PS #4 Answer Key

№1.

(a) Denote Q_{-i} the amount all firms but firm i produce. Than, $Q = Q_{-i} + q_i$, where q_i - the amount firm i produce.

$$\pi_i = (100 - q_i - Q_{-i})q_i - (256 + 20q_i)$$

$$\frac{\partial \pi_i}{\partial q_i} = 100 - 2q_i - Q_{-i} - 20q_i = 0$$

$$\Rightarrow q_i(Q_{-i}) = \frac{100 - Q_{-i}}{22};$$

Since all firms have identical cost function and production tecnologies $\Rightarrow$ in equilibrium they produce the same amount of output.

Therefore, in equilibrium:

$$Q_{-i} = (n-1)q^*$$

$$q_i = q^*$$

$$Q = n \cdot q^*.$$

$$\Rightarrow q^* = \frac{100 - (n-1)q^*}{22}$$

$$\Rightarrow n = \frac{100 - 21 \cdot q^*}{q^*} \quad (1).$$

Since, in the long run equilibrium each firm makes zero profit $\Rightarrow \pi_i^*(q_i^*, Q_{-i}^*) = 0 \quad \forall i = \overline{1,n}$

$$\Rightarrow (100 - q^* - (n-1)q^*)q^* - (256 + 20q^*) = 0$$

$$\Rightarrow n = \frac{100}{q^*} - \frac{256 + 20q^*}{q^*} \quad (2)$$

From (1), (2) $\Rightarrow$

$$n = \frac{100}{q^*} - \frac{256 + 20q^*}{(q^*)^2} = \frac{100 - 21q^*}{q^*}$$

$$\Rightarrow 21(q^*)^2 - 20q^* - 256 = 0$$ (Two solutions, positive and negative, but $q^* \geq 0$)

$$\Rightarrow q^* = 4.$$

$$\Rightarrow n^* = \frac{100 - 21 \cdot 4}{4} = 4.$$

So, in LR equilibrium there are 4 firms.

(b). $Q^* = n^* q^* = 4 \cdot 4 = 16$ – industry output

$P = 100 - Q^* = 84$ – price

$\pi_i^* = 0 \quad \forall i = \overline{1, n}$ – profit.

№ 10.8

a) $Q_H = 280 - 2p_H$

$Q_L = 160 - 2p_L$

$MC = 20$

$$\pi^H = (p_H - MC) Q_H = (p_H - 20) \cdot (280 - 2p_H)$$

$$\Rightarrow \frac{\partial \pi^H}{\partial p_H} = -4p_H + 320 = 0 \Rightarrow$$

$$p_H = \frac{320}{4} = 80$$ – price on "hot" day

$$\pi^L = (p_L - MC) Q_L = (p_L - 20)(160 - 2p_L)$$

$$\frac{\partial \pi^L}{\partial p_L} = -4p_L + 200 = 0$$

$$\Rightarrow p_L = 50$$ – price on "low" day.

(b) Equal number of "Hot" and "Cold" days ⇒
Prob("Hot") = Prob("Cold") = $\frac{1}{2}$.

⇒ Expected profit $\pi^e = (p - mc)\left(\frac{1}{2} \cdot Q_H + \frac{1}{2} \cdot Q_L\right) =$

$= (p - mc) \cdot (220 - 2p) = (p - 20)(220 - 2p)$

⇒ $\frac{\partial \pi^e}{\partial p} = -4p + 260 = 0$

⇒ $\underline{p = 65}$

(c). Profit under const price:

$\pi_H^c = (65 - 20)(280 - 2 \cdot 65) = 6750$ on "Hot" day

$\pi_L^c = (65 - 20)(160 - 2 \cdot 65) = 1350$ on "Cold" day

Profit under variable price

$\pi_H^v = (80 - 20) \cdot 120 = 7200$ on "Hot" day

$\pi_L^v = (50 - 20) \cdot 60 = 1800$ on "Cold" day

Assume the vending machine can last only for 2 days (you may make a different assumption here)

⇒ Constant price policy:

$\pi_H^c + \pi_L^c = 6750 + 1350 = 8100$

Variable price policy:

$\pi_H^v + \pi_L^v = 7200 + 1800 = 9000$

⇒ $9000 - 8100 = 900$

So, Coca-Cola would be willing to pay up to \$900 for vending machine that lasts for 2 days.

№10. 9

$D_a = 150{,}000 - 3p$

$D_{un} = 90{,}000 - 3p$ Capacity : $Q = 50{,}000$

$D_{im} = 240{,}000 - 3p$

a) Since the cost of operating the stadium is independent of the number of tickets sold $\Rightarrow$ profit maximization is equivalent to revenue maximization.

$\Rightarrow \ \pi_a = (150{,}000 - 3p_a) \cdot p_a = D_a \cdot p_a$

$\frac{\partial \pi_a}{\partial p_a} = 150{,}000 - 6p_a = 0$

$\Rightarrow p_a^* = \frac{150{,}000}{6} = 25{,}000$

$\Rightarrow D_a = 150{,}000 - 3 \cdot 25{,}000 = 75{,}000 > 50{,}000$

Since $\left.\frac{\partial \pi}{\partial q}\right|_{q<75{,}000} > 0$ $\Rightarrow p_a^* = \frac{150{,}000 - 50{,}000}{3} = \$33{,}333.33$ $\Rightarrow$ the price of ticket on average game.

$\pi_{un} = (90{,}000 - 3p_{un}) \cdot p_{un}$

$\frac{\partial \pi_{un}}{\partial p_{un}} = 90{,}000 - 6p_{un} = 0$

$\Rightarrow p_{un} = 15{,}000$

$D_{un} = 45{,}000 < 50{,}000$

$\Rightarrow p_{un} = 15{,}000$ - the price of ticket on not important game

$\pi_{im} = (240{,}000 - 3p_{im}) p_{im}$

$\frac{\partial \pi_{im}}{\partial p_{im}} = 240{,}000 - 6p_{im} = 0$

$\Rightarrow p_{im} = 40{,}000$

$\Rightarrow p_{im} = 240{,}000 - 3\cdot 40{,}000 = 120{,}000 > 50{,}000$

Since $\frac{\partial \pi}{\partial q}\Big|_{q<120{,}00} > 0$

$\Rightarrow$

$$p_{im} = \frac{240{,}000 - 50{,}000}{3} = 63{,}333.33$$

– the price of ticket on the important game.

6) Profit without capacity expansion:

$3\cdot \pi_a^* + 3\cdot \pi_{un}^* + \pi_{im} =$

$= 3\cdot 33{,}333.33 \cdot 50{,}000 + 3\cdot 15{,}000 \cdot 45{,}000 +$

$+ 63{,}333.33 \cdot 50{,}000 = 5\cdot 10^9 + 2025\cdot 10^6 + 3{,}166\cdot 10^9 =$

$= 10{,}191 \cdot 10^9 = 10{,}191\cdot 10^6.$

If university expands the stadium to 120,000 seats

$\Rightarrow$ Cost of expansion:

$C^{ex} = (120{,}000 - 50{,}000)\cdot 100 = 7\cdot 10^6.$

Revenue from selling tickets:

$R = 3\cdot p_a^* \cdot q_a + 3\cdot 15{,}000 \cdot 45{,}000 + 120{,}000 \cdot 40{,}000 =$

$3\cdot 25{,}000\cdot 75{,}000 + 2025\cdot 10^6 + 48\cdot 10^8 = 12.45\cdot 10^9$

$\Rightarrow$ profit after expansion:

$R - C^{ex} = 12.45\cdot 10^9 - 7\cdot 10^6 =$

$= 12{,}443\cdot 10^6 > 10{,}191\cdot 10^6$

$\Rightarrow$ I recommend that the University of California to expand the stadium.

№ 11.1

a) Since OEM is perfectly competitive =>

$p = MC = \$900 + \$100 + \$w = \$(1000 + w)$

$\Rightarrow Q = 50 \cdot 10^6 - 10 \cdot 10^3 \cdot p = 40 \cdot 10^6 - w \cdot 10^4.$

b) $\pi_{MCSF} = Q \cdot w = (40 \cdot 10^6 - w \cdot 10^4) \cdot w$

$\frac{\partial \pi_{MCSF}}{\partial w} = 40 \cdot 10^6 - 2 \cdot 10^4 \cdot w = 0$

$\Rightarrow w^* = 2000$

$\pi_{MCSF} = (40 \cdot 10^6 - 2 \cdot 10^3 \cdot 10^4) \cdot 2000 = \cancel{4 \cdot 10^{6}} \cancel{40{,}000}$

$= 4 \cdot 10^{10}$ - profit of Microsoft

$\pi_{OEM} = 0$ (Since $p = MC$).

$p = 10^3 + 2000 = 3 \cdot 10^3$ - price of computer.

c) $MC = 900 + 100 = 1000$

$\Rightarrow \pi = (p - MC) \cdot Q = (p - 1000)(50 \cdot 10^6 - 10 \cdot 10^3 p)$

$\frac{\partial \pi}{\partial p} = 50 \cdot 10^6 - 20 \cdot 10^3 p + 10^7 = 0$

$\Rightarrow p^* = 3000$ - price of computer

$\Rightarrow \pi = 4 \cdot 10^{10}$ - profit of vertically integrated firm

d). No, it could not. Since OEM is perfectly competitive, it will charge $p_{OEM} = MC = 1000$ for a computer with no operating system. So, the marginal cost for Microsoft is $1000 \Rightarrow p = 3000$

$\pi = 4 \cdot 10^{10}$