The Distributional Impacts of Minimum Wage Increases when Both Labor Supply and Labor Demand are Endogenous*

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Abstract

We develop and estimate a one-shot search model with endogenous entry, and therefore zero expected profits, by firms and endogenous labor supply by workers. Positive employment effects from an increase in the minimum wage can result as the employment level depends upon both the number of searching firms and the number of searching workers. Welfare implications are similar to the classical analysis: workers who most want the minimum wage jobs are hurt by the increase in the minimum wage with workers who were marginally interested in minimum wage jobs benefiting. We estimate the model using data on teenagers from the CPS and show that small changes in the employment level are masking large changes in labor supply and labor demand. Teenagers from well educated families see their employment probabilities increase as the positive labor supply effects outweigh the negative labor demand effects. In contrast, teenagers from less educated families have lower employment probabilities as they are pushed out of the market by their more privileged counterparts.

Keywords: Minimum wages, search, unemployment

JEL J6, J3

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1 Introduction

The classical analysis of the minimum wage operates from a simple labor supply and demand framework with the minimum wage serving as a price floor. When the labor market is competitive, a binding price floor leads to employment being determined solely by labor demand. With employment determined solely by labor demand an increase in the minimum wage must lead to a decrease in employment.

Empirical work by Card and Krueger (1994, 1995) has called the classical model into question. Their research suggests that an increase in the minimum wage may even have small positive employment effects. While there has been considerable controversy regarding their findings,[1] the evidence for strong negative employment effects from an increase in the minimum wage is surprisingly weak.[2]

The lack of strong negative employment effects from increasing the minimum wage has led some policy-makers to support minimum wage increases as a means of combating poverty. With no employment losses increasing the minimum wage involves transferring money from rich firms to poor workers. In this paper we show that changes in the employment level from a minimum wage increase may be masking much larger changes in labor supply and labor demand. Further, these larger changes imply employment losses for groups that most wanted the minimum wage jobs in the first place.

In particular, we develop a two-sided search model with endogenous labor supply and labor demand that can exhibit positive employment effects from an increase in the minimum wage. In the classical analysis, the number of searching workers has no effect on the number of matches. In a more general search model, the number of matches increases with the number of searching workers. Hence, increasing the minimum wage may induce search which can lead to higher employment levels even with the number of firms falling. However, these positive employment effects also lead to lower probabilities of matching at the individual level. As in Luttmer (2007) and Glaeser and Luttmer (2003), in expectation, those with the lowest reservation wages are hurt most by the increase in the minimum wage when workers are

[1] See in particular Neumark and Wascher (2000) and the reply by Card and Krueger (2000).

[2] Kennan (1995) suggests that the reason minimum wage effects have been difficult to quantify is that changes in the minimum wage are quite small relative to the cyclical variation and serial correlation in teenage employment.

randomly allocated to jobs.[3]

Our search model can therefore generate zero or positive employment effects while also having firms earn zero expected profits both before and after the minimum wage increase. The search model shows that the effect of a minimum wage increase may appear small because the variable used to measure this effect— the employment level— does not adequately capture the churning of the labor market. Individuals induced to enter the labor market result in more matches and may not lower the employment level. However, the new matches push out those who originally wanted minimum wage jobs. Therefore, there are possibly large negative welfare effects from a minimum wage increase, even if the employment level stays constant or increases.[4]

We show that the model developed in the theoretical section is estimable. Estimates of the model yield three sets of parameters: 1) the parameters of the wage generating process, 2) the parameters of the firm's zero profit condition (labor demand), and 3) the parameters of the search decision (labor supply). Although we do not observe firm behavior, we show that the firm's zero profit condition can be written as a function of the probability of a worker finding a match. The three sets of parameters can then be estimated from data on wages, employment, and search choices respectively.

We use a twelve year band (1989 to 2000) of 16 to 19 year old white teenagers from the basic monthly outgoing rotation CPS files. Minimum wages then vary across states and over time. We find that the employment elasticity with respect to a minimum wage increase is moderately negative for this group of teenagers. However, this is masking large increases in the probability of search coupled with large decreases in the probability of finding a job conditional on search.

Positive employment effects from a minimum wage increase then exist for sub-groups of

[3]Indeed, Glaeser (2002) suggests that non-competitive prices may lead to discrimination. If workers who have low reservation values also have characteristics that are unappealing but not related to productivity then the minimum wage provides even more scope for mis-allocation.

[4]An alternative search model by van den Berg (2003) shows welfare gains from an increase in the minimum wage in a model where firms are heterogeneous. Without a minimum wage, the model has two equilibria: one where low productivity firms survive and wages are low and one where there are only high productivity firms and wages are high. The second equilibrium is shown to dominate the first, with a minimum wage potentially serving to eliminate the bad equilibrium.

teenagers. In particular, those who come from highly educated families see their employment probabilities increase due to their increased probability of search. In contrast, those who come from less educated families see their employment probabilities fall. These teenagers were more likely to search in the first place and hence the decrease in labor demand outweighs the increase in labor supply. Teenagers from more privileged backgrounds have higher reservation values, but lower search costs as compared to teens from poorer, less educated families. Raising the minimum wage changes the incentives to entry for these two groups of teens in different ways, with the combination of higher expected wages and lower probabilities of employment being more attractive to the teens from well-educated families. Our results show that a minimum wage hike is then not a transfer from rich firms to poor workers, but from *poor* workers to *rich* workers.

These results are consistent with the reduced form results of Lang and Kahn (1998) and Neumark and Wascher (1995) who also show that the effects of minimum wage increases on the composition of the workforce may make raising the minimum wage unattractive. Lang and Kahn find that raising the minimum wage leads to a shift in the fast-food workforce from adults to teenagers,[5] while Neumark and Wascher's results suggest that minimum wage increases lead to a shift in the teenage workforce from those who have completed their schooling to those whose value of school is relatively high.

Estimation of structural search models have a rich history in labor economics.[6] While there is much variation in the types of search models estimated, all generally rely upon infinitely lived agents in a steady state equilibrium with reservation values determined in part by the continued value of search.[7]

This paper builds upon and is most related to Flinn (2002, 2006), which examine the

[5]Lang and Kahn also develop a theoretical model where minimum wage increases can lead to positive employment effects while still lowering welfare, though their model does not lend itself well to structural estimation.

[6]See Eckstein and van den Berg (2007) for a review

[7]Eckstein and Wolpin (1990) estimate a version of Albrecht and Axell (1984) where wage dispersion occurs because of heterogeneity in the reservation values of workers. Heterogeneity also exists in the productivity of firms with the marginal firm earning zero profits. van den Berg and Ridder (1998) estimate a version of the Burdett and Mortensen (1998) where now identical firms and workers can still lead to a heterogeneous distribution of wages. However, this heterogeneity comes at a cost as the number of firms and workers is exogenous and hence all firms earn positive profits.

welfare implications of minimum wage increases in a search model where firms and workers split a match-specific output. The main conclusions from Flinn (2006) is that it is possible for a minimum wage increase to be welfare-improving, and that modeling assumptions about the endogeneity of contact rates is crucial to the estimation of a welfare maximizing minimum wage. The equivalent of contact rates in our paper is the probability of a worker and a firm matching, and we explicitly allow this probability to be endogenous. Our model sacrifices the dynamics present in Flinn (2006) in treating search as a one-shot game. However, by making this sacrifice, we are able to develop a richer specification of labor supply as well as being able to consider equilibria across states and time. This in turn allows us to examine how minimum wages change the composition of the teenage workforce.

The rest of the paper proceeds as follows. Section 2 shows the classical model and how it does and does not relate to the matching model. Section 3 develops the two-sided search model, with welfare and employment analysis in section 4. Section 5 describes the data. The translation from the theoretical model to what is estimated is done in section 6. Section 7 presents the estimation results. Section 8 performs the policy simulations and Section 9 concludes.

2 The Classical Model

The classical analysis of the effects of a minimum wage can be found in most introductory economics textbooks. However, by first examining the classical model it is possible to see why our model is able to generate positive employment effects from an increase in the minimum wage while the classical model is not. Further the welfare implications of our model will turn out to be very similar to those of the classical model.

Figure 1 shows the implications of an increase in the minimum wage in the classical model. Employment here falls from Q^* to $\underline{Q}$. Note that the employment level only depends upon labor demand. How elastic, or inelastic, labor supply may be has no effect on the employment level. This is the primary difference between the classical model and matching models. Matching models rely on a 'matching function' which takes the number of searching workers and the number of searching firms and produces an employment level. Assuming one vacancy per firm, the matching function in the classical model is the minimum of the number of searching firms and the number of searching workers. The minimum must be the number of searching firms when there is a binding minimum wage. However, other matching functions that depend upon both the number of searching firms and the number of searching workers can produce increases in the employment level because the increased labor supply may more than compensate for the decreased labor demand.

The classical model requires an additional assumption as to how jobs are assigned because there is an excess supply of workers. In Figure 1 we have assumed that the probability of employment is the same across searching workers. This probability of finding a minimum wage job would be given by $\underline{Q}/\overline{Q}$ where $\overline{Q}$ is the number of individuals interested in working at the minimum wage. The area between the labor supply curve and the curve that kinks at $\overline{Q}$ gives the expected output over the reservation wage of the workers. Note that the expected output is smaller with the minimum wage increase for all workers below Q_c. These are the workers who were most interested in being employed and would be willing to trade a lower wage for a higher probability of employment.

The matching model described below has very similar welfare implications to the classical model. If there are losers because of a minimum wage increase, it will be those individuals who were most interested in being employed. Winners are then those individuals who would either not be interested or only marginally interested in being employed at the market clearing wage.

3 The Matching Model

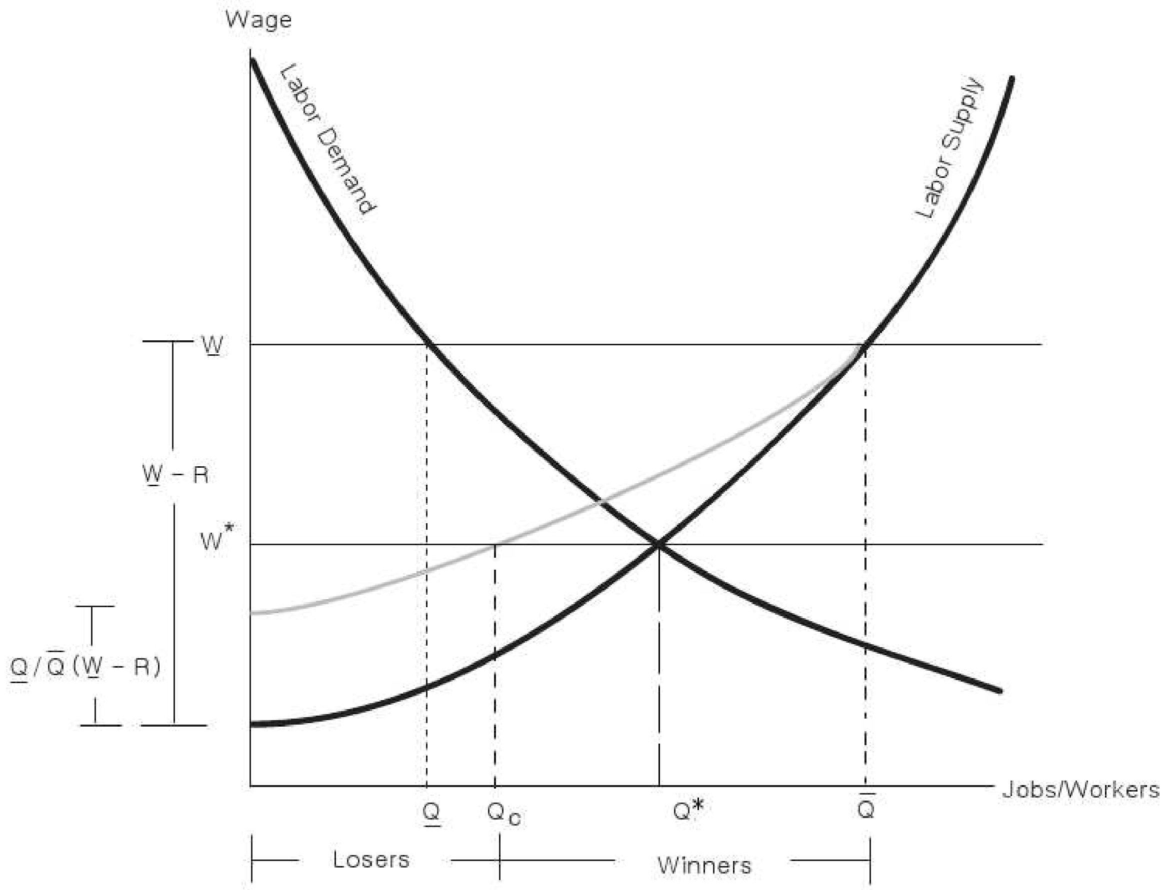
In this section we present a two-sided search model designed to highlight the effects of a minimum wage increase in the low wage market. All proofs are in the appendix. The model has four components:

1. The decisions by individuals regarding whether to search given their expectations regarding labor market outcomes and their value of leisure.

2. The decisions by firms to search such that, in equilibrium, a zero expected profit decision is satisfied.

3. The process by which workers and firms are paired.

Figure 1: Classical Employment Losses From a Minimum Wage Increase



4. The process governing wages.

each of which is described below.

3.1 Labor Force Participation

There are $\overline{N}$ individuals available to search and individuals live for one period.[8] Individuals are differentiated in their reservation values for not working. The ith individual has reservation value R_i, where R_i is drawn from the cumulative distribution function $F(R)$ and has support $[0, \infty)$. This reservation value can be leisure or any outside option for the individual. For instance, we may assume that R_i is the value of schooling for teenagers, with the treatment effect of education varying across the population.

Denote p as the probability that a worker matches with a firm conditional on searching, where p is the same for all individuals in a market.[9] Note that even if an individual does match with a firm, the match may be rejected by either the firm or the worker which will be discussed later in the paper. Denote K_i as the search cost for individuals that is paid whether the searching worker matches with a firm or not. K_i is drawn from the cumulative distribution function $H(K)$ and has support $[\underline{K}, \infty)$.

Individuals are risk neutral with the value of searching (not searching) for individual i denoted by V_{Si} (V_{Ni}). The payoff of matching with a firm is the wage, W, if the wage is above the individual's reservation value. If the wage is below the individual's reservation value, the match will be rejected and the payoff is the reservation value. There is uncertainty with regard to the wage which will be explained later in the paper. V_{Si} and V_{Ni} are then given by:

$$V_{Si} = pE\max\{W, R_i\} + (1 - p_i)R_i - K_i \quad (1)$$

$$V_{Ni} = R_i \quad (2)$$

Differencing the value of not searching from the value of searching yields the net expected value of searching, V_i, given by:

$$V_i = pE\max\{W - R_i, 0\} - K_i, \quad (3)$$

[8]For other theoretical models of job search with short time horizons see Pissarides (1992), Arcidiacono (2003), and Ahn and Arcidiacono (2004).

[9]Throughout this section market subscripts are suppressed. In the empirical section markets will be defined at the state, year, quarter, and age level.

The number of searching workers, N, is then endogenous with individuals searching when $V_i > 0$.

Note that search costs and reservation values affect the decision to search through different channels. See Figure 2 for an illustration. Consider two individuals, one with a high reservation value and a low search cost and another with a low reservation value and a high search cost. It is possible to find combinations of wages and probabilities of matching such that the first individual searches and the other does not. But it is also possible to find wage-probability combinations such that the second individual searches and the first does not. This case will occur at a higher probability of matching and a lower expected wage: individuals with relatively high search costs and low reservation wages are willing to take lower wages for higher probabilities of matching. The distinction between R and K is important, as we show that teens from disadvantaged backgrounds are more likely to have higher search costs and lower reservation wages compared to their more privileged counterparts in the empirical section. That is, teens from disadvantaged backgrounds care relatively more about being employed than the wage relative to their advantaged counterparts.

3.2 Firms

The number of firms, J, is endogenous. All firms within a market are identical and therefore have identical probabilities of matching with a worker, q. Production from a match is given by the random variable Y, which represents output and firms pay a search cost, C_1. Upon matching, the firm may pay an additional cost, C_2.[10]

We assume that the output of a match is given by:

$$Y_{ij} = Y^* \exp(\epsilon_{ij}) \tag{4}$$

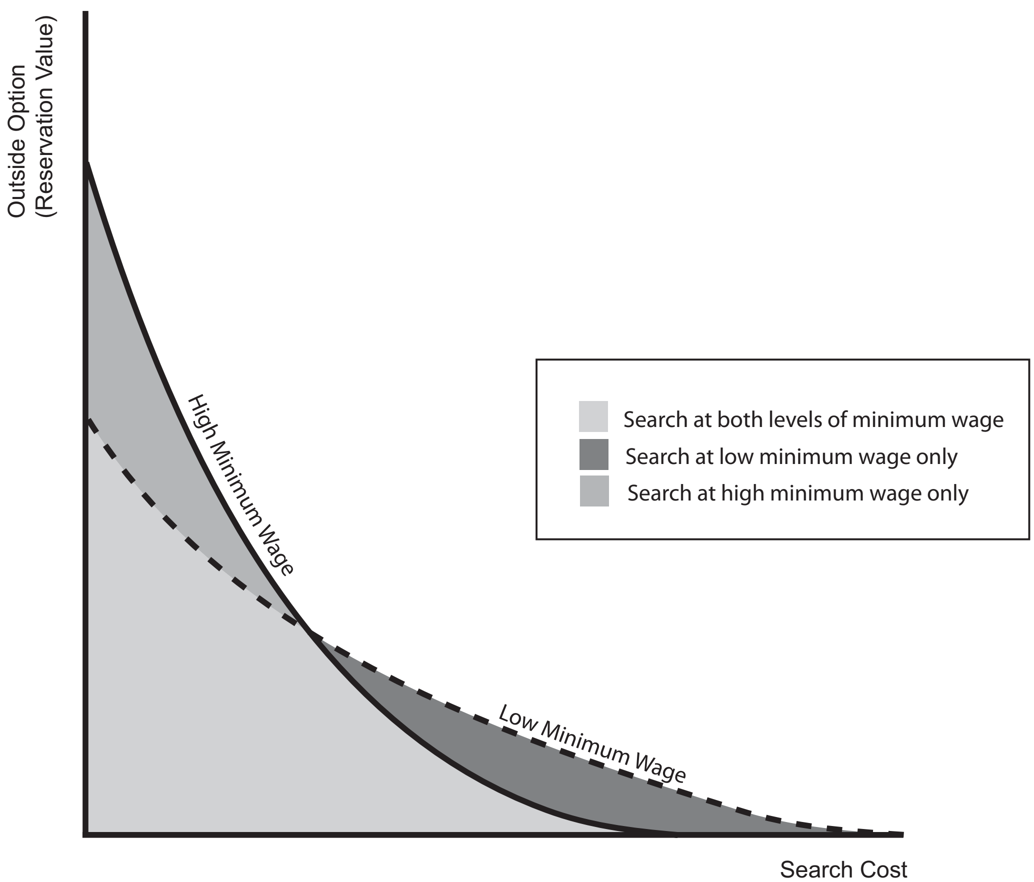
where Y^* is the median match value. ϵ_{ij} is then a match-specific component with zero median, is drawn from the cumulative distribution function $G(\epsilon)$, and has support $(-\infty, \infty)$.

Firms enter until all firms have zero expected profits. Expected profits are then given by:

$$qE(\max\{Y_{ij} - W_{ij}, 0\} - C_2) - C_1 = 0. \tag{5}$$

[10] C_2 turns out to be a negative in most cases. This parameter may be considered as a partial recoupment of the search cost upon matching. We introduce it here because it substantially improves the fit of the estimated model.

Figure 2: Effect of Reservation Values and Search Costs on Search Behavior



as firms will reject matches where $Y_{ij} < W_{ij}$.

3.3 Matching

With the search decisions for workers and firms defined above, we now describe how workers are allocated to firms. Workers and firms are matched using a Cobb-Douglas matching function with the restriction that the number of matches can be no greater than either the number of searching workers or the number of searching firms. Although many matching functions allow for positive employment effects from an increase in the minimum wage, we use a Cobb-Douglas matching function as in Pissarides (1992) to illustrate the result because of its prevalence in the literature.[11] We also use the Cobb-Douglas function in the empirical model.

The number of matches is then given by:

$$x = \min(AJ^{\alpha}N^{1-\alpha}, J, N), \tag{6}$$

where $\alpha \in (0, 1)$ and A is a normalizing constant. All workers and firms within a market have the same probability of finding a match implying that $p = \frac{x}{N}$ and $q = \frac{x}{J}$.

3.4 Wages

To close the model, we now specify the wage-generating process. Following Flinn (2006), wages conditional on matching following a bargaining process where the bargaining process produces a spike at the minimum wage. Matched pairs split Y_{ij} according to a Rubinstein bargaining game where the discount factors may vary for the firm and the worker. A successful match must pay at least the minimum wage, $\underline{W}$.

Building on Binmore, Shaked, and Sutton (1989) and Binmore, Rubinstein, and Wolinsky (1986), we show in the appendix that, under certain assumptions, there is a unique subgame perfect equilibrium of the bargaining game for all matches where $Y_{ij} \geq \max\{\underline{W}, R_i\}$.[12] The unique subgame perfect equilibrium outcomes yields the following expression for wages:

$$W_{ij} = \max\{\beta Y_{ij}, R_i, \underline{W}\} \tag{7}$$

[11]Indeed, most of the empirical literature is in agreement that there is a stable aggregate matching function of the Cobb-Douglas form and constant returns to scale in unemployment and job vacancies. See Petrongolo and Pissarides (2001) for a review.

[12]If $Y_{ij} < \max\{\underline{W}, R_i\}$ then the match will be rejected.

β, $\beta \in (0,1)$, may be interpreted as the worker's bargaining strength and represents the difference between the discount factors of the firm and worker. Matches yielding $Y_{ij} < \max\{\underline{W}, R_i\}$ are unsuccessful. The splitting of the output in this manner can generate the spike observed at the minimum wage in the data. All successful matches where the worker's share of the revenue would normally be below the minimum wage will earn the same wage even if their match-specific components differ. Note that the reservation value does not affect the match revenue division unless the reservation value is higher than both the minimum wage and β times the revenue of the match.

Proposition 1 establishes that an equilibrium for this model exists.

Proposition 1 *Given equations (3) - (7), $\{F(R)$, $G(\epsilon)$,$H(K)$, C_1, C_2, α, β, Y^*, $\underline{W}$, and $\overline{N}\}$, there exists an equilibrium in N and J.*

4 Implications of the Model

The model described above has a number of implications for a minimum wage increase. In this section we describe how a minimum wage increase affects the probability of matching and conditions under which a minimum wage increase positively affects the employment level. We further show conditions under which a minimum wage increases welfare for all searching workers and show which workers are hurt when these conditions are not met.

To keep the implications of the model simple, we make an assumption on the primitives to ensure that, in the presence of a minimum wage, any worker who searches will accept any match. That is, any worker who searches is willing to work for the minimum wage. As shown above, the wage generating process under Rubinstein bargaining yields: $W_{ij} = \max\{\beta Y_{ij}, \underline{W}, R_i\}$. Under certain conditions on the primitives, all workers who choose to search for a job will have reservation values less than the minimum wage implying that, $W_{ij} = \max\{\beta Y_{ij}, \underline{W}\}$. Namely, suppose the following condition holds:

$$\textbf{NR} \qquad \text{For all } R_i > \underline{W}, \qquad Pr(Y_{ij} \geq R_i) \left(E\left[\max\left\{\beta Y_{ij}, R_i\right\} | Y_{ij} \geq R_i\right] - R_i \right) - \underline{K} < 0$$

then we are able to establish the following lemma:

Lemma 1 *A worker who finds it optimal to search will accept any match.*

The expression on the left hand side of the inequality in NR represents the value of searching given the lowest search cost without the probability of matching. With the probability of matching ranging between zero and one, only workers who have reservation values below the minimum wage will search when NR holds.[13] This is effectively an assumption on the distribution of match revenues, Y, relative to the lowest search cost, $\underline{K}$. When the spread of possible revenues is small relative to the search costs, those with reservation values above the minimum wage do not find it worth the risk to search on the off-chance that, should they match, the draw on the match revenue will be at least as high as their reservation value.

We next establish that raising the minimum wage will always lower the probability of an individual being employed conditional on searching, even if the overall employment level increases.

Proposition 2 $\frac{dp}{d\underline{W}} < 0$, *regardless of the signs of* $\frac{dN}{d\underline{W}}, \frac{dJ}{d\underline{W}}$, *and* $\frac{dx}{d\underline{W}}$.

The intuition comes from examining the expected zero profit condition for firms which can be written as:

$$A\left(\frac{N}{J}\right)^{1-\alpha} E\max\left(Y_{ij}, W_{ij} - C_2\right) - C_1 = 0 \tag{8}$$

where $q = A(N/J)^{1-\alpha}$. In particular, increasing the minimum wage lowers profits conditional on matching, $E\max\left(Y_{ij}, W_{ij} - C_2\right)$. If the costs of the firm increase, the probability of finding a match for the firm must increase in order for the expected zero profit condition to hold. Since an increase in the match probability of the firm means an increase in the ratio of workers to firms, N/J, and the match probabilities of workers fall with an increase of N/J, we have the result. This holds whether or not the employment *level* has increased.

Although the probability of finding a job always falls with an increase in the minimum wage, the effect on the employment level is ambiguous. The next proposition shows that it is possible to simultaneously have an increase in the minimum wage, a decline in the probability of employment, and an increase in the level of employment. Proposition 3 outlines conditions on the labor demand and supply elasticities under which positive employment effects due to a minimum wage hike are possible.

[13]When $R > \underline{W}$, individuals search when $pPr(Y \geq R)[E(W|Y \geq R) - R] - \underline{K} > 0$ The value of p that leads to the highest value of the expression on the left hand side of the inequality is one. Setting $p = 1$ yields the left hand side expression in NR.

Proposition 3 $\frac{dx}{d\underline{W}} \geq 0$ *if* $\alpha\varepsilon_{LD} + (1-\alpha)\varepsilon_{LS} \geq 0$*, where* ε_{LD} *is the elasticity of labor demand and* ε_{LS} *is the elasticity of labor supply.*

Proposition 3 explicitly demonstrates that the direction of growth of employment is jointly dependent on the elasticities of labor supply and demand. Furthermore, since both elasticities depend on J and N, which are endogenous, as well as $\underline{W}$, the model can exhibit positive or negative employment effects. This is because an increase in $\underline{W}$ will generally pull J and N in opposite directions, which then leads to labor demand and supply elasticities being pulled in opposite directions. This dual effect on the employment level helps to explain not only why different studies have found positive and negative employment effects, but also why the magnitude of the effects has been so small. Since J and N are moving in opposite directions, α, or the measure of sensitivity of the matching function to a relative increase in J/N, helps to determine which effect is larger. As $\underline{W}$ increases, ε_{LD} and ε_{LS} continue to offset each other, which translates to a small movement in the employment level.

Finally, we show the conditions under which all workers experience a welfare increase from a minimum wage hike. Denote $E_1(W)$ and p_1 as the expected wage and probability of finding a match before the minimum wage increase. Denote $E_2(W)$ and p_2 as the corresponding values after the minimum wage increase. With reservation values for workers bounded below by zero, all workers are made better off in expectation by a minimum wage increase if:

$$p_1 E_1(W) < p_2 E_2(W).$$

Establishing when this holds is difficult when matches are rejected by the firm. That is, when $Y < \underline{W}$. However, if we bound the lowest value of productivity at some level $\underline{Y}$ such that $\underline{Y} > \underline{W}$, then no matches will be rejected by the firm. With this assumption, we have the following result:

Proposition 4 *When* $\underline{Y} > \underline{W}$*, all workers benefit from a marginal increase in the minimum wage if and only if* $1 - \alpha > \frac{E(W)}{E(Y) - C_2}$

If firms do reject matches because some match values are below the minimum wage, then the proposition only provides a necessary condition for increasing welfare. This is because raising the minimum wage would lead to more matches being rejected by the firm which is not taken into account in proposition 4.

If the conditions for proposition 4 are not met, then some workers are made worse off by the increase in the minimum wage. In particular, as discussed in section 3 and illustrated in Figure 2, it is those workers who most want the minimum wage jobs, those with the lowest reservation values, who are hurt by the increase. A minimum wage policy that measures its success by the employment level then misses the important distributional distortion caused by the increase. By increasing the minimum wage, more workers with higher reservation values enter the labor market. While these new workers are undoubtedly experiencing a welfare increase, workers who were already searching may be worse off, because these are the workers who were most willing to accept a lower paying job for a higher probability of employment.[14]

5 Data

We now describe the data used in the empirical analysis. We use a twelve year band of the basic monthly outgoing rotation survey files of the Current Population Survey (CPS) from 1989 to 2000. These twelve years cover four federal minimum wage changes as well as sixteen states[15] which changed their state minimum wage to out-pace the federal wage. The range of observed minimum wages in the twelve years run from \$3.35 to \$6.50. Our analysis covers white males

[14] An interesting extension to the model suggested by a referee involves introducing search intensity into the model. In the theory, p and K should now be functions of individual search intensity (s_i) and search intensity of all others (s_{-i}). Equation (1) changes to:

$$V_i = \max_{s_i} p(s_i, s_{-i})[E(W|W > R_i) - R_i] - K(s_i)$$

with $\frac{\partial p}{\partial s} \geq 0, \frac{\partial^2 p}{\partial s^2} < 0$, $\frac{\partial p}{\partial s-} \leq 0$, $\frac{\partial^2 p}{\partial s-^2} < 0$, $\frac{\partial K}{\partial s} > 0$, and $\frac{\partial^2 K}{\partial s^2} > 0$. The firm zero profit condition would change to:

$$q(s_{-i})E(\max\{Y - W, 0\} - C_2) - C_1 = 0$$

with $\frac{\partial q}{\partial s-} \geq 0, \frac{\partial^2 q}{\partial s-^2} < 0$. This would imply, given a distribution of Y, R, and some values of β, $\underline{W}$, s_{-i}, an optimal level of individual search intensity (s_i^*).We would expect $\frac{\partial s^*}{\partial \underline{W}} > 0$ and $\frac{\partial s-}{\partial \underline{W}} > 0$.

We would still expect $\frac{dN}{d\underline{W}} > 0$, $\frac{dJ}{d\underline{W}} < 0$, and all propositions would remain unchanged qualitatively. However, the search intensity would mitigate the incentive of new entrants into the labor market, as everyone already in the market would increase their search efforts in response to the minimum wage hike.

[15] There are actually eighteen states that paced ahead of the federal minimum wage, but we exclude Alaska and Hawaii from analysis as well as the District of Columbia.

who are between sixteen and nineteen years of age inclusive[16], using data from non-summer months.[17] From the CPS, we collect hourly wage, whether the individual is searching for work or not, whether the searching worker is employed or not, as well as a number of demographic variables that may affect an individual's reservation wage and search costs.

Table 1 presents descriptive statistics for three groups of teenagers: the population, job searchers, and those who are employed. Since search is one-shot game, job searchers refer to the sum of those who are unemployed and those who are currently working. Observations with employed individuals earning less than the minimum wage minus twenty-five cents were dropped, as well as individuals who reported earning more than $15 per hour.[18] As in Flinn (2006), we keep those earning less than a minimum wage but within twenty-five cents[19] because of measurement error in reported wages. These observations are treated as earning exactly at the minimum wage. One key variable to the analysis is the prime age male unemployment rate. This unemployment rate is calculated at the state-quarter level using CPS data for all males aged 30 to 39. This variable is assumed to affect job search of teenagers only through the expected wage and the probability of employment, having no effect on search costs or reservation values.

Table 1 shows that those who search are more likely to be older and out of school. The relationship between search and the other demographic variables, namely characteristics of the parents for those teenagers who are in school, shows no clear patterns. There is, however, a negative relationship between parental education and the probability of search once we condition on age. Parental characteristics are only calculated for those who are in school and nineteen year olds who are in school are more likely to have highly educated parents. This can be seen in Table 2 which breaks out the descriptive statistics by age.

The descriptive statistics by age show that older teenagers are more likely to participate in the labor market, be out of school, less likely to have their wages bind at the minimum, and

[16]We ran several specifications, restricting the data to those whose primary residence is with their parent(s) and/or who attended school in the last week, with no qualitative differences.

[17]We exclude June, July, and August.

[18]Less that 0.5% of employed workers were cut for making too much, while less than 6% of employed workers were cut for making too little.

[19]Within twenty-five cents refers to the nominal wage. After the sample selection, all wages and incomes are adjusted to 2000 dollars.

Table 1: Descriptive Statistics

Variable	Mean	Mean|Search	Mean|Employed
Age	17.48	17.76	17.82
	(1.12)	(1.08)	(1.06)
In School	0.750	0.614	0.613
Head Unemployed ‡	0.060	0.054	0.050
Head Other † ‡	0.130	0.104	0.097
Head Education HS or less ‡	0.497	0.493	0.484
Some College ‡	0.239	0.263	0.268
College Graduate ‡	0.158	0.154	0.156
Post-College ‡	0.106	0.091	0.091
Single Parent ‡	0.297	0.316	0.307
Prime Age Male Unemployment Rate	0.036	0.035	0.035
	(0.017)	(0.017)	(0.016)
Pr(Search)	0.552		
Pr(Employed|Search)		0.772	
Pr(Minimum Wage Binds|Employed)			0.167
E(Wage|Employed)			6.72
			(1.74)
Observations	83478	46085	35589

†Head Other is defined as a household head who cannot be categorized as employed or unemployed.

‡Calculated only for teenagers identified as living at home and attending school.

Table 2: Descriptive Statistics by Age

Variable	Age=16	Age=17	Age=18	Age=19
In School	0.953	0.905	0.675	0.455
Head Unemployed ‡	0.064	0.061	0.055	0.055
Head Other † ‡	0.136	0.134	0.123	0.117
Head Education HS or less ‡	0.521	0.509	0.480	0.445
Some College ‡	0.239	0.243	0.239	0.231
College Graduate ‡	0.145	0.150	0.169	0.190
Post-College ‡	0.096	0.098	0.113	0.134
Single Parent ‡	0.261	0.273	0.309	0.356
Prime Age Male Unemployment Rate	0.036	0.036	0.036	0.036
	(0.017)	(0.017)	(0.017)	(0.017)
Pr(Search)	0.361	0.503	0.624	0.731
Pr(Employed|Search)	0.673	0.758	0.795	0.815
Pr(Minimum Wage Binds|Employed)	0.275	0.230	0.144	0.099
E(Wage|Employed)	5.89	6.08	6.77	7.44
	(1.03)	(1.15)	(1.67)	(2.04)
Observations	21475	21002	20544	20457

†Head Other is defined as a household head who cannot be categorized as employed or unemployed.

‡Calculated only for teenagers identified as living at home and attending school.

have higher expected earnings than their younger counterparts. Because parental characteristics are calculated only for those identified as attending school, those who have an unemployed household head or whose head has low education are more likely to be younger. This is not true for single parent families as divorce is correlated with age of the child.

Table 3: State Minimum Wage from 1989 to 2000.

State	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
California	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.75	5.15	$5.75^{\diamond}$	5.75	5.75
Connecticut	4.25	4.25	$4.27^{\dagger}$	4.27	4.27	4.27	4.27	4.77°	5.18^{*}	5.18	$5.65^{\ddagger}$	$6.15^{\ddagger}$
Delaware	3.35	3.80	4.25	4.25	4.25	4.25	4.25	4.75	5.15	5.15	$5.65^{\S}$	5.15°
Iowa	3.35	$3.85^{\ddagger}$	$4.25^{\ddagger}$	$4.65^{\ddagger}$	4.65	4.65	4.65	4.75	5.15	5.15	5.15	5.15
Massachusetts	3.75	3.75	3.75	4.25	4.25	4.25	4.25	$4.75^{\ddagger}$	$5.25^{\ddagger}$	5.25	5.25	$6.00^{\ddagger}$
Maine	$3.75^{\ddagger}$	$3.85^{\ddagger}$	4.25	4.25	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
Minnesota	$3.85^{\ddagger}$	$3.95^{\ddagger}$	$4.25^{\ddagger}$	4.25	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
New Hampshire	$3.65^{\ddagger}$	3.80	$3.85^{\ddagger}$	$4.25^{\dagger}$	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
New Jersey	3.35	3.80	3.80	$5.05^{\dagger}$	5.05	5.05	5.05	5.05	5.15	5.15	5.15	5.15
New York	3.35	3.80	3.80	$4.25^{\dagger}$	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
Oregon	3.85^{*}	$4.25^{\ddagger}$	$4.75^{\ddagger}$	4.75	4.75	4.75	4.75	4.75	$5.50^{\ddagger}$	$6.00^{\ddagger}$	$6.50^{\ddagger}$	6.50
Pennsylvania	$3.70^{\ddagger}$	3.80	4.25	4.25	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
Rhode Island	$4.25^{\bigtriangledown}$	4.25	$4.45^{\dagger}$	4.45	4.45	4.45	4.45	4.75	5.15	5.15	$5.65^{\triangle}$	6.15^{*}
Vermont	$3.75^{\triangle}$	$3.85^{\triangle}$	4.25	4.25	4.25	$4.50^{\ddagger}$	4.50	4.75	5.25°	5.25	5.75°	5.75
Washington	$3.85^{\ddagger}$	$4.25^{\ddagger}$	4.25	4.25	4.25	$4.90^{\ddagger}$	4.90	4.90	5.15	5.15	$5.70^{\ddagger}$	$6.50^{\ddagger}$
Wisconsin	$3.65^{\triangle}$	3.80	3.80	4.25	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
Other States	3.35	$3.80^{\dagger}$	$4.25^{\dagger}$	4.25	4.25	4.25	4.25	4.75°	5.15^{*}	5.15	5.15	5.15

‡ Minimum wage change on 1/1 or 1/2. $\diamond$ Minimum wage change on 3/1. $\dagger$ Minimum wage change on 4/1. § Minimum wage change on 5/1.

$^{\triangle}$ Minimum wage change on 7/1 or 7/2. $\bigtriangledown$ Minimum wage change on 8/1. $*$ Minimum wage change on 9/1. $\circ$ Minimum wage change on 10/1.

We use the Monthly Labor Review to collect minimum wage at the state/month level. That is, from 1989 to 2000, we observe the minimum wage in each state, each month. Table 3 presents the minimum wage in each state, each month within the range of the collected CPS data. These minimum wages are nominal values. In the analysis, the wages and incomes are inflated to 2000 dollars.

6 Parameterizing the Model

In this section we show how to estimate the structural model. Estimation has three components. First, for those individuals who successfully match we observe wages. Second, we need to estimate the parameters of the zero profit condition. Although we do not observe the probability of a firm finding a match, we are able to rewrite the zero profit condition as a function of the individual's probability of finding a match. Finally, we observe decisions by individuals as to whether to search. We can use these decisions to estimate the supply side parameters. In practice, we estimate the model in two stages, first estimating the wage parameters and the zero profit parameters and then estimating the worker search parameters in a second stage.

6.1 Parameterizing Wages

Before specifying the distribution of wages, we first must specify the source of the wages: the output of the match, Y_{ij}. We assume that Y_{ij} is given by:

$$Y_{ij} = \exp(X_i\theta) \cdot \exp(\epsilon_{ij}) \tag{9}$$

where X_i are characteristics of individual i's market[20] and θ is the set of parameters to be estimated. Because the left tail of the wage distribution is so important to this analysis, we do not make the standard assumption of log-normality on the ϵ's.[21] Rather, we mix over two log-normal distributions allowing both the means and the variances of these distributions to vary. The probability of a draw coming from the rth distribution is then given by π_r where

[20] As discussed in the data section, a market is defined at the age, state, quarter, and year level.

[21] See Koning, van den Berg, and Ridder (2000) for an alternative approach to allowing for a flexible error distribution.

the rth distribution is distributed $\mathrm{N}(\mu_r, \sigma_r)$. In addition, we allow the variance terms to differ by age, defining σ^{2*} as $\sigma^{2*}_{rk} = \sigma^2_r \cdot A_k$, where A_k is an age-specific constant.[22]

We assume that condition NR holds implying that only firms reject matches in the presence of a binding minimum wage. That is, if a teenager finds it optimal search, he is willing to accept a minimum wage job.[23] With this condition, the wage generating process is given by: $W_{ij} = \min\{\beta Y_{ij}, \underline{W}\}$, implying that when the minimum wage does not bind log wages are given by:

$$\ln(W_{ij}) = X_i\theta + \ln(\beta) + \epsilon_{ij} \tag{10}$$

In the presence of a minimum wage the wage distribution is then distributed truncated log-normal with censoring at the minimum wage. The truncation occurs when the match value is so low that the firm rejects the match. This occurs whenever $\underline{W} > Y_{ij}$. There are then three relevant regions for the quality of the match:[24]

$$\begin{aligned} \beta Y_{ij} > \underline{W} \quad &\Rightarrow \quad \{W_{ij} = \beta Y_{ij}\} \\ Y_{ij} \geq \underline{W} > \beta Y_{ij} \quad &\Rightarrow \quad \{W_{ij} = \underline{W}\} \\ \underline{W} > Y_{ij} \quad &\Rightarrow \quad \{\text{No match}\} \end{aligned}$$

We then observe successful matches for those who are employed either at or above the minimum wage.

Let N_{11} and N_{12} indicate the number of individuals who have wage observations above and at the minimum wage respectively. Defining Φ and ϕ as the cdfs and pdfs of the standard

[22]We set $A_{16} = 1$, and estimate A_{17}, A_{18}, and A_{19}.

[23]We estimated reduced form wage equations to see if the factors that influenced the reservations values (for example, parental education) also influenced the wage. We found no evidence that higher (lower) reservation values were associated with higher (lower) wages. We also tested whether minimum wages had spillover effects by including a dummy variable for whether the minimum wage had been increased in the month prior. If spillovers exist, then we would expect a positive effect on log wages outside of the spike at the minimum wage. We found no evidence of spillover effects.

[24]Note this specification is quite similar to Meyer and Wise (1983a, 1983b) except that it results from the structural model and allows for a more flexible specification of the error distribution.

normal distribution, the likelihood for these observations then follows:

$$\mathcal{L}_1 = \left(\prod_{i=1}^{N_{11}} \frac{\sum_{r=1}^{2} \pi_r \phi \left(\frac{W_{ij} - X_i\theta - \ln(\beta) - \mu_r}{\sigma^*_{rk}} \right) / \sigma^*_{rk}}{\sum_{r=1}^{2} \pi_r \left(1 - \Phi \left(\frac{\ln(\underline{W}) - X_i\theta - \mu_r}{\sigma^*_{rk}} \right) \right)} \right) \times$$
$$\left(\prod_{i=1}^{N_{12}} \frac{\sum_{r=1}^{2} \pi_r \left(\Phi \left(\frac{\ln(\underline{W}) - X_i\theta - \ln(\beta) - \mu_r}{\sigma^*_{rk}} \right) - \Phi \left(\frac{\ln(\underline{W}) - X_i\theta - \mu_r}{\sigma^*_{rk}} \right) \right)}{\sum_{r=1}^{2} \pi_r \left(1 - \Phi \left(\frac{\ln(\underline{W}) - X_i\theta - \mu_r}{\sigma^*_{rk}} \right) \right)} \right)$$

The likelihood above is conditional on the firm not rejecting the match. The denominator in both expressions is one minus the probability that the revenue of the match is so low that the firm would rather not match than pay the minimum wage. The first expression then gives the conditional likelihood of wages above the minimum wage while the second expression is the conditional likelihood of receiving exactly the minimum wage. Note here that σ^*_{rk} is age-specific for the observed teenager.

6.2 Parameterizing Firms

Although we have no information on the firm, we can infer the parameters of the profit function by rewriting the zero profit condition as a function of the individual's probability of finding a match. To see this, note that the probability of finding a match for firms and workers is given by:

$$q = A \left(\frac{N}{J} \right)^{1-\alpha} \qquad p = A \left(\frac{J}{N} \right)^{\alpha}$$

implying that we can write q as:

$$q = A^{\frac{1}{\alpha}} p^{\frac{\alpha - 1}{\alpha}}$$

Substituting for q as a function of p in the zero expected profit condition yields:

$$A^{\frac{1}{\alpha}} p^{\frac{\alpha - 1}{\alpha}} E(\max\{Y - W, 0\} - C_2) - C_1 = 0$$

Solving for p yields:

$$p = \delta E(\max\{Y - W, 0\} - C_2)^{\frac{\alpha}{1-\alpha}} \tag{11}$$

where:

$$\delta = C_1^{\frac{-\alpha}{1-\alpha}} A^{\frac{-1}{1-\alpha}}$$

This zero expected profit condition is satisfied for every economy. That is, zero expected profits hold by age, state, quarter and year.

Given the assumed log-normal distribution of Y and the parameters of the wage-generating process, we can calculate $E(\max\{Y-W,0\})$, the expected surplus from matching. This surplus can be broken down into three parts: 1) when the match value is high enough such that the minimum wage does not bind, $\tilde{Y}_1$, 2) when the match value is such that the minimum wage binds, $\tilde{Y}_2$, and 3) when the match value is so low that the firm rejects the match. The last of these parts, $\tilde{Y}_3$ yields an expected revenue of zero. Since we estimate the wage distribution assuming all matches are successful, we have a natural test of this assumption from the zero expected profit function. In particular, we test whether the bargaining parameter is low enough such that this region has no observations. $\tilde{Y}_1$ and $\tilde{Y}_2$ are given by:

$$
\begin{aligned}
\tilde{Y}_1 &= \sum_{r=1}^{2} \pi_r \left[\exp(X_i\theta + \ln(1-\beta) + \mu_r + \sigma_{rk}^{*2}/2)\Phi\left(\frac{\sigma_{rk}^{*2} - \ln(\underline{W}) + X_i\theta + \ln(\beta) + \mu_r}{\sigma_{rk}^{*}}\right)\right] \\
\tilde{Y}_2 &= \sum_{r=1}^{2} \pi_r \left[\exp(X_i\theta + \mu_r + {\sigma_{rk}^{*}}^2/2) B_r \right. \\
&\quad \left. - \left(\Phi\left(\frac{\ln(\underline{W}) - X_i\theta - \mu_r - \ln(\beta)}{\sigma_{rk}^{*}}\right) - \Phi\left(\frac{\ln(\underline{W}) - X_i\theta - \mu_r}{\sigma_{rk}^{*}}\right)\right)\underline{W}\right]
\end{aligned}
$$

where B_r is given by:

$$
B_r = \left(\Phi\left(\frac{\sigma_{rk}^{*2} - \ln(\underline{W}) + X_i\theta + \mu_r}{\sigma_{rk}^{*}}\right) - \Phi\left(\frac{\sigma_{rk}^{*2} - \ln(\underline{W}) + X_i\theta + \ln(\beta) + \mu_r}{\sigma_{rk}^{*}}\right)\right)
$$

We then define $\tilde{Y}$ such that:

$$
\tilde{Y} = E(\max\{Y-W,0\}) = \tilde{Y}_1 + \tilde{Y}_2 \tag{12}
$$

implying that the probability of a working matching with a firm can be written as:

$$
p = \delta(\tilde{Y} - C_2)^{\frac{\alpha}{1-\alpha}} \tag{13}
$$

Here we can see the advantage of the additional parameter, C_2. Namely, if α is 0.5, then δC_2 serves as an intercept term with the slope given by δ itself. The model fit is substantially improved by adding this term.

However, in the data we do not observe whether an individual is matched with a firm, p, but only observe p times the probability that the match is successful, $p\psi$, where ψ is given by:

$$
\psi = 1 - \Phi\left(\frac{\ln \underline{W} - X\theta}{\sigma_{rk}^{*}}\right)
$$

Positive search outcomes for workers are then Bernoulli draws from $p\psi$. The likelihood function is then given by:

$$\mathcal{L}_2 = \prod_{i=1}^{N_2} \left(\psi\delta(\tilde{Y}_i - C_2)^{\frac{\alpha}{1-\alpha}}\right)^{m_i=1} \left(1 - \psi\delta\tilde{Y}_i^{\frac{\alpha}{1-\alpha}}\right)^{m_i=0}$$

where N_2 is the number of searching workers and m_i indicates whether or not the ith worker was matched. We allow the δ's and C_2's to vary by state.

6.3 Parameterizing the Individual

We now turn to the decision by individuals as to whether or not to search. Recall that an individual searches if:

$$p\psi(E(W) - R_i) - K_i > 0$$

which we can re-write to:

$$p\psi \cdot E(W) - p\psi \cdot R_i - K_i > 0$$

Because $p\psi$ is multiplicatively attached to R_i, but not to K_i, even if the observed variables are common across R_i and K_i, we can separately identify their coefficients. With the estimates from the previous two stages it is possible to calculate expected wages and the probability of employment for each individual.

We parameterize R_i such that all workers have positive reservation values:

$$R_i = \exp(Z_{1i}\gamma_1 + \eta_i)$$

Z_i is then a vector of demographic characteristics which affect the individual's outside option, the γ_1's are the coefficients to be estimated, and η_i is the unobserved portion of the reservation value. Search costs follow a similar specification where:

$$K_i = \exp(Z_{2i}\gamma_2)$$

Family background characteristics are allowed to operate through both the search costs and the reservations values while we include state, year, and quarter fixed effects only in the reservation values.

Individuals who come from privileged backgrounds may have high reservation values, making search less likely. However, these same individuals may also have lower search costs. What separately identifies search costs from reservation values is how individuals react to the probability of finding a job. In particular, those with low search costs but high reservation values will be more willing to trade off higher expected wages conditional on matching for lower probabilities of employment. In contrast, those with high search costs but low reservation values prefer lower wages coupled with higher match probabilities.

We allow the unobserved portion of the reservation value to be drawn from two different logistic distributions, one for teens who are attending school ($l = 1$) and another for teens who are not attending ($l = 0$). Substituting in and solving for η_{il} shows that an individual will search when:

$$\eta_{il} < \ln\left(E(W) - \frac{\exp(Z_{2i}\gamma_2)}{p\psi}\right) - Z_{1i}\gamma_1$$

We assume that the η_l's are logistically distributed with mean zero and variance σ_l^2. Since we do not observe the η's, the likelihood function is given by:

$$\begin{aligned}\mathcal{L}_3 \quad = \quad & \prod_{i=1}^{N_3} \Lambda\left((1/\sigma_l)\ln\left(E(W) - \frac{\exp(Z_{2i}\gamma_2)}{p\psi}\right) - Z_{1i}\gamma_1^*\right)^{s_i=1} \times \\ & \left(1-\Lambda\right)\left((1/\sigma_l)\ln\left(E(W) - \frac{\exp(Z_{2i}\gamma_2)}{p\psi}\right) - Z_{1i}\gamma_1^*\right)\Bigg)^{s_i=0}\end{aligned}$$

where $\Lambda = \exp(\cdot)/(1+\exp(\cdot))$, N_3 is the total number of potential searchers, and s_i is an indicator for whether the ith individual chose to search. In the standard logit, all coefficients are relative to the variance term. Here we can actually estimate σ_l as there is no other natural interpretation for the coefficient on the expression inside the log. The γ^*'s are then the γ's divided by the variance scale parameter, σ_l.

7 Results

Having specified the estimation strategy, we now turn to the results. The first stage of the estimation involves estimating the parameters of the wage generating process and the parameters of the zero profit condition. These estimates of the wage generating process are given in Table 4. In addition to the reported parameters, we also included state, year, and quarter fixed effects. The coefficient on the prime age male unemployment rate is negative and significant.

Table 4: Parameters of the Wage Generating Process[†]

Variable	Coefficient	Std. Error
Prime Age Male Unemployment Rate	-0.422	0.088
Age=17	0.038	0.003
Age=18	0.132	0.003
Age=19	0.222	0.003
μ_1	1.611	0.010
σ_1	0.111	0.003
μ_2	1.767	0.015
σ_2	0.203	0.029
π_2	0.365	0.029
A_{17}	1.076	0.019
A_{18}	1.425	0.025
A_{19}	1.695	0.032

[†]Estimated jointly with the parameters of the zero condition given in Table 5. Estimation also included state, year, and quarter fixed effects.

This will be important for the analysis of searching as this is our exclusion restriction: the adult unemployment rate only affects search through the expected wage and the probability of finding a match. Mixing over two log-normals shows that higher log wages are associated with higher variances. Variances on unobserved match-specific component increase with age, suggesting that having a common variance term would under-predict the fraction of nineteen year olds at the minimum wage while over-predicting the fraction of sixteen year olds at the minimum.

Estimates of the zero profit condition parameters are given in Table 5. These are β, the bargaining parameter, α, which measures how sensitive the number of matches are relative to the number of searching firms, a conglomerate parameter, δ, which is a function of the search cost (C_1) and the efficiency of the matching function, and C_2, the recoupment cost.

Table 5: Estimates of the Firm's Zero Profit Condition†

	Coefficient	Std. Error
$C_1^{\frac{-\alpha}{1-\alpha}} A^{-1}$	0.049	0.013
C_2	-14.755	7.970
α	0.462	0.038
β	0.336	0.020

†Estimated on 46,080 white male teenagers who were either employed or looking for a job. Average values for $C_1^{\frac{-\alpha}{1-\alpha}} A^{-1}$ and C_2, which were calculated for each state, are presented.

Note that δ and C_2 are allowed to vary by state. The relative weight of firms to workers in determining the probability of matching, α, is estimated at close to 0.45— a result that is in line the macroeconomics literature (Peterongolo and Pissarides 2001). The estimated β is around 0.34, which is similar to the estimates from Flinn (2006).

With the estimates of the log wage regression and the parameters of the zero profit condition, we calculate the probability of matching and the expected wages conditional on matching. We then use these estimates to estimate the value of search with the results presented in Table 7. The last two numbers in the table give $1/\sigma$ for those who are in school and those who are out of school. These numbers are crucial in estimating the wage elasticity. If the numbers are small, participation is driven primarily by unobserved reservation values. High values, in contrast, mean that individuals are very responsive to conditions in the labor market. The parameter estimates imply a labor supply elasticity of 2.48 for those who are in school with a corresponding labor supply elasticity of 0.75 for those who are out of school. The large gap in labor supply elasticities is driven part by the fact that the base level of labor force participation is actually quite high for those who are not in school. Note that these calculations hold the probability of finding employment fixed, which will not be the case when we simulate the effects of a minimum wage increase.

Reservation values and search costs are also reported in Table 6. In virtually all cases, a characteristic that leads to a higher reservation value also leads to a lower search cost. This makes sense. Those who have access to technologies that might lower search costs (computers, contacts, etc.) also are likely to be provided with more income from their parents, leading to higher reservation values. Higher parental education and coming from a two-parent family is then associated with lower search costs and higher reservation values. More advantaged backgrounds are then associated with individuals who are more likely to be willing to trade a lower probability of finding a job for a higher wage conditional on employment.

8 Elasticities

With the estimates of the model in hand, we now see how the minimum wage affects the probability of search, the probability of obtaining employment conditional on search, and the unconditional probability of employment. The elasticities of these three variables with respect to increasing the minimum wage are given in the first three columns of Table 7.

The table shows that with a minimum wage increase the probability of searching increases. However, this is counteracted by a decrease in the probability of finding a job conditional on searching leading to an overall employment elasticity of -0.143. This overall employment elasticity is masking much large changes in labor supply and demand. Namely, the search elasticity with respect to the minimum wage is 0.195 while the match elasticity, how the probability of employment conditional on search changes with an increase in the minimum wage, is -0.327. Therefore, while there is a large decrease in the probability of employment conditional on searching, the overall employment elasticity is buoyed by the increase in the number of searching workers.

The changes in employment and search are not uniform across the population. The next two rows show that the search elasticities are much higher for those who are in school than those who are out of school. These differences in search elasticities then lead to overall employment elasticities that are twice as large for those out of school than those in school. Hence, there is a shift in the composition of employment away from those who are out of school and towards those who are in school.

Compositional effects are also important within the in school population. Namely, higher search elasticities are seen for those with two-parent families with highly educated parents. Indeed, those who have a household head with more than a college degree have such large

Table 6: Estimates of the Search Parameters†

	Variable	Reservation Values	Search Costs
In School			
	Household Head Unemployed	-0.026	0.125
		(0.086)	(0.062)
	Household Head Other	0.790	-0.112
		(0.087)	(0.074)
	Household Head Some College	0.177	-0.222
		(0.090)	(0.087)
	Household Head College	0.971	-1.143
		(0.121)	(0.415)
	Household Head Post-College	1.473	-4.672
		(0.109)	(3.145)
	Single Parent	-0.187	0.107
		(0.048)	(0.040)
	$1/\sigma$	3.957	
		(0.517)	
Out of School‡			
	$1/\sigma$	3.569	
		(0.299)	

†Estimated on 83,478 white male teenagers. Estimation of reservation values also included age, state, year, and quarter fixed effects.

‡ Search costs are constant for teens out of school.

Table 7: Minimum Wage Elasticities

Group	Search Elasticity	Match Elasticity	Employment Elasticity	Share With Pos. Employment Elasticity
Population	0.195	-0.327	-0.143	0.228
Out of School	0.023	-0.243	-0.219	0.002
In School	0.302	-0.383	-0.093	0.303
Household Head HS or less	0.256	-0.386	-0.138	0.226
Household Head Some College	0.289	-0.391	-0.115	0.219
Household Head Four-year College	0.386	-0.365	-0.0002	0.438
Household Head Post Four-year College	0.462	-0.367	0.068	0.664
Single Parent	0.264	-0.376	-0.219	0.240

search elasticities that the overall employment effect for this group is positive. This is driven by the positive search elasticities for this group being 1.8 times larger than those whose who have a household head who dropped out before completing high school. These individuals are more responsive to the increase in the minimum wage, in part because they were less likely to search in the first place but also because these individuals are more willing to trade off a lower probability of employment for a higher expected wage conditional on employment.

The fourth column shows the share of individuals in particular groups who see their expected probability of employment increase with an increase in the minimum wage. Although almost 23% see their probability of employment increase, these are confined strictly to those who are in school. For those who are in school, we see that those with the most educated parents are three times as likely to experience a positive employment effect in expectation than those who have the least educated parents. Note that any increase in the probability of employment is being driven by the increased probability of searching as the probability of finding employment conditional on searching always falls with an increase in the minimum wage.

Although our methodologies are very different, these results echo the concerns raised by Lang and Kahn (1998) and Neumark and Wascher (1996) on the composition effects of minimum wage increases. The former find that increases in the minimum wage lead to substitution from adults to teenagers while the latter find substitution effects from those who are out of school to those who are in school. Here we find that substitution effects occur along multiple dimensions. Namely, we see an employment shift from those teenagers who are out of school to teenagers who are in school. For those who are in school, there is an employment shift from those who come from single parent families where the parent has little education to two parent families where the household head is highly educated.

9 Conclusion

This paper has developed two-sided matching model to explain the puzzling absence of a large impact on employment levels when the minimum wage is increased. In the classical framework, the exit by firms would dictate a decrease in employment. However, more general matching functions can generate positive employment effects from an increase in the minimum wage. In particular, if employment depends upon *both* the number of searching workers and the number of searching firms, the increase in the number of searching workers may more than offset the decrease in the number of searching firms. Even if positive employment effects result from a minimum wage hike, however, the probability of any individual worker finding a job has fallen. With employment probabilities falling, if any individuals are hurt from the minimum wage hike it will be those individuals who want the minimum wage jobs the most.

Estimating the structural model was made feasible by translating the firm's zero profit condition into a function of the probability of a searching worker finding a match. The estimates of the model allow us to decompose the employment effects into their labor supply and labor demand components. Consistent with the theory, we find small employment effects that are masking much larger changes in labor supply and labor demand.

While the employment effects are muted in the population of teenagers we considered, there are large disparities for certain sub-groups. Increases in the minimum wage lead to a shift from teenagers who are out school to teenagers who are in school. Further, those teenagers who are in school and have highly educated parents are likely to see positive employment effects as a result of their increased probability of search. In contrast, those who have less educated parents and/or come from single parent households see their employment probabilities fall. The overall composition of the low wage workforce then shifts away from those with disadvantaged backgrounds.

While this study has focused on teenagers, the potential effects of these teenagers on the market for adults in the low wage labor market are large. The small employment effects found in the previous literature may be masking much larger effects for those adults who find themselves in the low wage labor market. This group is likely to be searching for work regardless of the minimum wage and may be pushed out of the labor market by teenagers induced to search because of the higher minimum wage.

References

[1] Ahn T. and Arcidiacono P. "Paying to Queue: A Theory of Locational Differences in Nonunion Wages." *Journal of Urban Economics*, 55(3):564–579, 2004.

[2] Albrecht J.W. and Axell B. "An Equilibrium Model of Search Unemployment." *Journal of Political Economy*, 92:824–840, 1984.

[3] Arcidiacono P. "The Dynamic Implications of Search Discrimination." *Journal of Public Economics*, 87(7-8):1681-1707, 2003.

[4] Binmore, K., Rubinstein A., and Wolinsky A., "The Nash bargaining solution in economic bargaining," *RAND Journal of Economics* (17), 1986.

[5] Binmore K., Shaked A., and Sutton J. "An Outside Option Experiment." *Quarterly Journal of Economics*, 104(4):753–770, Nov. 1989.

[6] Brown C. "Minimum Wages, Employment, and the Distribution of Income." Handbook of Labor Economics, 2101– 2134, 1999.

[7] Burdett K. and Mortensen D.T. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review*, 39(2):257–273, May 1998.

[8] Card D. and Krueger A.B. "Minimum Wage and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania." *American Economic Review*, 84:772–793, 1994.

[9] Card D. and Krueger A.B. Myth and Measurement: The New Economics of the Minimum Wage. Princeton University Press, Princeton NJ., 1995.

[10] Card D. and Krueger A.B. "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania: Reply." *American Economic Review*, 90(5):1397–1420, 2000.

[11] Eckstein Z. and van den Berg G.J. "Empirical Labor Search: A Survey." *Journal of Econometrics*, 136(2):531-564, February 2007.

[12] Eckstein Z. and Wolpin K.I. "Estimating a Market Equilibrium Search Model From Panel Data on Individuals" *Econometrica*, 58:783–808, 1990.

[13] Flinn C. "Interpreting Minimum Wage Effects on Wage Distributions: A Cautionary Tale." *Annales d'Économie et de Statistique*, 67-68:309-355, 2002.

[14] Flinn C. "Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates." *Econometrica*, 74:571–627, 2006.

[15] Glaeser E. "Does Rent Control Reduce Segregation?" HIER Discussion Paper #1985, 2002.

[16] Glaeser E. and Luttmer E. "The Misallocation of Housing under Rent Control." *American Economic Review*, 93(4):1027-1046, 2003.

[17] Kennan J. "The Elusive Effects of Minimum Wages." *Journal of Economic Literature*, 33(4):1350–1365, 1995.

[18] Hosios, A. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economics Studies*, 57:279-298, 1990.

[19] Koning P., van den Berg G.J., and Ridder G. "Semi-nonparametric Estimation of an Equilibrium Search Model." *Oxford Bulletin of Economics and Statistics* 62(3):327–356, 2000.

[20] Lang K. and Kahn S. "The Effect of Minimum-wage Laws on the Distribution of Employment: Theory and Evidence." *Journal of Public Economics*, 69(1):67–82, 1998.

[21] Luttmer E. "Does the Minimum Wage Cause Inefficient Rationing?" *The B.E. Journal of Economic Analysis & Policy*, 7(1), (Contributions), Article 49, October 2007.

[22] Meyer R. and Wise D. "Discontinuous Distributions and Missing Persons: The Minimum Wage and Unemployed Youth." *Econometrica*, 51(6):1677–1698, 1983a.

[23] Meyer R. and Wise D. "The Effects of the Minimum Wage on Employment and Earnings of Youth." *Journal of Labor Economics*, 1:66–100, 1983b.

[24] Nelson R. "State labor legislation enacted in 2000." *Monthly Labor Review*, 124:12–24, 2001.

[25] Neumark D. and Wascher W. "Employment Effect of Minimum and Subminimum Wages: Reply to Card, Katz and Krueger." *Industrial and Labor Relations Review*, 47:497–512,1994.

[26] Neumark D. and Wascher W. "Minimum Wage Effects on Employment and School Enrollment." *Journal of Business and Economic Statistics*, 13(2):199–206, April 1995.

[27] Neumark D. and Wascher W. "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania: Comment." *American Economic Review*, 90(5):1362–1396, 2000.

[28] Petrongolo B. and Pissarides C. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, 39(2):390–431, June 2001.

[29] Pissarides C. "Loss of Skill During Unemployment and the Persistence of Employment Shocks." *Quarterly Journal of Economics*, 107(4):1371–1391, November 1992.

[30] van den Berg G.J. "Multiple Equilibria and Minimum Wages in Labor Markets with Informational Frictions and Heterogeneous Production Technologies." *International Economic Review*, 44:1337–1357, 2003.

[31] van den Berg G.J. and Ridder G. "An Empirical Equilibrium Search Model of the Labor Market." *Econometrica*, 66:1183–1221, 1998.

Appendix

Derivation of Wages from a Rubinstein Bargaining Game

Following the outlines of the proof in Binmore, Shaked and Sutton (1989) (from hereon referred to as BSS) and Binmore, Rubinstein, and Wolinsky (1986), we define m_f and M_f as the infimum and supremum payoffs for the firm, respectively, and m_w and M_w as the infimum and supremum payoffs for the worker, respectively. Match revenue is Y_{ij} and outside options are 0 and R_i for firms and workers, respectively.

In a Rubinstein bargaining game in which the firm moves first (in the absence of a minimum wage), the following inequalities hold:

$$\begin{aligned} m_f &\geq Y_{ij} - \max\{\tau_w M_w, R_i\} \\ Y_{ij} - M_f &\geq \max\{\tau_w m_w, R_i\} \\ m_w &\geq Y_{ij} - \tau_f M_f \\ Y_{ij} - M_w &\geq \tau_f m_f \end{aligned}$$

τ_w represents the worker's discount factor, and τ_f represents the firm's.

Inclusion of minimum wage means that the any bargaining offer (whether supremum or infimum) must be capped from below at the minimum wage, therefore, the inequalities change to:

$$\begin{aligned} m_f &\geq Y_{ij} - \max\{\tau_w M_w, \underline{W}, R_i\} \\ Y_{ij} - M_f &\geq \max\{\tau_w m_w, \underline{W}, R_i\} \\ m_w &\geq Y_{ij} - \tau_f M_f \\ Y_{ij} - M_w &\geq \tau_f m_f \end{aligned}$$

We will examine the case where $\underline{W} \geq R_i$ and $\underline{W} < R_i$ separately. First, when $\underline{W} \geq R_i$, we examine 3 regions, defined similarly to BSS:

$\underline{W} \leq \tau_w m_w$ (region 1), $\tau_w m_w < \underline{W} < \tau_w M_w$ (region 2), and $\underline{W} \geq \tau_w M_w$. (region 3)

Focusing on region 1, the inequalities change to:

$$
\begin{aligned}
m_f &\geq Y_{ij} - \tau_w M_w \\
Y_{ij} - M_f &\geq \tau_w m_w \\
m_w &\geq Y_{ij} - \tau_f M_f \\
Y_{ij} - M_w &\geq \tau_f m_f
\end{aligned}
$$

It is easy to show that:

$$\frac{(1-\tau_f)Y_{ij}}{1-\tau_f\tau_w} \leq m_w \leq M_w \leq \frac{(1-\tau_f)Y_{ij}}{1-\tau_f\tau_w}$$

Therefore, $M_w = m_w = \frac{(1-\tau_f)Y_{ij}}{1-\tau_f\tau_w}$.

Define $\beta = \frac{1-\tau_f}{1-\tau_f\tau_w}$. Then, $M_w = m_w = \beta Y_{ij}$, implying that $M_f = m_f = (1-\beta)Y_{ij}$.

We next show that region 2 yields a logical contradiction:

$$
\begin{aligned}
m_f &\geq Y_{ij} - \tau_w M_w \\
Y_{ij} - M_f &\geq \underline{W} > \tau_w m_w \\
m_w &\geq Y - \tau_f M_f \\
Y_{ij} - M_w &\geq \tau_f m_f
\end{aligned}
$$

which yields $\frac{(1-\tau_f)Y_{ij}}{1-\tau_f\tau_w} < m_w \leq M_w \leq \frac{(1-\tau_f)Y_{ij}}{1-\tau_f\tau_w}$.

For region 3, the inequalities are:

$$
\begin{aligned}
m_f &\geq Y_{ij} - \underline{W} \\
Y_{ij} - M_f &\geq \underline{W} \\
m_w &\geq Y_{ij} - \tau_f M_f \\
Y_{ij} - M_w &\geq \tau_f m_f
\end{aligned}
$$

This yields $m_w = M_w = (1-\tau_f)Y_{ij} + \tau_f\underline{W}$ and $m_f = M_f = Y_{ij} - \underline{W}$. Letting τ_f approach one, we have $m_w = M_w = \underline{W}$ and $m_f = M_f = Y_{ij} - \underline{W}$. When $\underline{W} \geq R_i$ and a worker successfully matches, his wage outcome is $\max\{\beta Y_{ij}, \underline{W}\}$.

Now, repeating the exercise with $\underline{W} < R_i$, we see that for regions 1 and 2, results are identical (since we just replace $\underline{W}$ with R_i), and region 3 changes to $m_w = M_w = R_i$ and

$m_f = M_f = Y_{ij} - R_i$. Therefore, when $\underline{W} < R_i$ and a worker successfully matches, his wage outcome is $\max\{\beta Y_{ij}, R_i\}$.

Combining these two results, when a worker successfully matches ($Y_{ij} > \underline{W}$), the unique subgame perfect equilibrium outcome of the bargaining game is a wage offer of $\max\{\beta Y_{ij}, \underline{W}, R_i\}$ which is accepted. QED

Proof of Proposition 1

Note that conditional on any $N \in [0, \overline{N}]$, as $J \to \infty$, $q \to 0$. There then exists a $\overline{J}$ such that for all N if $J' > \overline{J}$, profits are negative. Since the partial derivative of π is negative with respect to J,

$$\frac{\partial \pi}{\partial J} = -\frac{q\alpha\left(E\max\{Y_{ij}, W_{ij}\} - C_2\right)}{J} < 0$$

We know that for each value of N there is at most one value of J such that $\pi = 0$.

Similarly, define V as the search value. Taking the partial derivative with respect to N yields:

$$\frac{\partial V}{\partial N} = -\frac{p(1-\alpha)\left(E\max\{W - R_i, 0\}\right)}{N} + \frac{p\partial E\max\{W - R_i, 0\}}{\partial N} - \frac{\partial K_i}{N} < 0$$

as the second two terms must be negative when ordering the individuals according to V_i. We know that for each J there is at most one value of N such that $V = 0$.

We can then define the following mappings:

$$f_1 = \begin{cases} \pi(J, N) & \text{for } J \in (0, \overline{J}], N \in [0, \overline{N}] \\ \max\{\pi(0, N), 0\} & \text{for } J = 0, N \in [0, \overline{N}] \end{cases}$$

$$f_2 = \begin{cases} \min\{V(J, \overline{N}), 0\} & \text{for } J \in [0, \overline{J}], N = \overline{N} \\ V(J, N) & \text{for } J \in [0, \overline{J}], N \in (0, \overline{N}) \\ \max\{V(J, 0), 0\} & \text{for } J \in [0, \overline{J}], N = 0 \end{cases}$$

Then for each value of N, there exists a unique value of $J \in [0, \overline{J}]$ that satisfy $f_1 = 0$. Further, since π is continuous in N, this unique value is a continuous function of N. Similarly, for each J, there is a unique $N \in [0, \overline{N}]$ satisfying f_2 which is continuous in J. We can then use functions to define a continuous vector valued function mapping from $[0, \overline{J}] \times [0, \overline{N}]$ into itself. Then by Brouwer's fixed point theorem there exists a doublet $\{J^*, N^*\}$ where $f_1 = 0$ and $f_2 = 0$. QED.

Proof of Lemma 1

We show, given NR, if a worker searches, he accepts all matches. Assume $R_i > \underline{W}$.Then, individuals search when

$$pPr(Y_{ij} \geq R_i)[E(W|Y_{ij} \geq R_i) - R_i] > K_i$$

To derive the lower limit on K_i to make the condition above hold, set $p = 1$. The K_i that satisfies this condition is $\underline{K}$ for all searching workers, and yields the expression in NR. QED

Proof of Proposition 2

Consider the equilibrium before the minimum wage increase. The expected surplus for the firm conditional on matching is $E(\max\{Y_{ij} - W_ij, 0\}|\underline{W_1})$ and the probability of a firm matching is given by q_1. Note that the expected surplus for the firm conditional on matching is weakly decreasing in the minimum wage. The firm's expected zero profit condition is:

$$q_1(E(Y) - E_1(W) - C_2) - C_1 = 0$$

The firm's probability of matching must increase when the expected surplus conditional on matching fall in order for the zero profit condition to still bind. Note further that the probabilities of firms and workers matching is given by:

$$q = A\left(\frac{N}{J}\right)^{1-\alpha} \qquad\qquad p = A\left(\frac{J}{N}\right)^{\alpha}$$

The expression for the firm implies that $\frac{N}{J}$ must increase for the zero profit condition to bind. But if this fraction increases then p must fall. QED.

Proof of Proposition 3

Differentiating the matching function with respect to the minimum wage yields:

$$\frac{dx}{d\underline{W}} = \alpha q \frac{dJ}{d\underline{W}} + (1-\alpha)p\frac{dN}{d\underline{W}}$$

Rewrite as:

$$\begin{aligned}
\frac{dx}{d\underline{W}} &= \alpha \frac{x}{J}\frac{dJ}{d\underline{W}} + (1-\alpha)\frac{x}{N}\frac{dN}{d\underline{W}} \\
&= x\left(\alpha \frac{\frac{dJ}{J}}{d\underline{W}} + (1-\alpha)\frac{\frac{dN}{N}}{d\underline{W}}\right) \\
&= \frac{x}{\underline{W}}\left(\alpha \frac{\frac{dJ}{J}}{\frac{d\underline{W}}{\underline{W}}} + (1-\alpha)\frac{\frac{dN}{N}}{\frac{d\underline{W}}{\underline{W}}}\right) \\
&= \frac{x}{\underline{W}}\left(\alpha\varepsilon_{LD} + (1-\alpha)\varepsilon_{LS}\right)
\end{aligned}$$

Therefore, for the employment effect to be positive ($\frac{dx}{d\underline{W}} > 0$), it must be that $(\alpha\varepsilon_{LD} + (1-\alpha)\varepsilon_{LS}) > 0$, where ε_{LD} is the elasticity of labor demand and ε_{LS} is the elasticity of labor supply. QED

Proof of Proposition 4

In order for all workers to benefit from an increase in the minimum wage it is sufficient to show that the workers with the lowest reservation values, zero, are made better off by the increase. The value of search for these workers can be written as:

$$V = A\left(\frac{N}{J}\right)^{-\alpha} E(W) - K_i$$

Note that the zero profit condition for firms can be written as:

$$A\left(\frac{N}{J}\right)^{1-\alpha} (E(Y) - E(W) - C_2) - C_1 = 0$$

and that both of these conditions depend on N and J only through the ratio N/J. Further, the zero profit condition for the firm is an identity. Differentiating profits with respect to an increase in the minimum wage yields:

$$A\left(\frac{N}{J}\right)^{1-\alpha}\left((1-\alpha)(E(Y) - E(W) - C_2)\left(\frac{N}{J}\right)^{-1}\frac{\left(d\frac{N}{J}\right)}{d\underline{W}} - \frac{dE(W)}{d\underline{W}}\right) = 0$$

Solving for $d(N/J)/d\underline{W}$ yields:

$$\frac{d\left(\frac{N}{J}\right)}{d\underline{W}} = \frac{N}{(1-\alpha)(E(Y) - E(W) - C_2)J}\frac{dE(W)}{d\underline{W}}$$

We now have all components necessary to sign $dV/d\underline{W}$ for those with a reservation value of zero. Differentiating V with respect to $\underline{W}$ yields:

$$E(W)A(-\alpha)\left(\frac{N}{J}\right)^{-\alpha-1}\frac{d\left(\frac{N}{J}\right)}{d\underline{W}}+A\left(\frac{N}{J}\right)^{-\alpha}\frac{dE(W)}{d\underline{W}}$$

substituting in for $d(N/J)/d\underline{W}$ and rewriting yields:

$$\frac{pdE(W)}{d\underline{W}}\left[1-\frac{\alpha E(W)}{(1-\alpha)(E(Y)-E(W)-C_2)}\right]$$

Since $dE(W)/d\underline{W} > 0$, we have the result. QED