Final Exam Answer Key

№ 2.

$P_H = 16 - Q_H$

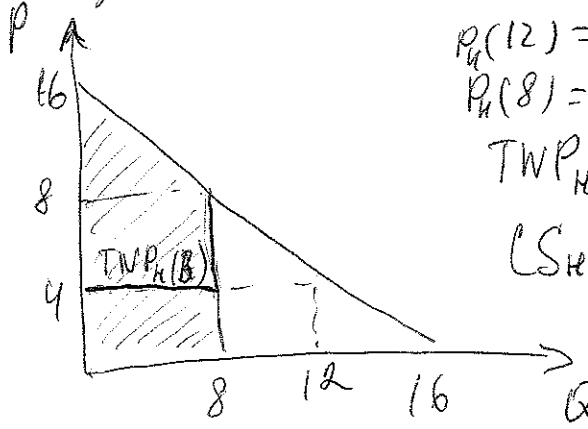
$P_L = 12 - Q_L$

$MC = 4$

In order for high demand consumer not to say he/she is low demand consumer, he/she has to get at least the same consumer surpleus from "menu" option designed for high demand consumer, as compared to CS extracted from "menu" option designed for low demand consumer. In the same way, low demand consumer has to extract at least the same or higher CS from "menu" option designed for him.

a) menu option A: choice of entry + 12 drinks for $20
menu option B: choice of entry + 8 desserts for $ 64.

High demand consumer



$P_H(12) = 16 - 12 = 4$

$P_H(8) = 16 - 8 = 8$

$TWP_H(A) = 12 \cdot 4 + \frac{1}{2} \cdot 12 \cdot 12 = 120$ – total willingness to pay of consumer H for "menu" option A.

$CS_H(A) = 120 - 120 = 0$ – consumer H surpleus from menu option A.

$TWP_H(B) = 8 \cdot 8 + 8 \cdot 8 \cdot \frac{1}{2} = 96$

$CS_H(B) = 96 - 64 = 32$

Since $CS_H(B) = 32 > 0 = CS_H(A)$ ⇒ high demand consumer would choose menu B

$P_L(12) = 12 - 12 = 0$

$P_L(8) = 12 - 8 = 4$

$TWP_L(A) = \frac{1}{2} \cdot 12 \cdot 12 = 72$

$\Rightarrow CS_L(A) = 72 - 120 < 0$

$TWP_L(B) = 4 \cdot 8 + \frac{1}{2} \cdot 8 \cdot 8 = 32 + 32 = 64$

$\Rightarrow CS_L(B) = 64 - 64 = 0 > CS_L(A)$

$\Rightarrow$ Low demand consumer will choose B.

$\Rightarrow$ This menu is not incentive competitive

b/ A: \$32 for entry + 8 tickets for \$4 each

B: \$72 for entry + 12 tickets for \$4 each drink

High demand consumer:

$TWP_H(A) = 96$

$CS_H(A) = 96 - 32 - 8 \cdot 4 = 32$

$TWP_H(B) = 120$

$CS_H(B) = 120 - 72 - 12 \cdot 4 = 0$

$\Rightarrow CS_H(A) > CS_H(B)$

High demand consumer will choose option A.

Low demand:

$TWP_L(A) = 64$

$CS_L(A) = 64 - 32 - 8 \cdot 4 = 0$

$TWP_L(B) = 72$

$CS_L(B) = 32 - 72 - 12 \cdot 4 < 0$

$\Rightarrow$ Low demand consumer would choose option A

=> This menu is not incentive compatible

c) A: entry + 8 drinks for $64
B: entry + 12 drinks for $88

H: $TWP_H(A) = 96$

$CS_H(A) = 96 - 64 = 32$

$TWP_H(B) = 120$

$CS_H(B) = 120 - 88 = 32$

=> $CS_H(A) = CS_H(B)$ => High demand consumer is indifferent between 2 options.

L: $TWP_L(A) = 64$;

$CS_L(A) = 64 - 64 = 0$

$TWP_L(B) = 72$

$CS_L(B) = 72 - 88 < 0$

$CS_L(A) > CS_L(B)$ => Low demand consumer will choose option A.

So, this menu of contracts is insentive competitive.

$N^0 = 12$.

(a)

	D	C
D	1, 1	5, 0
C	0, 5	4, 4

$1 > 0$, $5 > 4$ => "Defect" is the dominant strategy for each player.

b). From the table above, the Nash equilibrium of one-shot game is {Defect, Defect} = NE

(c) Since the one shot game has only one NE, (Defect, Defect), it is impossible to sustain (Cooperate, Cooperate) outcome in the first day of repeated game, because on the second day players will always play their single NE, (Defect, Defect). Therefore, they will play their single Nash equilibrium on the second day regardless of outcome of the game during first day.

So, the solution to the repeated game (s response to four possible outcome of the first day game

NE = { (Defect of the first day, (Defect, Defect, Defect, Defect)) ;
Strategy of the first player

(Defect on the first day, (Def., Def., Def., Def.)) }

Strategy of the second player

What to do on the second day (each strategy on every possible outcome of the first day game)

Suggame perfect NE!
The SPNE is found through backward induction.
See the explanation to this problem in your midterm # 1 exam answer key, Problem. 3.

SPNE = { (Defect, (Def., Def., Def., Def.)), (Def., (Def., Def., Def., Def.)) }.

So, SPNE = NE.

№3.

$P = 400 - 2Q$

$MC_1 = 25$

$MC_2 = 40$

a) $\pi_1 = (400 - 2(q_1 + q_2))q_1 - 25 \cdot q_1$

$$\frac{\partial \pi_1}{\partial q_1} = 400 - 4q_1 - 2q_2 - 25 = 0$$

$$\Rightarrow q_1^*(q_2) = \frac{375 - 2q_2}{4} \qquad (1)$$

$$\pi_2 = (400 - 2(q_1 + q_2))q_2 - 40q_2$$

$$\frac{\partial \pi_2}{\partial q_2} = 400 - 2q_1 + 4q_2 - 40 = 0$$

$$\Rightarrow q_2^*(q_1) = \frac{360 - 2q_1}{4} \qquad (2)$$

$$\Rightarrow q_2 = 90 - 0.5q_1 \overset{(1)}{=} 90 - \frac{1}{2} \cdot \left(\frac{375 - 2q_2}{4}\right)$$

$$8q_2 = 720 - 375 + 2q_2$$

$$q_2^* = 120 - 62.5 = 57.5$$

$$q_1^* = \frac{375 - 2 \cdot 57.5}{4} = \frac{260}{4} = 65$$

$$P = 400 - 2(57.5 + 65) = 400 - 115 - 130 = 155$$

(b) $\pi_1 = P \cdot q_1 - 25 \cdot q_1 = (155 - 25) \cdot 65 = 8450$

$\pi_2 = P q_2 - 40 \cdot q_2 = (155 - 40) \cdot 57.5 = 6612.5$

№4

$C(q) = 256 + 20q$

$P = 100 - Q$

a) $\pi = (100 - (n-1)q - \tilde{q}) \tilde{q} - 256 - 20\tilde{q}$

$\frac{\partial \pi}{\partial \tilde{q}} = 100 - (n-1)q - 2\tilde{q} - 20 = 0$

In equilibrium $q = \tilde{q} \Rightarrow$

$100 - (n+1)q = 20$

$\Rightarrow (n+1)q = 80 \quad (1)$

In long-run equilibrium: $\pi = 0$

$\Rightarrow (100 - nq) \cdot q = 256 + 20q$

$(80 - nq)q = 256 \quad (2)$

$\Rightarrow$ Plug (1) into (2) instead of nq

$\Rightarrow ((n+1)q - nq)q = 256$

$\Rightarrow q^2 = 256 \Rightarrow q^* = 16$

$\Rightarrow n^* = \frac{80}{q^*} - 1 = \frac{80}{16} - 1 = 4$

Equilibrium number of firms = 4.

(b) Lerner index:

$$L = \sum_{i=1}^{n} s_i \cdot \frac{P - MC}{P}$$

$MC = 20$

$P = 100 - 4 \cdot 16 = 36$

$s_i = \frac{1}{n} = \frac{1}{4} = 0.25$

$$\Rightarrow L = \sum_{i=1}^{4} \cdot \frac{1}{4} \cdot \frac{36-20}{36} = 4 \cdot \frac{1}{4} \cdot \frac{36-20}{36} = \frac{16}{36} = \frac{4}{9} \approx 0.444$$

(c) $$H = \sum_{i=1}^{4} s_i^2 = \sum_{i=1}^{4} \left(\frac{1}{4}\right)^2 = 4 \cdot \left(\frac{1}{4}\right)^2 = \frac{1}{4} = 0.25$$

$$C_{\frac{n}{2}} = C_2 = \sum_{i=1}^{2} s_i = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

$\underline{N^0 = 5}$

(a) Suppose there are two products A and B, and two consumers c and d.
Horizontal product differentiation is when consumer c prefers product A over product B, while consumer d prefers product B over product A.

Vertical product differentiation is when <u>both</u> consumers prefer one product, say A, over another, say B.

6)

	GB	SS	GHz	W	P
N_1	240	12	3	3	2000
N_2	120	15	1	6	500

	GB	SS	GHz	W	P
S	2	5	3	-1	-1
BP	5	2	5	-6	-1

Student's utility from notebook N_1:

$U_S(N_1) = 240 \cdot 2 + 12 \cdot 5 + 3 \cdot 3 + 3 \cdot (-1) - 2000 = -1454$

Student's utility from notebook N_2:

$U_S(N_2) = 2 \cdot 120 + 5 \cdot 15 + 3 \cdot 1 - 1 \cdot 6 - 500 = -188$

So $U_S(N_1) = -1454 < -188 = U_S(N_2)$

$\Rightarrow$ Student prefer notebook N_2 to N_1.

Bus. person's utility from notebook N_1:

$U_B(N_1) = 5 \cdot 240 + 2 \cdot 12 + 5 \cdot 3 + (-6) \cdot 3 - 2000 = -779$

Bus. person's utility from notebook N_2:

$U_B(N_2) = 5 \cdot 120 + 2 \cdot 15 + 5 \cdot 1 - 6 \cdot 6 - 500 = 99$

$\Rightarrow$ Since $-779 < 99$

$\Rightarrow$ business person prefer notebook N_2 to notebook N_1

$\Rightarrow$ Notebooks N_1 and N_2 are vertically differentiated.