Sequential Games

- When there is a sufficient lag between strategy choices our previous assumption of simultaneous moves may not be realistic.
- In these settings, the assumption of sequential decision making is more realistic.
- A prime example of this setting is when an industry is currently monopolized.
- Here, a second firm must decide whether or not to enter the industry.
- Given this decision, the monopoly decides whether to price aggressively or not.
Sequential Games

- The incumbent’s decision can be viewed as a function of the entrant’s decision.
- Specifically he observes whether or not the entrant firms does indeed enter,
- and then decides whether or not to price aggressively.
- In this setting we would have model the entrant moving first, and the incumbent moving second.
- We will model these types of games with a game tree.
Sequential Games

- This is basically a decision tree except there is more than one decision maker involved.
- In our notation, a circle will denote a decision node, with the number inside it denoting which player is making a decision.
- When the game ends, payoffs to each player are denoted in rectangles.
- Games represented in this way are referred to as games in extensive form.
- As we’ll see, many of these games will have multiple Nash equilibria.
Sequential Games

- Which is problematic as far as predicting what will happen.
- But as we’ll see, often it’s the case that one or more equilibria don’t “make sense”.
- One way we’ll eliminate “unreasonable” equilibria is solve the game “backwards.”
- This is effectively applying the principle of **backward induction**, which should help us predict how players will play.
- By solving the game backwards I mean we solve for optimal decisions at the lower nodes first, and working our way upwards to the top nodes.
Sequential Games

- Specifically, we will solve for Nash equilibria at lower nodes, which by themselves we will call a subgame.
- Then given this solution, solve for the entire game.
- Equilibria obtained this way, which are Nash at each subgame, are referred to as subgame-perfect equilibria.
- Equilibria that are dismissed because they are not subgame perfect can be resuscitated in different ways.
- Examples include enforceable and non-renegotiable contracts.
For example, in the entrant incumbent game, the contract would be such that the incumbent is forced to retaliate if the entrant enters.

Specifically, this can be done the incumbent paying a large fine if it does not retaliate.

We would model this with an extra node at the beginning of the game, where the incumbent chooses between writing the bond or not writing it.

Solving this more complicated game by backwards induction, we get the subgame equilibrium corresponding to the one we dismissed before, and the incumbent wins.
Sequential Games

- This has shown us that a **credible commitment** can have significant strategic value.
- Note that this can also be modeled by changing the order of the moves.
- In this case we would model it as the incumbent moving first.
Many realistic situations of strategic behavior are repeated over a period of time. Many times this cannot be modeled by a static model. One such example is the strategic action of retaliation. This is where one player changes its strategic variable in response to a rival’s action. This cannot be modeled with a static simultaneous-move game.
Repeated Games

- Instead we’ll model this as with a **repeated game**.
- When we consider a simultaneous choice game, when each player chooses one action only once, we’ll call it a **one shot game**.
- A **repeated game** is a one shot game which is repeated a number of times.
- In such settings, it is useful to distinguish between **actions** and **strategies**.
- We’ll define the latter as a complete player’s complete contingent plan of action for all possible occurrences in the game.
Repeated Games

- So, for example if a simple game is played twice a player has to indicate what action to take in the first period as well as what action to choose in period 2 as a function of what occurred in period 1.

- Thus the number of strategies can quickly become enormous, even if the game is only played twice.

- For example, in a simple $3 \times 3$ game played twice, each player has $3 \times 3^9$ strategies.

- This explosion in the number of strategies adds some interesting equilibria vis a vis the equilibria in the one shot game.

- We first note that the repeated play of equilibrium strategies in the one shot game form an equilibrium in the repeated game.
Repeated Games

- However, there are equilibria in the repeated game that do **not** correspond to equilibria of the one shot game.
- This is because players can react to other player’s past actions.
- For example, period 2 actions can be used to punish players in case they deviate from designated period 1 actions.