1. $V_c > V_o \implies \rho > \frac{\pi_p - \pi_n}{\pi_p - \pi_n}$

b.1. $V_c^e = V_i + V_c + V_3 = \rho (\pi_p + \rho V_c) + \alpha (1 - \alpha) (\pi_p + \rho V_c) + as(\pi_p - F + \frac{\rho}{1-\rho} \pi_n) = \pi_p \left( \frac{1 - \alpha s}{1-\rho} + as \right) + \alpha s \left( \frac{\rho}{1-\rho} \pi_n - F \right)$

b.2. $V_c^e > V_o$

$\pi_p \left( \frac{1 - \alpha s}{1-\rho} + as \right) + \alpha s \pi_n - as F \geq \pi_p + \frac{\rho \pi_n}{1-\rho}$

$\implies \rho \geq \frac{\pi_p - \pi_p + as F}{\pi_p - \pi_n + as (\pi_n + F - \pi_p)}$

$N^e = 1.3.$

a) Let $MC = \tau$; later assume $c = 0$.

$NE = \frac{1}{2} \rho \pi = \tau, \rho \pi = \tau^2$, and both make zero profit.

Proof. If one firm deviates from $\rho = c$ by increasing

the price, then it will get zero demand and zero

profit \implies not profitable to deviate.

If one firm deviates from $\rho = c$ by decreasing

the price, the it will make negative profit, because

the price is lower than marginal cost \implies not

profitable to deviate.) Here $\pi_n = 0$.

Note: You have to discuss/prove why you got

particular $NE$ for full credit.

b) Since problem is symmetric, joint profit

would be maximized if both firms collude

setting monopoly price!
\[ \pi = \frac{\partial \rho}{\partial \mu} (\rho - \mu) = (100 - \rho) / (\rho - \mu) \]
\[ \frac{\partial \pi}{\partial \rho} = 100 - 2\rho = 0 \Rightarrow \rho^* = 50 \]
\[ \pi^m = \rho^m = 50 \]
\[ \pi_M = \frac{1}{2} (100 - 50) / 50 - 50 = \frac{2500}{2} = 1250 \]

C. Here we have, \( S = \frac{1}{1 + r} = \frac{1}{1 + 0.1} = \frac{10}{11} \)

\[ V_C = \frac{\pi_M}{S} \]
\[ V_D = \frac{\pi_D}{S} \]

Let's compute \( \pi_D \):

If firm decreases, it captures the whole market demand.
So, if sufficient, do decrease price a little bit, and
make monopoly profit, \( \pi = 2500 = 2 \cdot \pi_M \),

\[ \Rightarrow V_0 = 2\pi_M \]

Since \( V_C = \frac{\pi_M}{S} \), and \( V_0 = 2\pi_M \), \( S < 1 \)

\[ \Rightarrow V_C \geq V_0 \bigg( \frac{1}{1 + \rho} \pi_M \geq 2\pi_M \bigg) \Rightarrow \]

It's not one repeated-game NE to charge
\( \rho_A = \rho_B = 50 \) given \( r = 10\% \).

If \( r = 10\% \) \( \Rightarrow \) \( S = \frac{1}{1 + 0.1} = \frac{10}{21} \)

\[ V_C = \frac{\pi_M}{1 + 0} = \frac{\pi_M}{1}, V_0 = 2\pi_M \]
\( \frac{21}{11}\pi_M < 2\pi_M \), \( V_C < V_0 \)

\[ \Rightarrow \text{It's not one repeated-game NE to charge} \]
\( \rho_A = \rho_B = 50 \) given \( r = 110\% \)
\[ V_c > V_0 \Rightarrow \]
\[ \frac{\Pi^M}{1-g} > 2\Pi^M \Rightarrow \]
\[ \frac{1}{1-g} > 2 \Rightarrow 1-g \leq \frac{1}{2} \Rightarrow g \geq \frac{1}{2} \]
\[ g = \frac{1}{1+r} \geq \frac{1}{2} \Rightarrow \]
\[ 1+r \leq 2 \Rightarrow r \leq 1 = 100\% \]

The highest interest rate is 100%.

d. If neither firm was certain that it would be able to detect changes in its rival's price, it would charge \( p = mc \). It is because it cannot threaten the other firm. However, if firms observe the demand, and condition their strategies on it, the answer to c does not change.

e. No, it is not. Given that A decreases, some year firm will make negative profit while setting \( p = 0 \) each period.

\[ JV = 8.15 \]

Let \( \Pi^M \) - monopoly profit of firm \( i \) in market \( i \), \( i=1,2 \)
\[ \Pi^M \] - maximum profit of firm \( i \) in market \( i \), given it unprices \( \Pi^M \)
\[ \Pi^o \] - profit of firm \( j \) set price = \( c \) in market \( i \), when \( \Pi^o < \Pi^M \).

Clearly, \( \Pi^M > \Pi^o \) since \( c > \overline{c} \)
\[ \Pi^o < \Pi^M \]

To make collusion possible:
\[ \frac{1}{1-g} > \Pi^M + \Pi^o + \frac{g}{1+r} \cdot \Pi_0 \Rightarrow \]
\[ \pi^M \geq (1 - \phi) (\pi^M - \pi_0^M) + \phi \pi_0 \]

\[ \Rightarrow \phi (\pi^M - \pi_0^M - \pi_0) \geq -\pi_0^M \]

\[ \Rightarrow \frac{\pi_0^M}{\pi_0^M + \pi_0 - \pi^M} \geq \phi \]