Paying to Queue:
A Theory of Locational Differences in Nonunion Wages

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May 7, 2003

Abstract

Traditional theories of the effect unions have on nonunion wages are difficult to reconcile with firm and worker mobility. We show how differences in nonunion wages can persist in a two-city search model. Nonunion wage differences across cities are driven by transition rates into the union sector. Should the queue for union jobs form in the nonunion sector, nonunion wages are lower in the city with the more powerful union as workers are willing to take a lower wage to line up for the union job. However, if the queue is formed from the unemployed sector, nonunion firms in the city with the more powerful union must pay a premium to workers for workers to be willing to leave the queue.

Keywords: Nonunion wage differentials, search, mobility
JEL J41, J61, R41.

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1 Introduction

How does a union affect wages in the nonunion sector? The economics literature is divided on the issue with two competing theories. The ‘spillover’ literature finds a tradeoff between union wages and employment.\(^1\) As union wages increase, there are fewer vacancies in the union sector, and the supply of nonunion labor increases. This outward shift in turns leads to a lower equilibrium wage for nonunion workers. The implicit assumption in the spillover literature is that the market for nonunion workers is competitive. In contrast, the ‘threat’ literature assumes that the market for nonunion workers is not competitive.\(^2\) The higher wages are in the union sector, the greater the incentive for workers in the nonunion sector to organize. High union wages then lead to high nonunion wages as nonunion firms increase the pay of workers to deter them from forming a union.

A sizable literature in recent years has focused on empirically testing the validity of the spillover and threat theories.\(^3\) A standard test is to estimate a wage regression with a measure of union power as a regressor. If the coefficient on the measure of union power is negative, the conclusion is that the spillover effect dominates. If positive, the threat effect dominates. Policymakers interested in maximizing the welfare of workers may then be more supportive of unions in industries where the threat effect dominates rather than the spillover effect, because threat causes no welfare loss in the labor sector and results in a distribution of profits more favorable to workers.

When firms or workers are mobile, the interpretation of the empirical findings for both theories is confounded.\(^4\) In the long run, firm and worker mobility should undo the wage effects generated from spillover and threat sources.

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\(^1\)See Friedman (1951), Johnson (1975), McDonald and Solow (1981), and Layard and Nickell (1990), among many others.
\(^2\)See Conant (1959), Rosen (1969), and Foulkes (1990), as well as many others.
\(^3\)Kahn (1979), Ichniowski, Freeman, Lauer (1989), Herz (1991), and Neumark and Wachter (1995) are some examples.
\(^4\)See Raphael and Riker (1999) and Ross (1998) for the importance of geographic mobility in explaining labor market outcomes.
Workers pushed out by the spillover effect will migrate to cities\textsuperscript{5} with higher nonunion wages while firms will enter the market to take advantage of the low wages. This entry by firms and exit by workers then leads to equalization in nonunion wages across cities. Similarly, if threat effects are present, firms will exit and workers enter until nonunion wages are again equalized.

How can persistent differences in nonunion wages exist across cities in the face of mobility? We develop a model in which how union jobs are obtained dictates the effect union power has on nonunion wages. The key assumption is that it is easier for workers to transition into union jobs in their city of residence than into union jobs in other cities. Then, whether the transition rates into union jobs are higher from the nonunion sector or the unemployed sector determines whether nonunion wages are lower or higher in the city with the stronger union. If workers find it easier to obtain a union job from the nonunion sector, nonunion firms recognize this and demand a share of the future benefits of queueing from the nonunion job. On the other hand, if it is more difficult for workers to find a union job from the nonunion sector, firms have to pay a premium to attract workers. We develop a two-city search model that generates persistent nonunion wage differences across cities due to transition probabilities into union jobs.

In certain markets it may be easier to obtain a union job if the worker is employed in a nonunion position in the same city. The nonunion job may then allow the worker to learn more easily of union job openings and send signals to the union about his productivity or his ability to ‘fit in’ with union members. These signals can be formal, such as performance reviews, or informal, such as social interactions with union members. Whether the signals are formal or informal, forward looking workers choose to enter nonunion jobs with the knowledge that they are queueing for union jobs. If the queue is in the nonunion sector, nonunion firms are able to extract some of the expected future benefits the worker has from potentially being employed by the union. Hence, lower nonunion wages result in the city with the more powerful union.

\textsuperscript{5}We use the term ‘cities’ and different labor markets synonymously.
In other markets, however, it may prove easier to obtain a union job if the worker is unemployed. Being unengaged in a nonunion job allows the worker to spend more time, collect more information, and generally invest more effort into the job search than his employed counterparts. For example, budding actors and actresses move to Los Angeles or New York to queue for acting positions. Forgoing the income as a waiter or busboy will allow the actor to practice his lines more and attend more auditions. In contrast to the case where the queue is from the nonunion sector, firms must pay workers a premium to leave the queue and this premium may lead to higher nonunion wages in the city with the stronger union. Whether the queue is from the nonunion sector or the unemployed sector then dictates how nonunion wages will differ across cities.

Whether the queue for union jobs is from the nonunion sector or the unemployment sector also affects unemployment rates. If firms have to pay a premium to workers to leave the queue, firms need to be compensated with a higher probability of matching with a worker. These premiums then lead to higher unemployment rates for workers in markets where the queue for union jobs is the unemployed sector. Hence, as the queue moves from the nonunion sector to the unemployed sector, workers tradeoff higher probabilities of employment for higher wages.

The next section presents the two-city search model. Section 3 provides comparative statics, showing how nonunion wages across cities differ due to stronger or weaker unions. Section 4 discusses the broader implications of the model. All proofs are in the appendix.

6 Also, advisors to Economics Ph.D. students often advise against taking too many teaching positions during the latter years of one’s dissertation process for short term monetary gain, and recommend using that time to improve the dissertation. Students who spend more time on their dissertation and publications then have a higher probability of transitioning into jobs at research universities.

7 A similar tradeoff is found in Sattinger (2002) where the longer consumers wait in line the lower the prices they face.
2 The Model

In this section we present a two-sided search model which is designed to highlight the effect queueing has on nonunion wages. There are two cities, with City 1 having both a union and a nonunion sector and City 2 only having a nonunion sector.\(^8\) There are \(N\) workers born each period, and these workers live for two periods.\(^9\) The timing for a particular cohort of workers follows:

1. Young workers choose to live in City 1 or City 2.
2. Nonunion firms and young workers search for jobs in the city where they live.\(^10\)
3. Matched workers negotiate a wage.
4. The union determines membership from the pool of old employed and unemployed workers in City 1.
5. All old workers in City 2 and old workers who did not receive a union job in City 1 take an outside option.

We assume that workers are risk neutral, do not discount the future, and endure no moving costs.\(^11\) All workers have identical abilities and are solely interested in maximizing their lifetime income.\(^12\) Young workers in City 1, \(N_1\), and City 2, \(N_2\), search for nonunion firms, and nonunion firms in each city search for workers. Entry by nonunion firms is endogenous with the number of vacancies posted in City 1 and City 2 denoted by \(J_1\) and \(J_2\). As in Pissarides (1992), the number of matches in City \(i\) is Cobb-Douglas on the interior and

\(^8\)All qualitative results hold when City 2 instead has a weaker union than City 1.
\(^9\)See Arcidiacono (forthcoming) and Pissarides (1992) for previous work on two period overlapping generations search models.
\(^10\)All qualitative results hold when we allow workers to search in both cities, as long as it is easier to match in the city in which they are located. See section 4 for a discussion.
\(^11\)These assumptions do not affect the qualitative results.
\(^12\)The comparative statics of the model do not change with heterogeneous match qualities as long as workers have ex ante identical expectations on the productivity of the match.
given by:

\[ x_i = \min(AJ_i^\alpha N_i^{1-\alpha}, J_i, N_i), \quad (1) \]

where \( \alpha \in [0, 1] \) and \( A \) is a normalizing constant. Conditional on the city, all workers have the same probability of finding a match. The probability of a worker in city \( i \) finding a match is then given by \( P_i = \frac{x_i}{N_i} \). All nonunion firms also have the same probability of finding a match, given by \( q_i = \frac{x_i}{J_i} \).

Union jobs are allocated on a strict seniority basis.\(^{13}\) Membership in the union is limited, and young workers must queue to obtain a union job when old. Within a cohort of old workers, the probability of transitioning into a union job depends not only on the ratio of workers to union jobs, but also on whether the worker is unemployed or employed at a nonunion firm. The probability of obtaining a union job when old conditional on being employed in City 1 when young is given by:

\[ P_{u|e} = \frac{\delta U}{\delta x_1 + (1 - \delta)(N_1 - x_1)}, \quad (2) \]

where \( U \) is the size of the union. The corresponding probability of obtaining a union job when old conditional on not matching in City 1 when young given by:

\[ P_{u|ne} = \frac{(1 - \delta)U}{\delta x_1 + (1 - \delta)(N_1 - x_1)}. \quad (3) \]

The \( \delta \) term indicates the degree of advantage a worker has in obtaining a union job when he is employed in a nonunion position in the first period. Structuring the transition probabilities in this way allows the probability of transitioning into the union job to be the same for unemployed and nonunion workers when \( \delta = 0.5 \). However, if \( \delta > 0.5 \), a worker in City 1 has a higher probability of transitioning into the union sector if he has a nonunion job than if he is unemployed. When \( \delta = 1 \) (0), the queue is only from the employed (unemployed) sector.

Wages in the union sector, \( W_u \), and the size of the union, \( U \), are taken as exogenous from the workers perspective. We do not model how \( W_u \) and

\(^{13}\)Allowing young workers to obtain union jobs does not affect the qualitative results as long as unions first decide membership and then those who do not find union jobs search for nonunion jobs.
$U$ are determined, and in particular do not model how the nonunion labor market influences the bargaining power of the union. While the nonunion sector undoubtedly influences the union sector, in this paper we are interested in what the general equilibrium must look like conditional on an observed union size and wage.

Old workers in City 1 who do not obtain a union job and all old workers in City 2 take an outside option $W_o$ when old. We impose $W_o < W_u$ to make the union job more attractive than the outside option. Having an outside option for old workers is equivalent to assuming that there is a separate labor market for old workers and the assumption has no effect on our qualitative results but does simplify the computations.

Wages in the nonunion sector are determined according to generalized Nash bargaining, with the worker’s share of the surplus denoted $\beta$, $\beta \in (0,1)$. The current period value of a match in city $i$ is given by $V_i$. Young workers who search in City 2 and are matched with firms receive:

$$W_2 = \beta V_2. \quad (4)$$

Nonunion wages in City 1 are more complex. Matches in City 1 may be worth more or less than the current period value as matching may influence the probability of getting into the union in the future. In particular, the total value of the match increases (decreases) if the probability of entering the union is higher (lower) from the nonunion sector than from the unemployed sector. It is this effect, the union queueing effect, which drives nonunion wage differences across cities. Nonunion wages in City 1 are then given by:

$$W_1 = \beta V_1 - (1 - \beta)P_D(W_u - W_o), \quad (5)$$

where

$$P_D = P_{u|e} - P_{u|ne}.$$ 

Note that $P_D(W_u - W_o)$ is the expected gain or loss in earnings when old from being matched with a firm in City 1 when young. Should the probability of obtaining a union job increase by being employed in the nonunion sector,
firms in City 1 extract a \((1 - \beta)\) share of the corresponding increase in expected future earnings. However, if by working in the nonunion sector the probability of transitioning into the union sector is smaller, firms in City 1 have to pay the workers a premium to take a job that lowers their expected future wages.

Young workers then move to City 1 or City 2 to maximize their life-time expected wages. For a young worker to be indifferent between moving to City 1 or City 2, we must have:

\[
P_1 W_1 + P_1 P_u | e W_u + (1 - P_1) P_u | ne W_u \\
+ P_1 (1 - P_u | e) W_o + (1 - P_1)(1 - P_u | ne) W_o = P_2 W_2 + W_o
\]

The left hand side has the expected wages for choosing City 1. The second and third terms represent the union wage times the probability of obtaining a union job conditional on being employed and unemployed, respectively. The last terms are then the corresponding probabilities and returns conditional on not obtaining a union job. The right hand side is the corresponding expected wages for choosing City 2: the probability of matching when young times the wage when young plus the value of the outside option when old. Rearranging the terms yields:

\[
P_2 W_2 - P_1 W_1 = P_1 P_u | e (W_u - W_o) + (1 - P_1) P_u | ne (W_u - W_o) \tag{6}
\]

Since the term on the right hand side is positive, we know that the combination of the probability of finding a job when young in City 2 times the wage in City 2 must be higher than the corresponding combination in City 1. That is, workers in City 1 take lower expected wages when young in the hopes of acquiring a union job, and thereby a higher wage, when old.

The difference in transition rates across the unemployed and nonunion sector also influences the entry decisions of nonunion firms. If \(P_D > 0\), nonunion firms in City 1 are able to extract a portion of the higher future wages the worker expects from working in the nonunion sector; making City 1 attractive to nonunion firms. If \(P_D < 0\), nonunion firms must pay workers a premium for
the inferior queue position. Nonunion firms in both cities will post vacancies in each city until the expected profits from posting a vacancy are zero. The expected zero profit conditions for firms operating in City 1 and City 2 are then:

\[
q_1 (1 - \beta)[V_1 + PD(W_u - W_o)] - K = 0 \\
q_2 (1 - \beta)V_2 - K = 0
\]

(7)

(8)

where \(K\) is the fixed cost of posting a vacancy and \(q_i\) is again the probability of a firm finding a match in city \(i\).

In order to close the model, we specify how the \(V_i\)'s, that is, the match values, are determined. We assume that:

\[
V_i = f(x_i),
\]

(9)

where:

\[
\frac{d[f(x_i) \cdot x_i]}{dx_i} = f'(x_i)x_i + f(x_i) > 0 \quad \text{while} \quad x_i \leq \bar{N} \tag{10}
\]

\[
\frac{d^2[f(x_i) \cdot x_i]}{d(x_i)^2} = f''(x_i)x_i + 2f'(x_i) < 0 \quad \text{while} \quad x_i \leq \bar{N} \tag{11}
\]

which is equivalent to assuming that the gross surplus function is concave in the number of matches. Hence, the more matches that occur the less surplus is available per match. This can be because of increased land prices or producing a good that is not traded across cities; in either case the surplus function is treated as outside the model. Concavity of the gross surplus function also implies downward sloping labor demand curves in each city.\(^{14}\)

This last assumption on the surplus function then closes the model. Proposition 1 establishes that an equilibrium for this model does exist.

**Proposition 1.** Given equations (1) - (11), \(\{K, A, \alpha, \beta, \delta, W_u, W_o, \bar{N}, U\}\), there exists an equilibrium in \(\{N_1, N_2, J_1, J_2, W_1, W_2\}\).

\(^{14}\)An equilibrium that is characterized by firms and workers locating in both cities in the presence of a constant returns to scale matching function when \(PD > 0\) necessitates a concave gross surplus function. Otherwise, both firms and workers would prefer to search in City 1.
3 Comparative Statics

Having shown that an equilibrium exists, we now establish the comparative statics results. We show how nonunion wages and unemployment rates across the two cities differ depending upon the location of the queue for union jobs.

We begin by noting that if the probabilities of finding a job and nonunion wages are equal across the two cities, workers prefer to search in City 1 due to the probability of transitioning into a union job. However, if the queue is from the nonunion sector, nonunion firms too have a preference for searching in City 1, all else equal, because firms can extract some of the expected future rents workers have from the union job. If the incentives for firms to search in City 1 are large enough, it may be possible for young workers to have a higher probability of matching in City 1 than City 2. Proposition 2 rules this out.

**Proposition 2.** $P_1 < P_2$ if $\delta < 1$ and $P_1 = P_2$ if $\delta = 1$.

The proof for proposition 2 shows that rearranging equations (4)-(8), we are left with the following expression:

$$\frac{J_1}{N_1} = \frac{J_2}{N_2} - \frac{1 - \beta}{\beta K} \frac{(1 - \delta)U(W_u - W_o)}{(2\delta - 1)x_1 + (1 - \delta)N_1}.$$

Note that because the matching function is constant returns to scale, the probabilities of matching for either firms or workers depend solely on the ratio of firms to workers. A higher ratio of firms to workers implies a higher probability of matching for a worker and a lower probability of matching for a firm. Since the second term on the right hand side is clearly negative, the probability of matching in City 2 must be higher than the probability of matching in City 1. For $\delta = 1$, however, the second term is zero; implying that the probabilities of matching in the two cities are equal. This is the case when the queue for union jobs is solely from nonunion jobs in City 1.

If the probabilities of matching are the same across the two cities then wages must be lower in City 1 for the worker indifference condition to hold. In fact, we show that any time the probability of transitioning into a union job is higher from the nonunion sector increasing either the size of the union or the union
wage leads to lower relative wages in City 1. Recalling that if $\delta > 0.5$ then $P_D > 0$,

**Proposition 3.** If $P_D > 0$, then $W_1 < W_2$, $\frac{d[W_1-W_2]}{dW_n} < 0$, and $\frac{d[W_1-W_2]}{dU} < 0$.

The larger union premium or union size, workers have a larger incentive to move to City 1. Firms too offer more positions in City 1 because of a better possibility of a match as well as being able to extract some of the union premium, which is larger due to the larger union size or union wages. The net result of these incentives will be a lower number of matches in City 2 and a higher number of matches in City 1. However, we emphasize that the probability of employment in City 2 is still higher than the probability of employment in City 1. While City 1 does have a higher number of matches, it also has a higher number of workers searching for work. The nonunion wage in City 1 will therefore be decreasing in the strength of the union as the number of matches increases, and the nonunion wage in City 2 will be increasing in the strength of the union, as the number of matches decreases. Nonunion workers in City 1 see their wages fall both because of the increased matches and also because of having to pay more to queue for the higher benefits of the union job. Hence, spillover-like results are found anytime the queue for union jobs come from the nonunion sector.

Since both the probability of matching and the nonunion wage are lower in City 1, we obtain results similar to the macroeconomics literature on the wage curve. This literature documents lower wages being associated with higher unemployment rates. This is true here as well; but results from a higher probability of transitioning into a better job in the future.

When $\delta < 0.5$ it is easier to transition to the union job from the unemployed sector than the nonunion sector. If this is the case, firms have to pay workers a premium to leave the queue. Proposition 4 then establishes that conditions exist where nonunion wages in City 1 are higher than nonunion wages in City 2, with the gap increasing as either union wages or union size increase.

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15See Blanchflower and Oswald (1994) and the references therein.
Proposition 4. If $P_D < 0$ conditions exist where $W_2 < W_1$, $\frac{d(W_1 - W_2)}{dW_u} > 0$, and $\frac{d(W_1 - W_2)}{dU} > 0$.

Nonunion wages may still be higher in City 2 even when $P_D < 0$. To see this, consider the zero profit condition for firms in City 1:

$$q_1(1 - \beta)[f(x_1) + P_D(W_u - W_o)] - K = 0.$$  

With an increase in union size or wages and $\delta < 0.5$, the second term inside the brackets becomes more negative. However, $q_1$ has increased as well as more workers are induced to work in City 1. The higher probability of matching may then lead to more matches in equilibrium, pulling down the surplus of the match, $f(x_1)$. The premium firms pay workers to leave the queue is then balanced against lower match values. The less sensitive the firm’s probability of matching is to changes in the number of searching workers, the more likely nonunion wages will be higher in City 1.

To show the tradeoff between wages and employment as $\delta$ changes, we simulate the model. Figures 1 and 2 plot changes in the relative probabilities of matching in the nonunion sectors in the two cities as well as the relative wages. As $\delta$ increases, we see the expected tradeoff between wages and employment. Increasing $\delta$ increases the probability of finding a match relative to City 2. However, the decrease in the relative probability gap comes at a cost: relative nonunion wages fall in City 1 as $\delta$ increases.

Whether the probability of matching depends more on firms (high $\alpha$) or on workers (low $\alpha$) also dictate how the tradeoff between the probability of matching and nonunion wages takes place. In particular, lowering the value of $\alpha$ flattens the curves in both graphs. This in turn reduces the region where threat-like effects ($W_1 - W_2 > 0$) exist.

16The firm’s probability of matching becomes less sensitive to the number of searching workers as the $\alpha$ parameter in the matching function increases.

17The parameters of the data generating process are: $\beta = 0.5$, $N = 1$, $K = 0.2$, $W_u = 0.2$, $W_o = 0.075$, $A = 0.75$, $U = 0.2$, $\gamma = 0.25$; where $f(x_1) = x_1^{\gamma}$.

18The empirical literature on matching functions has typically estimated $\alpha$ to be around .6. See Petrongolo and Pissarides (2001) for a review.
4 Discussion

This paper has developed a theory of persistent nonunion wage differentials across cities due to queueing for union jobs. If the queue for union jobs is from the nonunion sector in the same city as the union, nonunion firms in that city are able to extract a portion of the workers expected gains from having union jobs in the future. On the other hand, if the queue is from the unemployed sector, firms must pay workers in the city with a union a premium to leave the queue. These nonunion wage differentials persist despite expected zero profits for firms and workers having expected wages which are equal across cities.

Our treatment of union wage and size may be of some concern in a general equilibrium model, where variables of interest are expected to be endogenous. However, we emphasize that our comparative statics analysis is not about the ‘evolution’ of the nonunion and union sectors. In this paper we are not interested in measuring the responses of the union sector to the nonunion sector. Rather, we are interested in what the nonunion labor market must look like in
equilibrium given any union size or wage. No matter how a particular union size and wage came to be, the nonunion sector must behave in a particular way for firms to have zero expected profits and nonunion workers to be indifferent across locations.

A second strong assumption is the separability of the labor markets between City 1 and City 2. A worker who decides to move to City 2 in the first period must not be able to queue for the union jobs available in City 1. If this were possible, there would be no clear reason to move to City 1 in the first period. In fact, non-separability of labor markets would make City 1 and City 2 observationally equivalent. The probability of acceptance into a union would be equal across the two cities, and the nonunion wage across of the cities would also be equal, as workers migrate between the cities until the life-time expected income in the two cities become equal. It is certainly the case that workers may apply for union jobs in other geographic locations by such methods as telephone, internet, and word-of-mouth. However, we believe that a nonunion worker ‘closer’ to a particular union job is more likely to obtain that job in the
future than an otherwise identical worker.

‘Closer’ can signify similarity of job description as well as geographic proximity. The two-city model translates directly into a model with two industries. City 1 would then be an industry where a union did exist with City 2 being an industry where no union exists. This ease with which the ‘closer’ nonunion worker queues can be attributed to such factors as lower information cost and speed of information delivery in learning about such positions, and the ability to more easily lobby to obtain the position once the worker queues. Therefore, union membership should favor home-grown workers to nonunion workers from ‘farther away.’ What we expect to see in the data is not complete labor market separation, but a higher percentage of ‘closer’ workers being accepted into the union than workers who are ‘farther away.’ In fact, the union set-up used in this paper is relevant for any situation in which one market serves as an informal queue to gain entry into another, more desirable market.

The transitions that occur across jobs then lead to ripple effects throughout the whole economy. Wage increases in one job lead to lower wages in jobs which transition into the job with the wage increase, and correspondingly higher wages in jobs which do not transition into the job with the wage increase. Hence, the general equilibrium effects work to undo wage increases in particular industries through the entry and exit of workers and firms into the industry. This is consistent with Neumark and Wachter (1995) who find that industries with strong unions in particular locations have lower wages in the industry for nonunion workers, yet higher wages for other jobs in the city. The lower wages in the nonunion sector of an industry with a strong union is consistent with workers queueing for the union jobs by working in a nonunion job in the same industry. However, with workers queueing for a union job, there is a scarcity of workers for jobs outside of the union industry.

Appendix

Proof of Proposition 1

Note that conditional on any \( N_1 \in [0, N] \), as \( J_i \to \infty \), \( q_i \to 0 \) for \( i \in \{1, 2\} \).
Since \( f(x_i) \) and \( P_D \) are bounded above by \( W_u \) and 1 respectively, there exists a \( J_i \) such that for all \( N_1 \) if \( J_i' > J_i \), profits in City \( i \) are negative. Since the partial derivative of \( \pi \) is negative with respect to \( J_i \),

\[
\frac{\partial \pi}{\partial J_1} = \frac{q_1(1-\beta)(\alpha-1)}{J_1} [f(x_1) + P_D(W_u-W_o)] + \frac{q_1(1-\beta)\alpha}{J_1} \left[f'(x_1)x_1 - \frac{P_D^2x_1(W_u-W_o)}{U}\right] < 0
\]

\[
\frac{\partial \pi}{\partial J_2} = \frac{q_2(1-\beta)}{J_2} \left[(\alpha-1)f(x_2) + \alpha f'(x_2)x_2\right] < 0
\]

we know that for each value of \( N_1 \) there would be at most one value of \( J_i \) such that \( \pi_i = 0 \). Similarly, define \( S_1 \) as the expected surplus from choosing City 1 over City 2. Since \( \partial S_1/\partial N_1 \) is also negative,

\[
\frac{\partial S_1}{\partial N_1} = \frac{P_1\beta}{N_1}[\alpha f(x_1) + (1-\alpha)f'(x_1)x_1] + \frac{P_2\beta}{N_2}[\alpha f(x_2) + (1-\alpha)f'(x_2)x_2]
\]

\[
-(W_u-W_o)U[\beta P_1^2(2\delta-1)^2 + (1+\alpha)\beta P_1(2\delta-1)(1-\delta)
\]

\[
+(1-\alpha)P_1(2\delta-1)(1-\delta) + (1-\delta)^2]/d^2
\]

\[
< \frac{P_1\beta}{N_1}[\alpha f(x_1) + (1-\alpha)f'(x_1)x_1] + \frac{P_2\beta}{N_2}[\alpha f(x_2) + (1-\alpha)f'(x_2)x_2]
\]

\[
-(W_u-W_o)U[(2\delta-1)P_1 + (1-\delta)^2]/d^2
\]

\[
< 0
\]

where \( d = (2\delta-1)x_1 + (1-\delta)N_1 \), we know that for each pair \( \{J_1, J_2\} \) there is at most one value of \( N_1 \) such that \( S_1 = 0 \).

We can then define the following mappings:

\[
f_1 = \begin{cases} 
\pi_1(J_1, N_1) & \text{for } J_1 \in (0, J_1], N \in [0, N] \\
\max\{\pi_1(0, N_1), 0\} & \text{for } J_1 = 0, N_1 \in [0, N] 
\end{cases}
\]

\[
f_2 = \begin{cases} 
\pi_2(J_2, N_1) & \text{for } J_2 \in (0, J_2], N \in [0, N] \\
\max\{\pi_2(0, N_1), 0\} & \text{for } J_2 = 0, N_1 \in [0, N] 
\end{cases}
\]

\[
f_3 = \begin{cases} 
\min\{S_1(J_1, J_2, N_1), 0\} & \text{for } \{J_1, J_2\} \in [0, J_1] \times [0, J_2], N_1 = N \\
S_1(J_1, J_2, N_1) & \text{for } \{J_1, J_2\} \in [0, J_1] \times [0, J_2], N \in (0, N) \\
\max\{S_1(J_1, J_2, 0), 0\} & \text{for } \{J_1, J_2\} \in [0, J_1] \times [0, J_2], N_1 = 0 
\end{cases}
\]

Then for each value of \( N_1 \), there exists a unique value of \( J_1 \in [0, J_1] \) and \( J_2 \in [0, J_2] \) that satisfy \( f_1 = 0 \) and \( f_2 = 0 \). Further, since \( \pi_1 \) and \( \pi_2 \) are
continuous in $N_1$, this unique value is a continuous function of $N_1$. Similarly, for each pair \( \{J_1, J_2\} \), there is a unique $N_1 \in [0, \bar{N}]$ satisfying $f_3$ which is continuous in both $J_1$ and $J_2$. We can then use functions to define a continuous vector valued function mapping from $[0, J_1] \times [0, J_2] \times [0, \bar{N}]$ into itself. Then by Brouwer’s fixed point theorem there exists a triplet \( \{J_1^*, J_2^*, N_1^*\} \) where $f_1 = 0$, $f_2 = 0$, and $f_3 = 0$. QED.

**Proof of Proposition 2**

Substitute the zero profit conditions for firms in City 1 and City 2 into the worker indifference condition to obtain:

\[
\frac{\beta K}{1 - \beta} \frac{J_1}{N_1} + \frac{(1 - \delta)U(W_u - W_o)}{\delta x_1 + (1 - \delta)(N_1 - x_1)} - \frac{\beta K}{1 - \beta} \frac{J_2}{N_2} = 0
\]

Rearrange to:

\[
\frac{J_1}{N_1} = \frac{J_2}{N_2} - \frac{1 - \beta}{\beta K} \frac{(1 - \delta)U(W_u - W_o)}{\delta x_1 + (1 - \delta)(N_1 - x_1)}
\]

For the special case, where $\delta = 1$:

\[
\frac{J_1}{N_1} = \frac{J_2}{N_2}
\]

Since $P_1$ and $P_2$ are proportional to $\frac{J_1}{N_1}$ and $\frac{J_2}{N_2}$ respectively, $P_1 = P_2$. Note further that the second term on the left hand side is weakly negative, implying:

\[
\frac{J_1}{N_1} \leq \frac{J_2}{N_2}
\]

Therefore,

\[
P_1 = A \left( \frac{J_1}{N_1} \right)^\alpha \leq A \left( \frac{J_2}{N_2} \right)^\alpha = P_2
\]

QED.

**Proof of Proposition 3**

We first show that for $\delta > 0.5$, $W_1 < W_2$. Differencing the City 2 zero profit condition from the City 1 zero profit condition and simplifying yields:

\[
q_1(f(x_1) + P_D(W_u - W_o)) = q_2f(x_2)
\]
Solving for \( f(x_1) \) yields:

\[
 f(x_1) = \frac{q_2 f(x_2)}{q_1} - P_D(W_u - W_o)
\]

where \( P_D > 0 \) for \( \delta > 0.5 \). From Proposition 2, \( P_1 < P_2 \).

Since \( q_i = 1/F_i^{(\frac{1-\alpha}{\alpha})} \) for \( i \in \{1,2\} \), \( q_1 > q_2 \). Hence, \( f(x_2) > f(x_1) \) for the equality to hold. Now consider \( W_1 - W_2 \):

\[
 \beta f(x_1) - (1 - \beta)P_D(W_u - W_o) - \beta f(x_2).
\]

With \( f(x_2) > f(x_1) \) and the second term being negative, the expression itself must be negative. We now show that \( \frac{d(W_1 - W_2)}{dU} \) is negative as well for \( \delta > 0.5 \). Define:

\[
 F_1 = q_1(1 - \beta)[f(x_1) + P_D(W_u - W_o)] - K
\]

\[
 F_2 = q_2(1 - \beta)f(x_2) - K
\]

\[
 F_3 = \beta P_1[f(x_1) + P_D(W_u - W_o)] + P_u|ne(W_u - W_o) - \beta P_2 f(x_2)
\]

The Jacobian of the system of equations is then:

\[
 B = \begin{pmatrix}
 \frac{\partial F_1}{\partial J_1} & \frac{\partial F_1}{\partial J_2} & \frac{\partial F_1}{\partial N_1} \\
 \frac{\partial F_2}{\partial J_1} & \frac{\partial F_2}{\partial J_2} & \frac{\partial F_2}{\partial N_1} \\
 \frac{\partial F_3}{\partial J_1} & \frac{\partial F_3}{\partial J_2} & \frac{\partial F_3}{\partial N_1}
\end{pmatrix}
\]

Note that diagonal elements of the Jacobian are all negative:

\[
 \frac{\partial F_1}{\partial J_1} = \frac{q_1(1 - \beta)(\alpha - 1)}{J_1} [f(x_1) + P_D(W_u - W_o)] + \frac{q_1(1 - \beta)\alpha}{J_1} \left[ f'(x_1)x_1 - \frac{P_D^2}{U} x_1(W_u - W_o) \right] < 0
\]

\[
 \frac{\partial F_2}{\partial J_2} = \frac{q_2(1 - \beta)}{J_2} [(\alpha - 1)f(x_2) + \alpha f'(x_2)x_2] < 0
\]

\[
 \frac{\partial F_3}{\partial N_1} = \frac{P_1\beta}{N_1} [-\alpha f(x_1) + (1 - \alpha) f'(x_1)x_1] - (W_u - W_o)U[\beta P_1^2(2\delta - 1)^2 + (1 + \alpha)\beta P_1(2\delta - 1)(1 - \delta)
\]

\[+(1 - \alpha) P_1 (2\delta - 1)(1 - \delta) + (1 - \delta)^2]/d^2 < 0
\]

\[
 \frac{\partial F_3}{\partial N_1} = \frac{P_2\beta}{N_2} [-\alpha f(x_2) + (1 - \alpha) f'(x_2)x_2] < 0,
\]
where $\frac{\partial F_i}{\partial N_i}$ indicates the part of the partial derivative of $F_i$ that from the lifetime expected income of a young worker who moves to City $i$; $\frac{\partial F_i}{\partial N_i} = \frac{\partial F_1}{\partial N_1} + \frac{\partial F_2}{\partial N_1}$.

Note that $\frac{\partial F_1}{\partial N_1} = \frac{\partial F_2}{\partial N_2} = 0$. The other off-diagonal elements are:

$$
\frac{\partial F_1}{\partial N_1} = \frac{q_1(1 - \beta)(1 - \alpha)}{N_1} \left[ f(x_1) + f'(x_1)x_1 + P_D(W_u - W_o) \left[ 1 - \frac{(2\delta - 1)x_1}{d} - \frac{(1 - \delta)N_1}{d(1 - \alpha)} \right] \right]
$$

$$
\frac{\partial F_2}{\partial N_1} = \frac{q_2(1 - \beta)(\alpha - 1)}{N_2} \left[ f(x_2) + f'(x_2)x_2 \right] < 0
$$

$$
\frac{\partial F_3}{\partial J_1} = \frac{P_3\alpha\beta}{J_1} \left[ f(x_1) + f'(x_1)x_1 \right] - \frac{\alpha(1 - \beta)P_Dq_1(1 - \delta)(W_u - W_o)}{d}
$$

$$
\frac{\partial F_3}{\partial J_2} = -\frac{P_2\alpha\beta}{J_2} \left[ f(x_2) + f'(x_2)x_2 \right] < 0
$$

where the first and third expressions have ambiguous signs.

By the implicit function theorem:

$$
\begin{pmatrix}
\frac{\partial J_i}{\partial \theta} \\
\frac{\partial J_i}{\partial \theta} \\
\frac{\partial N_i}{\partial \theta}
\end{pmatrix}
= -B^{-1}
\begin{pmatrix}
\frac{\partial F_1}{\partial \theta} \\
\frac{\partial F_2}{\partial \theta} \\
\frac{\partial F_3}{\partial \theta}
\end{pmatrix}
$$

where:

$$
B^{-1} = \frac{1}{Det(B)} \begin{pmatrix}
\frac{\partial F_1 \partial F_3}{\partial J_1 \partial N_1} - \frac{\partial F_1 \partial F_2}{\partial J_2 \partial N_1} & \frac{\partial F_2 \partial F_1}{\partial J_2 \partial N_1} - \frac{\partial F_2 \partial F_3}{\partial J_1 \partial N_1} & -\frac{\partial F_3 \partial F_1}{\partial J_2 \partial N_1} - \frac{\partial F_3 \partial F_2}{\partial J_1 \partial N_1} \\
\frac{\partial F_1 \partial F_3}{\partial J_2 \partial N_1} - \frac{\partial F_1 \partial F_2}{\partial J_1 \partial N_1} & \frac{\partial F_2 \partial F_1}{\partial J_1 \partial N_1} - \frac{\partial F_2 \partial F_3}{\partial J_2 \partial N_1} & -\frac{\partial F_3 \partial F_1}{\partial J_1 \partial N_1} - \frac{\partial F_3 \partial F_2}{\partial J_2 \partial N_1} \\
-\frac{\partial F_3 \partial F_1}{\partial J_1 \partial N_1} - \frac{\partial F_3 \partial F_2}{\partial J_2 \partial N_1} & -\frac{\partial F_3 \partial F_1}{\partial J_2 \partial N_1} - \frac{\partial F_3 \partial F_2}{\partial J_1 \partial N_1} & \frac{\partial F_2 \partial F_1}{\partial J_1 \partial N_1} - \frac{\partial F_2 \partial F_3}{\partial J_2 \partial N_1}
\end{pmatrix}
$$

and $Det(B)$ can be written as:

$$
Det(B) = \frac{\partial F_2}{\partial J_2} \left[ \frac{\partial F_1 \partial F_3}{\partial J_2 \partial N_1} - \frac{\partial F_1 \partial F_2}{\partial J_1 \partial N_1} \right] + \frac{\partial F_1}{\partial J_1} \left[ \frac{\partial F_3 \partial F_2}{\partial J_1 \partial N_1} - \frac{\partial F_3 \partial F_1}{\partial J_1 \partial N_1} \right]
$$

We next show that $Det(B) < 0$ for $\delta > 0.5$. From $\frac{\partial F_1}{\partial N_1}$, let:

$$
[-\beta P_1^2(2\delta - 1)^2U - (1 + \alpha)\beta P_1(2\delta - 1)(1 - \delta)U \\
- (1 - \alpha)P_1(2\delta - 1)(1 - \delta)U](W_u - W_o)/d^2 = \phi < 0
$$

The first term of $Det(B)$ simplifies to:

$$
\frac{\partial F_2}{\partial J_2} \left( \frac{(1 - \beta)q_1}{J_1} [(\alpha - 1)[f(x_1) + P_D(W_u - W_o)] + \alpha f'(x_1)x_1] \times \left[ \phi - \frac{P^2_{u|ne}(W_u - W_o)}{U} \right] \\
- \frac{\alpha(1 - \beta)q_2^2P_D^2(W_u - W_o)}{U} \times \left[ 2\phi - \frac{P_{u|ne}(W_u - W_o)}{U} + \frac{\beta P_3}{N_1} [-\alpha f(x_1) + (1 - \alpha)f'(x_1)x_1] \right] \\
- \left( \frac{x_1}{N_1J_1} \right)^2 \beta(1 - \beta)f'(x_1)x_1f(x_1) + \alpha(1 - \beta)[1 - \alpha(1 - \beta)] \left[ \frac{q_1P_D(W_u - W_o)}{d} \right]^2 \right)
\right)
\right)
\right)
\right)
Each term inside the curly brackets is greater than zero. Since $\frac{\partial F_2}{\partial J_2} < 0$, the first term is less than zero. The second term of $Det(B)$ simplifies to:

$$-\frac{\partial F_1}{\partial J_1} \frac{P_2}{N_2} \frac{q_2}{J_2} \beta (1 - \beta) f'(x_2)x_2 f(x_2) < 0,$$

implying that $Det(B) < 0$.

Having shown that the determinant is less than zero, we next show that $\frac{\partial J_1}{\partial U} > 0$, $\frac{\partial N_1}{\partial U} > 0$ and $\frac{\partial J_2}{\partial U} < 0$.

Note that:

$$\frac{\partial F_1}{\partial U} = \frac{q_1(1 - \beta)(2\delta - 1)(W_u - W_o)}{d} \geq 0$$

$$\frac{\partial F_2}{\partial U} = 0$$

$$\frac{\partial F_3}{\partial U} = \frac{(W_u - W_o)[\beta P_1(2\delta - 1) + (1 - \delta)]}{d} \geq 0$$

Using the implicit function theorem:

$$\frac{\partial J_1}{\partial U} = -\frac{1}{Det(B)} \left[ \left( \frac{\partial F_3}{\partial N_1} \frac{\partial F_2}{\partial J_2} - \frac{\partial F_3}{\partial J_2} \frac{\partial F_2}{\partial N_1} \right) \frac{\partial F_1}{\partial U} - \left( \frac{\partial F_2}{\partial J_2} \frac{\partial F_1}{\partial N_1} \right) \frac{\partial F_3}{\partial U} \right]$$

$$= -\frac{1}{Det(B)} \left[ \frac{\partial F_2}{\partial J_2} \left( \frac{\partial F_3}{\partial N_1} \frac{\partial F_1}{\partial U} - \frac{\partial F_1}{\partial N_1} \frac{\partial F_3}{\partial U} \right) + \frac{\partial F_1}{\partial U} \left( \frac{\partial F_3}{\partial N_1} \frac{\partial F_2}{\partial J_2} - \frac{\partial F_3}{\partial J_2} \frac{\partial F_2}{\partial N_1} \right) \right]$$

Note that the last term inside the brackets is positive given (12) and $\frac{\partial F_3}{\partial U} > 0$.

With $-\frac{1}{Det(B)} > 0$, we need only show that the first expression inside the brackets is positive. Since $\frac{\partial F_2}{\partial J_2}$, we need to show $\frac{\partial F_3}{\partial N_1} \frac{\partial F_1}{\partial U} - \frac{\partial F_3}{\partial N_1} \frac{\partial F_3}{\partial U} < 0$. Note that:

$$\frac{\partial F_3}{\partial N_1} \frac{\partial F_1}{\partial U} - \frac{\partial F_3}{\partial N_1} \frac{\partial F_3}{\partial U} = \left[ \frac{q_1(1 - \beta)(2\delta - 1)(W_u - W_o)}{d} \right]$$

$$\times \left[ \frac{P_1 \beta}{N_1} \left[ f(x_1) + 2(1 - \alpha) f'(x_1)x_1 \right] \right]$$

$$- \left( \frac{1 - \delta)(1 - \alpha)}{(2\delta - 1)N_1} \left[ f(x_1) + f'(x_1)x_1 \right] \right]$$

$$- \left( \frac{1 - \alpha)(1 - \delta)^2(W_u - W_o)U}{d^2} \right)$$

$$\phi + \frac{P_1 \beta \alpha(1 - \delta)(2\delta - 1)(W_u - W_o)U}{d^2}.$$
negative. Hence, the whole expression is negative. Since $\frac{\partial F_2}{\partial J_2} < 0$, it must be the case that $\frac{\partial J_1}{\partial U} > 0$. We now establish that $\frac{\partial N_1}{\partial U} > 0$. Note that $\frac{\partial N_1}{\partial U}$ can be written as:

$$\frac{\partial N_1}{\partial U} = -\frac{1}{\text{Det}(B)} \frac{\partial F_2}{\partial J_2} \left[ \frac{\partial F_1}{\partial J_1} \frac{\partial F_3}{\partial U} - \frac{\partial F_3}{\partial J_1} \frac{\partial F_1}{\partial U} \right]$$

Since the term outside the brackets is negative, we need to establish that the term inside the brackets is negative as well. This term can be written as:

$$\frac{\partial F_1}{\partial J_1} \frac{\partial F_3}{\partial U} - \frac{\partial F_3}{\partial J_1} \frac{\partial F_1}{\partial U} = - \left[ \frac{q_1(1-\beta)(W_u-W_o)}{d} \right]$$

$$\times \left[ \frac{\beta P_1(2\delta-1)f(x_1)}{J_1} + \frac{(1-\delta)[(1-\alpha)f(x_1) - \alpha f'(x_1)x_1]}{J_1} \right]$$

$$+ \frac{(1-\alpha)P_D}{J_1} \left[ \beta P_1(2\delta-1) + (1-\delta) \right]$$

$$+ \frac{\alpha \beta P_D q_1(2\delta-1)}{d} \left[ (2\delta-1) + (1-\delta) \right]$$

Since for $\delta > 0.5$ all the terms inside both sets of large brackets are positive, the expression itself is negative. Hence, $\frac{\partial N_1}{\partial U} > 0$. Finally, we show that $\frac{\partial J_2}{\partial U} < 0$. Note that:

$$\frac{\partial J_2}{\partial U} = \frac{1}{\text{Det}(B)} \frac{\partial F_2}{\partial N_1} \left[ \frac{\partial F_3}{\partial J_1} \frac{\partial F_1}{\partial U} - \frac{\partial F_1}{\partial J_1} \frac{\partial F_3}{\partial U} \right].$$

With both $\text{Det}(B) < 0$ and $\frac{\partial F_2}{\partial N_1} < 0$ and the sign of the term inside the brackets established as negative from above, $\frac{\partial J_2}{\partial U} < 0$. Having determined that in equilibrium, with an increase in $U$, both $J_1$ and $N_1$ increase, we prove that $W_1$ must decrease using proof by contradiction. If both $J_1$ and $N_1$ increase, then $f(x_1)$ must decrease. Now suppose that $P_D$ decreases. Note that since $P_D$, $P_u|e$, and $P_u|ne$ are all related by a positive constant, a decrease in $P_D$ is equivalent to decreases in $P_u|e$ and $P_u|ne$. We can see from the zero profit condition for City 1 that $q_1$ must then increase. From the worker indifference condition, $f(x_1)$ decreases, and since $P_D$, $P_u|e$, and $P_u|ne$ decrease, $P_1$ must increase. However, by definition, $P_1$ decreases if $q_1$ increases. Therefore, $P_D$ must be increasing. Since $f(x_1)$ is decreasing and $P_D$ is increasing, $\frac{\partial W_1}{\partial U} < 0$. Since both $N_2$ and $J_2$ are decreasing in $U$, $x_2$ unequivocally decreases, and $W_2$ increases. Therefore, $\frac{\partial W_2}{\partial U} > 0$. Therefore, $\frac{\partial [W_1-W_2]}{\partial U} < 0$. QED.
Proof of Proposition 4

Let $\delta = 0$ and $\alpha = 1$. We first establish for this case that $W_1 - W_2 > 0$. Difference City 2 zero-profit condition from the City 1 zero-profit condition and simplifying yields:

$$f(AJ_1) = f(AJ_2) + P_{u|ne}(W_u - W_o)$$

implying that $f(AJ_1) > f(AJ_2)$. Differencing $W_1$ by $W_2$ yields:

$$\beta[f(AJ_1) - f(AJ_2)] + (1 - \beta)P_{u|ne}$$

since both terms are positive, $W_1 > W_2$. We now establish that $\frac{d(W_1 - W_2)}{d\ell}$ may be positive as well. Crucial to signing the expression is the determinant of the Jacobian,

$$Det(B) = \frac{\partial F_2}{\partial J_2} \left[ \frac{\partial F_1}{\partial J_1} \frac{\partial F_3}{\partial N_1} - \frac{\partial F_3}{\partial J_1} \frac{\partial F_1}{\partial N_1} \right]$$

where the sign of the determinant is now ambiguous. If the determinant is positive at a particular equilibrium triplet, however, we show that two other equilibria must exist where the determinant is negative. Further, we show that an equilibrium where the determinant is negative is unstable in that the next firm entering City 1 earns positive profits given the response by $N_1$ to the entry.

To show that there exists two other stable equilibria, note that $F_1$ spans $N_1$, and $F_3$ spans $J_1$ and $J_2$. Since $F_1$ is not a function of $J_2$, any value of $J_2$ will satisfy $F_1$. We can define $J_2$ implicitly as a function of $N_1$, which allows us to define $F_3(J_1, N_1)$. This compresses the problem into the $N_1 \times J_1$ space. If there exists a region of the space $N_1 \times J_1$ where $\frac{\partial N_1}{\partial J_1}|_{F_1} < \frac{\partial N_1}{\partial J_2}|_{F_3}$, then there must be at least two regions, one to the left and one to the right, where $\frac{\partial N_1}{\partial J_1}|_{F_1} > \frac{\partial N_1}{\partial J_2}|_{F_3}$, and $F_3$ must intersect $F_1$ in each of these regions for $F_3$ to span $J_1$. Therefore, since $\frac{\partial F_2}{\partial J_2} < 0$, there must exist at least two other equilibria where the determinant is negative. To show that an equilibrium where the determinant is positive is unstable note that in this case:

$$\frac{\partial F_1}{\partial J_1} \frac{\partial F_3}{\partial N_1} < \frac{\partial F_3}{\partial J_1} \frac{\partial F_1}{\partial N_1}$$
Dividing both sides by \( \frac{\partial F_1}{\partial N_1} \frac{\partial F_3}{\partial N_1} \) and using the implicit function theorem we have:

\[
\frac{\partial N_1}{\partial J_1} \bigg|_{F_1} < \frac{\partial N_1}{\partial J_1} \bigg|_{F_3}
\]

where the implicit function theorem in this case is for each equation, and not the entire system of equations. Increase \( J_1 \) an infinitesimal amount to \( J_1' \), holding \( N_1 \) fixed. Then, \( F_3 > 0 \). \( N_1 \) must increase to \( N_1' \) to restore the worker indifference condition, \( F_3 = 0 \). Let the \( N_1 \) value to restore the zero profit condition in City 1, \( F_1 = 0 \), be \( N_1'' \). Since \( \frac{\partial N_1}{\partial J_1} \bigg|_{F_1} < \frac{\partial N_1}{\partial J_1} \bigg|_{F_3} \), \( N_1' > N_1'' \). With \( N_1 = N_1' \), \( F_1 > 0 \), and expected profits for firms in City 1 are greater than zero. If expected profits are greater than zero, the \( J_1 \) will further increase. This process will continue to increase \( N_1 \) and \( J_1 \) sequentially, as long as \( \frac{\partial N_1}{\partial J_1} \bigg|_{F_1} < \frac{\partial N_1}{\partial J_1} \bigg|_{F_3} \). Therefore, the equilibrium point where the determinant is negative cannot be stable.

\[
\frac{\partial J_1}{\partial U} = - \frac{1}{\text{Det}(B)} \frac{\partial F_2}{\partial J_2} \left[ \frac{\partial F_3}{\partial U} \frac{\partial F_1}{\partial U} - \frac{\partial F_1}{\partial U} \frac{\partial F_3}{\partial U} \right]
\]

(13)

For the equilibria that do have a negative determinant, note that the relevant partial derivatives are:

\[
\frac{\partial F_1}{\partial U} = - \frac{q_1(1 - \beta)(W_u - W_o)}{d} < 0
\]

\[
\frac{\partial F_3}{\partial U} = \frac{(1 - \beta P_1)(W_u - W_o)}{d} \geq 0
\]

\[
\frac{\partial F_1}{\partial J_1} = \frac{q_1(1 - \beta)}{J_1} \left[ f'(x_1)x_1 - \frac{P^2x_1(W_u - W_o)}{U} \right] < 0
\]

\[
\frac{\partial F_2}{\partial J_2} = \frac{q_2(1 - \beta)f'(x_2)x_2}{J_2} < 0
\]

\[
\frac{\partial F_3}{\partial N_1} = - \frac{\beta P_1 f(x_1)}{N_1} - \frac{\beta P_2 f(x_2)}{N_2} + \frac{U(W_u - W_o)(\beta P_1 - 1)}{d^2} < 0
\]

Note that the term outside the brackets in (13) is negative. The term inside the brackets is positive implying \( \frac{\partial J_1}{\partial U} < 0 \). Note that for \( \alpha = 1 \) and \( \delta = 0 \),

\[
W_1 = \beta f(AJ_1) + (1 - \beta) P_u|\ne(W_u - W_o).
\]

Since the probability of the firms finding suitable workers (\( q_1 \)) does not change, the term inside the parenthesis in the zero profit condition cannot
change as well. Since the decrease in $J_1$ is increasing the value of $f(AJ_1)$, it must be that $P_{u/n}$ is also increasing to maintain the zero profit condition. Hence, both terms in the expression for $W_2$ are increasing. QED.

References


