PS#3

1. $V_C \geq V_D \quad \Rightarrow \quad \rho \geq \dfrac{\pi^D - \pi^M}{\pi^D - \pi^N}$

b.1. $V_C^{ex} = V_1 + V_2 + V_3 = (1-a)(\pi^M + \rho V_C) + a(1-s)(\pi^M + \rho V_C) + as(\pi^M - F + \dfrac{\rho}{1-\rho}\pi^N) = \pi^M\left(\dfrac{1-as}{1-\rho} + as\right) + as\left(\dfrac{\rho}{1-\rho}\pi^N - F\right)$

b.2. $V_C^{ex} \geq V_D$

$$\pi^M\left(\frac{1-as}{1-\rho} + as\right) + as\frac{\rho}{1-\rho}\pi^N - asF \geq \pi^D + \frac{\rho\pi^N}{1-\rho}$$

$$\Rightarrow \quad \rho \geq \frac{\pi^D - \pi^M + asF}{\pi^D - \pi^N + as(\pi^N + F - \pi^M)},$$

№8.3.

a) Let $MC = c$; later assume $c = 0$, and both make zero profit.

$NE = \{ p_A = c,\ p_B = c \}$

Proof. If one firm deviates from $p_A = c$ by increasing the price, then it will get zero demand and zero profit $\Rightarrow$ not profitable to deviate.

If one firm deviates from $p = c$ by decreasing the price, the it will make negative profit, because price will be lower than marginal cost $\Rightarrow$ not profitable to deviate.; Here $\pi_N = 0$.

Note: You have to discuss/prove why you got particular NE for full credit.

b) Since problem is symmetric, joint profit would be maximized if both firm collude setting monopoly price:

$\pi = Q(p - MC) = (100-p)(p-c)$

$\frac{\partial \pi}{\partial p} = 100 - 2p = 0 \Rightarrow p_M^* = 50$

$p_1^m = p_2^m = 50,$ $\pi_M = \frac{1}{2}(100-50)(50-c) = \frac{2500}{2} = 1250$

c. Here we have, $\delta = \frac{1}{1+r} = \frac{1}{1+0.1} = \frac{10}{11}$

$V_C = \pi_M + \sum_{t=1}^{\infty} \delta^t \pi_M = \left(\frac{1}{1-\delta}\right)\pi_M = \frac{\pi_M}{1-\delta}$

$V_D = \pi^D + \sum_{t=1}^{\infty} \delta^t \pi_N = \pi^D + \frac{\delta}{1-\delta} \cdot 0 = \pi^D$

Let's compute π^D:

If firm deviates, it captures the whole market deman

So, it sufficient ~~to~~ do decrease price a little bit, and make monopoly profit, $\pi = 2500 = 2 \cdot \pi_M$.

$\Rightarrow V_D = 2\pi_M$

Since $V_C = \left(\frac{1}{1-\delta}\right)\pi_M$, and $V_D = 2\pi_M$, $\delta < 1$

$\Rightarrow V_C \geq V_D \quad \left(\left(\frac{1}{1-\delta}\right)\pi_M \geq 2\pi_M \right) \Rightarrow$

It's ~~not~~ one repeated-game NE to charge

$p_A = p_B = 50$ given $r = 10\%$.

If $r = 110\% \Rightarrow \delta = \frac{1}{1+1.1} = \frac{10}{21}$

$V_c = \frac{\pi_M}{1-\delta} = \frac{21}{11}\pi_M$, $V_D = 2\pi_M$ $\frac{21}{11}\pi_M < 2\pi_M$, $V_c < V_D$

$\Rightarrow$ It's <u>not</u> one repeated-game NE to charge

$p_A = p_B = 50$ given $r = 110\%$

$V_C \geq V_D \Rightarrow$

$\frac{\pi_M}{1-\delta} \geq 2\pi_M \Rightarrow$

$\frac{1}{1-\delta} \geq 2 \Rightarrow 1-\delta \leq \frac{1}{2} \Rightarrow \delta \geq 1/2$

$\delta = \frac{1}{1+r} \geq \frac{1}{2} \Rightarrow$

$1+r \leq 2 \Rightarrow r \leq 1 = 100\%$

The highest interest rate is 100%,

d. If neither firm was certain that it would be able to detect changes in it's rival price, it would charge $p = mc$. It's because, it can not threaten the other firm. However, if firms observe the demand, and condition their strategies on it, the answer to c does not change.

e). No, it's not. Given that A decreases, ~~from~~ from some year firm will make negative profit while setting $p = 50$ each period.

№ 8.15.

Let π^M - monopoly profit of firm i in market i, $i = 1, 2$

π_0^M - ~~monopoly~~ maximum profit of firm i in market j, $i = 1, 2$, $j = 1, 2$, $i \neq j$, given it undercuts firm j price. $\bar{c} > \underline{c}$.

Clearly, $\pi^M > \pi_0^M > 0$, since

π_0 - profit of firm i in market i, when firm j set price $= \bar{c}$ in market i.

$\pi_0 < \pi^M$.

To make collusion possible:

$\frac{\pi^M}{1-\delta} \geq \pi^M + \pi_0^M + \frac{\delta}{1-\delta} \cdot \pi_0 \Rightarrow$

$$\pi^M \geq (1-\rho)(\pi^M - \pi_0^M) + \rho\,\pi_D$$

$$\Rightarrow \rho(\pi^M - \pi_0^M - \pi_D) \geq -\pi_0^M$$

$$\Rightarrow \rho \geq \frac{\pi_0^M}{\pi_0^M + \pi_D - \pi^M}.$$