Measuring Risk Aversion
Local Risk Aversion

**Definition:** Given a twice-differentiable Bernoulli utility function $u(\cdot)$, the *Arrow-Pratt measure of absolute risk aversion* at $x$ is defined as:

$$r_A = \frac{-u''(x)}{u'(x)}.$$

For two individuals, 1 and 2, with twice-differentiable, concave, utility functions $u_1(\cdot)$ and $u_2(\cdot)$, respectively, person 2 is *more risk averse* than person 1 at the level of income $x$ iff

$$r_A^1 = \frac{-u''_1(x)}{u'_1(x)} < \frac{-u''_2(x)}{u'_2(x)} = r_A^2$$

This measure allows us to compare attitudes towards risky situations whose outcomes are *absolute* gains or losses from current wealth $x$.

**NOTE:** this is only a *local* measure of risk aversion. For a different level of $x$, the comparison could switch.
Local Risk Aversion (Cont.)

Why not just use \( u''(x) \)? Two reasons:

1) No equivalence with other reasonable measures.

2) The AP measure is invariant to affine transformations of the utility function, but \( u'' \) is not.

Consider \( \succeq \) and suppose \( U \) represents \( \succeq \). The associated Bernoulli utility function is \( u(\cdot) \).

Take the family of utility functions \( v(x) = \beta u(x) + \gamma \). All these represent the same preferences.

We have \( v'(x) = \beta u'(x) \) and \( v''(x) = \beta u''(x) \).

The AP is then \( \frac{-u''(x)}{u'(x)} \), and is therefore the same for any function in this family.
Global Risk Aversion

Given two twice-differentiable Bernoulli utility functions $u_1(\cdot)$ and $u_2(\cdot)$, individual 2 is *globally more risk averse* than individual 1 if and only if there exists a concave function $\Psi(\cdot)$ such that

$$u_2(x) = \Psi(u_1(x)).$$

That is, $u_2(\cdot)$ is a concave transform of $u_1(\cdot)$. 
Risk Premium and Certainty Equivalent

Consider two individuals with utility functions $u_1(\cdot)$ and $u_2(\cdot)$. Individual 2 is more risk averse than individual 1 if and only if:

$$c(F, u_2) \leq c(F, u_1) \text{ for every lottery } F(\cdot).$$

Since $\rho = EV - CE$, equivalently individual 2 is more risk averse than individual 1 when 2’s risk premium is higher:

$$\rho(F, u_2) \geq \rho(F, u_1) \text{ for every } F(\cdot).$$
Pratt’s Theorem:

The three previous measures of risk aversion are all equivalent, given twice-differentiable utility functions.

That is, the property that a utility function $u_2$ is a concave transform of a utility function $u_1$ is equivalent to 2 having a greater A-P measure of absolute risk aversion for all $x$:

$$r_A(x, u_2) \geq r_A(x, u_1) \text{ for all } x \in [a, b],$$

and equivalent to 2 having a greater risk premium:

$$\rho(F, u_2) \geq \rho(F, u_1) \text{ for any } F(\cdot).$$
Risk Aversion and Wealth

**Definition:** The Bernoulli utility function \( u(\cdot) \) exhibits *decreasing* (*constant*) *increasing* *absolute risk aversion* if \( r_A(x, u) \) is a decreasing (constant) (increasing) function of \( x \).

e.g. consider two different wealth levels \( w_1 > w_2 \).

The set of possible outcomes involves a monetary payment \( x \).

A person’s utility function \( u \) exhibits decreasing absolute risk aversion (DARA) iff

\[
r_A(w_1 + x, u) < r_A(w_2 + x, u).
\]
Economists often model utility by an exponential function

\[ u(x) = -e^{-\lambda x} \quad \text{where } \lambda > 0. \]

This utility function exhibits constant absolute risk aversion (CARA).

\[ u'(x) = \lambda e^{-\lambda x} \quad u''(x) = -\lambda^2 e^{-\lambda x}, \]

\[ r_A = -u''/u' = \lambda. \]
Relative Risk Aversion

**Definition:** Given a twice-differentiable Bernoulli utility function $u(\cdot)$, the coefficient of *relative risk aversion* at $x$ is defined as:

$$ r_R = x \frac{-u''(x)}{u'(x)} = x r_A. $$

We can write it as follows:

$$ r_R = x \frac{-u''(x)}{u'(x)} = \frac{du'}{dx} \cdot \frac{x}{u'(x)} = \frac{du'(x)}{u'(x)} \cdot \frac{dx}{x} = \% \Delta u' / \% \Delta x. $$

If $r_R$ is decreasing with respect to $x$ we say that an individual exhibits decreasing relative risk aversion.

She becomes less risk averse with regard to gambles that are proportional to her wealth as her wealth increases.