

### Problem Set 3

1. In this question we will consider the basic collusive duopoly setting discussed in class but now introduce the potential for antitrust authorities to impose fines and breakup the cartel.

First, let's go back to the basic setup. Let  $\pi^M$  denote the profits in a period when the firms collude. If firms collude in all periods, the value of sticking to the agreement is:

$$V_C = \frac{\pi^M}{1 - \rho}$$

where  $\rho$  is the discount factor. Now let  $\pi^D$  denote the profits from deviating and  $\pi^N$  denote the profits when neither firm cooperates. From the strategy we discussed in class, the value to a firm to breaking the agreement is

$$V_D = \pi^D + \frac{\rho\pi^N}{1 - \rho}$$

- (a) For what values of  $\rho$  will the cartel arrangement be sustainable?
- (b) Now we will introduce an antitrust authority, which attempts to look for and prosecute cartels. In any given period, assume there is a probability  $a$  that the authority will investigate. If there is no investigation the cartel continues to the following year. The investigation takes 1 period and we assume there is a probability  $s$  that it leads to successful prosecution, in which case each cartel member is subjected to a fine of  $F$ , and the cartel breaks down. If the investigation is unsuccessful the cartel continues.

We will evaluate the expected present value of the profits of each cartel member. Again, let  $\pi^M$  denote the profits in a period when there is collusion. To evaluate the expected value, we will have to consider 3 scenarios on what the antitrust authority does.

- (i) No investigation in period 0, which happens with probability  $(1 - a)$ , so the expected profit is

$$V_1 = (1 - a)(\pi^M + \rho V_C)$$

where  $\rho, V_C$  are defined as above.

**(ii)** Unsuccessful investigation in period 0, which happens with probability  $a(1 - s)$ , in which case the expected profit is

$$V_2 = a(1 - s)(\pi^M + \rho V_C)$$

**(iii)** Successful investigation, which happens with probability  $as$ -each firm is fined  $F$ , and the collusion is broken, resulting in an expected profit of

$$V_2 = as(\pi^M - F + \frac{\rho}{1 - \rho}\pi^N)$$

**b.1** Determine the overall expected value of profit for a cartel member.

**b.2** At what range of values of  $\rho$  is the cartel arrangement sustainable?

2. Cabral 8.3.

3. Cabral 8.15.