PS #4 Answer Key

N = 1.

(a) Denote \( q_{-i} \) the amount all firms but firm i produce. Then, \( Q = q_{-i} + q_i \), where \( q_i \) the amount firm i produce.

\[ \frac{\partial \pi_i}{\partial q_i} = (100 - q_i - q_{-i})q_i - (25 - 6 + 20q_i) \]

\[ \frac{\partial \pi_i}{\partial q_i} = 100 - 2q_i - q_{-i} - 20q_i = 0 \]

\[ \Rightarrow q_i (Q_i) = \frac{100 - Q_i}{22} \]

Since all firms have identical cost function and production technologies, \( \Rightarrow \) in equilibrium, they produce the same amount of output.

Therefore, in equilibrium:

\[ Q_i = (n-1)q^* \]

\[ q_i = q^* \]

\[ Q = nq^* \]

\[ \Rightarrow q^* = \frac{100 - (n-1)q^*}{22} \]

\[ \Rightarrow q^* = n = \frac{100 - 21q^*}{q^*} \quad (1) \]

Since, in the long run equilibrium, each firm makes zero profit \( \Rightarrow \pi_i(q^*_i, Q) = 0 \) \( \forall i = 1, n \)

\[ \Rightarrow (100 - q^* - (n-1)q^*)q^* - (256 + 20q^*) = 0 \]

\[ \Rightarrow n = \frac{100}{q^*} - \frac{256 + 20q^*}{q^*} \quad (2) \]
From (11, 12) ⇒

\[ n = \frac{100}{q^*} + 256 + 20q^* = \frac{100}{q^*} \]

⇒ \[ 21(q^*)^2 - 20q^* - 256 = 0 \]

(Two solutions, positive and negative, but \( q^* > 0 \))

⇒ \[ q^* = 4 \]

⇒ \[ n^* = \frac{100 - 21q^*}{q^*} = 4 \]

So, in LR equilibrium there are 4 firms,

(6).

\[ q^* = \frac{4}{4} = 4 \text{, } q = 4 \text{ - industry output} \]

\[ p = 100 - q^* = 84 \text{ - price} \]

\[ \pi_i^* = 0 \text{, } i = 1, n \text{ - profit} \]

\[ N = 10.8 \]

a)

\[ q_m = 180 - 2p_m \]

\[ q_h = 150 - 2p_h \]

\[ MC = 20 \]

\[ \pi_m = (p_m - MC)q_m = (p_m - 20)(180 - 2p_m) \]

⇒ \[ \frac{\partial \pi_m}{\partial p_m} = -4p_m + 320 = 0 \Rightarrow \]

\[ p_m = \frac{320}{4} = 80 \text{ - price on that day} \]

\[ \pi^h = (p_h - MC)q_h = (p_h - 20)(160 - 2p_h) \]

\[ \frac{\partial \pi^h}{\partial p_h} = -4p_h + 200 = 0 \Rightarrow p_h = 50 \text{ - price on "low" day} \]
(b) Equal number of "Hot" and "Cold" days \\
\[ \text{Prob ("Hot") = Prob ("Cold") = } \frac{1}{2} \]
\[ \Rightarrow \text{Expected profit} = (p - mc) \left( \frac{1}{2} \cdot q_h + \frac{1}{2} \cdot q_c \right) = \\
= (p - mc) \cdot (220 - ap) = (p - 20)(220 - 2p) \]
\[ \Rightarrow \frac{d\pi}{dp} = -4p + 280 = 0 \]
\[ \Rightarrow p = 65 \]

(c). Profit under constant price:
\[ \pi^c_{\text{H}} = (65 - 20)(220 - 2 \cdot 65) = 6750 \quad \text{on "Hot" day} \]
\[ \pi^c_{\text{C}} = (65 - 20)(160 - 2 \cdot 65) = 1350 \quad \text{on "Cold" day} \]

Profit under variable price:
\[ \pi^v_{\text{H}} = (80 - 20) \cdot 120 = 9600 \quad \text{on "Hot" day} \]
\[ \pi^v_{\text{C}} = (50 - 20) \cdot 80 = 1600 \quad \text{on "Cold" day} \]

Assume the vending machine can last only for 2 days (you may make a different assumption here)

\[ \Rightarrow \text{Constant price policy:} \]
\[ \pi^c_{\text{H}} + \pi^c_{\text{C}} = 6750 + 1350 = 8100 \]

Variable price policy:
\[ \pi^v_{\text{H}} + \pi^v_{\text{C}} = 9600 + 1600 = 11200 \]

\[ \Rightarrow 11200 - 8100 = 3100 \]

So, Coca-Cola would be willing to pay up to $3100 for a vending machine that lasts for 2 days.
\[ N = 10, 9 \]

\[ \text{Capacity: } N = 50,000 \]

\[ D_a = 150,000 - 3p \]
\[ D_{un} = 90,000 - 3p \]
\[ D_{im} = 240,000 - 3p \]

(a) Since the cost of operating the stadium is independent of the number of tickets sold \( \Rightarrow \) profit maximization is equivalent to revenue maximization.

\[ P_a = \frac{D_a}{P_a} = \frac{150,000 - 3p}{P_a} \]
\[ \frac{\partial P_a}{\partial P_a} = 150,000 - 6P_a = 0 \]
\[ \Rightarrow P_a = \frac{150,000}{6} = 25,000 \]

Since \( 25,000 > 0 \) \( \Rightarrow \) \( P_a = 150,000 - 3 \times 25,000 = 75,000 > 59,000 \)
\[ P_a^* = \frac{150,000 - 50,000}{3} = 33,333 \text{.} \]
\( \Rightarrow \) the price of ticket on average game.

\[ D_{un} = (90,000 - 3P_{un})P_{un} \]
\[ \frac{\partial D_{un}}{\partial P_{un}} = 90,000 - 6P_{un} = 0 \]
\[ \Rightarrow P_{un} = 15,000 \]
\[ P_{un} = 45,000 < 50,000 \]
\[ \Rightarrow P_{un} = 15,000 - \text{the price of ticket on not important game} \]

\[ D_{im} = (240,000 - 3P_{im})P_{im} \]
\[ \frac{\partial D_{im}}{\partial P_{im}} = 240,000 - 6P_{im} = 0 \]
\[ \Rightarrow P_{im} = 40,000 \]
\[ P_{\text{im}} = \frac{240,000 - 50,000}{3} = 63,333.33 \] (Price of ticket on the important game.

6) Profit without capacity expansion:

\[ 3 \cdot P_{\text{im}}^* + 3 \cdot P_{\text{im}} + P_{\text{im}} = 2 \cdot 3.333,333.33 \cdot 50,000 + 3 \cdot 15,000 \cdot 45,000 + 63,333,333.33 \cdot 50,000 = 5.10^9 + 2025 \cdot 10^6 + 3.166 \cdot 10^9 = 2 \cdot 10^9 + 191 \cdot 10^6. \]

If university expands the stadium to 120,000 seats

3) Cost of expansion:

\[ C_{\text{ex}} = (120,000 - 50,000) \cdot 100 = 7 \cdot 10^6. \]

Revenue from selling tickets:

\[ R = 3 \cdot P_{\text{im}}^* \cdot N_{\text{a}} + 3 \cdot 15,000 \cdot 45,000 + 120,000 \cdot 40,000 = 3 \cdot 25,000 \cdot 75,000 + 2025 \cdot 10^6 + 48 \cdot 10^6 = 12,45 \cdot 10^9. \]

4) Profit after expansion:

\[ R - C_{\text{ex}} = 12,45 \cdot 10^9 - 7 \cdot 10^6 = 12,443 \cdot 10^6 > 10,191 \cdot 10^6. \]

\[ \Rightarrow \text{I recommend that the University of California expand the stadium.} \]
a) Since OEM is perfectly competitive =>
\[ P = MC = 900 + 100 + \omega = \$1000 + \omega \]
\[ \Rightarrow Q = 50 \cdot 10^6 - 10 \cdot 10^3 \cdot P = 40 \cdot 10^6 - \omega \cdot 10^3. \]

b) \[ \Pi_{\text{HSEF}} = Q \cdot \omega = (40 \cdot 10^6 - \omega \cdot 10^3) \cdot \omega \]
\[ \frac{\partial \Pi_{\text{HSEF}}}{\partial \omega} = 40 \cdot 10^6 - 2 \cdot 10^3 \cdot 10^3 \cdot \omega = 0 \]
\[ \Rightarrow \omega^* = 2000 \]
\[ \Pi_{\text{HSEF}} = (40 \cdot 10^6 - 2 \cdot 10^3 \cdot 10^4) \cdot 2000 = \frac{4 \cdot 10^{10}}{4 \cdot 10^6} = 4.10^{10} \quad \text{profit of Microsoft} \]
\[ \Pi_{\text{OEM}} = 0 \quad (\text{since} \quad P = MC), \]
\[ P = 10^3 + 2000 = 3 \cdot 10^3 \quad \text{price of computer} \]

\[ \Rightarrow (P - MC) \cdot Q = (P - 1000)(50 \cdot 10^6 - 10 \cdot 10^3 \cdot P) \]

\[ \frac{\partial \Pi}{\partial P} = 50 \cdot 10^6 - 20 \cdot 10^3 \cdot P + 10^3 = 0 \]
\[ \Rightarrow P^* = 3000 \quad \text{price of computer} \]
\[ \Rightarrow \text{profit of vertically integrated firm} \]

\[ \text{c) No, it would not. Since OEM is perfectly competitive, it will charge } P = MC = 1000 \]
\[ \text{for a computer with no operating system, so,} \]
\[ \text{the marginal cost for Microsoft } 11 \cdot 1000 \Rightarrow P = 3000 \]
\[ \Rightarrow \text{profit of vertically integrated firm} \]