Managers’ First Order Conditions

The manager’s Hamiltonian is

\[ H_i = \left[ e_{m,i} (1 - S_i) + S_i - c^S(S_i) \right] \cdot (\Pi_i + \Omega I_i) + q_i I_i, \]  

(A.1)

where \( q_i \) is the shadow value of the marginal increase in product quality. The first-order conditions with respect to \( P_i, I_i, Z_i \) and \( S_i \) are (dropping the \( s \) index of calendar time for simplicity):

\[ \frac{\partial \Pi_i}{\partial P_i} = 0; \]  

(A.2)

\[ \frac{\partial \Pi_i}{\partial I_i} + \frac{\partial \Pi_i}{\partial Z_i} + q_i = 0; \]  

(A.3)

\[ \frac{\partial \Pi_i}{\partial Z_i} = \frac{\partial}{\partial S_i} \left( \Pi_i + \Omega I_i \right) = -\hat{q}_i + rq_i; \]  

(A.4)

\[ \frac{\partial}{\partial S_i} \left( \Pi_i + \Omega I_i \right) = 0. \]  

(A.5)

Manipulating these expression gives us equations (12)-(15) in the text.

Proof of Proposition 1 (Equity Shares)

The founding shareholder chooses the equity allocation in order to

\[ \max_{e_{m,i}} (1 - e_{m,i}) \left[ 1 - S(e_{m,i}) \right] \]

subject to the manager’s stealing decision in (15), i.e., \( 1 - e_{m,i} = \frac{\partial c^S(S_i)}{\partial S_i} \). The shareholder’s first order condition reads

\[ -[1 - S(e_{m,i})] - (1 - e_{m,i}) S'(e_{m,i}) = 0, \]  

(A.6)

which implies

\[ -(1 - e_{m,i}) S'(e_{m,i}) = [1 - S(e_{m,i})]. \]

From the manager’s stealing condition (15), using the implicit function theorem, we have

\[ S'(e_{m,i}) = -\frac{1}{\partial^2 c^S(S_i)} < 0. \]  

(A.7)

Using (A.7) to substitute into (A.6), after algebraic manipulations, we obtain
\[ e_{m,i} = 1 - \frac{1 - S(e_{m,i})}{\frac{\partial^2 c^S(S_i)}{\partial^2 S_i}} = 1 - \frac{\partial^2 c^S(S_i)}{\partial^2 S_i} \left[ 1 - S(e_{m,i}) \right] \]  

(A.8)

Combining this with the stealing condition (15), we get

\[ 1 - S(e_{m,i}) = \frac{\partial c^S(S_i)}{\partial S_i}. \]  

(A.9)

We can use (A.8) and (A.9) to obtain the value of \( \Theta \equiv (1 - e_{m,i})(1 - S) \) as

\[ \Theta = \left( \frac{\partial c^S(S_i)}{\partial^2 S_i} \right)^2. \]

Proof of Proposition 2 (Steady State)

The saving behavior of the household yields

\[ r^* = \rho + \frac{\sigma \lambda}{1 - \sigma} + z^*. \]

We then solve for the other variables of interest by substituting this expression into the returns to investment (23) and to entry (24). We obtain:

\[ z^* = \frac{\alpha}{1 - \Omega} \left( \frac{1}{\theta} - 1 \right) x^* - \frac{\alpha}{1 - \Omega} \phi - \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right); \]

\[ z^* = \left[ \left( \frac{1}{\theta} - 1 \right) - \frac{\beta}{\Theta} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] x^* - \phi. \]

The first curve, which we call the corporate investment (CI) locus, describes the steady-state rate of investment \( z \equiv \dot{Z}/Z \) that incumbent intermediate firms generate given the firm size \( x \) that they expect to hold in equilibrium. The second curve, which we call the entry (EI) locus, describes the steady-state investment rate \( z \) that equalizes the return to entry and the return to investment given the value of \( x \) that both entrants and incumbents expect to hold in equilibrium. The steady state is the intersection of these two curves in the \((x, z)\) space. After some algebra, we obtain:

\[ x^* = \frac{(1 - \frac{\alpha}{1 - \Omega}) \phi - \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right)}{(1 - \frac{\alpha}{1 - \Omega}) \left( \frac{1}{\theta} - 1 \right) - \frac{\beta}{\Theta} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right)}; \]

\[ z^* = \left[ \frac{\alpha}{1 - \Omega} \phi + \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] \frac{\beta}{\Theta} - \frac{\alpha}{1 - \Omega} \left( \frac{1}{\theta} - 1 \right) - \frac{\beta}{\Theta} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right). \]
The steady state rate of firm entry (variety growth) that guarantees that the firm’s size, \( x \equiv \frac{X}{Z} = \theta Z L / N^{1-\sigma} \), is constant in the long run equals
\[
N^* \equiv \left( \frac{\bar{N}}{N} \right)^* = \frac{\lambda}{1-\sigma}.
\]
From equation (25), we obtain the growth rate of the per capita final output and GDP as
\[
\left( \frac{\dot{Y}}{Y} \right)^* - \lambda = \left( \frac{\dot{G}}{G} \right)^* - \lambda = \frac{\sigma \lambda}{1-\sigma} + z^*.
\]

**Proof of Proposition 3 (Consumption Ratio)**

We assume that the representative household provides managerial services to firms. Such services are not in units of labor and thus their provision does not come out of labor supply. In other words, we think of them as an additional endowment owned by the household. As discussed in the previous section, the provision of such services is remunerated with ownership shares. Moreover, it allows the extraction of rents, which is our main mechanism capturing corporate governance frictions. In our scheme such rents are capitalized in the wealth that accrues to the household and thus are embedded in the asset income flow. Since this is an important component of our analysis, it is best to develop it formally.

First, note that the present value of the income flow accruing to the founder is not the market value of the firm, which is instead
\[
V_{i,\text{market}}(t) = \int_{t}^{+\infty} e^{-\int_{t}^{s} r(v) dv} \left[ 1 - S_i(e_{m,i}(s), e_{a,i}(s)) \right] \Pi_i(s) ds.
\]
Household wealth in our economy is
\[
A = \int_{0}^{N} A_i di,
\]
which is the aggregate of the wealth generated by each firm. The wealth generated by each firm, in turn, is the sum of two components that accrue, respectively, to manager and founder (for simplicity, in this accounting exercise we denote \( S_i(e_{m,i}(s)) = S_i(s) \)):
\[
A_i(t) = \int_{t}^{+\infty} e^{-\int_{t}^{s} r(v) dv} \left[ e_{m,i}(s) \left( 1 - S_i(s) \right) + S_i(s) \right] \Pi_i(s) ds
+ \int_{t}^{+\infty} e^{-\int_{t}^{s} r(v) dv} \left[ 1 - e_{m,i}(s) \right] \left[ 1 - S_i(s) \right] \Pi_i(s).
\]
Consolidating:
\[
A_i(t) = \int_{t}^{+\infty} e^{-\int_{t}^{s} r(v) dv} \Pi_i(s) ds = V_i(t).
\]
Thus, wealth in our economy is still defined as the fundamental value of the \( N \) existing firms. The upshot of this discussion is that from the household side of the economy we obtain the same labor supply, budget constraint and, therefore, saving behavior as in the benchmark frictionless model. What changes with frictions are the returns to investment and entry and, therefore, the paths
of market structure and economic growth. Crucially, because of these changes, the path of net profit of the typical firm, $\Pi_t(s)$, changes as well, so that corporate governance frictions change the magnitude of the wealth, $A(t)$, generated by the market. We now develop this part of the analysis.

The terms $e_{m,i}$, $S_i$ are constant. Accordingly, the value of $\Theta$ is also constant. For clarity, we refer to this baseline value of $\Theta$, which remains constant throughout the transition, as $\Theta_0$. When $n > 0$, the free entry condition yields

$$\beta X = V_i^{\text{founder}} = \Theta_0 V_i$$

so that, imposing symmetry,

$$V = \frac{\beta \theta Y}{\Theta_0 P N}.$$  

Consequently, assets market equilibrium requires

$$A = NV = \frac{\beta \theta}{\Theta_0 P} \cdot Y,$$  

which says that the wealth ratio $A/Y$ is constant. This result and the saving schedule,

$$r = \rho - \lambda + \hat{C}/C,$$  

allow us to rewrite the household budget,

$$\dot{A} = r A + wL - C,$$

as the following differential equation in $c \equiv C/Y$

$$\frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} = (-\rho + \lambda) - (1 - \theta) \frac{1}{\zeta} + \frac{1}{\zeta} \frac{C}{Y}, \quad \zeta \equiv \frac{\beta \theta}{\Theta_0 P}.$$  

This differential equation indicates that the $C/Y$ ratio has a unique steady state. In addition, because $\zeta > 0$ it is also unstable, implying that an initial condition different from the steady state value will result in a tendency for the ratio to accelerate or decelerate and eventually violate the transversality condition. Therefore, the equilibrium $c$ must jump immediately to the constant value

$$c^*_{n>0} = \zeta (\rho - \lambda) + 1 - \theta,$$

which, by setting $\Theta = \Theta_0$ and $P = \frac{1}{\beta}$ is the bottom line of the function $c(x)$ in the text of the proposition.

When $n = 0$ assets market equilibrium still requires $A = NV$ but it is no longer true that $NV = \frac{\gamma \beta \theta}{\Theta_0 P} \cdot Y$ since by definition the free-entry condition does not hold. This means that the wealth ratio $A/Y$ is not constant. However, the relation

$$r = \frac{\Pi_i}{\hat{V}_i} + \frac{\hat{V}_i}{V_i},$$  

(A.12)
holds, since it is the arbitrage condition on equity holding that characterizes the value of an existing firm regardless of how it came into existence in the first place. Imposing symmetry and inserting the definition of net profit,

$$\Pi = \left( P - 1 \right) \left( \frac{\theta}{P} \right) \frac{1 - \sigma}{N} L - \phi \right] Z - I,$$

(A.12), and (A.10) into the household budget (2) yields

$$0 = N \left[ (P - 1) X - \phi Z - I \right] + (1 - \theta) Y - C$$

$$= N Z \left[ (P - 1) \frac{X}{Z} - \phi - z \right] + (1 - \theta) Y - C$$

$$= \frac{N Z}{Y} \left[ (P - 1) \frac{X}{Z} - \phi - z \right] + 1 - \theta - C \frac{Y}{Y}.$$

The definition $x = X/Z = \theta \frac{Z}{P} L/N^{1-\sigma}$ allows us to rewrite this expression as

$$c = 1 - \theta + \theta^2 \left[ \left( \frac{1}{\theta} - 1 \right) - \frac{\phi + z}{x} \right],$$

which is the top line of the function $c(x)$ in the text of the proposition.

**Proof of Proposition 4 (General Equilibrium)**

We start with the expressions for the returns to investment and to entry, reproduced here for convenience

$$r_Z = \frac{\alpha}{1 - \Omega} \left[ \left( \frac{1}{\theta} - 1 \right) x - \phi \right],$$

(A.13)

$$r_N = \frac{\Theta}{\beta} \left( \left( \frac{1}{\theta} - 1 \right) - \frac{\phi + z}{x} \right) + z + \frac{x}{x}.$$

(A.14)

Proposition 3 says that $c$ is constant when there is entry, i.e., when $n > 0$, and that in such a case the return to saving becomes $r = \rho - \lambda + \dot{Y}/Y$. Therefore, we can use the expression for the return to entry (A.14) and the definition $x = X/Z = \theta \frac{Z}{P} L/N^{1-\sigma}$ to obtain

$$n = \frac{\Theta}{\beta} \left[ \left( \frac{1}{\theta} - 1 \right) - \frac{\phi + z}{x} \right] - \rho + \lambda, \quad z \geq 0,$$

(A.15)

which holds for positive values of the right-hand side. The saving schedule (A.11) and the reduced-form production function,

$$Y = \left( \frac{\theta}{P} \right) \frac{1 - \sigma}{N} Z L,$$

(A.16)

yield

$$r = \rho - \lambda + \dot{Y}/Y = \rho + z + \sigma n.$$
Combining this expression with the return to investment (A.13) yields

\[ \frac{\alpha}{1 - \Omega} \left[ \left( \frac{1}{\theta} - 1 \right) x - \phi \right] = \rho + z + \sigma n. \]  

(A.17)

Setting \( z = 0 \) and solving for \( n \), this expression yields the middle branch of the function \( n(x) \) in the text of the proposition. Moreover, combining (A.17) with the rate of entry in (A.15) and solving for \( z \) yields

\[ z(x) = \frac{\left( \frac{1}{\theta} - 1 \right) x - \phi \left( \frac{\alpha}{1 - \Omega} - \frac{\sigma \theta}{\beta x} \right) - (1 - \sigma) \rho - \sigma \lambda}{1 - \frac{\sigma \theta}{\beta x}}, \]

(A.18)

which is the bottom branch of the function \( z(x) \) in the text of the proposition. Substituting \( z(x) \) back into (A.15) yields the bottom branch of the function \( n(x) \) in the text of the proposition.

With these expressions in hand, we focus on the thresholds. The definition of \( x \) and the reduced-form production function (A.16) yield

\[ \frac{\dot{x}}{x} = \frac{\dot{Y}}{Y} - n - z = \lambda - (1 - \sigma) n(x). \]

Suppose that the threshold for entry is smaller than the threshold for investment. Then, according to (A.15), \( n(x) > 0 \) for

\[ \frac{\Theta}{\beta} \left[ \left( \frac{1}{\theta} - 1 \right) + \frac{\phi + z}{x} \right] - \rho + \lambda > 0, \]

since \( z = 0 \), which yields

\[ x > x_N \equiv \frac{\phi}{\left( \frac{1}{\theta} - 1 \right) - \frac{\beta (\rho - \lambda)}{\Theta}}. \]

Assumption (32) in the text of the proposition guarantees that this value is finite. On the other hand, according to (A.18), \( z(x) > 0 \) for

\[ \left[ \left( \frac{1}{\theta} - 1 \right) x - \phi \right] \left( \frac{\alpha}{1 - \Omega} - \frac{\sigma \theta}{\beta x} \right) > (1 - \sigma) \rho + \sigma \lambda, \]

because entry is already active, which yields

\[ x > x_Z \equiv \text{arg solve} \left\{ \left[ \left( \frac{1}{\theta} - 1 \right) x - \phi \right] \left( \frac{\alpha}{1 - \Omega} - \frac{\sigma \theta}{\beta x} \right) = (1 - \sigma) \rho + \sigma \lambda \right\}. \]

This equation has always a finite solution \( x_Z \) and thus we do not need a condition equivalent to (32). The assumption

\[ z(x_N) = \frac{\left( \left( \frac{1}{\theta} - 1 \right) x_N - \phi \right) \left( \frac{\alpha}{1 - \Omega} - \frac{\sigma \theta}{\beta x_N} \right) - (\rho - \sigma \rho + \sigma \lambda)}{1 - \frac{\sigma \theta}{\beta x_N}} < 0, \]

moreover, ensures that \( x_N < x_Z \) because it says that at \( x_N \) the value of \( z \) that agents would need to choose to equalize returns is negative. The non-negativity constraint thus binds and agents choose \( z = 0 \). This is assumption (33) in the text of the proposition.
To understand whether the solution just found is a stable Nash Equilibrium, we use (A.15) to rewrite (A.14) as
\[
\begin{align*}
\tau_N &= \frac{\Theta}{\beta} \left[ \left( \frac{1}{\theta} - 1 \right) - \frac{\phi + z}{x} \right] + z + \lambda - (1 - \sigma) n(x) \\
&= \frac{\sigma \Theta}{\beta} \left[ \left( \frac{1}{\theta} - 1 \right) - \frac{\phi + z}{x} \right] + z + \lambda + (1 - \sigma) (\rho + \lambda).
\end{align*}
\]

Given \( x \), an equilibrium with both entry and investment — that is stable in the Nash sense that agents have no incentives to deviate from it — exists if in the \((z, \tau)\) space this line intersects the line given by (A.13) from below. This requires that the line just derived is positively sloped, that is, \( 1 > \sigma \Theta / \beta x \) for \( x > x_Z \). A sufficient condition for this to be true is \( 1 > \sigma \Theta / \beta x_N \), which is assumption (34) in the text of the proposition.

**Proof of Proposition 5 (Dynamics)**

For \( x \leq x_N < x_Z \) we have \( \dot{x}/x = \lambda \) and the economy crosses the threshold for entry in finite time. For \( x_N < x < x_Z \) we have, after rearranging terms,
\[
\frac{\dot{x}}{x} = \sigma \lambda + (1 - \sigma) \rho - (1 - \sigma) \frac{\Theta_0}{\beta} \left( \left( \frac{1}{\theta} - 1 \right) - \frac{\phi}{x} \right).
\]

The economy, therefore, crosses the threshold for investment in finite time since firm profitability is still growing at \( x = x_Z \) in light of assumption (40). To guarantee that a solution \( \Psi(x) = 0 \) exists, we assume
\[
\lim_{x \to \infty} \Psi(x) = (1 - \sigma) \cdot \lim_{x \to \infty} \left[ \frac{\lambda}{1 - \sigma} - n(x) \right]
\]
\[
= (1 - \sigma) \cdot \lim_{x \to \infty} \left[ \frac{\lambda}{1 - \sigma} - \frac{\Theta_0}{\beta} \left( \left( \frac{1}{\theta} - 1 \right) - \frac{\phi + z(x)}{x} \right) + (\rho - \lambda) \right]
\]
\[
= (1 - \sigma) \cdot \lim_{x \to \infty} \left[ \rho + \frac{\sigma \lambda}{1 - \sigma} - \frac{\Theta_0}{\beta} \left( \left( \frac{1}{\theta} - 1 \right) - \frac{\phi}{x} - \frac{z(x)}{x} \right) \right] < 0.
\]

Since
\[
\lim_{x \to \infty} z(x) = \lim_{x \to \infty} \frac{((1/\theta - 1) x - \phi) \left( \frac{\alpha}{1 - \Omega} - \frac{\sigma \Theta_0}{\beta x} \right) - (1 - \sigma) \rho - \sigma \lambda}{1 - \sigma \Theta_n/\beta x}
\]
\[
= \lim_{x \to \infty} \frac{((1/\theta - 1) x - \phi) \left( \frac{\alpha}{1 - \Omega} - \frac{\sigma \Theta_0}{\beta x} \right) - (1 - \sigma) \rho - \sigma \lambda}{1}
\]
\[
= \frac{\alpha}{1 - \Omega} \left( \frac{1}{\theta} - 1 \right) x,
\]

we have
\[
\lim_{x \to \infty} \Psi(x) = \lim_{x \to \infty} \left[ \rho + \frac{\sigma \lambda}{1 - \sigma} - \frac{\Theta_0}{\beta} \left( 1 - \frac{\alpha}{1 - \Omega} \right) \left( \frac{1}{\theta} - 1 \right) \right] < 0.
\]
Proof of Proposition 6 (Welfare)

In this section, we derive Equations (40), (43) and (44). Let \( \varphi_1 \equiv (P - 1) \) and \( \varphi_2 \equiv \frac{\Theta_0}{\beta} \). Using the function \( n (x) \), we write the law of motion of \( x \) in (37) as

\[
\frac{\dot{x}}{x} = (1 - \sigma) \left[ \frac{\lambda}{1 - \sigma} - n (x) \right] = (1 - \sigma) \left[ \rho + \frac{\sigma \lambda}{1 - \sigma} - \frac{\varphi_2}{x} (\varphi_1 x - \phi - z (x)) \right].
\]

Using the function \( z (x) \), after some algebra, we can rewrite this expression as

\[
\frac{\dot{x}}{x} = (1 - \sigma) \left[ \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) x - \varphi_2 (\varphi_1 x - \phi - z (x)) \right] = (1 - \sigma) \frac{\varphi_2 \phi (1 - \frac{\alpha}{1 - \Omega}) - (\rho + \frac{\sigma \lambda}{1 - \sigma})}{1 - \frac{\varphi_2}{x}}.
\]

This differential equation is linear if we approximate \( \frac{\sigma \Theta_0}{\beta x} \approx 0 \) in the denominator. So, finally, we write

\[
\frac{\dot{x}}{x} = (1 - \sigma) \left[ \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) - \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] \frac{\phi (1 - \frac{\alpha}{1 - \Omega}) - (\rho + \frac{\sigma \lambda}{1 - \sigma})}{\varphi_2 (1 - \frac{\alpha}{1 - \Omega}) - (\rho + \frac{\sigma \lambda}{1 - \sigma})} = x
\]

and define

\[
x^* \equiv \frac{\phi (1 - \frac{\alpha}{1 - \Omega}) - (\rho + \frac{\sigma \lambda}{1 - \sigma})}{\varphi_2 (1 - \frac{\alpha}{1 - \Omega}) - (\rho + \frac{\sigma \lambda}{1 - \sigma})} = \frac{\phi (1 - \frac{\alpha}{1 - \Omega}) - (\rho + \frac{\sigma \lambda}{1 - \sigma})}{(1 - \frac{\alpha}{1 - \Omega}) (P - 1) - \frac{\beta}{\Theta_0} (\rho + \frac{\sigma \lambda}{1 - \sigma})}.
\]

\[
\nu \equiv (1 - \sigma) \left[ \varphi_2 \phi (1 - \frac{\alpha}{1 - \Omega}) - (\rho + \frac{\sigma \lambda}{1 - \sigma}) \right] = (1 - \sigma) \left[ \left( 1 - \frac{\alpha}{1 - \Omega} \right) (P - 1) \frac{\Theta_0}{\beta} - \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right].
\]

This gives us the expression

\[
\dot{x} = \nu \cdot (x^* - x),
\]

and the solution

\[
x (t) = x_0 e^{-\nu t} + x^* \left( 1 - e^{-\nu t} \right),
\]

which is reported in (40). To compute the utility flow, we proceed in three steps. For simplicity, we omit time arguments unless necessary. Consider first

\[
\frac{C}{L} = \left[ 1 - \theta + \frac{(\rho - \lambda) \beta \theta}{\Theta_0 \frac{\Theta_0}{P}} \right] \cdot \frac{Y}{L} = \left[ 1 - \theta + \frac{(\rho - \lambda) \beta \theta}{\Theta_0 \frac{\Theta_0}{P}} \right] \left( \frac{\Theta_0}{P} \right) \frac{\theta}{\frac{\Theta_0}{P}} \cdot N^\sigma Z.
\]

Let

\[
\left[ 1 - \theta + \frac{(\rho - \lambda) \beta \theta}{\Theta_0 \frac{\Theta_0}{P}} \right] \left( \frac{\Theta_0}{P} \right) \frac{\theta}{\frac{\Theta_0}{P}} \equiv \Lambda.
\]
Then,

\[
\log \left( \frac{C}{L} \right) = \log \Lambda + \sigma \log N_0 + \sigma \log \frac{N}{N_0} + \log Z.
\]

From the definition of \( x \) we have

\[
x = \theta^{\frac{2}{1-\sigma}} L / N^{1-\sigma} \Rightarrow N = \left( \frac{\theta^{\frac{2}{1-\sigma}} L}{x} \right)^{\frac{1}{1-\sigma}} .
\]

Then, recalling our assumptions on population dynamics,

\[
\log \left( \frac{C}{L} \right) = \log \Lambda + \sigma \log N_0 + \frac{\sigma}{1-\sigma} \log \left( \frac{x_0 L}{x L_0} \right) + \log Z
\]

\[
= \log \Lambda + \sigma \log N_0 + \frac{\sigma}{1-\sigma} \log \left( \frac{L_0 e^{\lambda t}}{L_0} \right) + \frac{\sigma}{1-\sigma} \log \left( \frac{x_0}{x} \right) + \log Z.
\]

Also, we approximate

\[
z = ((P - 1) x - \phi) \frac{\alpha}{1 - \Omega} - (1 - \sigma) \rho - \sigma \lambda.
\]

Adding and subtracting \( z^* \) from \( z(s) \),

\[
\log Z(t) = \log Z_0 + \int_0^t z(s) \, ds
\]

\[
= \log Z_0 + z^* t + \alpha \int_0^t [z(s) - z^*] \, ds
\]

\[
= \log Z_0 + z^* t + \alpha \frac{1}{1 - \Omega} (P - 1) \int_0^t [x(s) - x^*] \, ds
\]

\[
= \log Z_0 + z^* t + \alpha \frac{1}{1 - \Omega} (P - 1) (x_0 - x^*) \int_0^t e^{-\nu s} \, ds
\]

\[
= \log Z_0 + z^* t + \frac{1}{\nu} \alpha \frac{1}{1 - \Omega} (P - 1) (x_0 - x^*) (1 - e^{-\nu t}).
\]

Approximating the log, we can write

\[
\log \left( \frac{x(t)}{x_0} \right) = \log \left( 1 + \frac{x(t)}{x_0} - 1 \right)
\]

\[
= \left( \frac{x(t)}{x_0} - 1 \right)
\]

\[
= \frac{x(t) - x_0}{x_0}
\]

\[
= \frac{x^* - x_0}{x_0} (1 - e^{-\nu t}).
\]
These results yield, after rearranging terms,

\[
\log \left( \frac{C}{L} \right) = \log \Lambda + \sigma \log N_0 + \log Z_0 \\
+ \left( \frac{\sigma}{1 - \sigma} \lambda + z^* \right) t \\
+ \left[ \frac{\alpha}{1 - \Omega} \frac{x_0}{\nu} (P - 1) + \frac{\sigma}{1 - \sigma} \right] \left( 1 - \frac{x^*}{x_0} \right) (1 - e^{-\nu t}),
\]

which corresponds to Equation (43) in the text. Without loss of generality, we set

\[
\sigma \log N_0 + \log Z_0 = 0.
\]

This is just a normalization; it does not affect the results. We then substitute the expression derived above into the welfare functional and integrate to obtain the level of welfare associated to the transition from a generic initial condition \( x_0 \) (Equation (44)):

\[
U_0 = \frac{\log \Lambda}{\rho - \lambda} + \frac{\sigma \lambda + z^*}{(\rho - \lambda)^2} + \frac{\alpha}{1 - \Omega} \frac{x_0}{\nu} \left( \frac{1}{\delta} - 1 \right) + \frac{\sigma \nu}{\rho - \lambda + \nu} \left( 1 - \frac{x^*}{x_0} \right).
\]

**Proposition 7 (Stealing-Monitoring)**

The first-order condition of the manager (reproduced here for convenience),

\[
(1 - e_{m,i}) \frac{\partial \Sigma_i (M_i, S_i)}{\partial S_i} = \frac{\partial c^S(S_i)}{\partial S_i},
\]

defines in implicit form the reaction function of the manager (dropping the \( s \) index of calendar time for simplicity):

\[
S_i (M_i; e_{m,i}), \quad \frac{\partial S_i (M_i; e_{m,i})}{\partial M_i} < 0, \quad \frac{\partial S_i (M_i; e_{m,i})}{\partial e_{m,i}} > 0.
\]

This function characterizes his optimal stealing effort given (i) the monitoring effort of the monitor and (ii) the path of the equity shares set by the founder. Similarly, the first-order condition of the monitor (also reproduced here for convenience),

\[
-e_{a,i} \frac{\partial \Sigma_i (M_i, S_i)}{\partial M_i} = \frac{\partial c^M(M_i)}{\partial M_i},
\]

defines in implicit form the reaction function of the monitor:

\[
M_i (S_i; e_{a,i}), \quad \frac{\partial M_i (S_i; e_{a,i})}{\partial S_i} < 0, \quad \frac{\partial M_i (S_i; e_{a,i})}{\partial e_{a,i}} > 0.
\]

This function characterizes his optimal monitoring effort given (i) the stealing effort of the manager and (ii) the path of the equity shares set by the founder.

The properties of these reaction functions follow solely from the concavity of the stealing function \( \Sigma_i (M_i, S_i) \) with respect to each argument, holding the other constant, and the convexity of
The stealing-monitoring Nash equilibrium

the stealing and monitoring cost functions $c^S(S_i)$ and $c^M(M_i)$. To characterize the NE we need more structure, specifically assumptions on the second cross-partial derivatives. We assume

$$\frac{\partial}{\partial M_i} \left( \frac{\partial \Sigma_i (M_i, S_i)}{\partial S_i} \right) \leq 0$$

to ensure that the reaction function of the manager is (weakly) decreasing in monitoring because monitoring (weakly) reduces the marginal benefit to stealing. A well-defined, stable, NE then only requires that in the $(S_i, M_i)$ space the reaction function of the manager be steeper than that of the monitor. A sufficient condition for this to be the case is that the reaction function of the monitor be increasing or flat (i.e., weakly increasing), that is,

$$\frac{\partial}{\partial S_i} \left( -\frac{\partial \Sigma_i (M_i, S_i)}{\partial M_i} \right) \geq 0.$$ 

Economically, this assumption says that the marginal benefit of monitoring is (weakly) increasing in the manager’s stealing effort. Figure (1) of this Appendix illustrates the resulting NE and the comparative statics effects of the equity shares $e_{m,i}$ and $e_{a,i}$. 

Figure 1: Manager and Monitor Reaction Functions
Proof of Proposition 8 (Equity Shares with Monitoring)

The assumption that the ownership shares are set at time $t$ and then held constant for $s > t$ allows us to write the objective function of the founder as

$$V^\text{founder}_i(t) = \int_t^{+\infty} e^{-\int_t^s r(v)dv} [1 - e_{m,i}(t) - e_{a,i}(t)] \left[1 - \Sigma_i (e_{m,i}(t), e_{a,i}(t))\right] \Pi_i(s) ds$$

$$= [1 - e_{m,i}(t) - e_{a,i}(t)] \left[1 - \Sigma_i (e_{m,i}(t), e_{a,i}(t))\right] \cdot \int_t^{+\infty} e^{-\int_t^s r(v)dv} \Pi_i(s) ds.$$

Moreover, it allows us to rewrite the manager’s choice of $I_i$ as

$$[e_{m,i}(t) (1 - \Sigma_i (e_{m,i}(t), e_{a,i}(t))) + \Sigma_i (e_{m,i}(t), e_{a,i}(t)) - c^S(S_i (e_{m,i}(t), e_{a,i}(t)))] \cdot (1 - \Omega) = q_i(s).$$

The key to this expression is that the left-hand side is constant for $s > t$ and therefore $\dot{q}(s) = 0$ for $s > t$. Consequently, the return to investment reduces to

$$r_Z = \frac{\alpha}{1 - \Omega} \left[\left(1 - \frac{1}{\theta} - 1\right) \frac{X_i}{Z_i} - \phi \left(\frac{Z}{Z_i}\right)^{1-\alpha}\right],$$

which is independent of the equity shares. Accordingly, the paths $P_i(s), I_i(s), Z_i(s), Z(s), X_i(s)$ — and thus, crucially, the path $\Pi_i(s)$ — do not depend on the shares. More precisely, the founder’s problem (i) does not have a dynamic constraint, and thus reduces to a sequence of identical problems, and (ii) features $\partial \Pi_i(s)/\partial e_{m,i}(t) = \partial \Pi_i(s)/\partial e_{a,i}(t) = 0$. Therefore, the founder solves (dropping the now redundant $t$ argument for simplicity):

$$\max_{e_{m,i}, e_{a,i}} [1 - e_{m,i} - e_{a,i}] \left[1 - \Sigma_i (e_{m,i}, e_{a,i})\right].$$

We thus have the following two conditions:

$$1 - \Sigma_i (e_{m,i}, e_{a,i}) = [1 - e_{m,i} - e_{a,i}] \cdot \left(-\frac{\partial \Sigma_i (e_{m,i}, e_{a,i})}{\partial e_{m,i}}\right);$$

$$1 - \Sigma_i (e_{m,i}, e_{a,i}) = [1 - e_{m,i} - e_{a,i}] \cdot \left(-\frac{\partial \Sigma_i (e_{m,i}, e_{a,i})}{\partial e_{a,i}}\right).$$

On the left-hand sides are the marginal costs of incentivizing the manager and the monitor through ownership shares. On the right are the marginal benefits through the reduction (direct and indirect) of the manager’s stealing. Combining the two conditions we obtain:

$$-\frac{\partial \Sigma_i (e_{m,i}, e_{a,i})}{\partial e_{m,i}} = -\frac{\partial \Sigma_i (e_{m,i}, e_{a,i})}{\partial e_{a,i}},$$

which says that the founder is indifferent between obtaining a marginal reduction in stealing directly, by giving ownership shares to the manager, or indirectly, by giving ownership shares to the monitor.
Consumption Ratio with Monitoring

We show that the consumption-output ratio is still represented by Equation (31) even in the presence of a monitoring technology. We assume that the rents extracted by monitoring agents are capitalized in the wealth that accrues to the households as much as the rents extracted by the managers. The market value of the firm is

\[ V_{i}^{\text{market}}(t) = \int_{t}^{+\infty} e^{-f_{r}(v)dv} [1 - \Sigma_{i} (e_{m,i}(s), e_{a,i}(s))] \Pi_{i}(s)ds \]

Household wealth is \( A = \int_{0}^{N} A_{i}di \), which is the aggregate of the wealth generated by each firm. The wealth generated by each firm, in turn, is the sum of three components that accrue, respectively, to manager, the monitoring agent, and founder (for simplicity, in this accounting exercise we denote \( \Sigma_{i} (e_{m,i}(s), e_{a,i}(s)) = \Sigma_{i}(s) \)):

\[ A_{i}(t) = \int_{t}^{+\infty} e^{-f_{r}(v)dv} [e_{m,i}(s) (1 - \Sigma_{i}(s)) + \Sigma_{i}(s)] \Pi_{i}(s) ds + \]
\[ + \int_{t}^{+\infty} e^{-f_{r}(v)dv} [e_{a,i}(s) (1 - \Sigma_{i}(s))] \Pi_{i}(s) ds + \]
\[ + \int_{t}^{+\infty} e^{-f_{r}(v)dv} [1 - e_{m,i}(s) - e_{a,i}(s)] (1 - \Sigma_{i}(s)) \Pi_{i}(s). \]

Consolidating:

\[ A_{i}(t) = \int_{t}^{+\infty} e^{-f_{r}(v)dv} \Pi_{i}(s) ds = V_{i}(t). \]

The terms \( e_{m,i}, e_{a,i}, \Sigma_{i} \) are constant. Accordingly, the value of \( \Theta \) is also constant. We call it \( \Theta_{0} \). When \( n > 0 \), the free entry condition yields

\[ \beta X = V_{i}^{\text{founder}} = \Theta_{0} V_{i} \]

so that, imposing symmetry,

\[ V = \frac{\beta}{\Theta_{0}} \frac{\theta Y}{NP}. \]

The rest of the proof is the same as that of Proposition 3.

Monitoring: Calibration

Assume that \( \mu_{S} > \mu_{M} \), and that \( 1 - (\mu_{S} - \mu_{M}) - \mu_{S} \log \left( \frac{\mu_{S}}{\eta_{S}} \right) + \mu_{M} \log \left( \frac{\mu_{M}}{\eta_{M}} \right) + \mu_{M} \log \left( \frac{\mu_{M}}{\eta_{M}} \right) < 0 \). One can show that the equilibrium with an interior solution in which \( e_{m,i}, e_{a,i}, S_{i}, M_{i} \) and \( \Sigma_{i} \) are all non-negative implied by (54)-(55) is characterized by the following expressions for \( e_{m,i} \) and \( e_{a,i} \)

\[ e_{m,i} = 1 - \exp \left\{ \frac{1 - (\mu_{S} - \mu_{M}) - \mu_{S} \log \left( \frac{\mu_{S}}{\eta_{S}} \right) + \mu_{M} \log \left( \frac{\mu_{M}}{\eta_{M}} \right) + \mu_{M} \log \left( \frac{\mu_{M}}{\eta_{M}} \right)}{\mu_{S} - \mu_{M}} \right\} \in (0,1) \]; (A.19)
\[ e_{a,i} = (1 - e_{m,i}) \frac{\mu_M}{\mu_S} \in (0, 1) . \] (A.20)

Conversely, if for the given set of parameters, the above expressions for \( e_{m,i} \) and \( e_{a,i} \) imply \( \Sigma_i < 0 \), then the equilibrium is characterized by \( S_i = M_i = e_{m,i} = e_{a,i} = 0 \) and, hence, by \( \Theta = 1 \) (no diversion). We choose the parameters \( \mu_M, \eta_M, \mu_S \) and \( \eta_S \) assuming a target \( \Sigma = 0.016\% \) and \( e_m = 10\% \) as in the baseline case in section (7.1). In addition, we target the monitoring equity share \( e_a = 2\% \). Three parameters are sufficient to match the three targets. Nevertheless, there are also two non-negative constraints on \( M \) and \( S \) that need to be satisfied. We start with a parameter \( \mu_S \) close to 1 and then use equations (A.19) and (A.20) and to obtain the remaining two parameters. If any of the two non-negative restrictions is not satisfied we try a different initial value for \( \mu_S \). The solution is reported in Table 2, that also shows the steady state parameters and quantities of the economy with the monitoring technology. Because the implied \( \Theta \) in the economy with monitoring is different from that shown in the non-monitored economy of Section 7 (for instance we now have a 2\% share attributed to the monitoring shareholders), the parameter \( \phi \) is recalibrated to target the same long-run growth rate.

**Derivation of appropriation factor with debt**

The founding shareholder’s free entry condition is

\[ V_i^{\text{founder}}(t) = \int_t^{+\infty} e^{-\int_t^s r(v)dv} \left( (1 - e_{m,i}(s))(1 - S_i(e_{m,i}(s))) \right) \left[ \Pi_i(s) - R_{d,i}(s) \right] ds = (1 - \gamma_d) \beta X(t). \]

This can also be rewritten as

\[
(1 - e_{m,i})(1 - S_i(e_{m,i})) \int_t^{+\infty} e^{-\int_t^s r(v)dv} \Pi_i(s)(s) ds - (1 - e_{m,i})(1 - S_i(e_{m,i})) \int_t^{+\infty} e^{-\int_t^s r(v)dv} R_{d,i}(s) ds = (1 - \gamma_d) \beta X(t).
\]

Next, observe that, with a competitive credit market, the creditors’ participation constraint holds with equality. Hence, on the left hand side we can replace \( \int_t^{+\infty} e^{-\int_t^s r(v)dv} R_{d,i}(s) ds \) with \( \gamma_d \beta X(t) \), obtaining

\[
(1 - e_{m,i})(1 - S_i(e_{m,i})) \int_t^{+\infty} e^{-\int_t^s r(v)dv} \Pi_i(s) ds - (1 - e_{m,i})(1 - S_i(e_{m,i})) \gamma_d \beta X(t) = (1 - \gamma_d) \beta X(t)
\]

or

\[
(1 - e_{m,i})(1 - S_i(e_{m,i})) V_i(t) - (1 - e_{m,i})(1 - S_i(e_{m,i})) \gamma_d \beta X(t) + \gamma_d \beta X(t) = \beta X(t)
\]

where we have also replaced \( \int_t^{+\infty} e^{-\int_t^s r(v)dv} \Pi_i(s) ds \) with \( V_i(t) \). After simple algebra, the above can be rewritten as

\[
(1 - e_{m,i})(1 - S_i(e_{m,i})) V_i(t) - \left[ S_i(e_{m,i}) + e_{m,i}(1 - S_i(e_{m,i})) \right] \gamma_d \beta X(t) = \beta X(t)
\]

or, also, \( \Theta'V(t) = \beta X(t) \).
Normative analysis of welfare

In this section we present welfare analysis by studying three social planner (henceforth SP) arrangements. In one the SP looks for the first-best solution. In a second scenario, the social planner is constrained by market mechanisms, namely the SP takes as given the firm’s investment policy function.

First Best

We now study the unconstrained social planner problem. Recall the household has preferences

\[ U(0) = \int_0^\infty e^{-\rho t} L(t) \log \left( \frac{C(t)}{L(t)} \right) dt, \quad L(t) = e^{\lambda t}, \quad \rho > \lambda > 0. \] (A.21)

The resource constraint is

\[ Y = C + NX + N\phi Z + NI + \beta X \cdot \dot{N}. \] (A.22)

Also,

\[ \dot{Z} = I. \] (A.23)

Finally, note that

\[ Y = NX^\theta \left( \frac{Z}{N^{1-\sigma} L} \right)^{1-\theta} = X^\theta N^{1-(1-\sigma)(1-\theta)} (ZL)^{1-\theta}. \] (A.24)

At this stage it is already possible to see that the social planner problem is much harder than the characterization of the decentralized market equilibrium. First, note that we no longer have a relation of the type \( NX = \beta^2 Y \) coming out of the price-quantity decision of firms because the social planner internalizes all interactions. As a consequence, we cannot work with the reduced-form production function obtained in the decentralized equilibrium, but must rewrite the resource constraint as:

\[ \frac{Y - C - NX - N\phi Z - NI}{\beta X} = \dot{N}. \]

This is because the social planner takes into account that devoting resources to setting up new firms entails the opportunity cost that such resources are no longer available for other activities.

The social planner Hamiltonian is:

\[ H_{\text{planner}} = L \log \left( \frac{C}{L} \right) + d_N \frac{Y - C - NX - N\phi Z - NI}{\beta X} + d_Z I. \]

What can we say here, before taking \( \text{foc} \)?

1. The SP internalizes that consuming the marginal unit of final output reduces the flow of resources available for other uses. The DME compares the marginal utility of that unit of
consumption to the private shadow value wealth. So, a distortion exists if the private value of wealth does not signal the social opportunity cost of consumption. Formally, the SP calculates

\[
\frac{\partial H_{\text{planner}}}{\partial C} = 0 \Rightarrow \frac{1}{C} = \frac{d_N}{\beta X} \Rightarrow d_N = \frac{\beta X}{C}.
\]

The reason why on the right-hand side we have the shadow value of the mass of firms is that the SP internalizes that entry, \( \hat{N} \), is gross output minus all other uses of the final good. The DME treats entry as a residual and determines it according to the free-entry condition instead of a foc that internalizes the opportunity cost of allocating a unit of the final good to entry instead of to the other uses. More precisely, the DME takes the shadow value of entry to be \( d_N = 0 \). For the SP such shadow value is strictly positive. We see immediately, therefore, that this distortion is larger the larger the marginal utility of consumption and the larger the technological entry cost.

2. An additional consideration is that the SP calculates

\[
\frac{\partial H_{\text{planner}}}{\partial d_N} = \hat{N},
\]

obtaining back the resource constraint of the economy. The representative household in the DME makes a similar calculation and obtains back the budget constraint that in equilibrium gives exactly the economy’s resource constraint. So, there are no distortions from this channel. More precisely, the distortion discussed above stems from the joint consideration of consumption and entry behavior. The DME treats them as separate decisions, the SP treats them as two sides of the same decision.

3. The SP internalizes that allocating the marginal unit of output to producing an intermediate good, \( X \), has a social opportunity cost. The DME instead follows only from private value-maximizing consideration. Therefore, a distortion exists because the SP calculates

\[
\frac{\partial H_{\text{planner}}}{\partial X} = 0
\]

\[
\Rightarrow d_N \cdot \frac{(\partial Y/\partial X - N) X - (Y - C - N X - N\phi Z - NI)}{X^2} = 0
\]

\[
\Rightarrow \theta Y - NX = Y - C - NX - N\phi Z - NI
\]

\[
\Rightarrow (1 - \theta) Y = C + N\phi Z + NI.
\]

Note that this is simply the resource constraint written with GDP on the right-hand side. The DME yields \( NX = \theta^2 Y \), which simply ignores the interdependence of all uses of GDP due to the resource constraint.

4. The SP internalizes that allocating the marginal unit of output to firm investment, \( I \), has a social opportunity cost. The DME instead equates the private cost of investment to the private
marginal value, which follows only from private value-maximizing consideration. Therefore, a distortion exists because $d_Z \neq q_i$. Formally, the SP calculates

$$\frac{\partial H_{\text{planner}}}{\partial I} = 0 \Rightarrow \frac{d_N N}{\beta X} = d_Z,$$

while the DME features (in the baseline case)

$$q_i = [e_{m,i} (1 - S_i) + S_i - e^S(S_i)] \cdot (1 - \Omega).$$

5. The SP calculates the shadow value of firm-specific knowledge and of the mass of firms according to:

$$\frac{\partial H_{\text{planner}}}{\partial Z} = (\rho - \lambda) d_Z - \dot{d}_Z \Rightarrow d_N \frac{\partial Y}{\partial Z} - N \phi - \frac{d_N}{\beta X} = (\rho - \lambda) d_Z - \dot{d}_Z;$$

$$\frac{\partial H_{\text{planner}}}{\partial N} = (\rho - \lambda) d_N - \dot{d}_N \Rightarrow d_N \frac{\partial Y}{\partial N} - X - \phi Z - I - \frac{d_N}{\beta X} = (\rho - \lambda) d_N - \dot{d}_N.$$

The corresponding expressions for the DME are the ones characterizing the market returns $r_Z$ and $r_N$. Furthermore, the DME satisfies the arbitrage condition $r_Z = r_N = r_A$. More precisely, we have:

$$\rho - \lambda = \frac{\alpha}{1 - \Omega} \left[ \left( \frac{1}{\theta} - 1 \right) \frac{X}{Z} - \phi \right] - \frac{\dot{C}}{C};$$

$$\rho - \lambda = \frac{\Theta}{\beta} \left( \left( \frac{1}{\theta} - 1 \right) - \frac{\phi Z + I}{X} \right) - \frac{\dot{N}}{N}.$$

The corresponding expressions for the SP are, after some manipulations:

$$\rho - \lambda = \frac{\partial Y}{\partial Z} - N \phi - \frac{\dot{N}}{N} - \frac{\dot{C}}{C};$$

$$\rho - \lambda = \frac{\partial Y}{\partial N} - X - \phi Z - I - \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} - \frac{\dot{C}}{C}.$$

Comparing the expressions for the returns to firm-specific knowledge, we obtain the distortion

$$\left( \frac{\partial Y}{\partial Z} - N \phi - \frac{\dot{N}}{N} - \frac{\dot{C}}{C} \right)_{SP} \neq \left( \frac{\alpha}{1 - \Omega} \left[ \left( \frac{1}{\theta} - 1 \right) \frac{X}{Z} - \phi \right] - \frac{\dot{C}}{C} \right)_{DME}.$$

Doing the same for the returns to the mass of firms, we obtain the distortion

$$\left( \frac{\partial Y}{\partial N} - X - \phi Z - I + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} - \frac{\dot{C}}{C} \right)_{SP} \neq \left( \frac{\Theta}{\beta} \left( \left( \frac{1}{\theta} - 1 \right) - \frac{\phi Z + I}{X} \right) - \frac{\dot{N}}{N} \right)_{DME}.$$

A key item in this comparison is that, as argued above, the marginal products for the social planner obtain from the expression

$$Y = X^\theta N^{1-(\sigma)(1-\theta)} Z^{1-\theta}.$$
Hence,
\[
\frac{\partial Y}{\partial Z} = (1 - \theta) \frac{Y}{Z} \quad \text{and} \quad \frac{\partial Y}{\partial N} = (1 - (1 - \sigma)(1 - \theta)) \frac{Y}{N}.
\]

A few remarks are in order.

1. One can recover the comparison with the DME without corporate governance distortions by setting \(\Omega = 0\) and \(\Theta = 1\).

2. We are leaving aside for brevity a discussion of the role of the possible corner solutions. It is clear that the thresholds at which the SP activates firm investment and entry must be different from those that apply for the DME.

We can make further progress if we focus on the steady state. The condition on \(X\) and the resource constraint yield:

\[
\frac{\theta Y}{NX} = 1 + \beta n.
\]

This equation, moreover, says that if \(x\) and \(n\) are constant then \(Y/NZ\) is constant. The condition on \(C\) yields

\[
\frac{1}{c} = d_N N \cdot \frac{Y}{\beta NX}.
\]

This now says that \(d_N N\) must be constant since we want \(c\) constant. The condition for \(I\) says that \(\frac{\dot{d}_Z}{d_Z} = -\frac{\dot{X}}{X}\). The conditions for \(Z\) and \(N\) then yield:

\[
\frac{\partial Y}{\partial Z} \frac{Y}{NZ} - \phi = \rho - \lambda - \frac{\dot{d}_Z}{d_Z} \Rightarrow (1 - \theta) \frac{Y}{NZ} - \phi = \rho - \lambda + z;
\]
\[
\frac{\partial Y N Y}{\partial N Y N} - X - \phi Z - I \beta X = \rho - \lambda - \frac{\dot{d}_N}{d_N} \Rightarrow (1 - (1 - \sigma)(1 - \theta)) \frac{Y}{NZ} - x - \phi - z \beta X = \rho - \lambda + n.
\]

In summary, we have:

\[
\frac{\theta Y}{NX} = 1 + \beta n \quad \text{vs} \quad \frac{\theta Y}{NX} = \frac{1}{\theta};
\]
\[
\frac{1}{C} = d_N \frac{1}{\beta X} \quad \text{vs} \quad \frac{1}{C} = \frac{1}{Y} \frac{1}{1 - \theta + \frac{(\rho - \lambda)\beta}{\Theta}};
\]
\[
(1 - \theta) \frac{Y}{NZ} - \phi - z = \rho - \lambda \quad \text{vs} \quad \frac{\alpha}{1 - \Omega} \left[ \frac{1}{\beta} - 1 \right] \frac{X}{Z} - \phi = \sigma n - z = \rho - \lambda;
\]
\[
\frac{(1 - (1 - \sigma)(1 - \theta))}{\beta X} \frac{Y}{NZ} - x - \phi - z \beta x = \rho - \lambda \quad \text{vs} \quad \frac{\Theta}{\beta} \left[ \frac{1}{\beta} - 1 \right] \frac{\phi + I}{X} = \rho - \lambda;
\]
\[
\frac{Y}{NZ} = x^\theta \left[ \frac{L}{N^{1-\sigma}} \right]^{1-\theta}.
\]

Can we solve for these equations for our key variables? Use the first equation to get rid of \(Y/NX = (1 + \beta n)/\theta\):

\[
(1 - \theta) x (1 + \beta n)/\theta - \phi = \rho - \lambda + z;
\]
\[
\frac{(1 - (1 - \sigma)(1 - \theta))}{\beta x} x (1 + \beta n)/\theta - x - \phi - z = \rho - \lambda + n.
\]
Also, in steady state \( n = \lambda / (1 - \sigma) \). Therefore, we have:

\[
z = \frac{1 - \theta}{\theta} x (1 + \beta n) - \phi - (\rho - \lambda) \quad \text{vs} \quad z = \frac{\alpha}{1 - \Omega} \left( \frac{1}{\theta} - 1 \right) x - \alpha \phi - \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right);
\]

\[
z = \left[ \frac{\sigma (1 - \theta)}{\theta} (1 + \beta n) - \beta (\rho - \lambda) \right] x - \phi \quad \text{vs} \quad z = \left[ \frac{1}{\theta} - 1 - \beta \frac{\theta}{\Omega} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] x - \phi.
\]

These are two lines that intersect in \((x, z)\) space as for the DME. One can thus carry out a straightforward graphical comparison. Solving for the SP, moreover, yields:

\[
x_{SP} = \frac{\rho - \lambda}{(1 - \sigma)(1 - \theta)} (1 + \beta n) + \beta (\rho - \lambda);
\]

\[
z_{SP} = \frac{\sigma (1 - \theta)}{\theta} (1 + \beta n) - \beta (\rho - \lambda) \frac{(\rho - \lambda) - \phi}{(1 - \sigma)(1 - \theta)} (1 + \beta n) + \beta (\rho - \lambda)
\]

These expressions can be compared to those that we obtained for the decentralized market equilibrium, reported here for convenience:

\[
x^* = \frac{\left( 1 - \frac{\alpha}{1 - \Omega} \right) \phi - \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right)}{\left( 1 - \frac{\alpha}{1 - \Omega} \right) \left( \frac{1}{\theta} - 1 \right) - \beta \frac{\theta}{\Omega} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right)};
\]

\[
z^* = \frac{\alpha}{1 - \Omega} \phi + \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \left( \frac{1}{\theta} - 1 \right) - \beta \frac{\theta}{\Omega} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right).
\]

Additional thoughts. Can we compare the position of the CI and EI loci for SP and DME? Since \( n = \lambda / (1 - \sigma) \) is the same in steady state:

\[
z = \frac{1 - \theta}{\theta} x (1 + \beta n) - \phi - (\rho - \lambda) \quad \text{vs} \quad z = \frac{\alpha}{1 - \Omega} \left( \frac{1}{\theta} - 1 \right) x - \alpha \phi - \left( \rho + \sigma n \right);
\]

\[
z = \left[ \frac{\sigma (1 - \theta)}{\theta} (1 + \beta n) - \beta (\rho - \lambda) \right] x - \phi \quad \text{vs} \quad z = \left[ \frac{1}{\theta} - 1 - \beta \frac{\theta}{\Omega} \left( \rho + \sigma n \right) \right] x - \phi.
\]

Is CI locus for SP higher or lower than that for DME for all \( x \)? Condition:

\[
\left( 1 + \beta n - \frac{\alpha}{1 - \Omega} \right) \left( \frac{1}{\theta} - 1 \right) x > \phi (1 - \alpha) - n.
\]

Sufficient to set RHS<0. But we do not have strong reasons to say that the curves are in any specific relative position.

Is EI locus for SP higher or lower than that for DME for all \( x \)? Condition:

\[
\frac{\sigma (1 - \theta)}{\theta} (1 + \beta n) - \beta (\rho - \lambda) > \frac{1}{\theta} - 1 - \beta \frac{\theta}{\Omega} \left( \rho + \sigma n \right).
\]
Note: The plots compare the first-best social planner allocation (SP) with the decentralized equilibrium (DME) for economies that have a different set of parameters. In general the position of the SP relative to DME depends on the underlying technology, preferences, entry costs, and the like.

This is a restriction on parameters only. Working along these lines we can illustrate what parameters yield which outcome. With no corporate governance distortion, $\theta = 1$, we have

$$\beta\lambda \left(1 + \frac{1}{\theta} \frac{1}{1 - \sigma}\right) > \left(\frac{1}{\theta} - 1\right) (1 - \sigma).$$

Assume this condition holds. We can construct two examples (and more that we do not report for brevity).

**Empire Building: Alternative Specifications**

In the variation of the model of Section 10.1, the manager’s utility flow is (57). The manager forms the following Hamiltonian

$$H = [e_{m,i} (1 - S_i) + S_i - c^S(S_i)] \cdot \Pi_i + \Omega I_i + qI_i,$$
The optimal price set by the manager is the same as in the baseline case, \( P_i = \frac{1}{\theta} \) and the stealing condition remains (15). The condition on investment now instead implies

\[
q_i + \Omega = \Theta_{m,i},
\]

while the dynamic equation on \( Z_i \) is

\[
\Theta_{m,i} \frac{\partial \Pi_i}{\partial Z_i} = -\dot{q}_i + rq_i.
\]

Combining, we obtain

\[
r = \frac{\alpha \Theta_{m,i}}{\Theta_m - \Omega} \left\{ \left[ \frac{1}{\theta} - 1 \right] \frac{X_i}{Z_i} - \phi \left( \frac{Z_i}{Z} \right)^{\alpha - 1} \right\} + \frac{\dot{q}_i}{q_i}.
\]

Let us now derive the CI and EI loci. The returns to investment and to entry become

\[
r_Z = \frac{\alpha \Theta_m}{\Theta_m - \Omega} \left[ \left( \frac{1}{\theta} - 1 \right) x - \phi \right] + \frac{\dot{q}}{q},
\]

\[
r_N = \frac{\Theta}{\beta} \left[ \left( \frac{1}{\theta} - 1 \right) - \phi + z \right] + \frac{\dot{x}}{x} + z
\]

and the loci become

\[
z = \frac{\alpha \Theta_m}{\Theta_m - \Omega} \left[ \left( \frac{1}{\theta} - 1 \right) x - \phi \right] - \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right),
\]

\[
z = \left[ \frac{1}{\theta} - 1 - \frac{\beta}{\Theta} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] x - \phi.
\]

The steady state values of firm size and investment become

\[
x^* = \frac{\left( 1 - \frac{\Theta_m}{\Theta_{m-1}} \right) \phi - \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right)}{\left( 1 - \frac{\Theta_m}{\Theta_{m-1}} \right) \left( P - 1 \right) - \frac{\gamma \beta}{\Theta} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right)},
\]

\[
z^* = \frac{\left[ \frac{\Theta_{m-1}}{\Theta_{m-1}} \phi + \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right) \right] - \frac{\alpha \Theta_{m-1}}{\Theta_{m-1}} \left( P - 1 \right)}{\left( 1 - \frac{\Theta_{m-1}}{\Theta_{m-1}} \right) \left( P - 1 \right) - \frac{\gamma \beta}{\Theta} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right)} \left( \rho + \frac{\sigma \lambda}{1 - \sigma} \right).
\]
Figure 3: Gains from Growth in Manager’s Equity Shares

- Note: The curve represents gains from deviations from a zero growth in the Manager’s Equity share, $e_m$. This is at its optimal value of 10%. The zero-growth guess appears to be warranted. The numerical search is run on a grid of is 0.0001%.