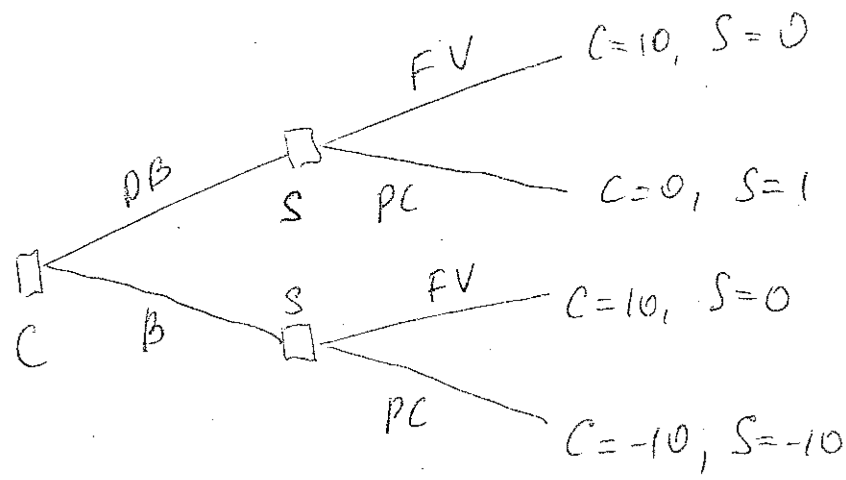
HW #2

1.



a. Soldiers have four strategies:

$S_1 = (FV(DB), FV(B))$

$S_2 = (PC(DB), FV(B))$

$S_3 = (FV(DB), PC(B))$

$S_4 \ (PC(DB), PC(B))$

Cortes has two strategies:

$C_1 = (DB)$

$C_2 = (B)$

Two find all Nash equilibria, construct a table:

S

C	S_1	S_2	S_3	S_4
DB	10, 0	0, 1	10, 0	0, 1
B	10, 0	10, 0	-10, -10	-10, -10

So, there are three Nash equilibria:

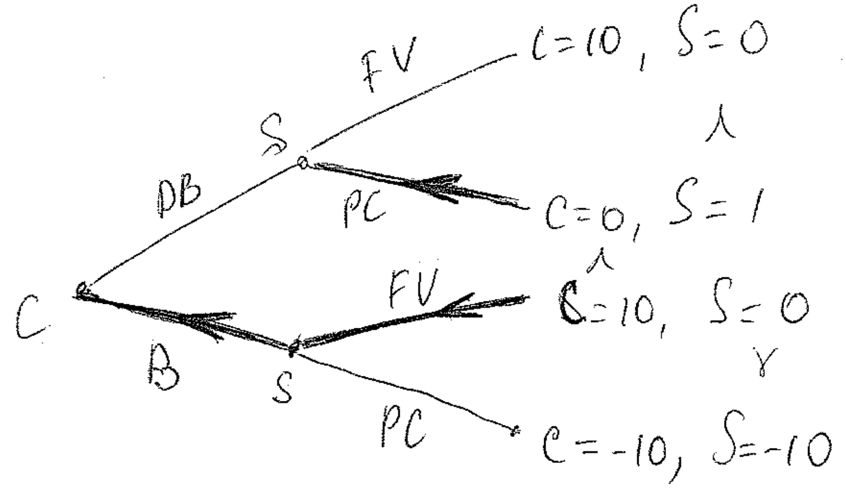
NE = { 1. C goes B; S goes FV if C goes DB, and S goes FV if C goes B;

2. C goes B; S goes PC if C goes DB, and S goes FV if C goes B;

3. C goes DB; S goes PC if C goes DB, and S goes PC if C goes B. }.

Note! In Nash equilibrium strategies not actions have to be specified.

b). We solve the problem by backward induction when searching for subgame perfect NE.



SPNE = { C goes B; S goes FV if C goes B
S goes PC if C goes DB }.

Note! There is only one SPNE, but when specifying SPNE, we have to specify strategy of the player, not his/her action.

№ 2

a) $mc(q) = P$

$$Q^d = \frac{6000 - 50P}{9} \Rightarrow 6000 - 50P = 9(n \cdot q) = 450q \Rightarrow$$

$$P = \frac{6000 - 450q}{50} = 120 - 9q$$

$$\Rightarrow 2q + 10 = 120 - 9q \Rightarrow q^*_{pm} = 10$$

б) $p = 2q + 10 \Rightarrow q = \frac{p-10}{2}$

Industry supply curve:

$$Q^s = nq = \begin{cases} 50\,\frac{p-10}{2} = 25p - 250, & p \geq 10 \\ 0, & p < 10. \end{cases}$$

c) $Q^s = Q^d \Rightarrow \frac{6000 - 50p}{9} = 25p - 250$

$$\Rightarrow \underline{p^* = 30}$$

$$Q^* = 25 \cdot 30 - 250 = \underline{500}.$$

№ 3.

a) $R = P(Q) \cdot Q = \left(120 - \frac{9}{50}Q\right)Q$

$$\Rightarrow mR = \frac{\partial R}{\partial Q} = 120 - \frac{18}{50}Q = 120 - \frac{9}{25}Q;$$

б) $mR = mc \Rightarrow 120 - \frac{9}{25}Q = 10 + \frac{Q}{25}$

$$\Rightarrow 110 = \frac{10}{25}Q$$

$$\Rightarrow Q^* = 275$$

$$P^* = 120 - \frac{9}{50}Q^* = 70.5$$

c) $P_2 = P\cdot Q - MC\cdot Q = 275\cdot 70.5 - (10 + \frac{275}{25})\cdot 275 =$

$= 275(70.5 - 21) = 13612.5$

Here I assume zero fixed costs.

№ 6.2.

Public is notified all detergents are all alike =>

- shifts to buying the cheapest detergent
- firms have no reason to pay differentiated part of cost (e.g. advertising cost) => total cost decreases => marginal cost decreases => firm will lower the price to attract demand

=> ~~at the~~ during this process profit is positive => new firms will enter the market

=> price will be decreasing till not equal to marginal cost.

=> demand will increase due to price increase

=> at the end, demand would be distributed uniformly among all firms

So, price decreases, total sales increases, number of firms increases.

№ 7.4.

In this problem, it is not stated what in terms of number of output capacity is equal to. Therefore, I assume that 1 capacity = 1 output of good (having 1 capacity means that you are able to produce at most 1 good in each period).

Since interest cost are zero, and capacity ~~is~~ can lasts for 4 years, then producing 1 good each year would lead to marginal cost $\frac{2400}{4} = 600.$

a) CS profit maximizing problem:

$$\max_q p\cdot q - 600\cdot q = \max_q \left((1200-(q+100))q - 600q\right)$$

Taking F.O.C:

$$1200 - 2q - 100 - 600 = 0$$

$$\Rightarrow q_{CS}(100) = \frac{500}{2} = 250$$

$\Rightarrow$ CS would want to build 250 units of capacity (here 1 good = 1 capacity) if jl were going to build 100 units of capacity.

If jl were going to build x units of capacity, the CS best response function is:

$$\max_q p\cdot q - 600q = \max_q \left((1200-(q+x))q - 600q\right)$$

FOC:

$$1200 - 2q - x - 600 = 0$$

$$\Rightarrow q_{BR}^{CS}(x) = \frac{600-x}{2} = 300 - \frac{x}{2}$$

$q_{BR}^{CS}(x) = 300 - \frac{x}{2}$ – CS best response function

b) Since the game is symmetric $\Rightarrow$ CS and jl best respone functions coincide

$$q_{BR}^{CS}(x) = q_{BR}^{jl}(x)$$

$\Rightarrow$

$$q_1 = 300 - \frac{300 - q_1/2}{2} \Rightarrow 2q_1 = 600 - 300 + q_1/2$$

$$\Rightarrow \frac{3}{4} q_1 = 300 \Rightarrow q_1 = 400$$

So, one-shot NE is

$q^{cs} = 400$ units of capacity
$q^{sl} = 400$ units of capacity.

№ 7.5.

$Q = 37.5 - P/4$; $mc^i = 40$, $i = \overline{1,2}$

a) $\pi_1 \cancel{= (p - mc)q = (150 - 4(q_1+q_2))q}$

$Rev_i = (150 - 4(q_1+q_2))q_1$

Marginal Revenue $= 150 - 8q_1 - 4q_2 = MR$

$MR = mc \Rightarrow$

$150 - 8q_1 - 4q_2 = 40$

$\Rightarrow q_1^*(q_2) = \dfrac{110 - 4q_2}{8}$ – best response of firm 1.

Symmetric game $\Rightarrow$ In equilibrium:

$$q_1^* = \frac{110 - 4q_2^*}{8} = \frac{110 - 4 \cdot \frac{110 - 4q_1^*}{8}}{8}$$

$\Rightarrow 8q_1^* = 110 - 55 + 2q_1^*$

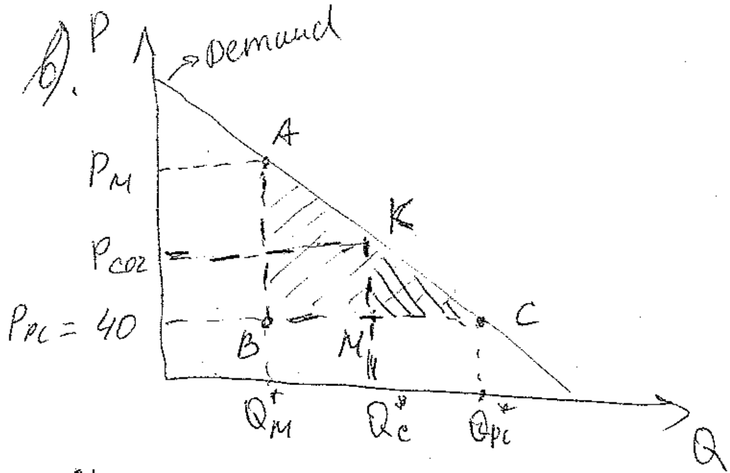
$\Rightarrow 6q_1^* = 55 \Rightarrow$

$q_1^* = q_2^* = \dfrac{55}{6} \Rightarrow Q_c^* = \dfrac{55}{3}$

Cournot equilibrium $= \{ q_1^* = \dfrac{55}{6},\ q_2^* = \dfrac{55}{6} \} \Rightarrow$

$p^*_{cor} = 150 - 4\left(\dfrac{55}{6} + \dfrac{55}{6}\right) = \dfrac{230}{3} \approx 76.67$

b).



Monopoly problem:

$$\max_Q (150-4Q)Q - TC$$

$$\Rightarrow \quad 150 - 8Q = MC = 40$$

$$\Rightarrow \quad Q_M^* = \frac{110}{8} = \frac{55}{4}$$

$$P_M^* = 150 - 4 \cdot \frac{55}{4} = 95$$

Perf. Competion: $P = MC = 40 \Rightarrow$

$$150 - 4Q = 40$$

$$\Rightarrow \quad Q_{PC}^* = \frac{110}{4} = \frac{55}{2}$$

$$\Rightarrow \quad P_{PC}^* = 150 - 4 \cdot \frac{110}{4} = 40$$

ABC – deadweight loss under monolopy

KMC – deadweight loss under Cournot

$$S_{ABC} = \frac{1}{2} BC \cdot AB = \frac{1}{2}(Q_{PC}^* - Q_M^*) \cdot (P_M - 40) =$$

$$= \frac{1}{2}\left(\frac{55}{2} - \frac{55}{4}\right) \cdot (95 - 40) = \frac{1}{2} \cdot \frac{55}{4} \cdot 55 = \frac{55^2}{8}$$

$$S_{KMC} = \frac{1}{2} KM \cdot MC = \frac{1}{2}(P_{COZ}^* - 40) \cdot \left(\frac{55}{2} - \frac{55}{3}\right) =$$

$$= \frac{1}{2}\left(\frac{230}{3} - \frac{120}{3}\right) \cdot \frac{55}{6} = \frac{1}{2} \cdot \frac{110}{3} \cdot \frac{55}{6} = \frac{55^2 \cdot 2}{6^2}$$

$$\Rightarrow \frac{S_{KMC}}{S_{ABC}} = \frac{55^2 \cdot 2}{6^2} \Big/ \frac{55^2}{8} = \frac{2 \cdot 8}{6^2} = \frac{16}{36} = \frac{4}{9} \approx 44.4\%$$

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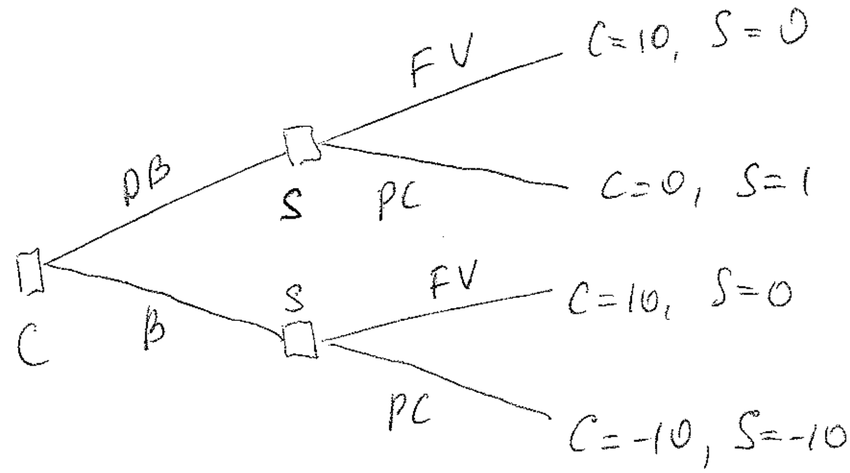

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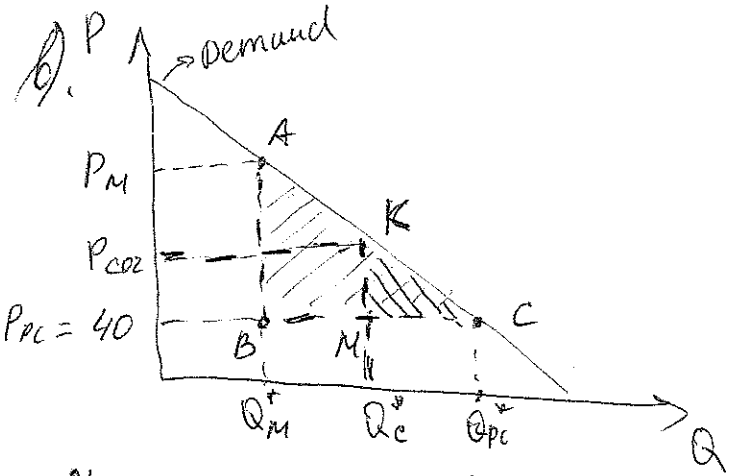

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