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ECON 604 Spring 2001 Lecture Notes

Topic 3:
Moral Hazard and Principal-Agent Problems

- The Principal-Agent Problem
  - Basic Economic Environment
  - The Interaction Between P and A
  - The Profit-Maximizing Contract
  - Full Information Case
  - Hidden Action Case
    - Risk-Neutral Agent
    - Risk-Averse Agent
  - Continuous Effort: The First Order Approach

- Principal-Agent with Hidden Information
  - Basic Environment
  - Benchmark: Public Information
  - Private Information Case

The Issue

Main Issue: Actions of agents are *hidden* from other parties to a transaction.

Principal-Agent problem: the *principal* wants the *agent* to perform an action costly to the agent, but the action is not directly observed by the principal.

Examples:

Owners (shareholders) of a firm and firm's manager.

Landowner and sharecropper.

Manufacturer and retailer.

The Principal-Agent Problem

Basic Economic Environment:

A *risk-neutral* firm owner (*the principal*) only cares about profits.

Let $e \in E$ be the level of effort exerted by the manager (*the agent*).

Profits are a random variable π, with (continuous) distribution function $F(\pi \mid e)$ on $[\underline{\pi}, \overline{\pi}]$, and associated density $f(\pi \mid e)$.

The Principal-Agent Problem (cont.)

Basic assumptions:

(i) $f(\pi \mid e) > 0$ for all $\pi \in [\underline{\pi}, \overline{\pi}]$ and $e \in E$.

(ii) For $e_1 > e_2$, $F(\pi \mid e_1) \leq F(\pi \mid e_2)$ for all $\pi \in [\underline{\pi}, \overline{\pi}]$.

Then:

$$\int_{\underline{\pi}}^{\overline{\pi}} \pi \cdot f(\pi \mid e_1) d\pi \geq \int_{\underline{\pi}}^{\overline{\pi}} \pi \cdot f(\pi \mid e_2) d\pi.$$

(iii) The manager has Bernoulli utility function

$$u(w,e) = v(w) - g(e)$$

with $v' > 0, v'' \leq 0,$ and $g' > 0$.

(iv) The manager has reservation utility of $\overline{u}$.

The Principal-Agent Problem (cont.)

Information assumptions:

(i) e is not observable to the principal (or if observable it is non-verifiable).

(ii) π is observable and verifiable.

Implication of (i) and (ii): the parties can only contract on π , not on e. We consider wage/compensation schedules of the form $w(\pi)$.

(iii) $F(\pi \mid e)$ is common knowledge.

(iv) $u(w, e)$ and $\overline{u}$ are common knowledge.

The Interaction Between P and A

1. The owner offers the manager a compensation schedule $w(\pi)$.

2. The manager accepts or rejects this schedule.

If the manager rejects, he earns his reservation value $\overline{u}$, the owner earns 0 profits and the interaction is over.

3. If the manager accepts, then he chooses a level of effort e, which generates a distribution over profits $F(\pi \mid e)$.

4. The profits of the firm are then realized and the manager is paid according to the schedule $w(\pi)$.

Manager's Effort Choice

If the manager accepts $w(\pi)$, her expected payoffs are:

$$\int_{\underline{\pi}}^{\overline{\pi}} [v(w(\pi)) \cdot f(\pi \mid e) d\pi] - g(e)$$

If the manager accepts she will choose $\hat{e}(w(\pi))$ where:

$$\hat{e}(w(\pi)) = \arg\max_{e} \int_{\underline{\pi}}^{\overline{\pi}} [v(w(\pi)) \cdot f(\pi \mid e) d\pi] - g(e)$$

Consequently, she will accept the contract iff:

$$\int_{\underline{\pi}}^{\overline{\pi}} [v(w(\pi)) \cdot f(\pi \mid \hat{e}) d\pi] - g(\hat{e}) \geq \overline{u}$$

Principal's Design of Contract

The principal must consider the agent's effort choice, since she will earn:

$$\int_{\underline{\pi}}^{\overline{\pi}} [\pi - w(\pi)] \cdot f(\pi \mid \hat{e}) d\pi.$$

The principal's problem is to find the contract $w(\pi)$ that maximizes her expected profits, taking into consideration:

(1) whether the agent will accept the contract.

(2) the agent's choice of effort given the contract; that is $\hat{e}(w(\pi))$.

Profit-Maximizing Contract for Two Effort Levels

Only two possible effort levels: $e_H > e_L$.

Assume $F(\pi \mid e_H) \leq F(\pi \mid e_L)$ and $g(e_H) > g(e_L)$.

The principal faces the following optimization problem:

$$\max_{w(\pi), e \in \{e_L, e_H\}} \int_{\underline{\pi}}^{\overline{\pi}} [\pi - w(\pi)] \cdot f(\pi \mid e) d\pi$$

s.t.

$$\int_{\underline{\pi}}^{\overline{\pi}} [v(w(\pi)) \cdot f(\pi \mid e) d\pi] - g(e) \geq \overline{u}.$$

(Individual Rationality (IR))

and

$$e = \arg\max_{\tilde{e}} \int_{\underline{\pi}}^{\overline{\pi}} [v(w(\pi)) \cdot f(\pi \mid \tilde{e}) d\pi] - g(\tilde{e})$$

(Incentive Compatibility (IC))

The Profit-Maximizing Contract (cont.)

Two-stage solution method:

First, for each effort level (e_H and e_L), find the $w(\pi)$ which maximizes the principal's expected profits subjet to:

(1) the agent will accept the contract (IR constraint),

(2) the agent will choose the intended effort level (IC constraint).

Second, compare the profits obtained in each case (e_H vs. e_L), and induce the agent to choose the effort level yielding the highest expected profits.