1. Consider the following principal-agent relationship. The principal's gross profit is $\Pi = e + \epsilon$, where $e$ is the agent's effort and $\epsilon$ is a random variable determined after effort is chosen. ($E(e) = 0$, $\text{Var}(e) = \sigma^2$). The principal is risk neutral and has the objective function $EV = E(\Pi - w)$ where $w$ is the wage. The agent is risk-averse and has objective function $U = E(w) - \lambda \text{Var}(w) - e^2/2$, and has a reservation utility of 0.

   (i) Suppose that effort is observable and verifiable. What is the optimal level of effort? Is the wage sensitive to risk?

   (ii) Suppose now that only $\Pi$ is observable and verifiable. For simplicity, restrict attention to linear compensation schemes: $w(\Pi) = a + b\Pi$. Determine the optimal contract $(a, b)$ for the principal. What is the slope of the incentive scheme, $b$, when $\lambda = 0$? Explain.

2. Consider a game in which there is a single buyer and a single seller of a quality-differentiated product. Let $q$ represent the quality of the product. The seller chooses $q$ which is not observable to the buyer (or any third party). The seller's marginal cost of improving quality is constant and normalized to 1.

   The buyer's valuation of the product depends on how well the product performs. The product's performance depends on both the quality level $q$ and the amount of effort, $e$, the buyer expends in caring for the product. The buyer's care-taking effort is unobservable to all but the buyer himself.

   The product's actual performance is observable and verifiable and is measured by the variable $x$. There is a deterministic relationship between performance and quality and effort: $x = X(q, e)$ where $X$ is increasing in both $q$ and $e$. The buyer's valuation of the product is then $v = V(q, e)$ where $V$ is increasing in both $q$ and $e$. Care-taking is costly for the buyer with the marginal cost of effort being a constant and equal to 1.

   Since only the actual performance is observable, the buyer and seller can write contracts only on $x$. Let $P_S(x)$ and $P_B(x)$ be the payment received by the seller and the payment made by the buyer, respectively, contingent on the outcome $x$. A pair $<P_S(x), P_B(x)>$ is a "contract" between the buyer and the seller.

   The payoff to the seller under a contract is then $\Pi = P_S(x) - q$.

   The payoff to the buyer is $U = V(q, e) - P_B(x) - e$.

   (a) Define the efficient choice of $q$ and $e$ as the one that maximizes the sum $\Pi + U$. Provide conditions that define the efficient choices $q^*$ and $e^*$.

   (b) Suppose seller and buyer choose $q$ and $e$ non-cooperatively given a contract. A contract is called "balanced" if $P_S(x) = P_B(x)$ for all $x$. (i.e., the amount the buyer pays is the amount the seller receives). Let the balanced contract price be $P(x)$. Show that it is not possible for a balanced contract to yield the efficient outcome.

   (c) A contract is called unbalanced if $P_S(x) \neq P_B(x)$ for some $x$. An example is a contract where the seller is required to pay a fine to a third party, where the amount of the fine depends on $x$. This contract would be given by the pair $<P_S(x) = r - L(x), P_B(x) = r>$ where $L(x)$ goes to the third party. Show a
contract of this form can implement the efficient outcome.

(d) Show that it is possible to arrange an unbalanced contract so that the buyer and the seller (and the third party) would prefer this contract to the best balanced contract.