \begin{align*}
\text{theory} & : \\
& \frac{\delta \theta}{R-1+\delta} \\
& = \frac{1 - \theta - (\bar{R} - 1) \frac{\bar{B}}{\bar{Y}}}{1 - \frac{\bar{X}}{\bar{Y}} - \frac{\bar{C}(C)}{\bar{Y}}} \\
& \geq \frac{\kappa (\eta_{cl,l}^{(W)})^2}{\eta_{cc}} \\
\text{econ.} & : \\
& \text{inv. share} \\
& \text{cons. share (W)} \\
& \text{cons. share (C)} \\
& \text{rel.exp.shares} \\
& \text{leis.risk.av.} \\
\text{calibr.} & : \\
& 25.7\% \\
& 60\% \\
& 14.3\% \\
& 0.5 \\
& 0, \infty
\end{align*} \] |
Cons. volatility of capitalist:

• To nail the Sharpe ratio with this model and a relative risk aversion of two, the consumption of the capitalist must be quite volatile,

\[ \sigma_c^{(C)} \geq \frac{\mathcal{SR}}{\eta_{cc}} = \frac{0.27}{2} = 13.5\% \text{ for } k = 4 \]

• Plausible? Evidence on luxury goods, see Aït-Sahalia - Parker - Yogo (2002).
## Results 1

<table>
<thead>
<tr>
<th></th>
<th>$\eta_{ll}(W) = 0$</th>
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<tr>
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<td>1.96</td>
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<td>$\sigma_{n,HP}$</td>
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<td>$\rho_{n,y}$</td>
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<td>n.a.</td>
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<tr>
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<td>1.00</td>
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## Results 2

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<td>$\sigma_{r_{capital}}$</td>
<td>2.39</td>
<td>9.02</td>
<td>9.45</td>
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</tbody>
</table>
Fixed labor, $\eta_{ll}^{(W)} = \infty$. 

Impulse responses to a shock in technology

- Percent deviation from steady state
- Years after shock

- cons.cap.
- technology
- cons.worker
- capital
- labor
Flexible labor, $\eta_{ll}^{(W)} = 0$. 

Impulse responses to a shock in technology

Years after shock

Percent deviation from steady state

-1.5 -1 -0.5 0 0.5 1

-2 0 2 4 6 8

Years after shock

capital

cons.cap.

output

cons.worker

labor
Exogenous wages.

Impulse responses to a shock in technology

Percent deviation from steady state vs. Years after shock

- capital
- output
- cons worker
- technology
- cons cap.
Exogenous wages 2.

- Impulse responses to a shock in technology
  - Years after shock
  - Percent deviation from steady state
  - Variables: output, technology, capital, consumer, worker, wage, cons. worker

Macroeconomics and Asset Markets: some Mutual Implications. Harald Uhlig, Humboldt Universität zu Berlin. uhlig@wiwi.hu-berlin.de – p. 73/85
Two agents: summary.

- The model makes some progress towards a joint explanation.

- Consumption of “capitalists” must be extremely volatile to be consistent with the observed Sharpe ratio, at a risk aversion of 2.

- Fixing labor punts on important feature. Allowing for flexible labor and $\eta_{cl,l} = 0$ generates worse business cycle properties.

- Again key: alternative labor market paradigm. Exogenous law for wage movement does the trick!
Overview

1. Facts and a generic model
2. Asset pricing
3. Macroeconomic consequences
4. A two agent model.
5. Conclusions.
Conclusions 1.

- This paper: provides some analytics in a simple, but general benchmark economy.
- New results on preferences with nonseparabilities between consumption and leisure.
- Extension of techniques to two-agent economy.
Conclusions 2.

- **Mutual discipline** of asset market observations and macroeconomic observations:
  - **Economic choices** such as consumption and leisure ...
  - are taken as *exogenous* in asset pricing *literature* and suggest preference specifications, ...
  - which in turn may have undesirable *macroeconomic consequences*, once these economic choices are endogenized.

- **Key: alternative labor market paradigm.**
  Exogenous law for wage movement does the trick!
Campbell-Cochrane.

\[ E_0 \left[ \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1 - \gamma} \right] \]

- Surplus consumption ratio

\[ S_t^a \equiv \frac{(C_t^a - X_t)}{C_t^a} \]

- Use lower-case to denote logs.
Campbell-Cochrane 2.

• Assumption:

\[ s_{t+1}^a = (1 - \phi)\bar{s} + \phi s_t^a + \lambda(s_t^a) (c_{t+1}^a - c_t^a - g), \]

where \( \phi \in [0, 1) \), g and \( \bar{s} \) are parameters, and

\[ \lambda(s^a) = \begin{cases} \bar{S}^{-1} \sqrt{1 - 2(s^a - \bar{s})} - 1, & s^a \leq s_{max}; \\ 0, & s^a \geq s_{max}; \end{cases} \]

• Campbell-Cochrane: assume consumption to be an exogenous random walk.

• Explains lots of asset pricing facts.
Ljungqvist-Uhlig

• Ljungqvist-Uhlig: consider an economy with a constant stream of endowment. Let an agent with CC-preferences choose consumption, subject to consumption ≤ endowment.

• Analyze the social planners problem
A one-time destruction of consumption or periodic destruction of consumption vastly increases welfare.

Optimal decision rule look bizarre. Results preliminary.

CC preferences are nonconcave.

Habit decreases in consumption when increasing consumption by more than 20%.
Welfare gains

... from a one-time endowment destruction.
Welfare gains

<table>
<thead>
<tr>
<th>k</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
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<td>15.91</td>
<td>9.86</td>
</tr>
<tr>
<td>24</td>
<td>0.72</td>
<td>8.52</td>
<td>15.89</td>
<td>9.89</td>
</tr>
<tr>
<td>120</td>
<td>0.24</td>
<td>4.95</td>
<td>13.40</td>
<td>8.51</td>
</tr>
</tbody>
</table>

... from periodic destruction every $k$ periods.
Habit function.

Next-period habit as function of consumption.
Decision rule for consumption

\[ C(t) / Y(t) \]

\[ C(t-1) / Y(t-1) \]