Chapter 21

Externalities in Competitive Markets

At this point you may have gotten the impression that economists believe markets always and unambiguously result in efficient outcomes — with total surplus maximized when markets operate without interference from other institutions. If this were the case, there would be no efficiency role for non-market institutions in society, and their only justification would lie in concerns about the distribution of surplus — concerns about equity and fairness as these relate to the market allocation of scarce resources. But, while such issues do play an important role in justifying non-market institutions (including government), we will in this and the coming chapters investigate conditions under which non-market institutions are motivated by efficiency rather than equity concerns. These conditions include all the possible violations of the assumptions underlying the first welfare theorem (Chapter 15) — including the presence of market power and of asymmetric information.

Before we get to asymmetric information and market power, however, we will first take a look at yet a third set of conditions that lead to dead weight losses in the absence of other institutions — even when markets are perfectly competitive. These conditions are called externalities, and they arise whenever decisions of some parties in the market have a direct impact on others in ways that are not captured by market prices. When a firm’s production process emits pollution into the air, for instance, this pollution potentially has a direct impact on many. Put differently, the emission of pollution imposes on society costs that are typically not priced by the market and thus are not costs taken into account by producers unless some other institution imposes those costs on them. When I decide to get in the car and enter a congested road, I am similarly contributing to the overall congestion and thus am delaying others from getting to where they want to go, but I don’t think about others when I make the decision of whether to get in the car. When I play loud music on my patio at home, my neighbors get to “enjoy” the music as well. These are all examples of externalities — of “external costs or benefits” that markets do not internalize because the market participants do not have to pay for them.

1This chapter builds once again on a basic understanding of the partial equilibrium model from Chapters 14 and 15. Section 21B.3 also builds on the discussion of exchange economies in Chapter 16 but can be skipped if you have not yet read Chapter 16.
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The essential feature of an externality is then that either costs or benefits of production or consumption are directly imposed on non-market participants. Since non-market participants are neither demanders nor suppliers of goods, neither market demand nor market supply curves are affected by such externality costs or benefits. Thus, a competitive market composed of price-taking consumers and producers continues to produce in equilibrium where demand intersects supply. However, while the aggregate marginal willingness to pay curve still allows us to measure the benefits consumers receive from participating in markets and the supply curve still allows us to measure costs incurred by producers, there are now non-market participants that also incur benefits or costs. Thus, we can no longer simply use consumer and producer surplus to measure the net-gains for society from the existence of markets. Put differently, we have to include the externality costs and benefits that a competitive market ignores in our calculation of overall surplus.

Before we get started, I should note that we will treat consumers and producers as strictly separate in their roles as consumers and producers from their roles as individuals who may incur some damage or benefit from an externality. We generally lose nothing by making this assumption. Even if, for instance, a producer whose production causes pollution incurs health problems from pollution, no individual producer will take those costs into account in her production choices because, in competitive markets, each producer is so small relative to the market that her contribution to overall pollution is negligible. Thus, we will simply treat all producers as considering only their own production costs when making decisions, and then lump them in separately with all economic agents who are hurt by the aggregate level of pollution produced by the industry. In other words, we will treat producers as individuals who consider their own cost of production when making supply decisions, and then we will treat the part of that producer that is hurt by the overall level of pollution as a separate person.

21A.1 Production Externalities

Suppose, then, that we return to the example of an industry that produces “hero cards” but now we assume that the least-cost production process for producers involves the emission of green-house gases that contribute to environmental problems. Thus, in addition to the costs of production that are faced by each of the producers of hero cards, costs of pollution are imposed on others in society. We will then reconsider how many hero cards would be produced by a social planner who knows all the relevant costs and benefits and who seeks to maximize social surplus — how much production would take place if our omniscient and benevolent “Barney” from Chapter 15 would allocate resources. In our Chapter 15 analysis that excluded production externalities like pollution, it turned out that “Barney” could do no better than the competitive market. We will now see that this is no longer true when externalities become part of the analysis.

21A.1.1 “Barney” versus the Market

In Graph 21.1, we begin with the market demand and supply graph for hero cards in panel (a). Whether there are production externalities or not, the market will then produce \( x^M \) at price \( p^M \), with all consumers and all producers doing the best they can in equilibrium. Assuming tastes in hero cards are quasilinear, consumers then get the shaded blue area in surplus while producers get the shaded magenta area. If the production of hero cards produces pollution, however, each hero
card that is produced imposes a pollution cost on society, a cost that is borne neither by those who consume nor those who produce hero cards.

Graph 21.1: Maximizing Social Surplus in the Presence of a Negative Production Externality

Panel (b) of the graph then inserts a green curve labeled “SMC”. This curve represents the social marginal cost of producing hero cards. It includes the producers’ marginal costs that are captured in the market supply curve, but it also includes the additional cost of pollution that is imposed on others. Thus, the social marginal cost curve must lie above the supply curve since it includes costs in addition to those incurred by producers. It may be that the SMC curve is parallel to the supply curve — implying a constant marginal cost of pollution for each hero card produced, or that it diverges from the supply curve — implying that each additional hero card results in a greater additional pollution cost than the last one. Regardless of how exactly it is related to supply, however, it is this curve that accurately reflects the society-wide cost of production.

As a result, our omniscient and benevolent “Barney” would then decide to continue to produce so long as the benefits from production as represented by the marginal willingness to pay of consumers outweighs the overall cost of additional production for society. Put differently, Barney would certainly produce the first hero card because there is some consumer to whom this card is worth more than all the costs incurred by society as measured by SMC, and he would continue to produce until the green SMC crosses the blue marginal benefit curve. He would not, however, produce any more than that — because once SMC is higher than the marginal willingness to pay of consumers, the society-wide cost of additional hero cards is larger than the benefit. Barney then would choose to produce \(x^B\), resulting in an overall surplus for society represented by the shaded green area.

We can already see that the social planner who seeks to maximize overall surplus will therefore choose less production than will occur in the market. This implies that the market will produce an inefficiently high level of output in the absence of any non-market institutions that curtail production. This is clarified even further in panel (c) where we have labeled some areas in the graph that can now be used to calculate the dead weight loss society incurs under market production. Area \((a + b + c)\) is equal to the blue consumer surplus (assuming the uncompensated demand is equal to marginal willingness to pay) in panel (a) while area \((d + e + f)\) is equal to the magenta producer surplus from panel (a). Producers and consumers are, in their roles as producers and
consumers, unaffected by the pollution and therefore receive the same surplus as if there was no pollution. However, we also know that, in the presence of pollution, we have to take into account the overall cost of the pollution that is produced when the market quantity \( x^M \) is produced. That area is the difference between the costs incurred by producers and the costs as represented in the SMC curve — an area equal to \( (b + c + e + f + g) \). Thus, we have to subtract that from consumer and producer surplus to get overall social surplus \( (a + d - g) \) under market production. Under Barney’s benevolent dictatorship, on the other hand, society gets an overall surplus of \( (a + d) \) equal to the green area in panel (b). The market therefore produces a deadweight loss equal to \( (g) \).

**Exercise 21A.1** Suppose that the “pollution” emitted in the production of hero cards is of a kind that has no harmful effects for humans but does have the benefit of killing the local mosquito population — i.e. suppose the pollution is good rather than bad. Would the market produce more or less than Barney?

**Exercise 21A.2** Would anything fundamental change in our analysis if we let go of our implicit assumption that the aggregate demand curve is also equal to the aggregate marginal willingness to pay curve? (Your answer should be no. Can you explain why?)

### 21A.1.2 Another Efficient Tax

Our analysis thus far tells us that competitive markets will produce too much in the presence of negative pollution externalities. As a result, there exists the potential for government policy to enhance efficiency — and thus reduce or eliminate the deadweight loss from market overproduction. And we have already seen in Chapter 18 that taxation of goods is one policy tool that can reduce market output. In the absence of externalities, this is inefficient because the market allocation of resources was efficient to begin with. Now, however, this reduction of an otherwise inefficient output level can reduce rather than increase deadweight loss.

Suppose, for instance, you knew both the market demand and supply curves as well as the optimal production level \( x^B \) that Barney would choose. This information is depicted in panel (a) of Graph 21.2. Based on what we learned about taxes and tax incidence in Chapter 18, you can then easily determine the tax rate \( t \) required to reduce market output from \( x^M \) to \( x^B \) by simply letting \( t \) per unit be equal to the green vertical distance in the graph. As a result, buyers in the market would face the higher price \( p_B \) while sellers would receive the lower price \( p_S \) with the difference between the two prices representing the payment \( t \) per unit in taxes. A tax such as this that is intended to reduce market output to its efficient quantity because of the presence of a negative production externality is called a Pigouvian Tax.\(^2\)

In panel (b) of the Graph we can then analyze more directly how this tax is efficient. In the absence of the tax, the market produces output \( x^M \) at price \( p^M \). You can check for yourself, in a way exactly analogous to what we did in panel (c) of Graph 21.1, that the competitive market on its own will produce overall surplus equal to \( (a + b + e + h - j) \), with the triangle \( j \) once again representing deadweight loss. Under the tax \( t \), however, consumer surplus \( (a) \) and producer surplus \( (h + i) \) combine with a positive tax revenue \( (b + c + e + f) \) and a social cost from pollution \( (c + f + i) \) to produce an overall surplus \( (a + b + e + h) \). This is exactly equal to the green maximum surplus achieved by benevolent Barney in Graph 21.1b — and eliminates the deadweight loss \( (j) \).

Put differently, the reason we found taxes to be inefficient in Chapter 19 was that they distorted the price signal that coordinated efficient cooperation between producers and consumers — but, in

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\(^2\)The tax is named after Arthur Cecil Pigou (1877-1959), a British economist and student of Alfred Marshall (who succeeded Marshall as Professor of Political Economy at Cambridge University). Pigou developed the distinction between private and social marginal cost in his most influential work entitled *Wealth and Welfare*. 
The presence of externalities, the price signal is already distorted insofar as it does not efficiently coordinate production and consumption. The tax then removes the distortion and causes the market to “internalize the externality”.

In order for the government to be able to impose an efficient Pigouvian tax, it must however know the optimal quantity \( x^B \) it wants the market to reach and it must know the difference between the market demand and supply curve at that quantity. Put differently, the government must know the marginal social damage caused by pollution at the optimum quantity. If it possesses this information, the government can achieve the maximum social surplus by simply setting the per-unit tax equal to this marginal social damage of pollution.

Exercise 21A.3 What if the government only knows the marginal social damage of pollution at the equilibrium output level \( x^M \) and sets the tax rate equal to this quantity? Will this result in the optimal quantity being produced? If not, how do the the SMC and the supply curve have to be related to one another in order for this method of setting the tax to work?

It may in principle not look too difficult for the government to gather sufficient information to implement a Pigouvian tax that causes markets to once again produce efficiently. However, suppose that there are now many different industries, each causing pollution. In order to set optimal Pigouvian taxes, the government now has to know this same information for each industry and set the per unit tax in each industry, letting taxes vary across polluting industries as the marginal social damage of pollution at the optimum is different everywhere. This would then result in a complex system of different Pigouvian taxes across all polluting industries. As technology changes, these rates would have to be continuously adjusted. And, perhaps worst of all, unless the government adjusts Pigouvian taxes whenever firms find ways of reducing pollution on their own, individual firms in each industry would gain no benefit from applying pollution-abating technologies in their own firms — because they would still face the same taxes. Thus, while it may look easy in principle to impose Pigouvian taxes, it is much more difficult to do so in practice and to simultaneously encourage those industries for whom it is easy to reduce pollution to do so in ways other than simply cutting production due to the tax.

It is for this reason that economists have largely turned away from recommending Pigouvian taxes on output and have instead turned to alternatives that focus more directly on forcing pro-
ducers to confront the tradeoff between reducing pollution (through less production or through the development of pollution abating technologies) or paying for its social costs. This shift in focus has also been made possible by new technologies that allow governments to pinpoint who is producing pollution — and thus to require polluters to pay for pollution directly. This can be done either through a pollution tax (as opposed to a Pigouvian tax on output), or through the design of market-based environmental policy. We will discuss the latter first and then briefly compare it to the former.

Exercise 21A.4 In Chapter 18, we discussed the efficiency losses from government mandated price ceilings or price floors. Could either of these policies be efficiency enhancing in the presence of pollution externalities (assuming the government has sufficient information to implement these policies)?

21A.1.3 Market-based Environmental Policy

The most common market-based environmental policy works as follows: The government determines an overall level of pollution (of each kind) that it finds acceptable and then issues pieces of paper that permit the owner to emit a certain quantity of different types of pollutants per week (or month or year). These pieces of paper, known as pollution vouchers or tradable pollution permits, thus represent the “right to pollute” by some amount. Then the government releases these rights — either by auctioning them off or by simply giving them to different firms in different industries. It turns out that it does not matter which precise way the government uses to distribute such permits — the important feature for our analysis is that individuals who own such permits can sell them to others if they so choose (and thus transfer the “right to pollute” to someone who is willing to pay more than it is worth to the original owner). In essence, the policy therefore “caps” the overall pollution level by fixing the number of pollution permits — and then allows “trade” in permits to determine who uses them. For this reason, it has come to be known as a cap-and-trade policy.

Pollution vouchers have value to producers because they permit producers to emit pollution in their production process. At the same time, whenever a producer chooses to use such a voucher, she incurs an economic (or opportunity) cost — because she could have chosen to sell (or rent) the voucher to someone else instead. Each producer therefore has to weigh the costs and benefits of using a pollution voucher — and each producer knows that she will have to use fewer vouchers the less she produces and the more she takes advantage of pollution-abating technologies. Since some production processes lend themselves to pollution-abating technologies more easily than others, firms in some industries will have a greater demand for such vouchers than firms in other industries. As a result, by introducing pollution vouchers into an economy (and prohibiting the emission of pollution when firms do not own such vouchers), the government has created a new market — the market for pollution vouchers.

Exercise 21A.5 Explain how firms face a cost for pollution regardless of whether the government gives them tradable pollution vouchers or whether firms have to purchase these.

This market is depicted in Graph 21.3 where pollution vouchers appear on the horizontal axis and the price per voucher appears on the vertical. By introducing only a limited quantity of such vouchers, the government has set a perfectly inelastic supply at precisely that quantity which results in the level of overall pollution across all industries. Firms that emit pollution in their production processes are the demanders of such vouchers, with demand depending on how much pollution is involved in producing different types of goods and how easy it is for firms to find ways of reducing
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the pollution emitted in production. Put differently, those firms that find it difficult to reduce their pollution will be willing to pay more for the right to pollute than those who can easily put a filter on their smokestacks. In equilibrium, pollution vouchers will then sell at price $p^*$. 

Graph 21.3: A Market for Pollution Vouchers

Assuming the government can monitor polluting industries effectively (which is becoming increasingly easy as pollution monitors are widely distributed by the Environmental Protection Agency across different regions and as satellite technology is becoming increasingly effective at detecting pollution emissions from very precise locations), a system of pollution vouchers then achieves the following: First, it imposes a cost on polluters by requiring that they purchase sufficient pollution rights for the pollution they emit. This, then, causes an upward shift in firm $MC$ curves as pollution vouchers become an input into the production process, and with it a shift in the market supply curve in polluting industries. Such a shift will result in less production of output in such polluting industries. Second, the system introduces an incentive for firms to search for (and invest in) pollution-abating technologies. So long as it costs less to reduce pollution from my firm than the pollution vouchers would cost me, I now have an incentive to reduce my pollution emissions. Third, the system creates an incentive for new firms to arise and to independently invest in research and development of pollution-abating technologies because the system has increased the demand for such technologies in light of the fact that polluters would otherwise have to pay for vouchers in order to produce.

As a result, the system achieves an overall reduction in pollution at the least social cost and without the government adjusting any policy to changing conditions. The government does not have to be in the business of picking which industry reduces which type of pollution by how much, and it does not have to adjust those policies as pollution-abating technologies (that are more applicable to some industries than to others) are produced. All the government has to do is to set an overall pollution target and print a corresponding quantity of pollution vouchers. The newly created pollution voucher market then rations who gets the vouchers and who does not get them — with those for whom reductions in pollution are most costly choosing to use vouchers and others choosing to reduce pollution cheaply. Put differently, pollution vouchers are government interventions that harness the power of a newly created market to generate the information required to reduce pollution at the lowest possible cost without any further government interference.

Exercise 21A.6 If the government, after creating the pollution voucher market, decides to tax the sale of pollution vouchers, will there be any further reduction in pollution? (Hint: The answer is no.)
And there is one final check on the system: While we have said thus far that polluters are the ones who will form the demand curve in the pollution voucher market, it is in principle possible to allow anyone at all to participate in that market. If, for instance, a group of deeply concerned citizens feels that the government is permitting too much pollution to be emitted into the air, they could pool resources and purchase some quantity of the vouchers — thus increasing the price (and raising the cost to polluting) while lowering the supply (if they simply store away the pollution vouchers). As we will see in a later chapter on public goods, such groups face a difficult free rider problem that they need to overcome, but if they can, they are able to impact the overall level of pollution without lobbying the government.

One last clarifying caveat, however: While pollution vouchers offer a mechanism to reduce pollution to a target level in the least costly way, there is nothing in a pollution voucher system that guarantees we will have set the socially optimal target for pollution to begin with. If the political process that determines this target is efficient, then the target will be set optimally. But otherwise, the target might be too high or too low — all that the cap-and-trade system does for us is to get us to the target in the least costly way.

21A.1.4 Pollution Taxes, Pigouvian Taxes and Cap-and-Trade

While the idea of taxing output in polluting industries — as originally proposed by Pigou — has lost considerable favor among economists, the very technology that allows the establishment of markets in tradable pollution permits now enables governments to tax pollution (rather than output) directly. One suspects that, had Pigou thought it possible to detect pollution where it is emitted, he would most likely have favored taxing pollution rather than output as well. Taxing pollution directly has the same advantages over Pigouvian taxes that we have pointed out for cap-and-trade systems, and a per-unit-of-pollution tax is in fact equivalent to establishing tradable pollution permits if the tax rate is set at the same level as the price-per-unit-of-pollution that emerges in cap-and-trade systems. Both systems provide incentives for firms to invest in pollution abating technologies; neither requires governments to adjust industry tax rates as circumstances change (as it the case under Pigouvian taxes on output); overall pollution is reduced in the least cost ways as firms for whom it is easy to reduce pollution will do so rather than incur the cost of pollution (by either paying a pollution tax or using pollution vouchers); and neither system automatically results in full efficiency unless the government has lots of information on what the efficient tax rate or the efficient number of pollution permits is.

Exercise 21A.7 In one of the 2008 Presidential Primary debates, one candidate advocated the cap-and-trade system over a carbon tax on the grounds that the carbon tax would be partially passed onto consumers in the form of higher prices. Another candidate who also supported the cap-and-trade system corrected this assertion — suggesting that, to whatever extent a carbon tax would be passed onto consumers, the same is true of costs (of tradable permits) under the cap-and-trade system. Who was right?

While pollution taxes and cap-and-trade systems are therefore quite similar, environmental policy makers nevertheless debate their relative merits. Some consider it important to set precise target levels for pollution — with cap-and-trade systems allowing an easy way of establishing such targets while then letting the market for tradable permits determine the per-unit-of-pollution price required to implement the target. Others believe it is more important to specify the per-unit-of-pollution cost directly through a tax in order to allow firms to plan accordingly — leaving the

3The exchange took place in the January 5, 2008 Democratic Presidential Primary Debate held at St. Anselm College. The first candidate was New Mexico Governor Bill Richardson; the second was then-Senator Barak Obama.
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level of pollution reduction that result to arise from firm responses to the tax. Again, if the per-
unit-of-pollution tax is set at the same rate as the per-unit-of-pollution price that emerges under
a particular “cap” in a cap-and-trade system, the two policies have identical effects – but one gets
there by being precise about the target pollution level up front while the other gets there by being
precise about the per-unit-pollution cost up front.

A second issue that is raised in policy debates regarding cap-and-trade versus pollution taxes
relates to politics and implementation. Some fear that a nation-wide – or even world-wide – cap-and-
trade system would involve excessive government bureaucracy to administer the various markets for
different types of pollution vouchers while others argue that administering pollution taxes would
involve similar issues. In practice, however, there appears to be one important political reason for
environmental policy makers to favor the cap-and-trade system: It has a built in mechanism for
overcoming concentrated opposition from industries that are particularly affected. Such industries
would face increased marginal costs under both the pollution tax and the cap-and-trade system, but
pollution vouchers could be given away for free to some industries in order to “buy” their political
support. In essence, this involves a transfer of wealth (in the form of pollution vouchers that can be
traded) without a change in the increased opportunity cost of emitting pollution. Under pollution
taxes, one could similarly “buy off” industry opposition through transfers of taxpayer money, but
this appears to be politically more controversial.

Exercise 21A.8 Suppose that advocates of pollution taxes proposed a reduction in such taxes for key in-
dustries that would otherwise be opposed to the policy. How is this different than giving pollution vouchers
away for free to such key industries in a cap-and-trade system?

Finally, to the extent to which the pollution problem to be addressed is global (as in the case of
greenhouse gases) rather than local (as in the case of acid rain), policy makers may favor the cap-
and-trade system as it permits the establishment of global markets in tradable pollution permits
to achieve global reductions in pollution while allowing an initial establishment of country-specific
“caps” through negotiated international agreements. Such a system does not enshrine country-
specific caps because permits could be traded across national boundaries, but – much as support
form particular industries can be gained by giving some pollution permits away – international
support for such agreements could be facilitated by initially allocating relatively more pollution
permits to some countries rather than other countries.

Exercise 21A.9 Less developed countries often point out that countries like the U.S. did not have to
confront the fact that they caused a great deal of pollution during their periods of development – and thus
suggest that developed countries should disproportionately incur the cost of reducing world-wide pollution
now. Can you suggest a way for this to be incorporated into a global cap-and-trade system?

21A.2 Consumption Externalities

We have thus far considered only externalities generated in the production of goods and, with the
exception of the externality considered in within-chapter-exercise 21A.1, we have limited ourselves
to externalities that have negative impacts on others — or what we have referred to as negative
externalities. Externalities can, however, arise in production and consumption, and they can be
positive or negative. We will now illustrate the impact of an externality on the consumer side, and,
to differentiate it further from what we have done so far, we will consider a positive rather than a
negative externality.
Suppose, for instance, that production of hero cards entails no pollution whatsoever but, whenever a consumer purchases hero cards for children, the world becomes a better place. In particular, suppose that for each child that is exposed to hero cards, future crime falls and good citizens emerge. This may sound silly because of the context of the example, but such arguments are often made in markets like children’s programming on television or markets involving the arts. The essential nature of the argument is always the same: In addition to the private benefits that consumers obtain directly from consumption, others in society benefit indirectly in ways that are not priced by the market.

21A.2.1 Positive Externalities from Consumption

Graph 21.4 then presents a series of graphs for positive externalities that is exactly analogous to the series of graphs in Graph 21.1 for negative externalities. Panel (a) simply illustrates consumer and producer surplus along market supply and demand curves (once again under the assumption that demand can be interpreted as marginal willingness to pay). Panel (b) introduces a new curve called "SMB" or social marginal benefit. This curve includes all the benefits society gains from each unit of consumption. It therefore includes all the private benefits that consumers get (and that are measured by the demand curve), plus it includes additional social benefits that are gained by others. As in the case of SMC and supply, SMB and demand can be related to each other in a variety of ways, but under positive externalities SMB must certainly lie above demand (or private marginal willingness to may).

Graph 21.4: Underproduction in the Presence of a Positive Externality

Our benevolent social planner would then use this SMB to measure the marginal benefit of each hero card that is produced (while measuring the marginal cost along the supply curve in the absence of negative externalities.) He would therefore choose the production level \( x^B \) in panel (b) of the graph, giving the shaded green area as overall social surplus. Thus, the market produces an inefficiently low quantity of a good that exhibits a positive consumption externality. We can derive the exact deadweight loss from the areas labeled in panel (c) of Graph 21.4. At the competitive market equilibrium, consumer surplus is simply area \( \text{a} \) (equivalent to the blue area in panel (a)) and producer surplus is area \( \text{b} \) (equivalent to the magenta area in panel (a)). Since the market
produces an output level $x^M$, the additional social benefit from the externality is given by area ($c$). Thus, the market achieves an overall social gain equal to area $(a + b + c)$. Our social planner, on the other hand, achieves that plus area $(d)$ — implying that society incurs a deadweight loss of $(d)$ in the absence of non-market institutions that induce additional production.

21A.2.2 Pigouvian Subsidies

One non-market institution that we already know from our previous work can raise the level of output in the market is a government subsidy. Suppose that the government knows it wants to raise output in the hero card market to $x^B$ above the market quantity $x^M$. In panel (a) of Graph 21.5, this implies that the government can accomplish its goal by imposing a subsidy $s$ equal to the green vertical distance, thus lowering the price for buyers to $p_B$ and raising the price for sellers to $p_S$. Our discussion of the economic incidence of a subsidy in Chapter 18 treats this in more detail and illustrates that the degree to which prices faced by buyers and sellers change depends on the relative price elasticities of market demand and supply curves. When such a subsidy is used to “internalize a positive externality”, it is known as a Pigouvian subsidy. As in the case of a Pigouvian tax, it can restore efficiency by removing the externality-induced distortion in market prices.

Graph 21.5: An Efficient Pigouvian Subsidy

Suppose again (for simplicity) that tastes for hero cards are quasilinear and that we can therefore treat the market demand curve as the aggregate marginal willingness to pay curve for consumers. In panel (b) of the graph we can then calculate the areas that make up total surplus before and after the subsidy. Before the subsidy, consumer and producer surplus simply sum to $(a + b + c + d + e + f + g + k)$. Under the subsidy, consumer surplus is $(a + b + c + d + e + f + g + k)$, producer surplus is $(b + c + d + f + i)$ and surplus for non-market participants is $(e + f + h + i + j)$. From the sum of these areas we then need to subtract the cost of the subsidy, which is $(b + c + f + g + i + j + k)$ — giving us a total surplus of $(a + b + c + d + e + f + h + i + j + k)$. Thus, total surplus under the subsidy is now equal to the green area in Graph 21.4b which we concluded was the maximum social gain possible, with the subsidy having eliminated the deadweight loss $(h + i)$ that occurred under a pure market allocation.

Exercise 21A.10 Suppose that, instead of generating positive consumption externalities, hero cards actually
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divert the attention of children from studying and thus impose negative consumption externalities. Can you see how such externalities can be modeled exactly like negative production externalities?

21A.2.3 Charitable Giving, Government Policy and Civil Society

In the case of a negative production externality of pollution, we illustrated next how government could, instead of attempting to calculate all the “right” Pigouvian taxes each year, create a new market of pollution vouchers that can efficiently reduce pollution to some level set by the government. In the case of positive consumption externalities, I can’t offer a similar market-based policy that is currently under discussion, but we should note that the market outcome we have predicted in the model may not necessarily be the actual outcome if markets operate within the context of non-governmental and non-market institutions that we referred to in Chapter 1 as civil society. The word “civil society” does not have a clear definition and is often used to mean many different things. In this text, I will refer to an institution as a “civil society” institution whenever it is not clearly set up by the government nor does it operate strictly on the self-interested motives that generate explicit prices in markets. Civil society institutions are then the sets of interactions among individuals that occur outside the context of government and outside the context of explicit market prices. Such institutions tend to arise as individuals try to use persuasion rather than the political process to address issues of concern that are not addressed in the market. The existence of positive consumption externalities offers an example — because, as we have seen, it is a case when the market in the absence of non-market institutions produces too little of goods that are valued in society beyond their simple consumption value.

As you are no doubt aware, many organizations spend substantial energy in attempts to make people aware of many social concerns in an attempt to persuade them to voluntarily contribute money or time to organized efforts aimed at addressing such concerns. In the case of television programming for children, for instance, we have all seen appeals on television for private donations to increase funding for such programs. Such efforts to appeal for charitable donations run into difficulties involving “free riding” that we will address more explicitly in Chapter 27 and thus offer no guarantee of achieving a fully efficient outcome, but they appear to play an important role in many circumstances where positive externalities would make markets by themselves produce too little.

At this point, we will simply leave the issue with the observation that all three types of institutions that we have discussed — government, markets and civil society, face obstacles in achieving efficient outcomes. Markets, as we have seen, will tend to underproduce in the presence of positive externalities and overproduce in the presence of negative externalities; governments may face difficulties in ascertaining the information necessary for implementing optimal outcomes through taxes, subsidies (or other means), especially as circumstances within societies change, and they face political hurdles that we will treat more explicitly in Chapter 28. And civil society efforts that rely on strictly voluntary engagement of non-market participants face difficulties in engaging those non-market participants fully as each will tend to rely on others to address the problem. Yet each appears to play a role in the real world.

Finally, just as the case of pollution vouchers represents an effort by government to engage market forces in finding efficient solutions to excessive pollution, government policies are often aimed at engaging civil society institutions more. The most obvious example of this can be found in the U.S. income tax code that offers tax deductions to individuals who voluntarily give to charitable causes — thus subsidizing such causes without the government making the explicit decision of which
charities will end up engaging non-market participants. Thus, when the government faces too many hurdles in designing explicit subsidies for each industry that generates positive externalities, it can offer such general subsidies aimed at reducing the hurdles faced by civil society organizations in finding non-market, non-governmental solutions.

**Exercise 21A.11** In what sense does the tax-deductibility of charitable contributions represent another way of subsidizing charities?

**Exercise 21A.12** In a progressive income tax system (with marginal tax rates increasing as income rises), are charities valued by high income people implicitly favored over charities valued by low income people? Would the same be true if everyone could take a tax credit equal to some fraction \( k \) of their charitable contributions?

**Exercise 21A.13** We did not explicitly discuss a role for civil society institutions in correcting market failures due to negative externalities. Can you think of any examples of such efforts in the real world?

### 21A.3 Externalities: Market Failure or Failure of Markets to Exist?

Thus far, we have seen that markets by themselves will produce inefficient quantities of goods that exhibit positive or negative consumption and production externalities. In the absence of government intervention, civil society efforts may contribute to greater efficiency. Alternatively, government policies can be designed to change market output directly (as in the case of Pigouvian taxes and subsidies) or to indirectly harness the advantages of market forces (as in the case of cap-and-trade policies) or civil society institutions (as in the case of the tax deductibility of charitable contributions) to increase efficiency and lower deadweight losses. After we have explored more fully (in the upcoming chapters) the many hurdles faced by markets, governments and civil society institutions in implementing optimal outcomes for society, we will return in the last part of the book to a general model of how we can ascertain the appropriate balance of markets, government and civil society depending on the particulars of the social problem that is to be solved.

In the meantime, however, we can see yet another efficiency-enhancing policy tool the government has at its disposal by exploring a little more deeply the fundamental problem created by the presence of externalities. We have seen that markets by themselves will tend to “fail” in the presence of externalities — and this has often led economists to refer to externalities as one (of several) potential market failures. In this section, we will see how this market failure arises because of the fact that, whenever there is an externality generated in competitive markets, we can trace the over or under production that arises from this externality to the lack of a market or the non-existence of a market somewhere else.

#### 21A.3.1 Pollution and Missing Markets

Consider again the case of a market in which pollution is a by-product of production. The fundamental reason that a market will overproduce in this case (relative to the efficient quantity) is that producers are not forced to face the full costs they impose on societies when making production decisions. In particular, if the pollution that is generated is air pollution, the producer escapes paying for the input “clean air” that is used in the production process unless some mechanism (like Pigouvian taxes, pollution taxes or pollution vouchers) is implemented. Were there a market for each of the inputs used in production — including the input “clean air”, the producers would have to fully pay for all the costs they impose. *Air pollution therefore arises as a problem that keeps markets from producing efficiently because one of the inputs into production is not bought and sold.*
Chapter 21. Externalities in Competitive Markets

I know that this sounds rather silly — how could there possibly be a “market for clean air” when no one owns the air and therefore no one can sell clean air to firms that use it in the production process. It sounds silly because it is silly. Nevertheless, if we can suspend disbelief for a moment, we can see the conceptual point that the externality is a problem precisely because we have not found a way to create a market in clean air. If there was such a market, and if all air was owned by different people, then each user of clean air would have to pay for it as it is being used. Consumers of clean air — including producers who use clean air as an input — would have to pay for clean air just as firms have to pay for labor and capital. Such a market for clean air would therefore result in a market price which would, in the absence of any other externalities, result in maximum social surplus in the clean air market. As producers contemplate production that involves pollution, they would then face a price for clean air — shifting their marginal cost curves up and thus shifting market supply up to be equal to the social marginal cost ($SMC$) of production rather then the marginal private cost that excludes the social cost of pollution. This would then result in the efficient quantity of the pollution-generating output, with social surplus once again maximized purely by market forces.4

In an abstract conceptual sense, the market failure generated by the presence of externalities can then be traced to the failure of a market to exist. Does recognizing this get us any closer to solving the problem? In the case of pollution, it is that recognition that has led economists to come up with the proposal for creating markets in pollution vouchers. Pollution voucher markets are not the same as markets in clean air, but they represent an attempt to resolve a problem created by the non-existence of a market (for clean air) through the creation of a different type of market that can help. Recognizing the market failure generated by externalities as a failure of a market to exist can therefore create the opportunity for innovative government interventions that may, at least in some cases, work better than other government solutions we might otherwise implement.

21A.3.2 The Tragedy of the Commons

This insight then points toward a huge role that governments more generally have to play in order for markets to function efficiently. Throughout our treatment of the efficiency of markets in Chapters 15, 16 and 17, for instance, we made the implicit assumption that markets for all sorts of inputs such as labor and capital actually exist. Presuming that such markets exists presumes that individuals own resources that they can trade — and this presumes that there is some mechanism in place that protects the property rights of owners of resources. Firms cannot just take my leisure and use it for labor inputs — they are required to persuade me to sell my leisure to them by offering me a wage that I consider sufficient. Similarly, they cannot just take my savings or retirement account and use the money to buy labor, land and equipment — they have to pay for using my financial capital by paying me interest. All this requires a well-established system of legally enforced property rights, and such a system has in practice typically required government protection and a well-functioning court system to enforce property rights.5

Externalities, as we have seen, arise when such property rights have not been established. Pollution is a problem because there does not exist a system of property rights to clean air that forces

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4It is noteworthy that it does not actually matter who owns the right to clean air — whether individuals or firms own this right, a market that prices the use of clean air in production would form. If the polluter herself owns the right to the air, she is still facing the cost of polluting because her opportunity cost of using the clean air in her own production is to sell the clean air to someone else in the market. We will say more on this later on.

5Most of us, including me, take for granted that such protection of property rights must be provided by government. And it usually is. But there are contrarian voices among some economists and philosophers that maintain government is not necessary for protection of private property to emerge. We will say a bit more about this in Chapter 30.
firms to pay for using clean air as an input into production. In effect, without some other institution in place, firms are simply able to take clean air for free as they produce goods — something we do not permit for inputs like labor and capital. Were they to similarly be able to take my leisure and capital — were there no legal system of property rights in those input markets, we would have even worse externality issues to deal with. Whenever a resource is not clearly owned by someone, it therefore becomes possible for economic agents to take those resources without incurring a cost — even though this imposes costs on society. It is then a logical consequence that, if it is feasible for the government to establish a system of property rights in resources that are not currently owned by anyone, such government interference can create additional markets that reduce the problem of externalities by forcing market participants to face the true social cost of what they are doing.

For this reason, economists have come to refer to externality problems that arise from the non-existence of markets as the “Tragedy of the Commons” — the “tragedy” of social losses that emerges when property is “commonly” rather than privately owned. We could say, for instance, that clean air is owned by everyone, but that simply means it is owned by no one in particular. Parents know this tragedy well. When we give toys to our children as common property to be shared without any guidance or rules, our children tend to fight like cats and dogs as they try to get those toys for themselves. Most parents therefore quickly learn that conflict is reduced if clear ownership of toys is established, with each child knowing (to the extent that children fully internalize this) that they have to get permission from the other child when seeking to use that child’s toys. When parents realize this, they act as economists who understand the tragedy of the commons.

More generally, much human suffering in the world can be directly traced to societies not heeding the lessons of the Tragedy of the Commons. Entire societies have been set up in attempts to abolish private property and replace the mechanism of markets with some alternative mechanism. It takes only a quick glance at 20th century history, for instance, to see how much societies that have protected private property and (thus established markets) have economically thrived while societies that have attempted to do the opposite have failed. A full understanding of externalities suggests that such societies failed because they created huge externalities by eliminating markets without finding an alternative government or civil society mechanism to generate social surplus. In short, by not supporting markets, they have created large “tragedies of the commons.”

Exercise 21A.14 Large portions of the world’s forests are publicly owned – and not protected from exploitation. Identify the tragedy of the commons – and the externalities associated with it – that this creates.

Exercise 21A.15 Why do you think there is a problem of over-fishing in the world’s oceans?

21A.3.3 Congestion on Roads

We do not, however, have to dig into historical examples of non-market based societies or reach for the pie in the sky of “markets in clean air” to see the relevance of an understanding of the Tragedy of the Commons in thinking about solutions to externality problems. Economists who have estimated the social cost of externalities in the U.S., for instance, have found that the social cost of time wasted on congested roads rivals the social cost of environmental damage from pollution. Think of your own experiences being stuck in traffic — it is mind-numbing to be stuck in traffic even for short periods of time because the opportunity cost of our time is large. In some of our larger cities, commuters routinely spend significant amounts of time in precisely such a position.

The problem of congested roads is an example of a Tragedy of the Commons. Roads, by and large, are commonly or publicly owned — which is to say that they are not owned by anyone. As you and I get on the road, we may think about the cost of taking the drive into the city — the
cost of our time, the gasoline we use and the depreciation of our car. We do not, however, think about the cost we are imposing on everyone else that is also taking a trip. Put differently, there is a negative externality each of us imposes on everyone else who is on the road at the same time as we add to the congestion of the road. In the absence of a mechanism that makes us face this social cost of our private actions, we therefore will tend to take too many trips — and we will be on the road at the “wrong” times. You may say that surely my own contribution to the congestion of the roads is minor, but all of us together are causing the congestion problem that wastes billions of dollars worth of time each week on the congested roads of larger cities. If my entry onto the road causes thousands of others to take even one more second to get to where they are going, I am imposing quite a social cost on others without paying any attention to it.

**Exercise 21A.16** Can you think of another cost that we do not think about as we decide to get onto public roads?

Solutions for this particular Tragedy of the Commons are still evolving, and changes in technology are playing a large part in shaping these solutions just as new technologies that permit detection of pollution have shaped new environmental policies (such as pollution taxes and cap-and-trade systems). The difficulty in finding a way for individuals to internalize the social cost they are imposing on others on the road lies in the difficulty of establishing a market that will price that social cost. In the past, economists have often proposed somewhat blunt policies falling into two general categories: First, we can impose a tax on gasoline that will raise the cost of driving and therefore reduce the amount of driving individuals will undertake; and second, when there are sufficiently many individuals in sufficiently dense geographic areas, governments can design public transportation systems like subways that are expensive to build but that, once built, can offer attractive alternative means of transportation within cities.

The building of public transportation may alleviate congestion, but it does not in itself address the Tragedy of the Commons that remains on public roads — and it may create a different Tragedy of the Commons if public transportation is priced in such a way as to cause congestion in buses, subways and so forth. Nevertheless, it has represented an important element of addressing crowding on roads in some urban areas. Taxation of gasoline is appealing in that it does raise the cost of driving and bring it more into line with the social cost of individual decisions during peak traffic hours, but it also raises the cost of driving during non-peak hours when congestion is not a problem — thus creating deadweight losses during those hours just as it reduces deadweight losses during peak hours.

**Exercise 21A.17** Are there other externality-based reasons to tax gasoline?

In recent years, however, it has become possible to price driving on congested roads more directly through tolls. Before the advent of electronic equipment that has made this easier, such tolls have involved toll booths which themselves can contribute to congestion around the booths as traffic slows down even as it keeps individuals off the roads. As technology improves, however, we are beginning to see increasingly efficient mechanisms for tolls to be imposed, mechanisms that do not require individuals to stop, reach into their wallets and pay a toll-booth attendant. As a result, we are seeing cities increasingly use electronic tolls that can vary with the time of day that individuals choose to use roads. User fees in the form of tolls then represent an attempt to make individuals face the social cost of driving during peak hours. At least in principle, such technology also permits the more direct establishment of markets in roads — markets in which road networks are privately owned and the use of the road is priced within markets. As technology and our understanding of the
underlying causes of externalities on roads is changing, we therefore see the emergence of new ways for government policy to interact with markets to reduce the social costs of an important externality. If such topics are of interest to you, you might consider taking an urban or transportation economics course at some point.

Exercise 21A.18 Some have argued against using tolls to address the congestion externality on the grounds that wealthier individuals will have no problem paying such tolls while the poor will. Is this a valid argument against the efficiency of using tolls?

21A.4 Smaller Externalities, the Courts and the Coase Theorem

We have thus far focused primarily on externalities that affect many individuals – such as pollution and congestion. But many of the externalities that we are most aware of in our daily lives are much less grand – the loud music in the dorm-room next to yours, the odor from the student who insists on sitting next to you in class but who also insists on showering infrequently, the insensitivity of the person on the bus who appears to be talking loudly to himself but is actually speaking on his well-hidden cell-phone, or that baby that just stopped screaming only to have switched from an externality that affects the auditory nerve to one that affects our sense of smell. These are all negative externalities – but we could think of positive ones as well. When I smile in the hallway at work, a few people a day might derive direct benefits from my cheerful disposition, or when I open the door for a student carrying heavy books (such as the one you are reading – sorry, I don’t know how to be brief!), that student’s life might be just a bit better today – even more so if I happen at the moment to be offering a rousing rendition of “O Sole Mio”. If you think about it, externalities are everywhere that people operate within close proximity to one another – in the workplace, in restaurants, in neighborhoods. And sometimes these externalities cause us to take each other to court.

21A.4.1 The Case of the Shadow on your Swimming Pool

Consider, for instance, the following example: You and I live next to each other in peace and harmony. Suddenly I win some money in the lottery and decide that I want to add to my house. So I draw up some plans to add an additional floor to my existing house. Normally you would not care about this, but it turns out that the additional floor will cast a long shadow onto your property – and in particular the area of your property that currently contains a beautiful (and sunny) swimming pool. You get very upset that your swimming pool will suddenly be in the shade all the time, and so you go to court and ask the judge to stop my building plans. Your legitimate argument is that I am imposing a negative externality that I am not taking into account. “He must be stopped,” you insist to the judge.

The judge sees your point but he wants to be careful and is trying to figure out whether it would or would not be efficient to build the addition to my house despite the adverse effect this will have on you. Maybe I get a lot more enjoyment from the addition than you lose from the shade on your swimming pool, or maybe it’s the other way around. Maybe it would cost you very little to move to a different house and have someone that does not care about the shade on the swimming pool move into your house (thereby eliminating the very externality we are worried about). Or maybe it would be easy for me to find a bigger house elsewhere and relatively costless for me to move. It’s hard to tell without the judge figuring out a lot of details about the case. And, one might argue, that there isn’t an easy way to judge this on a basis other than efficiency. After all, we both are
equally to blame for the existence of the externality – it would not exist if I were not trying to build an addition, but it also would not exist if you were not so insistent on having the sun shining on your silly pool!

21A.4.2 The Coase Theorem

Ronald Coase, an economist at the University of Chicago, came along and had a neat insight that might, under certain conditions, make the judge’s life a lot easier. He thought that the reason you are taking me to court is that we are confused about who has what “property rights” – and this ambiguity is making it difficult for us to come up with the optimal solution to our problem on our own. Suppose, for instance, you knew the judge would rule that I had the right to build regardless of the damage this does to you. You might then invite me for coffee and ask if there isn’t a way you could convince me to not build my addition. If the damage that is done to you is greater than the pleasure I get from my addition – i.e. if it would be efficient for me not to build the addition – you would in fact be willing to pay me an amount that will make me stop the addition. Perhaps I would find another way to add to my house, or perhaps I would move with the money you gave me to make me stop. If, on the other hand, your pain from the addition is less (in dollar terms) than my pleasure – i.e. if it is efficient for me to go ahead with the addition – you would discover over coffee that you aren’t willing to pay me enough to stop the addition. Perhaps you will just stay and suffer in a shaded pool, or perhaps you’ll move elsewhere. But notice that, once you know that I have every right to build the addition, you have an incentive to figure out whether you can pay me to stop – and once you figure this out, you will assure that the efficient outcome happens.

The same is true in the case where I know that you have the “property right” – that is, that you have a right to block my addition. In that case, I have an incentive to have you over for coffee to my house to see if I could persuade you to let me go ahead. If the addition means more to me than the pain it causes you, then you will be willing to accept a payment that I am willing to pay in order to get you to drop your objections. If, on the other hand, my gain from the addition is less than your pain, then I won’t be willing to pay you enough to get you to stop your objections. Thus, if the initial “property right” rests with you, then I am the one who has an incentive to figure out whether my gain is greater than your pain – and in the process get us to do what is efficient. Note that neither one of us actually cares about efficiency – but, once we know who has what rights, our private incentives make it in our interest to find the efficient outcome.

Exercise 21A.19 True or False: While it might not matter for efficiency which way the judge rules, you and I nevertheless care about the outcome of his ruling.

To the extent to which we find this reasoning persuasive, Coase has just gotten the judge who cares only about efficiency off the hook: No matter what the judge decides, you and I will arrive at the efficient outcome – the most important thing is that he needs to define the property rights so that we can have coffee and know what we are negotiating about. I know this problem well in my house where I am frequently called upon to be the judge that adjudicates cases of property rights disputes involving my two eight year old daughters. Knowing about Coase, I don’t even listen to

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Ronald Coase (1910-), who won the 1991 Nobel Prize in part for his contribution to this area, has the rare quality of being both an economist and a person so averse to math that it has been said of him (which is probably not true) that he will not number the pages of his manuscripts. The article in which he put forth the Coase Theorem – “The Problem of Social Cost,” The Journal of Law and Economics 3 (1960), 1-44 – is therefore quite readable by those with math phobias – and incidentally is one of the most cited articles in all of economics.
their arguments. I just flip a coin to decide who gets the property rights this time and then send them off to negotiate with each other.\footnote{My wife thinks that makes me a bad parent. Weird.}

### 21A.4.3 Bargaining, Transactions Costs and the Coase Theorem

The Coase Theorem then says it is essential that property rights be clearly defined in cases when there are negative externalities but it is not necessarily essential how those rights are defined. This should have a familiar ring – we just emphasized in the previous section that the absence of “markets” for the externality is the real underlying problem with externalities. Coase’s argument is similar, except that he does not insist that we have to have a competitive “market” in the externality – all we need to do is establish who has what rights and then let people solve the problem on their own by bargaining with one another. In our example of me building an addition to my house that will then cast a shadow on your swimming pool, there is no hope of establishing a real (competitive) market, but we can clarify property rights sufficiently to give us an incentive to figure out how to solve the externality problem.

Coase was not, however, naive, and he recognized that there might be barriers that keep people from getting together to bargain their way out of an externality problem once property rights are fully defined. These barriers are called transactions costs, and if they are sufficiently high, you and I might never have that coffee to talk about how to proceed. If we just can’t stand each other’s presence in the same room, then there is a transactions cost to getting together – and when this is the case, the judge’s decision suddenly matters a great deal more. If the efficient outcome is for me to build my addition and the judge rules in your favor, these transactions costs would keep me from getting together with you to offer you the payment necessary to let me proceed. Similarly, if the efficient outcome is for me to not build the addition and the judge rules in my favor, transactions costs again keep us from getting together in order for you to offer me the payment necessary not to build. Thus, in the presence of sufficiently high transactions costs, the judge needs to figure out what the efficient outcome is and then rule accordingly so that it is not necessary for us to get together to solve the problem through side payments among each other. The full Coase Theorem can then be stated as follows: In the presence of sufficiently low transactions costs, the efficient outcome will arise in the presence of externalities so long as property rights are sufficiently clear.

We can then see that the Coase Theorem offers us a decentralized way out of externality problems so long as transactions costs are low, and transactions costs will tend to be lower the fewer individuals are affected by an externality. If it’s just you and me arguing about whether I should or should not build an addition that only affects the two of us, we might think that transactions costs are in fact sufficiently low and we will bargain our way to a solution if the assignment of property rights requires such bargaining. For this reason, we might not worry about all the every-day externalities that affect only small numbers of people – chances are probably better that individuals themselves will figure out the efficient outcome than that a government with limited information can dictate the efficient outcome. Put differently, as long as people in “small externality settings” have reasonable expectations about how the law will treat externality issues if such issues were to be adjudicated in a court room, such problems are best handled in the “civil society” in which people interact voluntarily outside the usual price-governed market setting.

**Exercise 21A.20** Use the Coase Theorem to explain why the government probably does not need to get involved in the externality that arises when I play my radio sufficiently loud that my neighbors are adversely affected, but it probably does need to get involved in addressing pollution that causes global warming.
21A.4.4 Bees and Honey: The Role of Markets and Civil Society

The Coase Theorem applies to all types of externalities – positive or negative. So far, we have been sticking with the example of the negative externality of the shadow cast on your pool by the addition to my house. A classic example of positive externalities involves bee keepers and apple orchard owners. It turns out, however, that – although the example was originally given as motivation for Pigouvian subsidies, this is a case were Coase’s insights – as well as our more general insights on markets and property rights – have held true in the real world, and there appears to be no need for further Pigouvian interventions.8

Externalities in the case of bees and apple orchards abound. In order for apple trees to produce fruit, bees need to travel from tree to tree to carry pollen from “male” to “female” trees. And in order for bees to produce honey, they need some blossoms to visit. (You probably remember all this from the “birds and the bees” talk that I recently had to have with my children.) Bee keepers that let their bees roam therefore impose a positive externality on apple orchard growers (who benefit from the cross-polination services), and apple orchard growers bestow a positive externality on bee keepers (by providing them with the means for apple honey production). Even if we can figure out a way for markets to solve this problem in general, there is the second problem: bees have a way of not staying on the precise properties on which they were released. So if one orchard owner hires cross-polination services (or invests in his own bees), the bees will cross into neighboring orchards and provide services there – while also contributing to honey production.

In the absence of markets that can price all these externalities, our theory predicts that there would be too few bees on apple orchards – resulting in too little cross-polination and too little honey. As it turns out, however, none of this is a surprise to bee keepers and orchard growers. Fairly sophisticated markets for bee-keepers to release their bees on orchards have emerged “spontaneously” – markets that established “themselves” in an environment where government’s only role has been to guarantees the integrity of contracts and thus the property rights that are defined in those contracts. The flowers on apple trees, it turns out, do not produce much honey — causing the externality to go almost entirely from bee keepers to apple orchard growers. (The “apple honey” that you can find on your supermarket shelves has precious little honey produced from apple trees – it’s mainly the product of wild flowers that grow in the area of the orchards.) Clover, on the other hand, produces tons of honey. Thus, growers of clover produce a net-positive externality for bee keepers. While apple growers pay bee keepers to release their bees on the orchard, bee keepers pay clover growers for permitting them to release their bees on the clover farms. This is an example of competitive markets resolving an externality problem when property rights are well established.

Exercise 21A.21 In what sense do you think the relevant property rights in this case are in fact well established?

This does not, however, resolve the more “local” externalities between orchard owners. If one owner hires bee-services, those same bees cross over into other orchards – benefitting those growers (while also benefitting the bee-keepers). Another economist has looked at this closely, and he identifies a social custom that has emerged within the civil society – that is to say, outside the realm

8This externality between bee keepers and orchard growers was pointed out by the economist James Meade (1907-95) who argued in 1952 that Pigouvian subsidies were needed to remedy the problem. Meade shared the Nobel Prize in 1977 for contributions to the theory of international trade – which only goes to prove that even Nobel Laureates can get it wrong (as Meade did in the case of subsidies in the bee keeping business.) To his credit, Meade wrote eight years before Coase published his insights that came to be known as the Coase Theorem.
of explicit market-based transactions and outside the realm of government intervention. This has been dubbed the “custom of the orchards” – and it takes the form of an implicit understanding among orchard owners in the same area that each owner will employ the same number of bee hives per acre as the other owners in the area. While the Coase Theorem literally interpreted suggests that individuals will resolve these “local” externalities through bargaining, this illustrates another possible way for the theorem to unfold: Sometimes it is easier to converge on some local understanding of appropriate behavior that can be sustained among small groups within the civil society – rather than negotiate all the time about how many bee hives everyone is going to hire this time around. (In part B of exercise 24.17, we investigate a game theoretic explanation for the “custom of the orchards”).)

21B The Mathematics of Externalities

We will begin our mathematical exploration of externalities in competitive markets (as in Section A) with the motivating example of a polluting industry in partial equilibrium. Using linear supply and demand curves, we can demonstrate how to calculate the optimal Pigouvian tax. Furthermore, we will explore how the establishment of pollution permit markets can in principle achieve the same efficiency gains as an optimally set Pigouvian tax and that, in fact, there exists a cap-and-trade policy that is equivalent to any Pigouvian tax policy in the absence of pollution abating technologies. In the presence of such technologies, however, we will suggest, as we have in Section A, that pollution voucher markets (as well as direct pollution taxes) have an inherent advantage over Pigouvian taxes on output. While we won’t cover positive externalities (and accompanying Pigouvian subsidies) in detail, the mathematics is virtually identical to that underlying Pigouvian taxes and is therefore left as an end-of-chapter exercise.

We then turn to a more in-depth analysis of how externalities and the inefficiencies they give rise to are fundamentally problems of missing markets. In particular, we’ll demonstrate how new markets can be defined in an exchange economy that contains consumption externalities — and how establishment of these new markets should in principle resolve the inefficiency from externalities. We then demonstrate this in an extension of our example of a 2-person exchange economy from Chapter 16 before concluding with a discussion of the Coase Theorem.

21B.1 Production Externalities

In the presence of sufficient information, it is not mathematically difficult to determine the extent of deadweight losses from pollution or to arrive at an optimal Pigouvian tax. We will demonstrate this here briefly with an example in which we use linear demand, supply and social marginal cost \( SMC \) curves, and we will assume for convenience that the (uncompensated) market demand curve is in fact also the appropriate marginal willingness to pay curve along which to measure consumer surplus.

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9The economist is Steven Cheung – who was also the one who uncovered the contracts made by clover and apple growers with bee keepers. This is discussed in considerably more detail in “The Fable of the Bees: An Economic Investigation,” Journal of Law and Economics 17 (1973), 53-71.
21B.1.1 “Barney” versus the Market

We can begin with linear market demand and supply functions we have used in previous chapters, with

\[ x_d = \frac{A - p}{\alpha} \quad \text{and} \quad x_s = \frac{B + p}{\beta}, \tag{21.1} \]

and we have previously calculated the competitive market equilibrium in this case as

\[ p^M = \frac{\beta A - \alpha B}{\alpha + \beta} \quad \text{and} \quad x^M = \frac{A + B}{\alpha + \beta}. \tag{21.2} \]

Now suppose that each unit of output \( x \) produces \( \delta \) units of carbon dioxide pollution, and suppose that the damage from this pollution increases quadratically with additional pollution dumped into the air. In particular, suppose the externality cost is given by

\[ C_E(x) = (\delta x)^2. \]

Then the marginal externality cost for each unit of \( x \) is the derivative of \( C_E(x) \) with respect to \( x \), or

\[ MC_E = 2\delta^2 x. \]

The inverse of the supply curve in equation (21.1) is the industry’s marginal cost curve — i.e. \( MC_S = -B + \beta x \). Added together, these two curves make up the social marginal cost curve

\[ SMC = -B + (\beta + 2\delta^2)x. \tag{21.3} \]

**Exercise 21B.1** Suppose \( A = 1,000 \), \( \alpha = 1 \), \( \beta = 0.5 \), \( \delta = 0.5 \) and \( B = 0 \). Illustrate the market demand and supply as well as the \( SMC \) curves in a graph with \( x \) on the horizontal axis.

The efficient or optimal output level \( x^{opt} \) — the output our mythical “Barney” would choose — then occurs at the intersection of the \( SMC \) and the inverse demand curve \( p = A - \alpha x \). Solving the equation \(-B + (\beta + 2\delta^2)x = A - \alpha x \) for \( x \), we get

\[ x^{opt} = \frac{A + B}{\alpha + \beta + 2\delta^2}, \tag{21.4} \]

which you can immediately see is less than the competitive equilibrium quantity \( x^M \) in equation (21.2).

**Exercise 21B.2** Suppose the “pollution” emitted is actually not harmful and simply kills the mosquito population in the area. The \( SMC \) of the pollution might then be negative — i.e. this kind of pollution might actually produce social benefits. Will the efficient quantity now be greater or less than the market quantity? Show this within the context of the example.

21B.1.2 The Efficient Pigouvian Tax

We can now determine the optimal Pigouvian tax \( t^{opt} \) that will insure that the market produces the efficient level of output. In order for consumers to want to buy \( x^{opt} \), they must face a price \( p_d \) such that \( x^{opt} = x_d(p_d) = (A - p_d)/\alpha \). Similarly, in order for producers to supply \( x^M \) in equilibrium, they must face a price \( p_s \) such that \( x^{opt} = x_s(p_s) = (B + p_s)/\beta \). Solving these equations and plugging in our solution for \( x^{opt} \) from equation (21.4), we then get

\[ p_d = \frac{(\beta + 2\delta^2)A - \alpha B}{\alpha + \beta + 2\delta^2} \quad \text{and} \quad p_s = \frac{\beta A - (\alpha + 2\delta^2)B}{\alpha + \beta + 2\delta^2}, \tag{21.5} \]
and subtracting \( p_s \) from \( p_d \) gives us the optimal Pigouvian tax \( t_{\text{opt}} \) required to get this difference in consumer and producer prices; i.e.

\[
t_{\text{opt}} = p_d - p_s = \frac{2\delta^2(A + B)}{\alpha + \beta + 2\delta^2}.
\]  

(21.6)

**Exercise 21B.3** Complete exercise 21B.1 by illustrating and labeling the Pigouvian tax for this example.

**Exercise 21B.4** Using the graph from the previous exercise, calculate consumer surplus, producer surplus, the externality cost and overall surplus in the absence of the Pigouvian tax. Then calculate these again under the Pigouvian tax, taking into account the tax revenue raised. What is the deadweight loss from not having the Pigouvian tax?

### 21B.1.3 Cap-and-Trade

Now suppose that instead of imposing a tax \( t \) on output, the government requires that producers hold a pollution voucher for each unit of carbon dioxide emitted in the production process. Since every unit of output \( x \) produces \( \delta \) units of pollution, a producer must therefore hold \( \delta \) pollution vouchers for every unit of output she produces. If the rental price of a voucher is \( r \), this implies that the industry marginal cost goes from \(( -B + \beta x)\) to

\[
MC_S = -B + \beta x + \delta r.
\]  

(21.7)

**Exercise 21B.5** Illustrate how this shifts the supply curve in your graph (where you assume \( A = 1,000 \), \( \alpha = 1 \), \( \beta = 0.5 \), \( \delta = 0.5 \) and \( B = 0 \)).

Setting this equal to the (inverse) demand curve (which is \( p = A - \alpha x \)) and solving for \( x \), we get the new equilibrium quantity (given a voucher rental rate of \( r \)) as

\[
x^*(r) = \frac{A + B - \delta r}{\alpha + \beta}.
\]  

(21.8)

It is immediately clear from this equation, as it should be from the graph you drew in exercise 21B.5, that the market will produce less so long as the rental price of vouchers is greater than zero. But this does not yet answer the question of how the rental price of vouchers is determined in the first place. This price is, as demonstrated in Graph 21.3, determined in the new market for pollution vouchers that the government creates when it limits the quantity of vouchers to some level \( V \).

Every unit of output causes \( \delta \) in pollution — and thus requires \( \delta \) pollution vouchers. In the absence of any new introduction of pollution abating technologies, a total voucher level of \( V \) then implies that the market will reduce its output to \((1/\delta)V \). Substituting this into equation (21.8) on the left hand side and solving for \( r \), we get the equilibrium rental price for vouchers (given an overall supply of vouchers fixed at \( V \)) as

\[
r(V) = \frac{\delta(A + B) - (\alpha + \beta)V}{\delta^2}.
\]  

(21.9)

Now notice what happens if the government provides exactly enough vouchers to allow the market quantity \( x^M = (A + B)/(\alpha + \beta) \) to be produced; i.e. suppose the government sets \( V = \delta(A + B)/(\alpha + \beta) \). Plugging this into equation (21.9), we get an equilibrium voucher price of zero —
the voucher giving me the right to pollute ceases to be worth anything. For any level of $V$ below this, equation (21.9) tells us we will have a positive rental price for vouchers.

We can then relate this directly to Graph 21.3 in which we argued that firms will form a demand curve in the market for vouchers while the government will set a perfectly inelastic supply by setting a fixed voucher (and thus a fixed pollution) amount. In fact, equation (21.9) is the demand curve (or the inverse demand function) for vouchers by polluting firms, and the inverse of this equation, 

$$v(r) = \frac{\delta(A + B) - \delta^2 r}{\alpha + \beta}.$$  \hspace{1cm} (21.10)

is the demand function that relates the rental price $r$ to the quantity demanded by producers.

**Exercise 21B.6** Verify that a voucher price of zero results in the market output according to this demand function.

**Exercise 21B.7** Illustrate the demand curve for pollution vouchers and label its slope and intercept.

Here is an alternative (and perhaps more intuitive) derivation of the demand for pollution vouchers: The maximum amount that a firm is willing to pay to be allowed to produce one more unit of output depends on how much the firm thinks it can sell its output for and what the firm’s other costs are. In panel (a) of Graph 21.6, for instance, if the government limited the quantity in the market to $x_1$, the marginal firm would be willing to pay an amount equal to the blue distance in order to produce — because this is the difference between the marginal cost of production for this firm and the marginal willingness to pay for the output by the marginal consumer. Similarly, if the government limited total output to $x_2$, the marginal firm would be willing to pay at most an amount equal to the magenta distance.

Graph 21.6: Going from the Market for $x$ to the Pollution Voucher Market

Now suppose we converted the units in which we measure $x$ to voucher units — knowing that we will have to have $\delta$ vouchers for 1 unit of output $x$. The marginal benefit (or inverse demand) function when units are measured in terms of $x$ is just the demand curve $MB = A - \alpha x$. If we now measure $x$ in $1/\delta$-units, for instance, then the vertical intercept term of this function will have to be divided by $\delta$ — giving a vertical intercept of $A/\delta$ for our new marginal benefit curve. The
The horizontal intercept of our original marginal benefit curve, on the other hand, has to change from $A/\alpha$ to $\delta A/\alpha$. From this, we can calculate the slope of our new marginal benefit curve as the (negative) vertical intercept divided by the horizontal intercept, or $-A/(\delta A/\alpha) = -\alpha/\delta^2$. Thus, the marginal benefit curve when output is expressed in voucher-units is

$$MB(v) = \frac{A}{\delta} - \frac{\alpha}{\delta^2} v = \frac{\delta A - \alpha v}{\delta^2}.$$  \hspace{1cm} (21.11)

and, applying similar logic to the producers’ marginal cost curve $MC = -B + \beta x$, the marginal cost curve when output is expressed in voucher units is

$$MC(v) = -\frac{\delta B + \beta v}{\delta^2}.$$  \hspace{1cm} (21.12)

Panel (b) of Graph 21.6 illustrates these marginal benefit and cost curves which are equivalent to those in panel (a) except that the units on the horizontal axis are $1/\delta$ the units in panel (a).

Exercise 21B.8 What is the relationship between the length of the blue and magenta lines in panels (a) and (b)?

Exercise 21B.9 Implicitly we are assuming $\delta = 2$ in panel (b) of Graph 21.6. How would this graph change if $\delta < 1$ — i.e. if each unit of output produces less than one unit of pollution?

The most that the marginal firm is willing to pay for a voucher is then simply the difference between $MB(v)$ and $MC(v)$,

$$MB(v) - MC(v) = \frac{\delta (A + B) - (\alpha + \beta) v}{\delta^2}.$$  \hspace{1cm} (21.13)

exactly the expression for the voucher demand curve in equation (21.9). This function is graphed in panel (c) of Graph 21.6, and the equilibrium price $r^*$ in the voucher market is then simply determined by the intersection of this demand curve with the inelastic supply set at $V$ by the government or, mathematically, by substituting $V$ for $v$ in equation (21.13).

Exercise 21B.10 If $V = \delta x_2$, which distance in panels (a) or (b) of Graph 21.6 is equal to $r^*$?

Exercise 21B.11 For the case when $A = 1,000$, $\alpha = 1$, $\beta = 0.5$, $\delta = 0.5$ and $B = 0$ (as you have assumed in previous exercises), what is the rental rate of the pollution voucher when $V = 250$? What is the price of a pollution voucher if the interest rate is 0.05?

### 21B.1.4 Pollution Vouchers versus Taxes

In Section A we drew a distinction between Pigouvian taxes (that are levied on output) and pollution taxes which are levied on pollution that a firm emits. In our mathematical example here, there is a one-to-one relationship between taxing output and taxing pollution — because we have assumed that each unit of output produces $\delta$ units of pollution. Thus, in our simplified example, the Pigouvian tax on output is not that different from a pollution tax — and, as a result, we can illustrate that, for every pollution voucher cap under a cap-and-trade policy, there exists a tax that achieves the same outcome. Keep in mind, however, that the real world introduces complexities that create a real distinction between Pigouvian and pollution taxes, an issue we return to after demonstrating the equivalence of tax and cap-and-trade policies for our example.
Suppose the government knows the optimal level of output \( x^{\text{opt}} \) in equation (21.4) as well as the amount \( \delta \) of pollution emitted by each unit of production. The information, combined with our knowledge of supply and demand curves, is then sufficient to set the optimal voucher level at

\[
V^{\text{opt}} = \delta x^{\text{opt}} = \frac{\delta(A + B)}{\alpha + \beta + 2\delta^2}.
\]  

(21.14)

Plugging \( V^{\text{opt}} \) into equation (21.9), this implies an equilibrium rental rate for vouchers of

\[
r^*(V^{\text{opt}}) = 2\delta \left(\frac{A + B}{\alpha + \beta + 2\delta^2}\right).
\]  

(21.15)

In order to produce one unit of output, we have to rent \( \delta \) vouchers – which implies that the marginal cost of production has increased by

\[
\delta r^*(V^{\text{opt}}) = \frac{2\delta^2(A + B)}{\alpha + \beta + 2\delta^2}.
\]  

(21.16)

Note that this is exactly equal to the optimal Pigouvian tax \( t^{\text{opt}} \) we derived in equation (21.6); i.e.

\[
t^{\text{opt}} = \delta r^*(V^{\text{opt}}).\]  

(21.17)

Thus, so long as the government sets the number of pollution vouchers correctly, the market for these vouchers will result in a price equal to the tax the government would have liked to impose had it chosen to use a Pigouvian tax instead. In fact, as illustrated in Table 21.1 for the example that you have worked with in many of the within-chapter exercises, for any tax imposed on outputs, there exists and equivalent voucher level that will result in a voucher rental rate that has the same impact on producers as the tax.

**Exercise 21B.12** Suppose the government simply gives away the pollution vouchers. Why is the deadweight loss the same under tax and cap-and-trade policies that satisfy \( t = \delta r^*(V) \) (even though one makes revenue for the government while the other does not)?
Exercise 21B.13 Illustrate on a graph where the deadweight loss falls when \( t = 400 \) in Table 21.1. What about when it falls at \( t = 100 \)?

As noted above, however, our mathematical example obscures within its simplicity a difference between taxing output in polluting industries and taxing pollution emissions directly. This is because we illustrated the case of a single industry, ignoring the fact that many industries engage in pollution — and we have not introduced the potential for pollution abating technologies to play a role. Even within a single industry, a Pigouvian tax on output differs from a pollution tax in that the latter allows firms to reduce their tax obligations by introducing pollution abating technologies while the former does not. Thus, the equivalence of a Pigouvian tax to a pollution tax within an industry only survives if we assume that the government will adjust the Pigouvian tax on output as firms introduce pollution abating technologies. When considering pollution across industries, this is further complicated by the fact that industries will differ in terms of the ease with which they can introduce pollution abating technologies — with any equivalence between Pigouvian taxes and pollution taxes then assuming that the government continuously adjusts Pigouvian per-unit taxes as pollution abating technologies are introduced in different settings. The equivalence between cap-and-trade and pollution taxes, on the other hand, is robust to the introduction of such real-world complications.

21B.2 Consumption Externalities

The mathematics behind our graphical development of consumption externalities is almost identical to that behind production externalities. As a result, we will treat this in end-of-chapter exercise 21.1 rather than developing it fully here. Instead, we will proceed next to considering the problem of consumption externalities in a general equilibrium setting where we will be able to illustrate more precisely what we mean when we say that the presence of an externality necessarily implies the absence of a market that, if established, would eliminate the inefficiency that arises from the externality.

21B.3 Externalities and Missing Markets

The idea of using (pollution voucher) markets to solve the externality problems created by pollution is closely linked to a more general understanding of externalities as a problem of “missing markets” (or, as we put it in Section A, of a failure of markets to exist.) The intuition behind this is not difficult to see once we see how the missing markets could be defined — and how pricing within those markets will then lead those who emit externalities to face the costs (or benefits) they impose on others. Using our tools from Chapter 16, however, we can be a little more precise about what we mean by missing markets and how an establishment of those markets resolves the inefficiency from externalities under competition. We will do so below for the case of externalities in an exchange economy, but one could similarly illustrate this in an economy with production.\(^\text{10}\)

\(^{10}\)This approach to illustrating the “missing market” aspect of externalities was introduced by Kenneth Arrow (1921-) (whom we previously mentioned in Chapter 16 as the 1972 Nobel Laureate who co-founded modern general equilibrium theory) in “The Organization of Economic Activity: Issues Pertinent to the Choice of Market versus Non-Market Allocations,” in *Public Expenditure and Policy Analysis* (Havenman, R. and J. Margolis, eds.), Chicago: Markham (1970). A subsequent literature that we allude to in the Appendix to this chapter points out a technical problem in this way of modeling externality markets, a problem we will for now glance over.
21B.3.1 Introducing Consumption Externalities into an Exchange Economy

In Chapter 16, we defined an exchange economy as a set of consumers denoted \( n = 1, 2, ..., N \), with each consumer characterized fully by her endowments of each of \( M \) different goods as well as her tastes summarized by utility functions defined over \( M \) goods (denoted \( m = 1, 2, ..., M \)). An exchange economy was then given simply by

\[
\left( \{(e^n_1, e^n_2, ..., e^n_m)\}_{n=1}^N, \{u^n: \mathbb{R}^M \to \mathbb{R}^1\}_{n=1}^N \right) \tag{21.18}
\]

Because each consumer cares only about her own consumption of each of the goods (and because there are no other agents like producers), there is no externality in this exchange economy. An externality \( (\text{in the absence of production}) \) then arises when one consumer’s consumption directly enters the utility function of another consumer. In principle, such consumption externalities in an exchange economy could arise in every direction — with every consumer’s consumption of each good entering every other consumer’s utility function.

We could then think of consumer \( n \) as consuming some of each of the \( M \) goods and being affected by her “impression” of each other consumer’s consumption of each of the \( M \) goods. Suppose, for instance, that we let \( x^n_{ij} \) denote “person \( n \)’s impression of person \( j \)’s consumption of good \( i \)”. If \( x^n_{ij} \) enters person \( n \)’s utility function, then person \( j \) is generating a consumption externality when consuming good \( i \). But if each person’s consumption of each good potentially enters each person’s utility function, then each person is in essence consuming \( NM \) different goods rather than \( M \) goods as before. For instance, if \( N = 2 \) and \( M = 2 \), consumer 1 consumes \((x^1_{11}, x^1_{21}, x^1_{12}, x^1_{22})\).

Exercise 21B.14 Which two of these four goods represent the consumption levels \( x^1_1 \) and \( x^1_2 \) that exist for person 1 in an exchange economy without externalities?

We have therefore taken an economy with \( M \) goods and defined, for each person, \( NM \) goods that enter her utility function. The exchange economy defined in equation (21.18) can then be rewritten with consumption externalities as

\[
\left( \{(e^n_1, e^n_2, ..., e^n_m)\}_{n=1}^N, \{u^n: \mathbb{R}^{NM} \to \mathbb{R}^1\}_{n=1}^N \right) \tag{21.19}
\]

21B.3.2 The Missing Markets in an Exchange Economy with Externalities

We have now introduced “impressions of other individuals’ consumption” explicitly as new goods. But this implies that we have implicitly introduced production into the exchange economy — because each time a consumer makes a decision to consume some of the \( M \) goods, she is “producing” \((N - 1) \) of these newly defined goods. When I consume good 1, I am producing an impression of my consumption of good 1 that now potentially enters everyone else’s utility function. But our exchange economy has no markets that set prices for such goods and thus no market mechanism to govern my production decisions!

Suppose, for instance, that person \( j \)’s consumption of good \( i \) enters individual \( n \)’s utility function in a positive way. In this case, person \( j \) is a producer of an output \( x^n_{ij} \), an output that consumers like \( n \) would be willing to pay for but don’t since there is no market and no price. Alternatively, suppose that \( x^n_{ij} \) enters person \( n \)’s utility function negatively — implying consumer \( j \) emits a negative consumption externality by consuming good \( i \). In this case, we can view consumer \( j \) as

\[\text{If you are uncomfortable with this notation, please review the discussion surrounding expression (16.1) in Chapter 16.}\]
using $x_{ij}^n$ as an input into the production of her own consumption of good $i$. But, once again because there is no market for this input and thus no price, consumer $j$ does not need to purchase the input $x_{ij}^n$ when deciding how much of the good $i$ to consume.

**Exercise 21B.15** If there are two consumers and two goods, how many missing markets are there potentially? More generally, how many missing markets could there be when there are $M$ goods and $N$ consumers?

In some cases, externalities will take a form where the externality affects every consumer whereas in other cases the externality may affect only some consumers. Suppose, for instance, that consumer $j$ is choosing good $i$ that represents the number of car rides she takes — and each car ride emits pollution that contributes to global warming. In that case, her car rides enter each consumer’s utility function in the same quantity (even though different consumers will feel differently about how bad this externality is). Put differently, in this example

$$x_{ij}^n = x_{ij} \text{ for all } n \neq j; \quad (21.20)$$

i.e. each individual other than $j$ experiences the impact of $j$’s car rides in the same quantity. In other cases, an externality is more “local” — affecting some individuals differently than others. For instance, if $j$ chooses good $i$ that represents music played in the backyard, her immediate neighbors are affected more than more distant neighbors. In this case, $x_{ij}^n$ will differ depending on the distance between individual $j$ and $n$.

### 21B.3.3 Introducing Property Rights and New Markets

In order establish the new markets that can price the externality effects within this exchange economy, we have to begin by specifying a set of new property rights. If my car rides cause pollution, do I have the right to pollute or do others have the right not to have pollution inflicted on them? If I play loud music on my patio, do I have the right to do as much of this as I want to, or do others have the right to not be bothered by my music? For efficiency purposes, however, it turns out that what matters most is that property rights be established so that markets can form. For now, we will illustrate in a simple example how markets price externalities when markets are established, and we will return to a discussion of the extent to which it matters how property rights are assigned in Chapter 27.

One way to think of how property rights are established in the new markets is to extend the endowments for individuals to include endowments of the new goods. In this way, rights could be distributed in a variety of ways — although we will typically think of rights being established strictly one way or another; i.e. either someone has the right to pollute or the polluters have the right not to be bothered by pollution unless they sell their rights. But once we have established a system of property rights, we have arrived at an exchange economy that simply has more goods than before. And none of the goods now appears in more than one utility function — which means there is technically no more externality in the economy with the expanded set of markets. Since we know that exchange economies without externalities are such that competitive equilibria are efficient regardless of how many goods and consumers there are in the economy, the establishment of these new markets therefore leads to an economy in which competitive equilibria are efficient, with prices of the newly defined goods causing the emitters of externalities to take full account of the marginal (social) costs and benefits of their actions.
21B.3.4 A Numerical Example

In Chapter 16, we worked through an example of a 2-person, 2-good exchange economy in which \((e_1^1, e_2^1) = (3, 6), (e_1^2, e_2^2) = (10, 4)\), \(u^1(x_1, x_2) = x_1^{3/4}x_2^{1/4}\) and \(u^2(x_1, x_2) = x_1^{1/4}x_2^{3/4}\). Given that only each individual’s own consumption appears in her utility function, this represents an exchange economy without externalities. Suppose now, however, that consumption of good 1 by individual 1 enters individual 2’s utility function. Using our notation above, this implies that the good \(x_1\) — individual 2’s perception of individual 1’s consumption of good 1, enters \(u^2\). To keep our notation in this example as simple as possible, let’s define \(x_3 = x_1\), and let individual 2’s utility function be re-defined as

\[
u^2(x_1, x_2, x_3) = x_1^{1/4}x_2^{3/4}x_3^\gamma. \tag{21.21}\]

Depending on whether \(\gamma\) is greater or less than zero, individual 1 is therefore now imposing a positive or negative consumption externality on individual 2. When \(\gamma = 0\), the example reduces to our example from Chapter 16 with no externality.

We can first ask what the competitive equilibrium of this exchange economy will be. In the absence of a market for \(x_3\), however, nothing fundamental has changed from the way we calculated the equilibrium of this economy in Chapter 16: Individual 1 will maximize the same utility function subject to the same budget constraint as before and will thus have the same demand equations. Individual 2 will maximize the new utility function in equation (21.21) subject to the same constraints as before, but \(x_3\) will simply cancel out as we solve for her demand equations — resulting in the same demands as in Chapter 16. With both individuals exhibiting the same demands, we get the same competitive equilibrium as before, with \(p^2/p^3 = 3/2\), \((x_1^1, x_2^1) = (9, 2)\) and \((x_1^2, x_2^2) = (4, 8)\).

Exercise 21B.16 Verify that individual 2’s demand functions for \(x_1\) and \(x_2\) are unchanged as a result of the inclusion of \(x_3\) in her utility function.

Exercise 21B.17 Do you think the conclusion (in exercise 21B.16) that demands for \(x_1\) and \(x_2\) do not change will hold regardless of what form the utility function takes?

But now suppose that a market is introduced for the good \(x_3\) (with price \(p_3\)). Let’s begin by thinking of the externality as negative (i.e. \(\gamma < 0\)) — and suppose that property rights are assigned such that individual 2 has the right to not experience the externality unless she agrees voluntarily to do so. This implies that individual 1 will have to pay not only \(p_1\) for each unit of \(x_1\) she consumes — but also \(p_3\) (since \(x_1 = x_3\)). The optimization problem for consumer 1 then becomes

\[
\max_{x_1, x_2} u^1(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \text{ subject to } p_1e_1^1 + p_2e_2^1 = (p_1 + p_3)x_1 + p_2x_2. \tag{21.22}\]

Solving this in the usual way, we get

\[
x_1^1 = \frac{\alpha(p_1e_1^1 + p_2e_2^1)}{p_1 + p_3} \text{ and } x_2^1 = \frac{(1-\alpha)(p_1e_1^1 + p_2e_2^1)}{p_2}. \tag{21.23}\]

Individual 2, on the other hand, will receive \(p_3\) for every unit of \(x_3\) that individual 1 emits, but, since individual 2 is given the “property rights” to \(x_3\), individual 2 chooses how much of \(x_3\) to sell. The optimization problem for individual 2 then becomes

\[
\max_{x_1, x_2, x_3} u^2(x_1, x_2, x_3) = x_1^\beta x_2^{1-\beta}x_3^\gamma \text{ subject to } p_1e_1^2 + p_2e_2^2 + p_3x_3 = p_1x_1 + p_2x_2. \tag{21.24}\]
Solving this, we get

\[ x_1^2 = \frac{\beta(p_1e_1^2 + p_2e_2^2)}{(1 + \gamma)p_1}, \quad x_2^2 = \frac{(1 - \beta)(p_1e_1^2 + p_2e_2^2)}{(1 + \gamma)p_2} \quad \text{and} \quad x_3 = \frac{-\gamma(p_1e_1^2 + p_2e_2^2)}{(1 + \gamma)p_3}. \tag{21.25} \]

Exercise 21B.18 Verify these demand functions. (Hint: It becomes significantly easier algebraically to first take logs of the utility function.)

Exercise 21B.19 Do the demand functions converge to those we derived in the absence of an externality as the externality approaches zero (i.e. as \( \gamma \) approaches zero?)

We can now solve for equilibrium prices. As in Chapter 16, we will be able to solve only for relative prices and can therefore set one of the prices to 1. Suppose, then, we set

\[ p_1 = 1. \tag{21.26} \]

Setting demand equal to supply in the market for good 2 — i.e. setting \( x_1^2 + x_2^2 = e_1^2 + e_2^2 \), we can then solve for \( p_2 \) as

\[ p_2 = \frac{(1 - \alpha)(1 + \gamma)e_1^2 + (1 - \beta)e_2^2}{\alpha(1 + \gamma)e_1^2 + (1 + \beta)e_2^2}. \tag{21.27} \]

In addition, it must be true that demand is equal to supply in the \( x_3 \) market — where the amount of \( x_3 \) consumer 2 is willing to sell must be equal to the amount of \( x_1 \) that consumer 1 wants to consume — i.e. \( x_1^2 = x_3 \). Solving this, we can get \( p_3 \) in terms of \( p_2 \) (with \( p_1 \) again set to 1) —

\[ p_3 = \frac{-\gamma(e_1^2 + p_2e_2^2)}{\alpha(1 + \gamma)(e_1^2 + p_2e_2^2) + \gamma(e_1^2 + p_2e_2^2)}. \tag{21.28} \]

In Table 21.2, we then calculate the competitive equilibrium prices and quantities when the market for good \( x_3 \) has been established. The table begins with negative values for \( \gamma \) — i.e. with the case where individual 1’s consumption of good 1 imposes a negative externality on individual 2. As you move down the table, the externality becomes less severe, with no externality when \( \gamma = 0 \). Finally, the table moves into positive values for \( \gamma \) — implying a positive externality on individual 2 from the consumption of good 1 by individual 1. Notice that \( p_3 \) is positive whenever the consumption externality is negative — implying that the presence of a negative externality results in individual 2 receiving compensation for suffering the negative effects of individual 1’s consumption. But when the externality becomes positive, \( p_3 \) becomes negative — implying that now individual 2 compensates individual 1 for the positive effect \( x_1^2 \) has on individual 2. Thus, the establishment of the missing market results in individual 2 imposing a “tax” on individual 1’s consumption of good 1 when the externality is negative and a “subsidy” when the externality is positive.

Of course, just as in Chapter 16, it would not be reasonable to expect market prices to govern exchange — either in the presence or in the absence of externalities — when there is literally only one individual on each side of the market. The 2-person exchange economy simply provides a useful tool with which to illustrate how markets set prices in general equilibrium. But the analysis above continues to hold exactly the same way if we assume that there are many “type 1” and many “type 2” individuals when competitive price taking behavior becomes more realistic. And for the “two-person case” we have the Coase Theorem to fall back on — a theorem already mentioned in Section A, and one we now examine a bit more closely.
Chapter 21. Externalities in Competitive Markets

Equilibrium with “Missing Market” Established

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
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<th>( x_2^1 )</th>
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<td>2.28</td>
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</table>

Table 21.2: \( \alpha = 3/4, \beta = 1/4, (e_1^1, e_2^1) = (3, 6), (e_1^2, e_2^2) = (10, 4) \)

21B.4 Small “Markets” and the Coase Theorem

In Section A, we introduced the insight of Ronald Coase with respect to the types of externalities that make us mad enough to take each other to court. We gave the example of me building an addition to my house and you taking me to court because my addition would cast a shadow on your beautiful swimming pool. It is precisely in such “small” settings that, even if we established “markets” of the types we have discussed, there would not be much of a “market” since only one or a few people would be operating on each side of the market. And, while we can theoretically investigate what market prices would look like if they in fact arose, it is more realistic to think of “bargaining” as the way in which externality issues would be resolved in such “markets”.

21B.4.1 Bargaining under Complete and Incomplete Information

Bargaining by definition does not happen in competitive settings – since in competitive settings each consumer and producer is a price taker. We are therefore jumping a bit ahead of ourselves as we think about bargaining under the Coase Theorem. You and I are decidedly not price takers during our coffee as we discuss the level of compensation that you have to offer me to stop building (if the judge ruled in my favor) or the level of compensation I will pay you to let me build (if the judge ruled in your favor). Put differently, we are jumping ahead because we are thinking of a “strategic” setting, one in which you and I have some real control over our economic environment.

Economists (and particularly game theorists) have, over the past few decades, arrived at a well-defined theory of bargaining, some of which was directly inspired by Coase’s confidence that bargaining in an atmosphere in which property rights have been fully clarified will lead to efficient outcomes when externalities are involved. Some of that theory (just as some of the development of game theory in Chapter 24) assumes that you and I have perfect information about each other’s costs and benefits of my addition to my house. And under such circumstances, Coase appears to be on solid ground: The theory predicts that you and I will in fact reach a bargain that will lead to the efficient outcome under the conditions envisioned by Coase.12 Intuitively, this is not hard to see – if

12In cases where income (or endowment) effects are important (as when tastes are not quasilinear), we have to be slightly more careful because “the” efficient outcome may differ depending on how property rights are assigned. This is explored further in exercise 21.4.
the story I told in Section A about how we will bargain our way to efficiency made sense, you have the basic intuition. We can demonstrate this more formally once we have developed some game theory tools – and illustrate how two individuals arrive at bargains under complete information in end-of-chapter exercise 24.9.

21B. The Mathematics of Externalities

21B.4.2 Bargaining under Incomplete Information

In Section A, however, we implicitly assumed what we have just made explicit: that you and I both know what the costs to you are (relative to you solving your shaded pool problem in other ways) of me adding to my house in a way that casts a shadow on your swimming pool and what the benefits (relative to other ways of solving my need for additional housing) are to me of building the addition in this way. Let’s denote your costs as \( c \) and my benefit as \( b \). Efficiency dictates that I go ahead with my addition if \( b > c \), and we argued that, so long as property rights have been specified and transactions costs are low, the efficient outcome will happen.

But suppose that you are not sure what \( b \) is and I am not sure what \( c \) is. Rather, you have beliefs about \( b \) and I have beliefs about \( c \). Let my beliefs be represented by \( 0 \leq \rho(c) \leq 1 \) for any \( c > 0 \), with \( \rho(c) \) equal to the probability I place on your costs being less than or equal to \( c \). Similarly, let your beliefs be represented by \( 0 \leq \delta(b) \leq 1 \) for any \( b > 0 \), with \( \delta(b) \) equal to the probability that you place on my benefits being less than or equal to \( b \). Now suppose the judge rules in your favor – i.e. you now have the right to a shadow-free pool and I cannot build my addition unless you agree to it.

I will therefore come to coffee and offer you compensation based on my beliefs of what your costs are. To arrive at an offer I make to you, I will have to calculate the offer \( p \) that maximizes my expected payoff. My expected payoff from any offer \( p \) is the probability that the offer will be accepted times the benefit I receive from having my offer accepted. For any offer \( p \), I believe that the probability that your true costs are less than or equal to \( p \) is \( \rho(p) \) – which implies that I believe that the probability of you accepting my offer is \( \rho(p) \). The benefit I receive if the offer is accepted is my benefit \( b \) from having the addition built minus the payment \( p \) I have to make to you – i.e. the benefit I receive if the offer is accepted is \((b - p)\). I therefore solve the following optimization problem as I calculate my optimal offer given the beliefs I have:

\[
\max_p \rho(p)(b - p). \tag{21.29}
\]

I will obviously not make an offer \( p > b \), and you will not accept an offer \( p < c \). But, depending on what my beliefs are, I may well make an offer \( p^* \) that maximizes my expected payoff but where \( p^* < c \) even though \( b > c \). Thus, depending on my beliefs about your true underlying costs, the addition may not get built if the judge rules in your favor despite the fact that building the addition is efficient.

Exercise 21B.20 Suppose the judge rules in my favor instead. What optimization problem do you solve as you come over to have coffee in order to offer me a payment for not building the addition? Can it again be the case that the efficient outcome does not happen for certain beliefs \( \delta \) you might have about my true benefit from the addition?

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\(^{13}\)In Section B of Chapter 24, you will learn more about the equilibrium concept that we have implicitly just applied, which we will call a Bayesian Nash Equilibrium. An example of this is also presented in end-of-chapter exercise 21.2.
Depending on how we define what we mean by “transactions costs”, we now may or may not have to amend the Coase Theorem. As stated in Section A, the theorem says that so long as property rights are sufficiently specified in the presence of externalities, the efficient outcome will occur from decentralized decisions if transactions costs are sufficiently low. As we have just seen, strategic bargaining between individuals who understand the assignment of property rights in the presence of externalities may not result in efficiency even when there are no transactions costs keeping the individuals from getting together and bargaining. But the cost of obtaining information about the relative costs and benefits from the externality may in itself be considered a transactions cost, in which case we can leave the Coase Theorem as stated before.

Conclusion

This chapter is the first to have introduced an economic force that causes our First Welfare Theorem to break down: Markets, by themselves, cease to be efficient maximizers of social surplus in the presence of externalities. While previous chapters may have given the impression that microeconomists see all forms of government intervention as inherently inefficient, we have now seen that markets cannot operate efficiently in isolation. First, the very existence of markets presumes an underlying system of property rights that, in practice, has almost always required the explicit involvement of government. In the absence of such property rights, we are faced with what we have called the “Tragedy of the Commons” when individual incentives lead to over-use of resources. And where externalities arise, it is precisely because of the “Tragedy of the Commons”: We all “own” the air (or, alternatively, none of us own it) – and as a result no one makes sure we pay for the pollution we cause. We all “own” the roads, and so no one is charging us for the congestion we contribute to when we drive during peak hours. The market failure that arises from externalities is therefore caused by the “failure of a market to exist”.

This is not to say that markets themselves can always solve externality problems. Air pollution is a problem because no market for air exists, but it is not exactly easy to establish such markets. But policies aimed at correcting inefficiencies from externalities must ultimately do what markets would do if they could be established fully: They must cause individual actors in the economy to face the full marginal costs (and benefits) of their actions. We saw that this could in principle be done through Pigouvian taxes and subsidies which force individuals to confront the larger social costs and benefits of their private choices. We saw it could be done through the creative establishment of markets like those for pollution vouchers which, this time through the need to purchase a voucher if one intends to pollute, again forces polluters to pay for at least some of the social cost of their production choices. Or it could be done through such policies as electronic tolls on roads. Or, as Coase tells us, it could be done in the case of smaller externalities simply by clarifying through property rights cases who in fact has a right to do what – and then rely on interested individuals to bargain their way to efficiency. The key in all these policies, however, is to bring private and social marginal costs in line with one another. Government policies (such as Pigouvian taxes and subsidies), fostering of new markets (such as pollution vouchers) and clarifications of property rights in the civil society (that can have individuals bargain outside the price-based market system) can thus all contribute to greater efficiency in the presence of externalities.

In the upcoming chapter, we will see another important instance when competitive markets by themselves will not result in efficient outcomes: the wide-spread case where information is not shared uniformly by market participants. We will see that such asymmetric information results in a new form of externality that can prevent important markets from forming — and that offer
opportunities for non-market institutions to enhance efficiency.

Appendix: Fundamental Non-Convexities in the Presence of Externalities

In our treatment of how the establishment of missing markets can restore efficiency in the presence of externalities, we glanced over a technical problem that has become known as the problem of fundamental non-convexities. The essence of the problem is this: Suppose we reconsider our numerical example of an exchange economy with a negative consumption externality from consumer 1’s consumption of good 1 (as we did in the chapter). Suppose further that we take the assumption that consumer 2 has a right to not experience the externality and must be persuaded to sell that right by accepting payment in proportion to the externality that is emitted. We know that if the price $p_3$ is zero, consumer 2 will not sell any rights to consume good 1 to consumer 1 (since consumer 2 would then experience a negative externality without compensation). Now suppose that $p_3 > 0$ (as in the equilibria we described in Table 21.2). What is to keep consumer 2 from wanting to sell an infinite number of rights to pollute — thus making an infinite income to spend on consumption of goods 1 and 2? Put differently, if there is no limit on the number of rights that individual 2 can sell, a positive price will cause the consumer to want to sell an infinite quantity of $x_3$ while a non-positive price will cause her to want to sell zero. No matter what $p_3$ is set at, consumer 2 therefore prefers a corner solution.\(^4\)

But if consumer 2 will sell only zero or an infinite amount of $x_3$, no equilibrium in the $x_3$ market exists — and the establishment of the $x_3$ market with all rights assigned to the victim of the negative externality does not in fact lead to a competitive equilibrium that eliminates the inefficiency from the externality. In order for the equilibria that we discuss in Table 21.2 to emerge, there must therefore be some limit to the number of rights that consumer 2 can sell.

The solution to this fundamental non-convexity problem lies in finding ways of “bounding” the property rights in externality markets such that, for instance, victims of pollution cannot in fact sell large or infinite amounts of these rights when the price is positive. While this is not easily done in the context of defining externality markets in the way that we have done in our exchange economy example, we have already shown how this in fact can be done when “rights” are defined along the lines of pollution vouchers. Here, a limited number of these rights are allocated in the economy, thus eliminating the problem of fundamental non-convexities.\(^5\)

Exercise 21B.21 Why did our mathematical methods of solving for consumer 2’s demand for $x_3$ not uncover this problem?

End of Chapter Exercises

21.1 Consider the case of a positive consumption externality.

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\(^4\)This is referred to as a “fundamental non-convexity” because it represents a non-convexity in the production set for pollution rights. The problem of fundamental non-convexities in externality markets was first pointed out by Starrett, D. (1972): “Fundamental Nonconvexities in the Theory of Externalities,” *Journal of Economic Theory* 4, 180-99.

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A: Suppose throughout this exercise that demand and supply curves are linear, that demand curves are equal to marginal willingness to pay curves and that the additional social benefit from each consumption unit is \( k \) and is constant as consumption increases.

(a) Draw two graphs with the same demand curve but one that has a fairly inelastic and one that has a fairly elastic supply curve. In which case is the market output closer to the optimal output?

(b) Does the Pigouvian subsidy that would achieve the optimal output level differ across your two graphs in part (a)?

(c) Draw two graphs with the same supply curve but one that has a fairly inelastic demand curve and one that has a fairly elastic demand curve. In which case is the market output closer to the optimal output?

(d) Does the Pigouvian subsidy that would achieve the optimal output level differ across your two graphs in part (c)?

(e) True or False: While the size of the Pigouvian subsidy does not vary as the slopes of demand and supply curves change, the level of under-production increases as these curves become more elastic.

(f) In each of your graphs, indicate who benefits more from the Pigouvian subsidy – producers or consumers.

B: Suppose demand is given by \( x_d = (A - p)/\alpha \) and supply is given by \( x_s = (B + p)/\beta \).

(a) Derive the competitive equilibrium price and output level.

(b) Suppose that the marginal positive externality benefit is \( k \) per unit of output. What is the function for the social marginal benefit \( SB \) curve?

(c) What is the optimal output level?

(d) What is the Pigouvian subsidy? Show the impact it has on prices paid by consumers and prices received by producers – and illustrate that it achieves the optimal outcome.

(e) Next, suppose that the total externality social benefit is given by \( SB = (\delta x)^2 \). Does the market outcome change? What about the optimal outcome?

(f) Derive the Pigouvian subsidy now – and illustrate again that it achieves the social optimum.

21.2 The Coase Theorem is often applied in court cases where the parties seek to clarify who has the right to do what in the presence of externalities. Consider again (as in the text discussion) the case of the addition to my house that will then cast a shadow on your swimming pool. Suppose that my benefit from the addition is \( b \), and the cost you incur from my shadow is \( c \). Suppose throughout this exercise that transactions costs are zero.

A: In this part of the exercise, suppose that you and I both know what \( b \) and \( c \) are.

(a) If we both know \( b \) and \( c \), why don’t we just get together and try to settle the matter over coffee rather than ending up in court?

(b) If the judge (who has to decide whether I have a right to build my addition) also knows \( b \) and \( c \), propose a sensible and efficient rule for him to use to adjudicate the case.

(c) Judges rarely have as much information as plaintiffs and defendants. It is therefore reasonable for the judge to assume that he cannot easily ascertain \( b \) and \( c \). Suppose he rules in my favor. What does Coase predict will happen?

(d) What if he instead rules in your favor?

(e) In what sense will the outcome always be the same as it was in part (b) – and in what sense will it not?

B: Next, assume that I know \( b \) and you know \( c \) – but I do not know \( c \) and you do not know \( b \).

(a) Suppose the judge rules in your favor, and I now attempt to convince you to let me build the addition anyhow. I will come to your house and make an offer based on my belief that your cost is less than \( \tau \) with probability \( \rho(\tau) = \tau/\alpha \). What offer will I make?

(b) For what combinations of \( b \) and \( c \) will the outcome be inefficient?

(c) Suppose instead that the judge ruled in my favor. You therefore come to my house to convince me not to build the addition even though I now have the right to do so. You will make me an offer based on your belief that my benefit from the addition is less than or equal to \( \bar{\tau} \) with probability \( \delta(\bar{\tau}) = \bar{\tau}/\beta \). What offer will you make?

(d) For what combinations of \( b \) and \( c \) will the outcome be inefficient?

(e) Explain how the cost of obtaining information might be considered a transactions cost – and the results you derived here are therefore consistent with the Coase Theorem.
21B. The Mathematics of Externalities

21.3 We discussed in the text that the “market failure” that emerges in the presence of externalities can equally well be viewed as a “failure of markets to exist”, and we discussed the related idea that establishing property rights may allow individuals to resolve externality issues even when markets are not competitive.

A: We will explore this idea a bit further by asking whether there is a “right way” to establish property rights in the case of pure consumption externalities.

(a) Suppose we consider the case where your consumption of music in your dorm room disturbs me next door. Let \( x \) denote the number of minutes you choose to play music each day, and let \( e \) be the number of minutes you are allowed to play music. If \( e \) is set at 0, who is given the “property rights” over the air on which the soundwaves travel from your room to mine?

(b) What if \( e \) is set to 1,440 (which is equal to the number of minutes in a day)?

(c) The assignment of \( e \) in part (a) represents the extreme case where you have no right to play your music while the assignment in (b) represents the polar opposite extreme where I have no right to peace and quiet. Review the logic behind the Coase Theorem that suggests the efficient outcome will be reached regardless of whether \( e = 0 \) or \( e = 1,440 \), so long as transaction costs are low.

(d) Draw a graph with minutes of music per day on the horizontal axis – ranging from 0 to 1,440. Draw a vertical axis at 0 minutes and another vertical axis at 1,440 minutes. Then illustrate your marginal willingness to pay for minutes of music (measured on the left vertical axis) and my marginal willingness to pay for reductions in the number of minutes of music (measured on the right axis) – and assume that these are invariant to how \( e \) is set. What is the efficient number of minutes \( m^* \)?

(e) Since \( e = 0 \) and \( e = 1,440 \) are two extreme assignments of property rights, we can now easily think of many cases in between. Does the Coase Theorem apply also to these in between cases? Why or why not?

(f) From a pure efficiency standpoint, if the Coase Theorem is right, is there any case for any particular assignment of \( e \)?

B: Suppose that your tastes can be described by the utility function \( u(x, y) = \alpha \ln x + y \), where \( x \) is the number of minutes per day of music and \( y \) is a composite consumption good. My tastes, on the other hand, can be described by \( u(x, y) = \beta \ln(1440 - x) + y \), with \((1440 - x)\) representing the number of minutes per day without your music. Both of us have some daily income level \( I \), and the price of \( y \) is 1 given that \( y \) is a composite good denominated in dollars.

(a) Let \( e \) be the allocation of rights as defined in part A – i.e. \( e \) is the number of minutes that you are permitted to play music without my permission. When \( x < e \), I am paying you \( p(e - x) \) to play less than you are allowed to – and when \( x > e \), you are paying me \( p(x - e) \) for the minutes above your “rights”. What is your budget constraint?

(b) What is my budget constraint?

(c) Set up your utility maximization problem using the budget constraint you derived in (a) – then solve for your demand for \( x \).

(d) Set up my utility maximization problem and derive my demand for \( x \).

(e) Derive the \( p^* \) we will agree to if transaction costs are zero – and derive the number of minutes of music you will play. Does your answer depend on the level at which \( e \) was set?

(f) According to your results, how much music is played if I don’t care about peace and quiet (i.e. if \( \beta = 0 \)?

(g) True or False: The total number of minutes of music played does not depend on \( e \) – but you and I still care how \( e \) is assigned.

21.4 In exercise 21.3, we began to investigate different ways of assigning property rights in the presence of externalities.

A: Consider again the case of you playing music that disturbs me.

(a) Begin with the assumptions in exercise 21.3 that led to the graph you drew in part (d) of that exercise. Then suppose that the transaction cost of getting together is \( k \). In your graph, indicate for what range of \( e \) such a transaction cost will prohibit the efficient outcome from being reached?

(b) If \( e \) is assigned outside that range, what will be the outcome?

(c) Next, suppose income – or endowment – effects are important; i.e. tastes are not quasilinear. Did we allow for that in exercise 21.3?
Everyday Exercise

whose demand can slope up without being Giffen goods.

Chapter 21. Externalities in Competitive Markets

willingness to pay for these goods thus falls as more people in their peer group consume the same goods.

other hand, like to be the center of attention and would like to consume goods that few others have. Their marginal

to pay is higher the more prevalent the toys are in their peer group. Some of my snooty acquaintances, on the

children are notorious for valuing toys more if their friends also have them – which implies their marginal willingness

to pay is higher the more prevalent the toys are in their peer group. This is not true for some goods. For instance,

Veblen goods

Suppose first that our tastes are again those given in part B of exercise 21.3.

(a) If you have not done exercise 21.3, do so now and check whether the level of music played will depend on

the assignment of property rights in the absence of transactions costs.

(b) Next, suppose that – instead of the tastes in exercise 21.3 – your tastes can be described by the utility

function \( u(x, y) = x^{\alpha}y^{(1-\alpha)} \) (where \( \alpha \) lies between 0 and 1). My tastes remain unchanged. How much

music will be played? Does your answer depend on \( e \) – and does the equilibrium price \( p^* \) depend on \( e \)?

(c) Next, suppose that my utility function is also Cobb-Douglas, taking the form \( u(x, y) = (1440 - x^3)^{\beta}y^{(1-\beta)} \).

Derive again the amount of music that will be played (assuming zero transactions costs). Does your answer

depend on \( e \)? Does the equilibrium price depend on \( e \)?

(d) Explain your results intuitively.

(e) In section 21B.3.4, we went through a numerical exercise to illustrate how the establishment of property

rights in the presence of externalities will resolve the “market failure” in a simple exchange economy. Review the example in the text prior to proceeding. Note that in the text we assigned the property rights in the new market to person 2 – the victim of the externality. But we could have assigned property rights in many other ways (as suggested in our music example). Define \( x_3 \) once again as the impression of person 1’s consumption of \( x_1 \) on person 2 i.e. \( x_3 = x_{11}^1 \). We can establish a market for the good \( x_3 \) by endowing individual 1 with \( e_3 \) units of \( x_3 \). This means that individual 1 can produce up to \( e_3 \) units of \( x_3 \) – which is the same as saying that individual 1 can consume up to \( e_3 \) units of \( x_1 \) – without having to pay the market price \( p_3 \). But if he wants to produce any more \( x_3 \), he must pay individual 2 \( p_3 \) for each additional unit above \( e_3 \). Similarly, under the endowment of \( e_3 \) for individual 1, individual 2 must pay \( p_3 \) per unit to individual 1 for any amount of \( x_3 \) that falls below \( e_3 \) – and receives \( p_3 \) for any amount of \( x_3 \) above \( e_3 \). In the numerical example of the text, what did we implicitly set \( e_3 \) to?

(f) Write down individual 1’s budget constraint when he is assigned \( e_3 \) in property rights. (Hint: If \( x_1 < e_1 \),

individual 1 will earn \( p_3(e_3 - x_1) \) but if \( x_1 > e_1 \), he will have to pay \( p_3(x_1 - e_3) \) which is equivalent to

saying he will earn \( p_3(e_3 - x_1) \).)

(g) Next, write down individual 2’s budget constraint.

(h) If you substitute your answer to (e) into the budget constraints in (f) and (g), you should end up with

the budget constraints we used in the numerical example of the text. Do you?

(i) Now suppose that \( u^1 = x_{11}^{12}(1-x_1^{1-\alpha}) \) and \( u^2 = (1440 - x_1)^{32}x_2^{(1-\beta)} \). Suppose further that \( p_1 = 0 \), \( p_2 = 1 \)

and \( p_3 = p \), and that \( e_1^1 = e_2^2 \). Can you now interpret the general equilibrium model as modeling our case

of you (person 1) bothering me (person 2) with music?

(j) Solve for \( p \) and \( x_3 \) (which is equal to \( x_1^{1} \)). Do you get the same answer as you got when you assumed

Cobb-Douglas tastes for both of us in part (c)?

21.5 Everyday Exercise: Children’s Toys and Gucci Products: In most of our development of consumer theory, we

have assumed that tastes are independent of what other people do. This is not true for some goods. For instance,

children are notorious for valuing toys more if their friends also have them – which implies their marginal willingness

to pay is higher the more prevalent the toys are in their peer group. Some of my snooty acquaintances, on the

other hand, like to be the center of attention and would like to consume goods that few others have. Their marginal

willingness to pay for these goods thus falls as more people in their peer group consume the same goods.\(^{16}\)

A: The two examples we have cited are examples of positive and negative network externalities.

\(^{16}\)Such goods are examples of Veblen goods. We previously mentioned these in an exercise in Chapter 7 as goods

whose demand can slope up without being Giffen goods.
(a) Consider children’s toys first. Suppose that, for a given number $N$ of peers, demand for some toy $x$ is linear and downward sloping — but that an increase in the “network” of children (i.e. an increase in $N$) causes an upward parallel shift of the demand curve. Illustrate two demand curves corresponding to network size levels $N_1 < N_2$.

(b) Suppose every child at most buys one of these toys which are produced at constant marginal cost. For a combination of $p$ and $x$ to be an equilibrium, what must be true about $x$ if the equilibrium lies on the demand curve for network size $N_1$?

(c) Suppose you start in such an equilibrium and the marginal cost (and thus the price) drops. Economists distinguish between two types of effects: A direct effect that occurs along the demand curve for network size $N_1$, and a bandwagon effect that results from increased demand due to increased network size. Label your original equilibrium $A$, the “temporary” equilibrium before network externalities are taken into account as $B$ and your new equilibrium (that incorporates both effects) as $C$. Assume that this new equilibrium lies on the demand curve that corresponds to network size $N_2$.

(d) How many toys are sold in equilibrium $C$? Connect $A$ and $C$ with a line labeled $\overline{AB}$. Is $\overline{AB}$ the true demand curve for this children’s toy? Explain.

(e) If you were a marketing manager with a limited budget for a children’s toy company, would you spend your budget on aggressive advertising early as the product is rolled out or wait and spread it out? Explain.

(f) Now consider my snooty acquaintances who like Gucci products more if few of their friends have them. For any given number of friends $N$ that also have Gucci products, their demand curve is linear and downward sloping — but the intercept of their demand curve falls as $N$ increases. Illustrate three demand curves for $N_1 < N_2$.

(g) Assume for convenience that everyone buys at most 1 Gucci product. Identify an initial equilibrium $A$ under which $N_1$ Gucci products are sold at some initial price $p$ — and then a second equilibrium $C$ at which $N_2$ Gucci products are sold at price $p' < p$. Can you again identify two effects — a direct effect analogous to the one you identified in (c) and a snob effect analogous to the bandwagon effect you identified for children’s toys? How does the snob effect differ from the bandwagon effect?

(h) True or False: Bandwagon effects make demand more price elastic while snob effects make demand less price elastic.

(i) In exercise 7.9, we gave an example of an upward sloping demand curve for Gucci products, with the upward slope emerging from the fact that utility was increasing in the price of Gucci products. Might the demand that takes both the direct and snob effects into account also be upward sloping in the presence of the kinds of network externalities modeled here?

B: Consider again the positive and negative network externalities described above.

(a) Consider first the case of a positive network externality such as the toy example. Suppose that, for a given network size $N$, the demand curve is given by $p = 25 N^{1/2} - x$. Does this give rise to parallel linear demand curves for different levels of $N$, with higher $N$ implying higher demand?

(b) Assume that children buy at most one of this toy. Suppose we are currently in an equilibrium where $N = 400$. What must the price of $x$ be?

(c) Suppose the price drops to $24. Isolate the direct effect of the price change — i.e. if child perception of $N$ remained unchanged, what would happen to the consumption level of $x$?

(d) Can you verify that the real equilibrium (that includes the bandwagon effect) will result in $x = N = 576$ when price falls to $24$? How big is the direct effect relative to the bandwagon effect in this case?17

(e) Consider next the negative network externality of the Gucci example. Suppose that, given a network of size $N$, the market demand curve for Gucci products is $p = (1000/N^{1/2}) - x$. Does this give rise to parallel linear demand curves for different levels of $N$, with higher $N$ implying lower demand?

(f) Assume again that no one buys more than one Gucci item. Suppose we are currently in equilibrium with $N = 25$. What must the price be?

(g) Suppose the price drops to $65. Isolate the direct effect of the price change — i.e. if people’s perception of $N$ remained unchanged, what would happen to the consumption level of $x$?

(h) Can you verify that the real equilibrium (that includes the snob effect) will result in $x = N = 62$? How big is the direct effect relative to the snob effect in this case?

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17 For a more detailed analysis of the quite interesting demand curve that arises under this network externality, see a similar example in exercise 21.8.
(i) Although the demand curves for a fixed level of $N$ are linear, can you sketch the demand curve that includes both direct and snob effects?

21.6 Business Application: Fishing in the Commons: In the text, we introduced the notion of the Tragedy of the Commons — and found its source in the emergence of externalities when property rights are not well established. This exercise demonstrates the same idea in a slightly different way.

A: Consider a self-contained lake which is home to fish that are sold on the market at $p$. Suppose the primary input into fishing this lake is nets that are rented at a weekly rate of $r$, and suppose the single input production frontier for fish has decreasing returns to scale.

(a) Draw a graph with fishing nets on the horizontal axis and fish on the vertical. Illustrate the marginal product of fishing nets.

(b) Recalling the relationship between “marginal” and “average” quantities, add the average product curve to your graph.

(c) If you own the lake, what is the relationship between the marginal product of fishing nets and prices ($p, r$) assuming you maximize profit?

(d) Illustrate the profit maximizing quantity of nets $n^*$ on your graph. Then, on a graph below it that plots the production frontier for fish, illustrate the number of fish $x^*$ that are brought to market.

(e) Suppose you instead charge a weekly fee for every fishing net that fishermen bring to your lake. Does the number of fish produced and nets used change?

(f) Next consider a nearby lake that is identical in every way except that it is publicly owned — with no one controlling who can come onto the lake to fish. Assuming all nets are used with the same intensity, each fishing net that is brought onto the lake can then be expected to catch the average of the total weekly catch. Illustrate on your graphs how many nets $\pi$ will be brought onto this lake — and how many fish $\pi$ this implies will be brought to market each week.

(g) Which lake yields more fish per week? Which lake is being harvested for fish efficiently?

(h) Suppose that what matters is not just the current crop of fish but also its implication for the future fish population of the lake. Explain how the privately owned lake is likely to house a relatively constant population of fish over time while the publicly owned lake is likely to run out of fish as time passes.

(i) The trade in elephant trunks — or ivory — has decimated much of the elephant population in some parts of Africa but not in others, with hunters often slaughtering entire herds, removing the trunks and leaving the rest. In some parts of Africa, the land on which elephants roam is public property — in other parts it is privately owned with owners allowed to restrict access. Can you guess from our lake example what is different about the parts of Africa where elephant herds are stable compared to those parts where they are nearing extinction?

(j) Why do you think that wild Buffalo in the American West are nearly extinct while domesticated cattle are plentiful in the same region?

B: Let $n$ again denote the fishing nets used in the lake and assume that $r$ is the weekly rental cost per net. The number of fish brought out of the lake per week is $x = f(n) = An^\alpha$ where $A > 0$ and $0 < \alpha < 1$, and fish sell on the market for $p$.

(a) Suppose you own the lake and you don’t let anyone other than yourself fish. How many fish will you pull out each week assuming you maximize profit?

(b) Suppose instead you allow others to fish for a fee per net — and you want to maximize your fees. Will more or fewer fish be pulled out each week?

(c) Next, consider the identical lake that has just been discovered near yours. This lake is publicly owned, and anyone who wishes to can fish there. How many fish per week will be pulled out from that lake?

(d) Suppose $A = 100, \alpha = 0.5, p = 10$ and $r = 20$. How many fish are harvested per week in (a), (b) and (c)? How many nets are used in each case?

(e) What is the weekly rental value of the lake? If we count all your costs — including the opportunity cost of owning the lake, how much weekly profit do you make if you are the only one to fish on your lake?

(f) How much profit (including the opportunity cost of fishing on the lake yourself) do you make if you allow others to fish on your lake for a per-net-fee? How much profit do the fishermen who pay the fee to fish on your lake make?

(g) How much profit do the fishermen who fish on the publicly owned nearby lake make?
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(h) If the government auctioned off the nearby lake, what price do you think it would fetch if the weekly interest rate is 0.12% or 0.0012?

(i) If the government auctioned off the nearby lake with the condition that the same number of fish per week need to be brought to market as before, what price would the lake fetch?

21.7 Business and Policy Application: The Externality when Fishing in the Commons: In exercise 21.6, we showed that free access to a fishing lake causes overfishing because fishermen will continue to fish until the cost of inputs (i.e., fishing nets, in our example) equals average rather than marginal revenue product.

A: Suppose that the lake in exercise 21.6 is publicly owned.

(a) What is the externality that fishermen impose on one another in this lake?

(b) Seeing the problem as one involving this externality, how would you go about setting a Pigouvian tax on fishing nets to remedy the problem? What information would you have to have to calculate this?

(c) Suppose instead that the lake is auctioned off to someone who then charges per-net fees to fishermen who would like to fish on the lake (as in A(e) of exercise 21.6). How do you think the fees charged by a profit maximizing lake-owner compare to the optimal Pigouvian tax?

(d) Do you think it is easier for the government to collect the information necessary to impose a Pigouvian tax in part (b) or for a lake-owner to collect the information necessary to impose the per-net fees in part (c). Who has the stronger incentive to get the correct information?

(e) How would the price of the lake that the government collects in (c) compare to the tax revenues it raises in (b)?

(f) Suppose instead that the government tries to solve the externality problem by simply setting a limit on per-net fishing licenses that fishermen are now required to use when fishing on the public lake. If the government sets the optimal cap on licenses and auctions these off, what will be the price per license?

(g) What do each of the above solutions to the Tragedy of the Commons share in common?

(h) Legislators who represent political districts (such as Congressmen in the U.S. House of Representatives) can be modeled as competing for pork barrel projects to be paid for by the government budget. Could you draw an analogy between this and the problem faced by fishermen competing for fish in a public lake? (This is explored in more detail in end-of-chapter exercise 28.2 in Chapter 28.)

B: * Let \( N \) denote the total number of fishing nets used by everyone and \( X = f(N) = AN^\alpha \) the total catch per week. As in exercise 21.6, let \( r \) be the weekly rental cost per net, let \( p \) be the market price for fish and let \( A > 0 \) and \( 0 < \alpha < 1 \).

(a) The lake is freely accessible to anyone who wants to fish. How much revenue does each individual fisherman make when he uses one net?

(b) What is the loss in revenue for everyone else who is fishing the lake when one fisherman uses one more net?

(c) Suppose that each fisherman took the loss of revenue to others into account in his own profit maximization problem when choosing how many nets \( n \) to bring. Write down this optimization problem. Would this solve the externality problem?

(d) A Pigouvian tax is optimally set to be equal to the marginal social damage an action causes when evaluated at the optimal market level of that action. Evaluate your answer to (b) at the optimal level of \( N \) to derive the optimal Pigouvian tax on nets.

(e) Suppose that all fishermen just consider their own profit but that the government has imposed the Pigouvian per-net tax you derived in (d). Write down the fisherman’s optimization problem and illustrate its implications for the overall level of \( N \). Does the Pigouvian tax achieve the efficient outcome?

(f) Suppose the government privatized the lake and allowed the owners to charge per-net fees. The owner might do the following: First, calculate the maximum profit (not counting the rental value of the lake) he would be able to make by simply fishing the lake himself with the optimal number of nets – then set the fee per net at this profit divided by the number of nets he himself would have used. What per-net fee does this imply?

(g) Compare your answer to (f) to your answer to (d). Can you explain why the two are the same?

(h) Suppose \( A = 100, \alpha = 0.5, p = 10 \) and \( r = 20 \). What is the optimal Pigouvian (per-net) tax and the profit maximizing per-net fee that an owner of the lake would charge?
Chapter 21. Externalities in Competitive Markets

21.8 Business Application: Network Externalities and the Battle between Microsoft and Apple: Many markets related to technology products operate in the presence of network externalities because the value of such products to consumers depends on how many other consumers are in the “network” of consumers. For instance, an internet connection would not be nearly as useful if no one else in the world was connected to the internet; a telephone becomes more useful the more other people also have telephones; and a computer operating system becomes more useful the more others use it – because then the market for software that runs on this operating system increases which in turn fosters greater software innovation for that platform. Assume throughout that we are analyzing the consumer market for computers and that a consumer buys at most one computer.

A: Consider the market for PC’s when the Microsoft Windows system first competed with the Apple Macintosh platform in the 1980’s. Microsoft and Apple pursued very different strategies: Microsoft licensed the Windows platform to lots of PC makers who competed with one another and thus drove down the price of PCs. Apple, on the other hand, did not license its Macintosh operating system – and sold it only with its own Apple computers that were more expensive.

(a) Suppose that people vary greatly in their interest for buying a personal computer, but their willingness to pay for a computer increases with the square root of the size of the “network” of others who use a computer with the same operating system; i.e. if someone’s willingness to pay for a computer is \( B \) when no one else is in the “network”, her willingness to pay for the same computer is \( BN^{1/2} \) when the network has \( N \) people. Pick three different levels of \( N \) – with \( N_1 < N_2 < N_3 \) – and illustrate the linear aggregate demand curves \( D_1, D_2 \) and \( D_3 \) – that correspond to these levels of \( N \) for a computer with a particular operating system.

(b) Suppose the demand curve \( D_1 \) tells us that \( N_1 \) computers are demanded at price \( p \). In what sense is this an equilibrium in which consumers are taking into account the network externality in their decision-making?

(c) Now suppose the price drops from \( p \) to \( p' \). If everyone assumes that the network size remains fixed at \( N_1 \), illustrate how many more computers will be sold. Why can this not be an equilibrium in the same way that our previous situation was an equilibrium?

(d) Now take into account that people will realize that the network is growing as price falls. What will happen if the number of computers demanded at \( p' \) on \( D_3 \) is \( N_3 \)? Illustrate the new equilibrium – and explain why some economists say that network externalities give rise to a bandwagon effect in addition to a direct price effect.

(e) How do you think the process of moving from our initial equilibrium to the final equilibrium unfolds over time as price falls from \( p \) to \( p' \)? True or False: Network externality of this kind cause demand to become more price elastic.

(f) Microsoft got a head start with its licensing policy that created competition and thus sharply falling prices in the PC market – while Apple’s computers were perpetually priced above PC’s. Can you use this model to explain how Microsoft’s Windows operating system became the dominant operating system?

(g) Suppose that the quality of Apple computers is now far better than any competing PC’s – and that it can be priced competitively. Why is this not enough for Apple to gain dominance in the computer market? How might you argue that the network externality you analyzed has led to an inefficient market outcome?

(h) Explain the following statement made by a technology company executive: “In the quickly moving tech market, it is usually better to be first rather than best.”

(i) In a recent update to its operating system, Apple introduced a new feature that allows users to switch between the traditional Macintosh operating system and the Microsoft Windows operating system. Do you think this was a good move in light of what this exercise has told us about network externalities?

B: Now consider the type of network externality described in part A more carefully. Suppose that the aggregate demand function for computers is given by \( x = (AN^{1/2} - p)/\alpha \).

(a) Does this demand function give rise to the parallel demand curves (for different levels of network size) you analyzed in part A?

(b) The consumer side of the market is in equilibrium if the network size \( N \) is equal to the number of computers sold. Use this to derive the actual demand curve \( P(x) \) that takes the network externality fully into account.

(c) Suppose \( A > 2\alpha \). What is the shape of this demand curve? Explain.

(d) Check your answer to (c) by graphing the demand function when \( A = 100 \) and \( \alpha = 1 \). Continue with these parameter values for the rest of the exercise.
(e) In models like this, we say that an equilibrium is stable if it does not lie on an upward sloping portion of the demand curve. Can you guess why? (Hint: Suppose that \( x^* \) is the equilibrium quantity on the upward sloping part of demand for some price \( p^* \). Imagine what would happen if slightly more than \( x^* \) were bought, and what would happen if slightly less than \( x^* \) would be bought.)

(f) Suppose the supply curve is horizontal at \( p = 2,000 \). Our model implies there are three equilibria – 2 that are stable and one that is not stable. What network sizes are associated with each of these equilibria?

(g) Suppose that we begin in the equilibrium in which no one owns a computer and the marginal cost of producing computers is $2,000. Why might firms launch an aggressive campaign in which they give away computers before selling them in stores? How many might they give away to “jump-start” the market?

21.9 Business Application: Pollution that increases firm costs – the Market Outcome: In the text, we assumed for convenience that the ill effects of pollution are felt by people other than producers and consumers. Consider instead the following case: An entire competitive industry is located around a single lake that contains some vital property needed for the production of \( x \). Each unit of output \( x \) that is produced results in pollution that goes into the lake. The only effect of the pollution is that it introduces a chemical into the lake – a chemical that requires firms to

A: We have now constructed an example in which the only impact of pollution is on the firms that are creating the pollution. Suppose that each unit of \( x \) that is produced raises every firm’s fixed cost by \( \delta \).

(a) Suppose all firms have identical decreasing returns to scale production processes, with the only fixed cost created by the pollution. For a given amount of industry production, what is the shape of an individual firm’s average cost curve?

(b) In our discussion of long run competitive equilibria, we concluded in Chapter 14 that the long run industry supply curve is horizontal when all firms have identical cost curves. Can you recall the reason for this?

(c) Now consider this example here. Why is the long run industry supply curve now upward sloping despite the fact that all firms are identical?

(d) In side-by-side graphs of a firm’s cost curves and the (long run) industry supply and demand curves, illustrate the firm and industry in long run equilibrium.

(e) Usually we can identify producer surplus – or firm profit – as an area in the demand and supply picture. What is producer surplus here? Why is your answer different from the usual?

(f) In chapter 14, we briefly mentioned the term decreasing cost industries – industries in which the long run industry supply curve is downward sloping despite the fact that all firms might have identical production technologies. Suppose that in our example the pollution causes a decrease rather than an increase in fixed costs for firms. Would such a positive externality be another way of giving rise to a decreasing cost industry?

B: * Suppose that each firm’s (long run) cost curve is given by \( c(x) = \beta x^2 + \delta X \) where \( x \) is the firm’s output level and \( X \) is the output level of the whole industry. Note that \( x \) is contained in \( X \) – and thus we could write the cost function as \( c(x) = \beta x^2 + \delta x + \delta X \) where \( X \) is the output produced by all other firms. When each firm is small relative to the industry, however, the impact of a single firm’s pollution output on its own production cost is negligible – and it is a good approximation (that makes the problem a lot easier to solve) to simply write a single firm’s cost curve as \( c(x) = \beta x^2 + \delta X \). Furthermore, if all firms are identical, it is reasonable to assume that all firms produce the same output level \( \overline{x} \). Letting \( N \) denote the number of firms in the industry, we can therefore write \( X = N \overline{x} \) and re-write the cost function for an individual firm as \( c(x) = \beta x^2 + \delta N \overline{x} \).

(a) How is our treatment of a producer’s contribution to her own costs similar to our “price-taking” assumption for competitive firms?

(b) Derive the marginal and average cost functions for a single firm (using the final version of our approximate cost function). (Be careful to realize that the second part of the cost function is, from the firm’s perspective, simply a fixed cost.)

(c) Assuming the firm is in long run equilibrium, all firms will make zero profit. Use your answer to (b) to derive the output level produced by each firm as a function of \( \delta \), \( \beta \), \( N \) and \( \overline{x} \).

(d) Since all firms are identical, in equilibrium the single firm we are analyzing will produce the same as each of the other firms – i.e. \( x = \overline{x} \). Use this to derive a single firm’s output level \( x(N) \) as a function of \( \delta \), \( N \), and \( \beta \). What does this imply about the equilibrium price \( p(N) \) (as a function of \( \delta \) and \( N \)) given that firms make zero profit in equilibrium?
(e) Since each firm produces \( x(N) \), multiply this by \( N \) to get the aggregate output level \( X(N) \) – then invert it to get the number of firms \( N(X) \) as a function of \( \beta, \delta \) and \( X \).

(f) Substitute \( N(X) \) into \( p(N) \) to get a function \( p(X) \). Can you explain why this is the long run industry supply curve with free entry and exit?

(g) Suppose the aggregate demand for \( X \) is given by the demand curve \( p_D(X) = A/(X^{0.5}) \). Set the industry supply curve equal to the demand curve to get the equilibrium market output \( X^* \) (as a function of \( A, \delta \) and \( \beta \)).

(h) Use your answer to (g) to determine the equilibrium price level \( p^* \) (as a function of \( A, \delta \) and \( \beta \)).

(i) Use your answer to (g) to determine the equilibrium number of firms \( N^* \) (as a function of \( A, \delta \) and \( \beta \)).

(j) Suppose that \( \beta = 1, \delta = 0.01 \) and \( A = 10,580 \). What are \( X^* \), \( p^* \) and \( N^* \)? How much does each individual firm produce? (Do exercise 21.10 to compare these to what is optimal.)

21.10 Policy Application: Pollution that increases firm costs – Barney’s Solution: Consider the same situation as the one described in exercise 21.9.

A: Assume again that the only impact of pollution is that it increases firm fixed costs by \( \delta \) for every unit of \( x \) that is produced in the industry.

(a) Suppose there are \( N \) firms in the equilibrium you described in exercise 21.9. What is the pollution related cost of firm \( i \) producing one more unit of \( x \)?

(b) How much of this pollution related cost does firm \( i \) not take into account? If firm \( i \) is one of a large number of firms, is it a good approximation to say that firm \( i \) does not take any of the pollution related cost into account? How is this similar to our “price-taking” assumption for competitive firms?

(c) Suppose that our benevolent social planner Barney can tell firms what to count as costs. Illustrate how Barney’s suggestion for each firm’s marginal cost curve is related to the marginal cost curve firms would otherwise use (given a fixed number \( N \) of firms in the industry)?

(d) What does your answer imply about the relationship between the firm’s AC curve and Barney’s suggestion for what the firm’s AC curve should be?

(e) True or False: If firms used Barney’s suggested cost curves, the long run industry supply curve would be upward sloping as you should have concluded in exercise 21.9 it is in the absence of Barney – but now it would lie above where it was in exercise 21.9.

(f) True or False: Under the efficient outcome, the industry would produce less at a higher price.

(g) If a single corporation acquired all the firms around the lake, would that corporation take the costs of pollution into account more like Barney or more like the individual competitive firms? (In parts of exercise 23.11, you’ll be asked to revisit this in the context of such a monopoly.)

B: Consider the same set-up as in part B of exercise 21.9. In the previous case where we derived the market equilibrium, we said that – in a model with many firms – it was reasonable to model each individual firm as not taking its own impact of pollution into account and to simply model the cost function as \( c(x) = \beta x^2 + \delta N \pi \) (where the latter entered as a fixed cost).

(a) Now consider the cost function that benevolent Barney would use for each firm: From the social planner’s perspective, the firm’s variable costs (captured by \( \beta x^2 \)) would still matter, as would the fixed cost from pollution (captured by \( \delta N \pi \) where \( \pi \) is the amount produced by each firm and \( N \) is the number of firms in the industry.) But Barney also cares about the following: each unit of \( x \) produced by firm \( i \) causes an increase in costs of \( \delta \) for each of the \( N \) firms – which implies that the pollution cost Barney would consider firm \( i \) as imposing on society is \( \delta Nx \). This implies that Barney’s cost function for each firm is \( c_B(x) = \beta x^2 + \delta N \pi + \delta Nx \). Derive from this the marginal and average cost functions that Barney would use for each firm (being sure to not treat the last term as a fixed cost.)

(b) Repeat parts (c) through (i) from exercise 21.9 using the cost functions Barney would use for each firm to arrive at \( X^* \), \( p^* \) and \( X^* \).

(c) Compare your answers to those from exercise 21.9. How do they differ?

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Note that the demand function is one that would emerge from utility maximization of the utility function \( U(x,y) = 2A x^{0.5} + y \) (where \( y \) is a composite good). Thus, it can be viewed as emerging from a representative agent with tastes that are quasilinear in \( x \) – and thus represents a true aggregate marginal willingness to pay as well as an uncompensated demand curve. See Chapter 15 for a review of this.
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(d) Suppose, as in part (j) of exercise 21.9 that $\beta = 1$, $\delta = 0.1$ and $A = 10,580$. What are $X^*$, $p^*$ and $N^*$? How much does each individual firm produce?

(e) Compare these to your answers in exercise 21.9. Can you give an intuitive explanation for why these answers differ despite the fact that pollution only affects the firms in the industry?

(f) What is the Pigouvian tax that is required in order for competitive firms to implement the equilibrium you just calculated in (d)? What price does this imply consumers would pay and what price does it imply producers would receive?

(g) Verify that your Pigouvian tax in fact results in prices for consumers and the industry that lead them to demand and supply the output level you calculated in part (d). (Note: You will need to refer back to your answers to exercise 21.9 to do this part.)

21.11 Policy Application: Pollution that increases firm costs – Policy Solutions

This exercise continues to build on exercises 21.9 and 21.10. Assume the same basic setup of firms located around a lake producing pollution that causes the fixed costs of all firms to increase.

A: Continue to assume that each output unit that is produced results in an increase of fixed costs of $\delta$ for all firms in the industry.

(a) Begin by illustrating the market demand and long run industry supply curves, labeling the market equilibrium as $A$.

(b) Next, without drawing any additional curves, indicate the point $B$ in your graph where the market would be producing if firms were taking the full cost of the pollution they emit into account.

(c) Illustrate the Pigouvian tax that would be necessary to get the market to move to equilibrium $B$.

(d) Suppose $N^*$ is the number of firms in the industry in the market outcome, $N^{opt}$ is the optimal number of firms and $\delta$ continues to be as defined throughout. What does the government have to know in order to implement this Pigouvian tax? Is what the government needs to know easily observable prior to the tax?

(e) Where in your graph does consumer surplus before and after the tax lie?

(f) Keeping in mind what you concluded in exercise 21.9, has (long run) producer surplus – or long run industry profit – changed as a result of the tax?

(g) True or False: The pollution cost under the Pigouvian tax is, in this example, equal to the tax revenue that is raised under the tax.

(h) Is there additional pollution damage under the market outcome (in the absence of the tax)?

(i) Is there a deadweight loss from not using the tax?

(j) Suppose the government instead wanted to impose a cap-and-trade system on this lake – with pollution permits that allow a producer to produce the amount of pollution necessary to produce one unit of output. What is the “cap” on pollution permits the government would want to impose to achieve the efficient outcome? What would be the rental rate of such a permit when it is traded?

(k) What would the government have to know to set the optimal cap on the number of pollution permits?

B: Continue with the functional forms for costs and demand as given in exercises 21.9 and 21.10. Suppose, as you did in parts of the previous exercises, that $\beta = 1$, $\delta = 0.1$ and $A = 10,580$ throughout this exercise.

(a) If you have not already done so in part (f) of exercise 21.9, determine the Pigouvian tax that would cause producers to behave the way the social planner would wish for them to behave. What price will consumers end up paying and what price will firms end up keeping under this tax?

(b) ** Calculate (for our numerical example) consumer surplus with and without the Pigouvian tax. (Skip this if you are not comfortable with integral calculus.) Why is (long run) producer surplus – or long run profit in the industry – unchanged by the tax?

(c) Determine the total cost of pollution before and after the tax is imposed.

(d) Determine tax revenue from the Pigouvian tax.

(e) What is the total surplus before and after the tax – and how much deadweight loss does this imply in the absence of the tax?

(f) Suppose next that the government instead creates a tradable pollution permits – or voucher – system in which one voucher allows a firm to produce the amount of pollution that gets emitted from the production of 1 unit of output. Derive the demand curve for such vouchers.

(g) What is the optimal level of vouchers for the government to sell – and what will be the rental rate of the vouchers if the government does this?
When asked to explain our actions, we sometimes simply respond by saying “it was the right thing to do.” The concept of “the right thing to do” is one that is often formed by observing others – and the more we see others “do the right thing,” the more we believe it is in fact “the right thing to do.” In such cases, my action “to do the right thing” directly contributes to the social norm that formed by observing others – and the more we see others “do the right thing,” the more we believe it is in fact “the right thing to do.”

### 21.12 Policy Application: Social Norms and Private Actions

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#### A: Consider for instance the use of observably “green” technology – such as driving hybrid cars. Suppose there are two types of car-buyers: (1) a small minority of “greenies” for whom green technology is attractive regardless of what everyone else does – and whose demand for green cars is therefore independent of how many others are using green cars; and (2) the large majority of “meanies” who don’t care that much about environmental issues but do care about being perceived as “doing the right thing.”

(a) Draw a graph with the aggregate demand curve $D_0$ for the “greenies.” Assume that green cars are competitively supplied at a market price $p^*$ – and draw in a perfectly elastic supply curve for green cars at that price.

(b) There are two types of externalities in this problem. The first arises from the positive impact that green cars have on the environment. Suppose that the social marginal benefit associated with this externality is some amount $k$ per green car and illustrate in your graph the efficient number of cars $x_1$ that this implies for “greenies.” Then illustrate the Pigouvian subsidy $s$ that would eliminate the market inefficiency.

(c) The second externality emerges in this case from the formation of social norms – a form of network externality. Suppose that the more green cars the “meanies” see on the road, the more of them become convinced that it is “the right thing to do” to buy green cars even if they are somewhat less convenient right now. Suppose that the “meanies’” linear demand $D_1$ for green cars when $x_1$ green cars are on the road has vertical intercept below $(p^* - k)$. In a separate graph, illustrate $D_1$ – and then illustrate a demand curve $D_2$ that corresponds to the demand for green cars by “meanies” when $x_2(> x_1)$ green cars are on the road. Might $D_2$ have an intercept above $p^*$?

(d) Does the subsidy in (b) have any impact on the behavior of the “meanies”? In the absence of the network externality, is this efficient?

(e) How can raising the subsidy above the Pigouvian level have an impact far larger than one might initially think from the imposition of the original Pigouvian tax? If the network externalities are sufficiently strong, might one eventually be able to eliminate the subsidy altogether and see the majority of “meanies” use green cars anyhow?

(f) Explain how the imposition of a larger initial subsidy has changed the “social norm” – which can then replace the subsidy as the primary force that leads people to drive green cars.

(g) Sometimes people advocate for so-called “sin taxes” – taxes on such goods as cigarettes or pornography. Explain what you would have to assume for such taxes to be justified on efficiency grounds in the absence of network externalities.

(h) How could sin taxes like this be justified as means of maintaining social taboos and norms through network externalities?

#### B: Suppose you live in a city of 1.5 million potential car owners. The demand curves for green cars $x$ for “greenies” and “meanies” in the city are given by $x_g(p) = (D - p)/\delta$ and $x_m(p) = (A + BN^{1/2} - p)/\alpha$, where $N$ is the number of green cars on the road and $p$ is the price of a green car. Suppose throughout this exercise that $A = 5,000$, $B = 100$, $D = 100,000$, $\alpha = 0.1$ and $\delta = 5$.

(a) Let the car industry be perfectly competitive, with price for cars set to marginal cost. Suppose the marginal cost of a green car $x$ is $25,000. How many cars are bought by “greenies”?

(b) Explain how it is possible that no green cars are bought by “meanies”?

(c) Suppose that the purchase of a green car entails a positive externality worth $2,500. For the case described in (a), what is the impact of a Pigouvian subsidy that internalizes this externality? Do you think it is likely that this subsidy will attract any of the “meanie” market?

(d) Would your answer change if the subsidy were raised to $5,000 per green car? What if it were raised to $7,500 per green car?

(e) ** Suppose that a subsidy of $7,500 per green car is implemented and suppose that the market adjusts to this in stages as follows: First, “greenies” adjust their behavior in period 0. Then, in period 1 “meanies” purchase green cars based on their observation of the number of green cars on the road in period 0. From then on, each period $n$, “meanies” adjust their demand based on their observation in period $(n - 1)$. Create a table that shows the number of green cars $x_g$ bought by “greenies” and the number $x_m$ bought by “meanies” in each period from period 1 through 20.
(f) Explain what you see in your tables in the context of network externalities and changing social norms.

(g) Now consider the same problem from a slightly different angle. Suppose that the number of green cars driven by “greenies” is $\mathcal{F}$. Then the total number of green cars on the road is $N = \mathcal{F} + x_m$. Use this to derive the equation $p(x_m)$ of the demand curve for green cars by “meanies” – and illustrate its shape assuming $\mathcal{F} = 16,000$.

(h) Relate this to the notion of “stable” and “unstable” equilibria introduced in exercise 21.8B(e). Given that you can calculate $\mathcal{F}$ for different prices, what are the stable equilibria when $p = 25,000$? What if $p = 22,500$ and when $p = 17,500$.

(i) Explain now why the $2,500 and $5,000 subsides would be expected to cause no change in behavior by “meanies” while a $7,500 would cause a dramatic change.

(j) Compare your prediction for $x_m$ when the subsidy is $7,500 to the evolution of $x_m$ in your table from part (e). Once we have converged to the new equilibrium, what would you predict will happen to $x_m$ if the subsidy is reduced to $2,500? What if it is eliminated entirely?