PS#5 Answer Key

N^0 12.7

a) First, in NE \( p_1 = p_2 \). If \( p_1 \neq p_2 \), assume \( p_1 > p_2 \)

\[ \Rightarrow \pi_1 = 0, \quad \pi_2 > 0 \Rightarrow \] for firm 1 it’s always profitable to lower its price and make positive profit.

Second, in NE, \( p_1, p_2 > 5 = MC \) (otherwise firm would make negative profit).

So, \( p_1 = p_2 = 5 \). (1).

Under the following condition the firm would not find it profitable to deviate to lower price

\[ \Rightarrow (p_i - 5) \cdot 5000 > (p_i - 11) \cdot 19000 \]

\[ \Rightarrow p_i \leq 7. \] (2)

From (1), (2), and conditions on whole-dollar amount pricing we get

\[ NE = \frac{1}{3} (p_1 = 5, p_2 = 5), (p_1 = 6, p_2 = 6), (p_1 = 7, p_2 = 7) \]

So, in this problem we have 3 Nash equilibria,

whereas in continuous pricing problem we would have just one \( NE = \frac{1}{3} p_1 = 5, p_2 = 5 \). This happens because in discrete-pricing case the firm is punished a lot from deviating as compared to the continuous price case, where punishment is absent.

b) When \( s = 2 \), suppose \( p_1 \neq p_2 \).

If \( |p_1 - p_2| > 2 \) \( \Rightarrow \) the firm that set max price

will get zero demand and make zero profit \( \Rightarrow \) it’s

not profitable for it to deviate. \( \Rightarrow \) if not an equilibrium.
If \(|p_1 - p_2| < 5 = \text{firms will have the same demand} \Rightarrow \text{the firm with lower price will find it profitable to increase its price and make positive profit.}

Therefore, in equilibrium, \(p_1 = p_2\).

As above, \(p_1 = p_2 \Rightarrow MC = 5\). \((1)\)

And \(p_1 = p_2 \leq 10\) \((2)\) (otherwise they will face \(0\) demand)

From the following condition, the firm will not find it profitable to lower its price from equilibrium price:

\[
(p-5) \cdot 5,000 \geq (p-2-1) \cdot 5 \cdot 10,000
\]

\[
\Rightarrow p = 12 \leq 11\]

\((3)\)

As was explained above, if \(p_1 = p_2 \in (5, 9)\) it was not always profitable for a firm to increase price by \(1\) and make higher profit. Only if \(p_1 = p_2 = 10\), firm can not increase the price.

\((4), (12) \Rightarrow \text{So, NE} = 4, p_1 = 10, p_2 = 10\).

\((5), (12) \Rightarrow \text{So, NE} = 4 (10, 10)\).

\((6), (12) \Rightarrow \text{in both (6) and (5) firm set price = 10 to equilibrium. Therefore, the expected profit in both parts is}

\[\Pi = 5,000 (10 - 5) = 25,000\]

\(\Rightarrow \text{The value of search matching costs is 200.}\)
a) Suppose in equilibrium there exist more than 2 different prices. Then, there exist \( p_1 \neq p_2 \neq p_3 \):

\[
p_1 < p_2 < p_3 \quad (1)
\]

Then, it should be

\[
(2) \quad p_2 \leq p_i + 44 \Rightarrow \text{ otherwise, if } p_i > p_1 + 44,
\]

\[
(11, 12) \quad \Rightarrow \quad 0 < p_3 - p_2 \leq 44 \Rightarrow \text{ but people with searching cost would not differentiate between } p_3 \text{ and } p_2. \text{ Therefore, it would be profitable for firm 2 to increase its profit price to be equal to } p_3 \text{ and make higher profit.}
\]

So, in equilibrium there could be no more than 2 different prices.

b) Suppose we have an equilibrium with \( p_1 \neq p_2 \)

and \( p_1 < p_2 \). Let's prove \( p_2 = 70 \).

Evidently, \( p_2 \leq 70 \), otherwise, firm 2 would face 0 demand.

If \( p_1 \neq p_2 \Rightarrow p_2 < p_2 - 44 \), otherwise people

If \( p_1 > 25 \) to make non-negative profit.

If \( p_1 = 25 \rightarrow \) the highest price firm 1 could set in order not to lose the demand is \( p_2 = 25 + 44 = 69 \)

But firm of type 1 can always increase its price to be without not decrease in demand \( \Rightarrow p_2 = 70 \).
C) So, we have 5 firms of type A that set price $p = 70$
and 20 firms of type B that set price $p = 45$. 
First, let's compute firms' profit.

- Consumers with a searching price buy the cheapest product from firm of type:

  20 firms with 1000 consumers
  \[ \frac{1000}{20} = 50 \]  each firm's number of buyers with a searching cost

- Consumers with $44$ searching cost buy from firm of type B, anyway $70 - 45 = 25 < 44$.

  \[ \frac{1000}{35} = 40 \] people with positive searching cost.

Therefore, the demand for type A firm is $15 = 40$,
and the demand for type B firm is $15 = 40 + 50 = 90$.
However, because of capacity constraint firm B
will sell only to 50 people, not 90.

Profit of each firm is:

- $T_A = (170 - 25) \cdot 40 = 95 \cdot 40 = 3800$
- $T_B = (45 - 25) \cdot 50 = 20 \cdot 50 = 1000$

And only $5 \cdot 40 + 20 \cdot 50 = 1200$ people will buy the product in this setup.

Since, there are people that could not buy the product, firm of type B would always find it profitable to increase its price and make greater higher profit.

Therefore, this is not an equilibrium.

Note: You can check that without capacity constraint this would be an equilibrium.
Assume firms in both countries have the access to the same technology $C = F + c_1$.

Before trade agreement, \( \hat{n}_1 = \left( a - c \right) \left( \frac{S_1}{F} \right)^2 - 1 \) - number of firms in country 1.

\( \hat{n}_2 = \left( a - c \right) \left( \frac{S_2}{F} \right)^2 - 1 \) - number of firms in country 2.

After completion of the single market, the demand is

\[ D = S_1 (a - p) + S_2 (a - p) = (S_1 + S_2) (a - p) \]

\[ \Rightarrow \hat{n}_{\text{new}} = \left( a - c \right) \left( \frac{S_1 + S_2}{F} \right)^2 - 1 \] .

Since, \( \sqrt{S_1} + \sqrt{S_2} > \sqrt{S_1 + S_2} \)

\[ \Rightarrow \hat{n}_{\text{new}} < \hat{n}_1 + \hat{n}_2 . \]

Thus, with the creation of single market, the number of firms are increased and the market becomes more competitive \( (\hat{n}_1 + \hat{n}_2 \text{ initially}) \Rightarrow \) market becomes more competitive.

\[ \Rightarrow \text{number of firms decreases} \Rightarrow \hat{n}_{\text{new}} \leq \hat{n}_1 + \hat{n}_2 . \]

\[ \Rightarrow 14.7 . \]

a) \( \hat{n}_i = F_i + c_i \), \( i = 1, 2 \)

\( \hat{n}_1 \) - number of firms with technology \( C_1 = F_1 + c_1 \),

\( \hat{n}_2 \) - \( 1-1-1 \) \( C_2 = F_2 + c_2 \).
\[ Q = n_1 q_1 + n_2 q_2 \quad \Rightarrow P = a - \frac{q_1 a + n_2 q_2}{S} \]

\[ T_1(q_1) = (a - \frac{(n_1 q_1 + q_1 + n_2 q_2)}{S}) q_1 - c_1 q_1 - F, \]

\[ \frac{\partial T_1(q_1)}{\partial q_1} = \frac{1}{\frac{(c_1 - \frac{n_1 q_1 + n_2 q_2}{S})}{S}} = 0 \]

\[ \Rightarrow q_1 = \frac{(a - c_1) S - n_2 q_2}{n_1 + 1} \quad (1) \]

In the same way, \[ q_2 = \frac{(a - c_2) S - n_1 q_1}{n_2 + 1} \]

\[ \Rightarrow (n_1 + 1) q_1 + n_2 q_2 = (a - c_1) S \quad \Rightarrow \text{Since } Q = n_1 q_1 + n_2 q_2 \]

\[ Q = (a - c_1) S - q_1 \]

\[ \Rightarrow Q = (a - c_1) S - q_1 \]

In the same way \[ Q = (a - c_2) S - q_2 \]

\[ q_1 + c_1 S = q_2 + c_2 S \quad \Rightarrow \frac{Q}{S} = q_1 + (c_2 - c_1) S \quad (2) \]

Plug (2) into (1):

\[ \Rightarrow \begin{cases} q_1 = \frac{(a - c_1) S - n_2 (c_1 - c_2) S}{n_1 + n_2 + 1} \\ q_2 = \frac{(a - c_2) S - n_1 (c_2 - c_1) S}{n_1 + n_2 + 1} \end{cases} \quad (3) \]

\[ \Rightarrow \frac{P}{S} = a - \frac{a - (a - c_1) S - q_1}{S} = c_1 - \frac{q_1}{S} = c_2 - \frac{q_2}{S} \]

\[ \Rightarrow \frac{q_1}{S} = c_1 + \frac{q_1}{S} - F_1 - c_1 q_1 = \frac{q_1^2}{S} - F_1 \]

\[ \Rightarrow \frac{q_2}{S} - F_2 \]
\[ \begin{align*}
  n_1 & = 0 \quad (4) \Rightarrow \quad q_1 = \frac{F_i S}{n_1 + n_2 + 1} \\
  n_2 & = 0 \quad (5) \Rightarrow \quad q_2 = \frac{F_2 S}{n_1 + n_2 + 1} 
\end{align*} \]

\[ (3, 15) \Rightarrow \]
\[ \begin{align*}
  q_1 & = \frac{F_1 S}{n_1 + n_2 + 1} = \frac{(a - c_1) - n_2 (c_1 - c_2) S}{n_1 + n_2 + 1} \\
  q_2 & = \frac{F_2 S}{n_1 + n_2 + 1} = \frac{(a - c_2) - n_1 (c_2 - c_1) S}{n_1 + n_2 + 1}
\end{align*} \]

System of equations (6) is the conditions for free entry equilibrium.

b) Let set \( F_1 = 1 \), \( F_2 = 4 \), \( c_1 = 2 \), \( c_2 = 1 \), \( s = 1 \), \( a = 10 \).

Then, from plugging in (6):
\[ \begin{align*}
  \sqrt{1.1} & = \frac{(10 - 2) - n_2 (2 - 1) \cdot 1}{n_1 + n_2 + 1} \\
  \sqrt{1.4} & = \frac{(10 - 1) - n_1 (1 - 2) \cdot 1}{n_1 + n_2 + 1}
\end{align*} \]

\[ \begin{align*}
  n_1 + n_2 + 1 & = 8 - n_2 \\
  2 (n_1 + n_2 + 1) & = 9 + n_1
\end{align*} \]

So, we have two unknowns \( n_1, n_2 \) and one equation \( n_1 = 7 - 2 n_2 \) (7).
We can check, that both $n_1 = 3, n_2 = 2$ and $n_1 = 1, n_2 = 3$ satisfy equation (7). Therefore, in this case there exist more than one equilibrium.