Cournot Model

- Suppose now that firms make output decisions simultaneously before prices are chosen.
- Firms know that for each level of output chosen q_1, q_2, equilibrium prices will be $P(q_1 + q_2)$.
- This means profit for firm i is

$$\pi_i = q_i(P(q_1 + q_2) - c)$$

  where c is the constant marginal cost.
- This is the Cournot Model.
- Firms simultaneously choose the output they want to produce, and then the market price is set at the level at which demand equals total quantity produced.

Cournot Model

- As before, we'll solve for the equilibrium outcome in two steps.
- First we'll derive the firm's optimal choice given its conjecture on what the other firm does (reaction curve).
- Next we put both reaction firms together and find a mutually consistent set of actions and conjectures.
- To derive the optimal output in the first stage, we first suppose Firm 1 believes Firm 2 will produce q_2.
- The demand curve facing Firm 1 is the residual demand curve $d_1(q_2)$).

Cournot Model

- Facing this residual demand curve, Firm 1 the optimal output is the same as in the monopolist setting.
- Optimal output will be the level where marginal revenue equals marginal cost.
- We will denote this optimal this conjecture based optimal output as $q_1^*(q_2)$.
- This varies with q_2. If $q_2 = 0$, Firm 1 is a monopolist, so $q_1^*(0) = q^M$.
- If Firm 2 produces the quantity corresponding to 0 profits, q^C, firm 1 produces nothing.

Cournot Model

- We can and will show that with a linear demand curve and constant marginal costs the function $q_1^*(q_2)$ is linear.
- We'll call this Firm 1's reaction function.
- Algebraic arguments:
- Let $Q = q_1 + q_2$, and let demand be given by

$$P(Q) = a - bQ$$

which implies profits are:

$$\pi_1 = (a - b(q_1 + q_2))q_1 - cq_1$$

Cournot Model

- Solving first order condition yields:

$$q_1^*(q_2) = \frac{a-c}{2b} - \frac{q_2}{2}$$

- As the last step in solving for an equilibrium, we find the point at which firms choose optimal quantities given their conjectures.
- An equilibrium corresponds to a pair (q_1, q_2) such that q_1 if Firm 1's optimal response given q_2 and vice versa.
- Graphically, this corresponds to where the two reaction curves intersecting.
- Algebraically, we have the system of equations:

$$\begin{aligned} q_1^N &= q_1^*(q_2^N) \\ q_2^N &= q_2^*(q_1^N) \end{aligned}$$

Cournot Model

- Because the two firms are identical, the equilibrium will be symmetric:

$$q_1^N = q_2^N = q^N$$

- So solving we get

$$\begin{aligned} q^N &= \frac{a-c}{2b} - \frac{q^N}{2} \\ &= \frac{a-c}{3b} \end{aligned}$$

Cournot Model

- What's particularly useful about this model is we get what we'd expect.
- Duopoly is a market structure intermediate to monopoly and perfect competition.
- One might expect equilibrium output and price to lie between what we got in sections 5.1 and 6.1.
- This follows from the equilibrium in the Cournot Model.
- Specifically, we found that Cournot equil. output is greater than the monopoly output and lower than the perfect competition output.
- And the Cournot equil. price is lower than the monopoly price and greater than the perfect competition price.