COMMODITY PRICES AND GROWTH*

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In this paper we propose an endogenous growth model of commodity-rich economies in which: (i) long-run (steady-state) growth is endogenous and yet independent of commodity prices; (ii) commodity prices affect short-run growth through transitional dynamics; and (iii) the status of net commodity importer/exporter is endogenous. We argue that these predictions are consistent with historical evidence from the 19th to the 21st century.

Historical evidence from the 19th to the 21st century suggests that in commodity-rich countries commodity prices are uncorrelated with growth in the long run but correlated with growth in the short run. This observation raises the question of what is the economic mechanism that causes the short-run co-movement between commodity prices and growth to vanish in the long run. To address it, we develop a general equilibrium model of innovation-led growth with endogenous market structure in which exogenous movements in commodity prices affect output growth in the short run, through transitional dynamics in total factor productivity (TFP), but have no effect on growth in the long run.

The model has three key ingredients: (i) entrant firms create new products; (ii) incumbent firms allocate resources to own productivity improvement; (iii) market structure is endogenous as firm size and number of firms are jointly determined in free-entry equilibrium.

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The interplay between entrants and incumbents regulates incentives to product and process innovation and sterilizes the effects of commodity price changes as the economy settles on a balanced growth path (BGP).

To understand this mechanism, note that, based on Peretto (1998, 1999), the model combines variety-expanding and productivity-improving innovation to provide a tractable characterization of an economy's dynamics.[1] Manufacturing is the engine of long-run growth. In this sector, incumbents engage in two activities: (i) they use labor and materials to produce intermediate goods supplied to the downstream consumption sector (materials are purchased from an upstream sector which uses labor and the commodity as inputs); (ii) they allocate labor to own productivity improvement. Movements in commodity prices affect the economy through two channels: (i) they change the value of the endowment thus inducing income/wealth effects ("commodity wealth channel"); (ii) they affect the demand for the commodity in the materials sector and, through the demand of materials in manufacturing and inter-sectoral labor reallocation, have cascade dynamic effects through all the vertical cost structure of production ("cost channel").

The economy features "long-run commodity price super-neutrality" in that steady-state TFP growth is independent of commodity prices. The mechanism driving this result is sterilization of market-size effects: given the number of firms, movements in commodity prices change the size of the manufacturing sector, firm size and so incentives to innovation. Everything else equal, this would have steady-state growth effects. However, as the size and thus profitability of incumbents change, firms' entry adjusts to bring the economy back to the initial steady-state level of firm size, thereby sterilizing the long-run growth effects of commodity price changes. We argue that neutrality of commodity prices for long-run growth is critical for the model to be consistent with two basic time-series observations: commodity prices exhibit large and persistent long-run movements (see Jacks, 2013), whereas trend growth in several commodity-rich economies (e.g., Western offshoots) does not.

[1] See Madsen (2008) for empirical evidence supporting the time-series predictions of this class of models.

In the short run, whether the economy experiences a growth acceleration or deceleration in response to exogenous movements in commodity prices depends on the overall substitutability between labor and commodity use. More precisely, the substitutability between labor and materials in manufacturing and between labor and commodity use in materials production drives the overall elasticity of demand for the commodity. After a commodity price increase, therefore, (i) the value of manufacturing production raises if commodity demand is inelastic ("global complementarity"), (ii) it falls if commodity demand is elastic ("global substitutability"), (iii) it remains unchanged if manufacturing and materials sectors have Cobb-Douglas production functions ("Cobb-Douglas-like economy"), and (iv) it raises or falls depending on the level of the price if materials and manufacturing display opposite complementarity/substitutability properties. Movements in manufacturing production in turn affect rates of return to firm entry and innovation by incumbents, which feeds into a temporary acceleration or deceleration of aggregate TFP.

The empirical literature provides a spectrum of findings ranging from little/no effect (see Gelb, 1988; Sala-i-Martin and Subramanian, 2003; Black *et al.*, 2005; Caselli and Michaels, 2013), positive (see Greasley and Madsen, 2010; Allcott and Keniston, 2017; Smith, 2014), to negative effect (see Ismail, 2010; Rajan and Subramanian, 2011; Harding and Venables, 2016; Charnavoki and Dolado, 2014). Our model identifies in the substitutability between labor and commodity a key conditioning factor capable of rationalizing this pattern. Importantly, the overall substitutability between labor and commodity is subsumed in three sufficient statistics such as (i) the price elasticity of the demand for materials in manufacturing, (ii) the price elasticity of the demand for the commodity in materials, and (iii) the commodity share in materials production costs, which can be either mapped into observables or estimated.[2]

[2]Note that in U.S. data the degree of substitutability largely varies across types of resources. For instance, Jin and Jorgenson (2010) document evidence of complementarity for several products of mining/harvesting activities (metal mining, oil and gas, coal mining, primary metals, non-metallic mining, tobacco products) and of substitutability for others (lumber and wood, stone and clay, non-tobacco agricultural products), but they generally reject the Cobb-Douglas unit elasticity specification.

1 Motivating Facts

In this section, we discuss the key empirical observations that motivate our paper.

A bird's-eye view of the data.—Empirical work on long-run trends in commodity prices and growth has been hindered by the shortness of the time period for which reliable data are available. Recently, however, it has become possible to combine the data compiled by Angus Maddison (see Bolt and van Zanden, 2014) for real GDP per capita with the data compiled by Jacks (2013) for commodity prices. The data span the 19th, 20th, and 21st century, approximately 150 years of data, and thus allows us to relate the long-run trend components in commodity prices and growth for several commodity-rich countries. This allows us to draw a marked distinction between the *steady-state* (long-run) and *transitional dynamics* (short-run) link between commodity prices and growth.

Consistently with the literature on commodity price super-cycles (see Cuddington and Jerrett, 2008; Jerrett and Cuddington, 2008; Erten and Ocampo, 2013; Jacks, 2013), we adopt the following definition of Long-Run (LR).

DEFINITION 1 (Long-run trend). *Given a time series, x_t, the Long-Run (LR) trend, x_t^{LR}, corresponds to the component of x_t with periodicity larger than 70 years.*

We use a band-pass filter, as implemented by Christiano and Fitzgerald (2003), to isolate the Short-Run (SR), x_t^{SR}, which corresponds to the component of x_t with periodicity between 2 and 70 years. The Long-Run (LR) trend component is then $x_t^{LR} \equiv x_t - x_t^{SR}$. The choice of the band-pass filter is dictated by our aim at contrasting the long-run (low-frequency) with the short-run (high-frequency) properties of the data. The first fact follows directly from applying Definition 1 to the data.

FACT 1. *Commodity prices exhibit large and persistent long-run movements whereas growth rates of real GDP per capita exhibit no such large persistent changes.*

Fact 1, that we take as one of the key observations for our analysis, posits an important

disconnect between the long-run properties of commodity prices and growth. Hence, we argue it provides a litmus test for endogenous growth models along the lines of Jones (1995): if long-run (steady-state) growth depended on commodity prices, then we would observe correlated swings in growth rates which is at odd with the data. This argument is analogous to that made by Stokey and Rebelo (1995) in the context of taxation and growth.

Figure 1 illustrates Fact 1 for the United States and energy prices. The LR component in real GDP per capita is almost a straight line implying that trend growth has been approximately constant for the last 150 years.[3] On the other hand, commodity prices exhibit large and persistent movements in the LR component. This observation is not specific to energy prices, but it is robust across several commodities, such as animal products, grains, metals, minerals, precious metals, and softs (see Jacks, 2013, and the Online Appendix).

The second fact that informs our analysis links commodity price movements and economic growth in the short run.

FACT 2. *Commodity prices and growth rates of real GDP per capita co-move in the short-run.*

Evidence for Fact 2 comes from a variety of sources. Despite the mixed evidence on the sign of the relationship, the empirical literature says that commodity prices are correlated with growth in the short-run. The first source of evidence is the literature on the curse of natural resources. For instance, Sachs and Warner (1995, 1999, 2001) find a statistically significant negative relationship between natural resource intensity (measured as exports of natural resources in percent of GDP) and average growth over a twenty-year period. The existence of such resource curse has been called into question by several papers (see Deaton and Miller, 1996; Brunnschweiler and Bulte, 2008; Alexeev and Conrad, 2009; Smith, 2015), but only in the sense that a resource boom has a positive instead of a negative growth

[3]See Online Appendix for time series of real GDP per capita in several other commodity-rich countries: Argentina, Australia, Brazil, Canada, Chile, Colombia, and New Zealand. Due to data limitations, we have been unable to extend the sample to other developing commodity-rich economies.

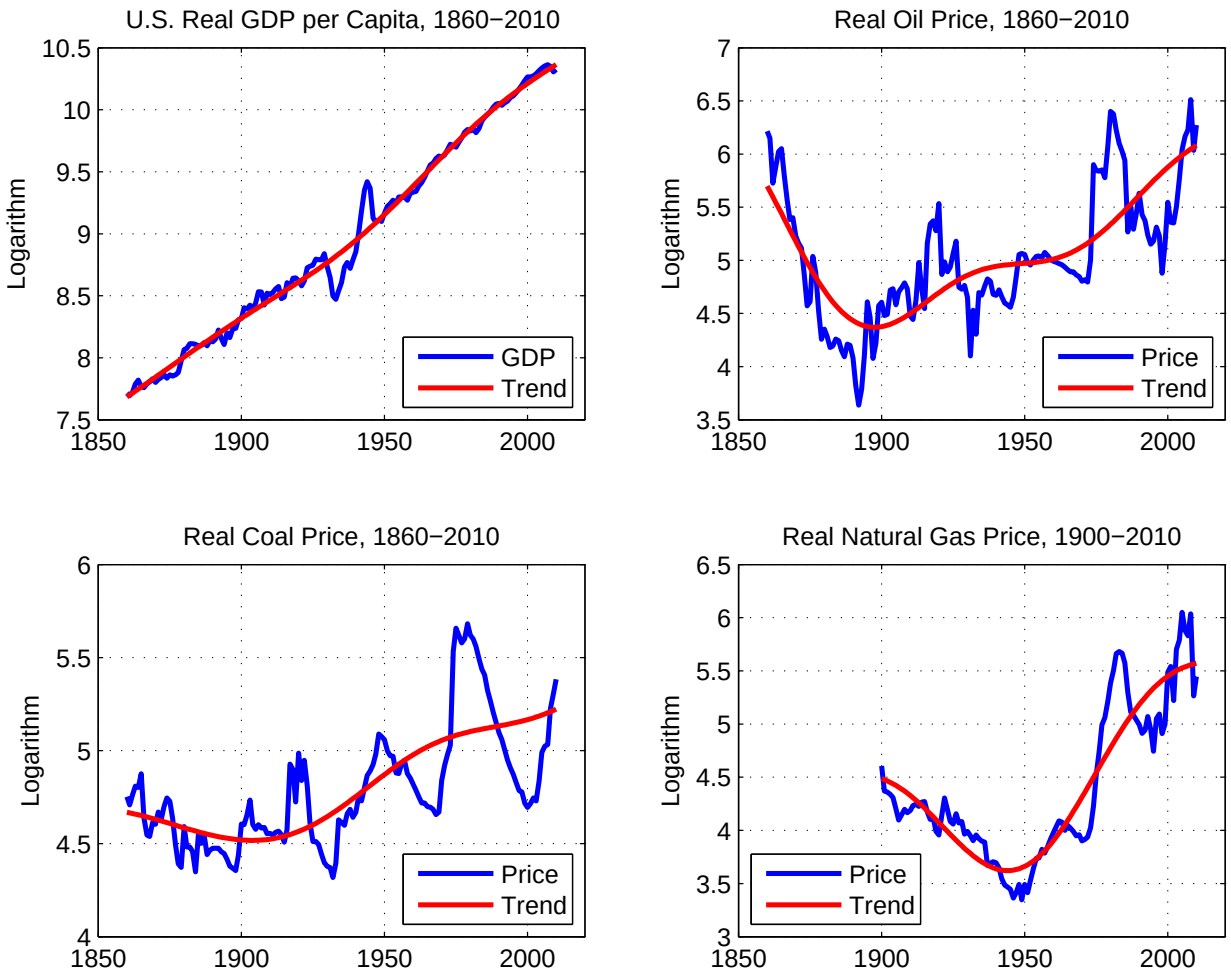

Figure 1: *U.S. Real GDP per Capita and Energy Prices*

Notes: Data for the U.S. real GDP per capita are from the Angus Maddison's dataset available at http://www.ggdc.net/maddison/maddison-project/home.htm. Real energy prices are available from David Jacks's website at http://www.sfu.ca/~djacks/data/boombust/index.html. Trend is the long-run (LR) trend component of the series as in Definition 1.

effect as the resource curse hypothesis would predict. We share this literature's focus on the low-frequency relationship between commodity prices and growth. However, we draw a sharp distinction between what we consider to be a long-run (steady-state) as opposed to a short-run (transitional dynamics) commodity price/growth relationship. The second source of evidence is the literature on oil prices and the business cycle (see Hamilton, 1996, 2003, 2009; Kilian, 2008a,b, 2009). This strand of literature focuses instead on the high-frequency relationship between oil prices and growth, as such it abstracts from the possibility of growth effects of oil prices in the long run.

Regression analysis: levels vs. growth rates.—Next, we provide more systematic evidence on the short- and long-run correlation between commodity prices and real GDP per capita. To this goal, we run ordinary least squares (OLS) regressions, both in levels and in growth rates. Table 1 shows estimation results for energy prices (oil, coal, and natural gas) and the United States. Table 2 reports results for all the Western Offshoots, but we replace energy prices with an equally-weighted index of forty commodity prices (including oil, animal products, grains, metals, minerals, precious metals, and softs).

In each table, panel A shows estimates from level regressions with a time trend:

$$\ln y_t = \alpha_0 + \alpha_1 t + \alpha_2 \ln p_t + e_t, \tag{1}$$

where y_t is real GDP per capita and p_t is the price of a specific commodity. In the estimating equation (1), let $\hat{y}_t \equiv \ln y_t - \hat{\alpha}_0 - \hat{\alpha}_1 t$ indicate percent deviations of the real GDP per capita from trend ("short-run" growth), such that $\mathbb{E}\left[\hat{y}_t | \mathcal{F}_t\right] = \hat{\alpha}_2 \ln p_t$ represents the expectation of such deviations from trend, conditional on the current realization of the information set $\mathcal{F}_t$. (Note that $\hat{\alpha}$ denote the OLS coefficient estimate of the corresponding parameter α.) Thus, a coefficient estimate of $\alpha_2 \neq 0$ implies that movements in commodity prices are correlated with the temporary deviations of the *level* of real GDP per capita from the long-run trend: commodity prices co-move with short-run growth in real GDP per capita.

Table 1: *Energy Prices and Growth — United States*

	Oil price	Coal price	Natural gas price
	(1)	(2)	(3)
A. Dependent variable: $\ln$ GDP			
ln price	0.060**	0.146**	0.051*
	(0.025)	(0.067)	(0.031)
Time trend	0.018***	0.017***	0.019***
	(0.000)	(0.000)	(0.001)
R^2	0.983	0.984	0.971
Observations	151	161	111
B. Dependent variable: $\Delta \ln$ GDP			
$\Delta \ln \text{GDP}_{-1}$	0.244*	0.245*	0.313**
	(0.136)	(0.133)	(0.138)
ln price	−0.001	−0.000	−0.001
	(0.005)	(0.010)	(0.007)
R^2	0.060	0.060	0.099
Observations	151	159	111

Notes: The dependent variable is the real GDP per capita from the Angus Maddison's dataset available at http://www.ggdc.net/maddison/maddison-project/home.htm. The independent variables are real energy prices available from David Jacks's website at http://www.sfu.ca/~djacks/data/boombust/index.html. All regressions include a constant. Newey-West standard errors reported in parentheses: lag length q is chosen based on the formula $q = \text{int}[4\,(n_T/100)^{1/4}]$, where "int" is a shorthand for "integer of" and n_T indicates the number of observations. ***, **, * indicate statistical significance at 1, 5, and 10 percent level, respectively.

Table 2: *Commodity Prices and Real GDP per Capita*

	United States	Australia	Canada	New Zealand
	(1)	(2)	(3)	(4)
A. Dependent variable: ln GDP				
ln price index	0.141	0.163*	0.334***	0.240***
	(0.090)	(0.090)	(0.094)	(0.091)
Time trend	0.020***	0.018***	0.022***	0.015***
	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.972	0.967	0.972	0.965
Observations	111	111	111	111
B. Dependent variable: Δ ln GDP				
Δ ln GDP$_{-1}$	0.310**	0.470***	0.355***	−0.031
	(0.139)	(0.103)	(0.090)	(0.109)
ln price index	0.014	−0.009	0.011	0.005
	(0.020)	(0.010)	(0.020)	(0.019)
R^2	0.103	0.229	0.135	0.001
Observations	111	111	111	111

Notes: The dependent variable is the real GDP per capita from the Angus Maddison's dataset available at http://www.ggdc.net/maddison/maddison-project/home.htm. The independent variable is a commodity price index available from David Jacks's website at http://www.sfu.ca/~djacks/data/boombust/index.html. All regressions include a constant. Newey-West standard errors reported in parentheses: lag length q is chosen based on the formula $q = \text{int}[4\,(n_T/100)^{1/4}]$, where "int" is a shorthand for "integer of" and n_T indicates the number of observations. ***, **, * indicate statistical significance at 1, 5 and 10 percent level, respectively.

We estimate a statistically significant $\hat{\alpha}_2 > 0$ for energy prices in all Western offshoots (see Online Appendix for estimation results for Australia, Canada, and New Zealand). As shown in Table 2, this result is robust to the use of a commodity price index. Commodity prices are positively correlated with short-run growth, thus confirming the empirical observation in Fact 2 above.

In each table, panel B shows estimates from growth regressions:

$$\Delta \ln y_t = \beta_0 + \beta_1 \Delta \ln y_{t-1} + \beta_2 \ln p_t + u_t, \tag{2}$$

where $\Delta \ln y_t \equiv \ln y_t - \ln y_{t-1}$. The estimating equation (2) implies that $\mathbb{E}\left[\Delta \ln y_t | \mathcal{F}_t\right] = \hat{\beta}_0 + \hat{\beta}_1 \Delta \ln y_{t-1} + \hat{\beta}_2 \ln p_t$. Hence, a coefficient estimate of $\hat{\beta}_2 = 0$ implies that, conditional on the past realization of the growth rate, movements in commodity prices are uncorrelated with the growth rate of real GDP per capita. In a nutshell, commodity prices do *not* Granger-cause growth. In addition, note that the long-run mean growth implied by (2) is $\mathbb{E}_\infty\left[\Delta \ln y_t\right] = \hat{\beta}_0/(1-\hat{\beta}_1) + \hat{\beta}_2 \mathbb{E}_\infty\left[p_t\right]$, such that a $\hat{\beta}_2 = 0$ also implies that long-run growth is independent of the commodity price. We estimate an economically and statistically insignificant $\hat{\beta}_2$ for energy prices in all Western offshoots. As shown in Table 2, these estimation results hold not only for the single energy prices, but also for the commodity price index.[4] Commodity prices are uncorrelated with long-run growth, thus confirming the empirical observation in Fact 1 above.

[4]Two remarks are in order here. First, we run a different specification of the estimating equation in (2), where we replace the independent variable in levels, $\ln p_t$, with that in growth rates, $\Delta \ln p_t \equiv \ln p_t - \ln p_{t-1}$. The coefficient estimate on $\Delta \ln p_t$ remains economically and statistically insignificant. Second, we note that the lack of statistically significance of $\ln p_t$ in (2) justifies the level specification in (1), where it is assumed that the time trend α_1 is constant and thus independent of commodity prices. Incidentally, the coefficient estimates of $\hat{\alpha}_2 > 0$ and $\hat{\beta}_2 = 0$ are consistent with the reduced-form implications of the theory.

2 Model

Time is continuous and indexed by $t \geq 0$. We consider a small open economy (SOE) populated by a representative household that supplies labor inelastically in a competitive labor market. The household chooses expenditures on home and foreign goods and savings by borrowing and lending in a competitive market for financial assets at the prevailing interest rate. Household income consists of returns on financial assets, labor income, profits, and commodity income. Commodity prices, then, directly affect household income, by changing the value of the commodity endowment. Such "commodity wealth channel" is the first of the two key transmission mechanisms of commodity price changes at play in the model.

The production side of the economy consists of four sectors: (1) final consumption good; (2) intermediate goods or manufacturing; (3) materials; (4) extraction. The consumption good sector consists of a representative competitive firm that combines intermediate goods to produce an homogeneous final good. Upon entry in the manufacturing sector, firms combine labor and materials to produce differentiated intermediate goods. Firms also engage in activities aimed at improving own productivity. Entry requires the payment of a sunk cost. Materials are supplied by an upstream competitive sector, which uses labor services and the commodity as inputs. Finally, the extraction sector sells the commodity endowment to the materials sector and potentially abroad. Changes in commodity prices propagate through the entire vertical cost structure of the economy, thus affecting relative prices, factor demands, and incentives to innovation. This "cost channel" is the second key transmission mechanism of changes in commodity prices at play in the model.

Manufacturing is the engine of endogenous growth. Specifically, the economy starts out with a given range of intermediate goods, each supplied by one firm. Entrepreneurs compare the present value of profits from introducing a new good with the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to losses. Once in the market, firms devote labor to productivity

improvement. As each firm strives to improve productivity, it contributes to the pool of public knowledge that benefits the future innovation activities of all firms. This process is self-sustaining and it allows the economy to grow at a constant rate in steady state, which is reached when entry stops and the economy settles into a stable industrial structure.

2.1 Household Sector

The representative household chooses expenditures on home and foreign goods to maximize lifetime utility:

$$U(t) \equiv \int_t^{\infty} e^{-\rho(s-t)} \log u\,(s)\, ds, \tag{3}$$

with

$$\log u = \varphi \log\left(\frac{Y_H}{P_H L}\right) + (1-\varphi)\log\left(\frac{Y_F}{P_F L}\right), \tag{4}$$

subject to the flow budget constraint,

$$\dot{A} = rA + WL + \Pi_H + \Pi_M + p\Omega - Y_H - Y_F, \tag{5}$$

where $\rho > 0$ is the discount rate, $0 < \varphi < 1$ controls the degree of home bias in preferences, A is assets holding, r is the rate of return on financial assets, W is the wage rate, L is population size, which equals labor supply since there is no preference for leisure, Y_H is expenditure on home consumption goods whose price is P_H, and Y_F is expenditure on foreign consumption goods whose price is P_F. In addition to asset, rA, and labor income, WL, the household receives the dividends paid out by the producers of the home consumption goods, Π_H, the dividends paid out by firms in the materials sector, Π_M, and the revenues from sales of the domestic commodity endowment, $\Omega > 0$, at the price p. The solution to the household's problem yields the optimal consumption/expenditure allocation rule,

$$\varphi Y_F = (1-\varphi)\, Y_H, \tag{6}$$

and the Euler equation governing saving behavior,

$$r = r_A \equiv \rho + \frac{\dot{Y}_H}{Y_H} = \rho + \frac{\dot{Y}_F}{Y_F}. \tag{7}$$

We interpret r_A in equation (7) as the reservation rate of return on financial assets at which the household is willing to trade current with future consumption. Note that movements in commodity prices directly affect the household sector by changing the value of the commodity endowment, thus inducing income/wealth effects ("commodity wealth channel").

2.2 Tradable Sector

The economy can be either an importer or exporter of the commodity. In the former case it sells the home consumption good in exchange for the commodity, in the latter it accepts the foreign consumption good as payment for its commodity exports. As in the SOE tradition, we posit the existence of a commodity market that accomodates any demand and/or supply at the exogenous price p. The foreign consumption good is imported at the exogenous and constant price P_F. Only final goods and the commodity are tradable. The balanced trade condition, which is also the market clearing condition for the consumption good market, is $Y_H + Y_F + p\,(O - \Omega) = Y$, where Y is the aggregate value of production of the home consumption good and O denotes the home use of the commodity. Using the consumption expenditure allocation rule (6), we can rewrite the balance trade condition as follows:

$$\frac{1}{\varphi} Y_H + p\,(O - \Omega) = Y. \tag{8}$$

Using equation (8), it yields that: (1) $O > \Omega$ (commodity importer) implies $Y > (1/\varphi)Y_H$, such that the economy exchanges home consumption goods for the commodity. Conversely, (2) $O < \Omega$ (commodity exporter) implies $Y < (1/\varphi)Y_H$, such that the economy exchanges the home commodity endowment for foreign consumption goods.

2.3 Final Good Sector

The home (homogeneous) consumption good is produced by a representative competitive firm with the following technology:

$$C_H = N^{\chi}\left[\frac{1}{N}\int_0^N X_i^{\frac{\epsilon-1}{\epsilon}}di\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{9}$$

where $\epsilon > 1$ is the elasticity of product substitution, X_i is the quantity of the non-durable intermediate good i, and N is the mass of goods. Based on Ethier (1982), we separate the elasticity of substitution between intermediate goods from the degree of increasing returns to variety, $\chi > 0$. The solution to the final producer's problem yields the isoelastic demand curve for intermediate goods:

$$X_i = \frac{YP_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon}di}, \tag{10}$$

where $Y = P_H C_H$ is the value of production of the home consumption good. Since the final good sector is perfectly competitive, $\Pi_H = 0$ for all $t \geq 0$.

2.4 Intermediate Goods Sector

The typical manufacturing firm produces intermediate goods with the following technology:

$$X_i = Z_i^{\theta} F\left(L_{X_i} - \phi, M_i\right), \tag{11}$$

where X_i is output, L_{X_i} is production employment, $\phi > 0$ is a fixed labor cost, M_i is the use of materials, and Z_i^{θ}, with $0 < \theta < 1$, is firm's TFP, which is a function of the stock of firm-specific knowledge, Z_i. $F(\cdot)$ is a standard production function, which is homogeneous of degree one in its arguments. Total production costs are:

$$W\phi + C_X(W, P_M)Z_i^{-\theta}X_i, \tag{12}$$

where $C_X(\cdot)$ is the unit-cost function, which is homogeneous of degree one in its arguments. Hicks-neutral technological change internal to the firm reduces unit production costs. The firm accumulates knowledge according to the following technology:

$$\dot{Z}_i = \alpha K L_{Z_i}, \tag{13}$$

where $\dot{Z}_i$ is the flow of firm-specific knowledge generated by productivity-enhancing activities, employing L_{Z_i} units of labor, and αK, with $\alpha > 0$, is labor productivity in such activities, which depends on the stock of public knowledge, K. Public knowledge accumulates over time as a result of spillovers across firms:

$$K = \sigma(N) \int_0^N Z_i di, \tag{14}$$

which posits that the stock of public knowledge K is the weighted sum of firm-specific stocks of knowledge, Z_i. The weight $\sigma(N)$ is a function of the number of existing varieties N and captures in reduced form the extent of spillovers effects. (Peretto and Smulders (2002) provide the micro-foundations for this class of spillovers function.) We use $\sigma(N) = 1/N$ which represents the *average* technological distance between differentiated products: when a firm i adopts a more efficient process to produce its own differentiated good X_i, it also generates not-excludable knowledge which spills over into the public domain. However, the extent to which this new knowledge can be used by another firm, say $j \neq i$, depends on how far in the technological space the differentiated products X_i and X_j are. This notion of technological distance is captured in reduced form by the term $\sigma(N)$=1/N, which formalizes the idea that as the number of varieties increases the average technological distance between existing products increases as well. This in turn translates into lesser spillovers effects from any given stock of firm-specific knowledge.

2.5 Materials Sector

A representative competitive firm uses labor services, L_M, and the commodity, O, as inputs to produce materials, M, which are purchased by the manufacturing sector at price P_M. The production technology of materials is $M = G(L_M, O)$, where $G(\cdot)$ is a standard production function, which is homogeneous of degree one in its arguments. Total production costs are:

$$C_M(W, p)\, M, \tag{15}$$

where $C_M(\cdot)$ is the associated unit-cost function, which is homogeneous of degree one in the wage, W, and commodity price, p.

2.6 Taking Stock: Vertical Cost Structure

Given the vertical structure of production, a commodity price change has cascade effects: (1) it directly affects production costs and so the price, P_M, in the upstream materials sector through the unit-cost function $C_M(W, p)$; (2) the change in P_M in turn affects production costs and so the price, P_i, in manufacturing through the unit-cost function $C_X(W, P_M)$; (3) the change in P_i finally affects production costs and so the price, P_H, in the consumption good sector through the demand for intermediate goods. Thus, the initial change in the commodity price affects the home Consumer Price Index (CPI). The materials sector competes for labor with manufacturing. This captures the inter-sectoral allocation problem of this economy.

3 Firms' Behavior and General Equilibrium

In this section, we first detail firms' behavior in manufacturing and materials sectors. Then, we impose general equilibrium conditions and study equilibrium dynamics and BGP.

3.1 Firms' Behavior in Manufacturing

We now turn to describe the problem faced by incumbents and entrant firms and then provide some insight into the transmission mechanisms of commodity price changes.

Incumbent firms.—Firm i chooses the time path for production employment, L_{X_i}, employment in productivity-improving activities, L_{Z_i}, and materials, M_i, to maximize

$$V_i(t) \equiv \int_t^{\infty} e^{-\int_t^s [r(v)+\delta]dv} \Pi_i(s) ds, \tag{16}$$

where $\delta > 0$ is a "death shock," which is required for the model to have symmetric dynamics in the neighborhood of the steady state. Using the total cost function in (12), instantaneous profits are:

$$\Pi_i \equiv \left[P_i - C_X(W, P_M) Z_i^{-\theta}\right] X_i - W\phi - W L_{Z_i}. \tag{17}$$

The firm maximizes $V_i(t)$ in (16) subject to the knowledge production technology (13) and the demand curve (10), taking as given the initial stock of knowledge, $Z_i(t) > 0$, and the rivals' knowledge accumulation paths, $Z_j(t')$ for $t' \geq t$.

Entrant firms.—An entrepreneur can employ $\beta Y/N$ units of labor to create a new firm that starts out its activity with productivity equal to the industry average. The associated sunk cost of entry is thus $\beta WY/N$.[5] Once in the market, the entrant firm solves a problem identical to the one outlined above for the incumbent firm. Thus, a free-entry equilibrium requires $V_i(t) = \beta W(t)Y(t)/N(t)$ for all $t \geq 0$.

Conditional factor demands.—The free-entry equilibrium is symmetric and manufac-

[5]See Peretto and Connolly (2007) for an interpretation of this assumption and alternative formulations of the sunk entry cost that maintain the fundamental equilibrium properties of the theory unchanged.

turing features conditional factor demands (see Online Appendix for further details):

$$WL_X = Y\left(\frac{\epsilon-1}{\epsilon}\right)S_X^L + \phi WN, \tag{18}$$

$$P_M M = Y\left(\frac{\epsilon-1}{\epsilon}\right)S_X^M. \tag{19}$$

The shares of the firm's variable costs due to labor and materials are:

$$S_X^L \equiv \frac{WL_{X_i}}{C_X(W,P_M)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W,P_M)}{\partial \log W}, \tag{20}$$

$$S_X^M \equiv \frac{P_M M_i}{C_X(W,P_M)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W,P_M)}{\partial \log P_M}. \tag{21}$$

Note that $S_X^L + S_X^M = 1$. We stress that commodity price changes affect the demand for labor services and materials through (i) equilibrium market-size effects, as captured by the aggregate value of manufacturing production, Y, and (ii) partial equilibrium effects, that play through the vertical cost structure of this multi-sector economy, as captured by the labor and materials shares of variable costs S_X^L and S_X^M.

Returns to innovation.—The rates of return to productivity improvement, r_Z, and entry, r_N, are:

$$r = r_Z \equiv \frac{\alpha}{W}\left[\frac{Y}{\epsilon N}\theta(\epsilon-1) - W\frac{L_Z}{N}\right] + \frac{\dot{W}}{W} - \delta, \tag{22}$$

$$r = r_N \equiv \frac{N}{\beta Y}\left[\frac{Y}{\epsilon N} - W\phi - W\frac{L_Z}{N}\right] + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} + \frac{\dot{W}}{W} - \delta. \tag{23}$$

Neither return depends directly on factors related to the commodity market. Explaining why this happens is key to understanding the short- and long-run transmission mechanism of commodity price changes. The technology in (11) yields a unit-cost function that depends

only on input prices and it is independent of the quantity produced, and thus of inputs use. Since the optimal pricing rule features a constant markup over unit costs, the firm's gross-profit flow rate, $x \equiv Y/\epsilon N$, is independent of input prices. The conditions in (22) and (23), then, capture the idea that investment decisions by both incumbents and entrants do not directly respond to conditions in the commodity market, because they are guided by the gross-profit rate only. Conditions in the commodity market have instead an indirect feedback effect through aggregate spending on intermediate goods, which is nevertheless sterilized by net entry/exit of firms. The interaction of entry and incumbents' innovation in productivity improvements is the key mechanism that makes the short-run relationship between commodity price movements and growth asymptotically vanish in the long-run.

3.2 Firms' Behavior in Materials Sector

The competitive producers of materials operate along an infinitely elastic supply curve—the price of materials equals marginal cost of materials production:

$$P_M = C_M\left(W, p\right). \tag{24}$$

In equilibrium, then, materials production is given by the conditional factor demand in (19), evaluated at the price P_M. Defining the commodity share in materials costs as

$$S_M^O \equiv \frac{pO}{C_M\left(W,p\right)M} = \frac{\partial \log C_M(W,p)}{\partial \log p}, \tag{25}$$

we can write the conditional demand for labor services and the commodity as follows:

$$WL_M = Y\left(\frac{\epsilon-1}{\epsilon}\right) S_X^M\left(1 - S_M^O\right), \tag{26}$$

$$pO = Y\left(\frac{\epsilon-1}{\epsilon}\right) S_X^M S_M^O. \tag{27}$$

3.3 General Equilibrium

We now turn to the general equilibrium of the model. The households' budget constraint (5) and the balanced trade condition (8) imply clearing in the labor market: $L = L_N + L_X + L_Z + L_M$, where L_N are labor services allocated to enter manufacturing, L_X and L_Z are employment in production and productivity-improving investment of incumbents, respectively, and L_M is employment in the materials sector. Equilibrium in the asset market requires rates of return equalization and that the value of the household's portfolio equals the total value of the securities issued by the corporate sector: $r = r_A = r_Z = r_N$ and that $A = NV = \beta Y$. We choose labor as the numeraire—that is, $W \equiv 1$—which is a convenient normalization since it implies that all expenditures are constant.

The following proposition characterizes the value of manufacturing production, balanced trade, and expenditures on home and foreign consumption goods.

PROPOSITION 1. *At any point in time, the equilibrium value of manufacturing production and the implied balanced trade condition are, respectively:*

$$Y(p) = \frac{L}{1 - \xi(p) - \rho\beta}, \quad \textit{with} \quad \xi(p) \equiv \left(\frac{\epsilon - 1}{\epsilon}\right) S_X^M(p)\, S_M^O(p)\,; \tag{28}$$

$$\frac{1}{\varphi} Y_H(p) - p\Omega = Y(p)\left(1 - \xi(p)\right). \tag{29}$$

The expenditures on home and foreign consumption goods are, respectively:

$$Y_H(p) = \varphi\left[\frac{L(1 - \xi(p))}{1 - \xi(p) - \rho\beta} + p\Omega\right], \quad \textit{and} \quad Y_F(p) = (1 - \varphi)\left[\frac{L(1 - \xi(p))}{1 - \xi(p) - \rho\beta} + p\Omega\right]. \tag{30}$$

Since $Y_H(p)$ and $Y_F(p)$ are constant, the real interest rate is $r = \rho$ for all $t \geq 0$.

Proof. See Online Appendix.

Proposition 1 states that the equilibrium value of manufacturing production depends on

the commodity price, p, through the co-share function $\xi(p)$. Hence, if, how, and to what extent, the level of home manufacturing production responds to changes in p depends on the properties of the underlying technologies of intermediate goods and materials production, subsumed in the cost shares $S_X^M(p)$ and $S_M^O(p)$. Perhaps not surprisingly, now, equilibrium expenditures on home and foreign consumption goods depend on the level of the commodity price as well, but via two distinct channels: (i) the price of the commodity, p, determines the value of the commodity endowment, $p\Omega$, thus contributing to the total income of the household sector; and (ii) the price of the commodity affects expenditures through $\xi(p)$, by determining the cost share of materials in manufacturing and that of the commodity in materials. Note also that the commodity price implicitly pins down the status of commodity importer/exporter through balance trade.

The following proposition characterizes equilibrium dynamics.

PROPOSITION 2. *Let $x \equiv Y/\epsilon N$ denote the gross profit rate. The general equilibrium of the model reduces to the following piece-wise linear differential equation in x:*

$$\dot{x} = \begin{cases} \frac{(\delta L/\epsilon N_0)e^{\delta t}}{1-\xi(p)-\frac{1}{\epsilon}} & \text{if} \quad \phi \leq x \leq x_N \\ \frac{\phi}{\beta\epsilon} - \left[\frac{1}{\beta\epsilon} - (\rho+\delta)\right] x & \text{if} \quad x_N < x \leq x_Z \\ \frac{\phi - \frac{\rho+\delta}{\alpha}}{\beta\epsilon} - \left[\frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho+\delta)\right] x & \text{if} \quad x > x_Z, \end{cases} \tag{31}$$

where $x_N = \phi/(1-\epsilon\rho\beta)$ and $x_Z = (\rho+\delta)/\alpha\theta(\epsilon-1)$. Assuming

$$\frac{\phi - (\rho+\delta)/\alpha}{1-\theta(\epsilon-1)-\beta\epsilon(\rho+\delta)} > \frac{\rho+\delta}{\alpha\theta(\epsilon-1)}, \tag{32}$$

the economy converges to the steady state

$$x^* = \frac{\phi - (\rho + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)} > x_Z. \tag{33}$$

The associated steady-state growth rate of productivity improvement is

$$\hat{Z}^* = \frac{(\phi\alpha - \rho - \delta)\theta(\epsilon - 1)}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)} - (\rho + \delta) > 0. \tag{34}$$

Proof. See Online Appendix.

Proposition 2 states the "long-run commodity price super-neutrality" result: the steady-state growth rate of productivity improvement, which is the only source of steady-state (long-run) growth in the model, is independent of the commodity price. The mechanism that drives this result is sterilization of market-size effects. To see this, (1) fix the number of firms at $\bar{N}$, then note that a change in the commodity price, p, affects the size of the manufacturing sector $Y(p)$, firm's gross profitability $x = Y(p)/\left(\epsilon\bar{N}\right)$, and thereby incentives to innovation. Everything else equal, this would have steady-state growth effects. (2) Now let the mass of firms vary as in the free-entry equilibrium; as the profitability of incumbent firms varies, the mass of firms endogenously adjusts (net entry/exit) to bring the economy back to the initial steady-state value of firm size. As a result, the entry process fully sterilizes the long-run growth effects of the initial price change.

4 Short-Run Effects of Commodity Price Changes

In this section, we discuss (i) how a permanent change in the commodity price affects the value of manufacturing production and (ii) how the status of commodity importer/exporter is endogenously determined within the model as function of the commodity endowment and price and of technological parameters.

4.1 Manufacturing Production

The following lemma derives a set of elasticities that are key determinants of the comparative statics of the value of manufacturing production with respect to the commodity price.

LEMMA 1. *Let the price elasticities of the demand for materials in manufacturing and for the commodity in materials be, respectively:*

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \left(\frac{\partial S_X^M}{\partial P_M} \times \frac{P_M}{S_X^M} \right), \tag{35}$$

$$\epsilon_M^O \equiv -\frac{\partial \log O}{\partial \log p} = 1 - \frac{\partial \log S_M^O}{\partial \log p} = 1 - \left(\frac{\partial S_M^O}{\partial p} \times \frac{p}{S_M^O} \right). \tag{36}$$

Then, the expression for the co-share function $\xi(p)$ in (28) yields:

$$\xi'(p) = \left(\frac{\epsilon - 1}{\epsilon} \right) \frac{\partial \left(S_M^O(p) S_X^M(p) \right)}{\partial p} = \frac{\xi(p) \Gamma(p)}{p}, \tag{37}$$

where

$$\Gamma(p) \equiv \left(1 - \epsilon_X^M(p) \right) S_M^O(p) + 1 - \epsilon_M^O(p). \tag{38}$$

Proof. See Online Appendix.

The key object in Lemma 1 is $\Gamma(p)$, which is the elasticity of $\xi(p) \equiv \left(\frac{\epsilon - 1}{\epsilon} \right) S_M^O(p) S_X^M(p)$ with respect to the commodity price, p. According to (27), $\Gamma(p)$ is the elasticity of the home demand for the commodity with respect to the commodity price, holding constant the value of manufacturing production. It thus captures the partial equilibrium effects of price changes in the manufacturing and materials sectors for given market size. Differentiating the log of $Y(p)$ in (28) with respect to p, rearranging terms, and using (38) yields:

$$\frac{d \ln Y(p)}{dp} = \frac{\xi'(p)}{1 - \xi(p) - \beta\rho} = \frac{\xi(p) \Gamma(p)}{p \left[1 - \xi(p) - \beta\rho \right]}. \tag{39}$$

The expression in (39) shows that the effects of a commodity price change critically depend on the overall pattern of substitutability/complementarity subsumed in the price elasticities of materials, ϵ_X^M, and commodity demand, ϵ_M^O, and in the commodity share of materials production costs, S_M^O. The following proposition states these results formally.

PROPOSITION 3. *Depending on the properties of the function* $\Gamma(p)$*, the comparative statics of* $Y(p)$ *with respect to* p *exhibit four cases:*

1. ***Global complementarity****. Suppose that* $\Gamma(p) > 0$ *for all* p*. Then, the value of manufacturing production* $Y(p)$ *in (28) is a monotonically increasing function of* p*.*

2. ***Global substitutability****. Suppose that* $\Gamma(p) < 0$ *for all* p*. Then, the value of manufacturing production* $Y(p)$ *in (28) is a monotonically decreasing function of* p*.*

3. ***Cobb-Douglas-like economy****. Suppose that* $\Gamma(p) = 0$ *for all* p*. This occurs when* S_M^O *and* S_X^M *are exogenous constants. Then, the value of manufacturing production* $Y(p)$ *in (28) is independent of* p*.*

4. ***Endogenous switching from complementarity to substitutability.*** *Suppose there exists a threshold price* $\tilde{p}$ *at which* $\Gamma(p)$ *changes sign, from positive to negative. Then, the value of manufacturing production* $Y(p)$ *in (28) is a hump-shaped function of* p *with a maximum at* $\tilde{p}$*.*

Proof. See Online Appendix.

The key result from Proposition 3 is that the sign of the comparative statics critically depends on the substitution possibilities between labor and materials in manufacturing and between labor and the commodity in materials. The equilibrium of the model suggests that a commodity price boom induces a decline in manufacturing activity when the economy exhibits overall substitutability between labor services and the commodity. The reason is that when demand is overall elastic, the commodity price change at the top of the vertical

production structure causes a large change in the quantity used; such change reflects the full set of adjustments, forward and backward, that take place in the economy. By contrast, a commodity price boom raises the value of manufacturing production when the economy exhibits overall complementarity between labor and the commodity. Note that changes in the commodity endowment have no effect on manufacturing activity but affect only expenditures on home and foreign consumption goods.

Available empirical estimates for the United states by Jin and Jorgenson (2010) point to overall complementarity for non-agricultural commodities and to overall substitutability for agricultural products. According to these estimates, then, our model predicts a qualitatively different comparative statics for, say, energy prices, as opposed to agricultural commodities. Specifically, a rise in energy-like prices would be associated with a boom in economic activity, featuring firms' entry and a temporary acceleration of TFP growth.

The Cobb-Douglas-like case in Proposition 3 occurs when the production technologies in materials and manufacturing are both Cobb-Douglas, such that $\epsilon_X^M = \epsilon_M^O = 1$. We do not discuss this case further as it is a knife-edge specification in which commodity price changes have no effect on the value of manufacturing production. Arguably, the most interesting case is when the function $\Gamma(p)$ switches sign as the model generates an endogenous switch from global complementarity to substitutability. This happens when production in materials and manufacturing displays opposite substitution/complementarity properties; for instance, when materials production exhibits labor-commodity complementarity while manufacturing exhibits labor-materials substitutability. In this latter case, there exists a threshold price $\tilde{p}$, such that $\Gamma(p) < 0$, for $p < \tilde{p}$, and $\Gamma(p) > 0$, for $p > \tilde{p}$. When the commodity price is low, the cost share $S_M^O(p)$ is relatively small and the function $\Gamma(p)$ is then dominated by the term $1 - \epsilon_M^O(p)$, which is positive since complementarity implies $\epsilon_M^O(p) < 1$ (inelastic commodity demand). Conversely, when the commodity price is high, the cost share $S_M^O(p)$ is relatively large and $\Gamma(p)$ is dominated by the term $1 - \epsilon_X^M(p)$, which is negative since substitutability implies $\epsilon_X^M(p) > 1$ (elastic materials demand).

4.2 Commodity Trade

An important building block of our model is that the commodity is used as an input in domestic production of materials. As a result, the status of commodity importer/exporter is determined within the model as a function of commodity endowment, Ω, commodity price, p, technological properties subsumed in the term $\xi(p)$, and other relevant parameters. The following proposition provides a clean result.

PROPOSITION 4. *The economy is an exporter of the commodity when*

$$\frac{\Omega}{L} > \frac{\xi(p)}{p[1-\xi(p)-\beta\rho]}. \tag{40}$$

Proof. See Online Appendix.

Proposition 4 formalizes the notion of "commodity supply dependence" captured by the model. For a given commodity price, p, there exists a threshold for the commodity-population ratio $\bar{\omega}$, such that: (i) for $\Omega/L < \bar{\omega}$ the economy is a commodity importer, i.e., $O > \Omega$; and conversely; (ii) for $\Omega/L > \bar{\omega}$ the economy is a commodity exporter, i.e., $O < \Omega$. This is a *specialization* result: the equilibrium features a trade-off between the rate at which the economy transforms the commodity endowment into home consumption goods—internal transformation rate (ITR)—and the rate at which it transforms the commodity endowment into foreign consumption goods—external transformation rate (ETR). Such trade-off depends on the country's own commodity endowment, the commodity price, and technological parameters of the domestic vertical structure of production. Our mechanism says that if ITR dominates ETR the economy is a commodity importer; otherwise, it is a commodity exporter. An alternative way to interpret commodity trade is to note that, for a given relative endowment Ω/L, there exists a commodity price threshold $\bar{p}$ such that for $p < \bar{p}$ the economy is a commodity importer whereas for $p > \bar{p}$ the economy is a commodity exporter. Economies with a larger commodity endowment are then commodity exporters for

a larger range of commodity prices, whereas economies with no commodity endowment are constrained to be commodity importers at all price levels.

5 Long-Run Effects of Commodity Price Changes

We now discuss the model's implications for aggregate TFP and welfare.

5.1 Firms' Innovation and Aggregate TFP

In the Online Appendix, we show that in this economy TFP is

$$T = N^{\chi} Z^{\theta}. \tag{41}$$

Accordingly, TFP growth is a weighted sum of the rates of vertical and horizontal innovation.

Steady-state TFP dynamics.—Using the solution for the steady-state growth rate of productivity improvement in (34), the steady-state growth rate of aggregate TFP is

$$\hat{T}^* = \theta \hat{Z}^* = \theta \left[\frac{(\phi\alpha - \rho - \delta)\,\theta\,(\epsilon - 1)}{1 - \theta\,(\epsilon - 1) - \beta\epsilon\,(\rho + \delta)} - (\rho + \delta) \right] \equiv g, \tag{42}$$

where $\hat{T}(t) \equiv \dot{T}(t)/T(t)$. Thus, the steady-state TFP dynamics is entirely driven by productivity-improving innovation, and thus fully insulated from the commodity price.

Transitional TFP dynamics.—Out-of-steady-state TFP dynamics is driven by firms' entry and productivity improvements, jointly, which are in turn regulated by the gross-profit rate. In the neighborhood of the steady-state $x^* > x_Z$, the dynamics of the gross-profit rate is governed by the following differential equation:

$$\dot{x} = \nu\,(x^* - x)\,, \tag{43}$$

where

$$\nu \equiv \frac{1-\theta\left(\epsilon-1\right)-\beta\epsilon\left(\rho+\delta\right)}{\beta\epsilon}, \quad \text{and} \quad x^{*} \equiv \frac{\phi-\left(\rho+\delta\right)/\alpha}{1-\theta\left(\epsilon-1\right)-\beta\epsilon\left(\rho+\delta\right)}. \tag{44}$$

The solution to equation (43) then yields:

$$x\left(t\right)=x_{0}e^{-\nu t}+x^{*}\left(1-e^{-\nu t}\right), \tag{45}$$

where $x_0 \equiv x(0)$ is the initial condition for $x(t)$. The following proposition characterizes the entire time path of aggregate TFP.

PROPOSITION 5. *Consider an economy starting at time $t=0$ with initial condition x_0. At any point in time $t>0$, the (log) level of TFP is*

$$\log T\left(t\right)=\log\left(Z_{0}^{\theta}N_{0}^{\chi}\right)+gt+\left(\frac{\gamma}{\nu}+\chi\right)\Delta\left(1-e^{-\nu t}\right), \quad \textit{where} \quad \Delta\equiv\frac{x_{0}}{x^{*}}-1. \tag{46}$$

Proof. See Online Appendix.

Equation (46) in Proposition 5 shows that commodity prices affect the level of aggregate TFP only through the displacement term, Δ. The steady-state growth rate of TFP, g, and the speed of reversion to the steady state, ν, are both independent of the commodity price. In a nutshell, unanticipated changes in the commodity price cause transitory deviations of TFP from its trend level, as captured by $\log T\left(t\right)-\log\left(Z_{0}^{\theta}N_{0}^{\chi}\right)-gt$, that eventually die out in the long-run. We associate these transitory deviations from trend to short-run growth. To provide further insight into the short-run propagation mechanism at play in the model, it is useful to describe in more detail the dynamic response to a commodity price shock.

TFP dynamic response to a commodity price shock.—Let us consider the case of an unanticipated and permanent change in the commodity price, that temporarily displaces the gross-profit rate, x, from its steady-state value, x^*, by Δ percent. Such a "gross-profit rate shock" forces the model to be in transition dynamics. To explain the mapping between

a commodity price shock and the displacement term in (46), we consider a permanent fall in the commodity price from p to $p' < p$, and an economy under global substitutability, such that $\Delta_Y \equiv Y(p') - Y(p) > 0$. The long-run super-neutrality result in Proposition 2 implies that $x^*(p') = x^*(p)$. Hence, after an unexpected (and permanent) fall in the commodity price, the value of manufacturing production jumps from $Y(p)$ to the new steady-state level $Y(p')$. In contrast, the mass of firms, N, is a predetermined variable, such that it does not respond on impact. The initial impact response, Δ_Y, is followed by transitional dynamics driven by net entry, $\dot{N} > 0$. Eventually, the mass of firms endogenously adjusts from $N(p)$ to $N(p')$, such that in steady state the initial jump Δ_Y is fully sterilized. Hence, firm size is the key driver of the economy's dynamic adjustment to the commodity price shock. The impact response is driven by the response of the value of manufacturing production, which instantaneously adjusts to the new equilibrium level. After the initial impact response, dynamics is driven by the adjustment of the mass of firms via net entry.

5.2 Welfare Implications

The analytical tractability of the model allows for a sharp characterization of the pass-trough mechanisms of commodity price changes to the overall welfare of the economy. The following proposition provides a useful result.

PROPOSITION 6. *Consider an economy starting at time $t = 0$ with initial condition x_0. At any point in time $t > 0$ the instantaneous utility flow is*

$$\log u(t) = \log \varphi \left(\frac{p\Omega}{L} + \frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} \right) - \varphi \log c(p) + \varphi g t + \varphi \left(\frac{\gamma}{\nu} + \chi \right) \Delta \left(1 - e^{-\nu t} \right), \quad (47)$$

where $\Delta \equiv x_0/x^* - 1$. The resulting level of welfare is

$$U(0) = \frac{1}{\rho} \left[\log \varphi \left(\frac{p\Omega}{L} + \frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} \right) - \varphi \log c(p) + \frac{\varphi g}{\rho} + \frac{\varphi \left(\frac{\gamma}{\nu} + \chi \right)}{\rho + \nu} \Delta \right]. \quad (48)$$

Proof. See Online Appendix.

Equation (48) in Proposition 6 identifies four channels through which commodity prices affect welfare: (1) the so-called "windfall effect" through the term $p\Omega/L$; (2) the commodity-labor substitutability effect through the term $\big(1-\xi(p)\big)/\big(1-\xi(p)-\rho\beta\big)$; (3) the "cost of living/CPI effect" through the term $c\,(p)\equiv C_X\big(W,C_M(W,p)\big)$; and (4) the "curse or blessing effect" through the transitional dynamics associated with the displacement term Δ and the steady-state growth rate, g.

Channel (1) captures static forces that the literature on the curse of natural resources has discussed at length. That is, an economy with a commodity endowment experiences a windfall when the price of the commodity raises. However, in our model this is not analogous to a lump-sum transfer from abroad in that the commodity is used for home production of materials and the value of manufacturing production adjust endogenously to the commodity price change; this adjustment is captured by the substitutability effect (2). In our environment the analogous of a pure lump-sum transfer corresponds to an increase in the commodity endowment, Ω. The cost of living/CPI effect (3) is due to the fact that the economy uses the commodity for the domestic production of materials; thus, an increase in the commodity price works its way through the vertical structure of production—from upstream materials production to downstream manufacturing—and it manifests itself as a higher price of the home consumption good (a higher CPI). The curse/blessing effect (4) captures instead dynamic forces that are critical for our analysis, and that we discussed in previous sections. The steady-state growth rate of TFP is independent of the commodity price due to the sterilization of market-size effects. However, there are transitional effects: (i) cumulated gains/losses from the acceleration/deceleration of the rate of quality improvement; (ii) cumulated gains/losses from the acceleration/deceleration of product variety expansion. These two transitional effects amplify the change in the value of manufacturing production induced by the change in the commodity price.

Commodity dependence, commodity price boom, and welfare.—Overall, the equilibrium of the model suggests that an economy with a positive commodity endowment can gain in terms of welfare from a commodity price boom *in spite of* being a commodity importer. The reason is that revenues from the sales of the endowment, $p\Omega$, go up one-for-one with p while commodity demand, pO, does not. Specifically, commodity consumption, O, responds negatively to an increase in p; this effect is strong if home commodity demand is elastic, thus under global substitutability.

The key insight derived from the equilibrium of the model is that what matters for welfare is not the commodity trade balance, but how manufacturing activity reacts to commodity price changes. Under global substitutability, the contraction of the commodity demand after a price boom mirrors the contraction of manufacturing activity, which is the manifestation of the specialization effect discussed above. The Schumpeterian mechanism at the heart of the model amplifies such a contraction—the instantaneous fall in Y—into a deceleration of the rate of TFP growth. The economy eventually reverts to the initial steady-state growth rate g, but the temporary deceleration contributes negatively to welfare.

With this narrative in mind, let us now consider a permanent increase in the commodity price from p to $p' > p$ at $t = t_0$, and the case of a commodity-importing economy under global substitutability. Since aggregate TFP is predetermined at $t = t_0$, the impact response of $\log u(t_0)$ is driven by the jump in the home CPI index, the windfall effect, and the commodity-labor substitutability effect. However, these forces work in opposite directions so that the initial jump in utility has an ambiguous sign. After the initial impact response, the transition path of $\log u(t)$ for $t > t_0$ is governed by the transitional dynamics of aggregate TFP. The permanent fall in manufacturing activity, from $Y(p)$ to $Y(p') < Y(p)$, produces a slowdown in TFP growth due to a slowdown of net entry and a reduction in firm-level innovation. As a result, a commodity price boom is welfare improving if and only if the windfall effect through $p\Omega$ is large enough to compensate for the commodity-labor substitutability effect, the cost of living effect, and the curse effect through $\Delta < 0$. The closed-form solution for

welfare (48) in Proposition 6 shows how model's parameters determine the relative strength of these effects.

6 Conclusion

In this paper we studied the relationship between commodity prices, commodity trade, and growth within an endogenous growth model of commodity-rich economies that possesses an important property. Namely, long-run growth is endogenous and yet independent of commodity prices, whereas commodity prices affect short-run growth through transitional dynamics in aggregate TFP. We argue that these predictions are consistent with historical data from the 19th to the 21st century: commodity prices exhibit large and persistent long-run movements whereas trend growth in real GDP per capita in the Western Offshoots (Australia, Canada, New Zealand, and United States) has been approximately constant for the last 150 years. The novel insight of the analysis is that changes in commodity prices induce movements in real income, and thereby market-size effects that alter incentives to innovation. As a result, commodity prices and growth co-move in the short run. However, such market-size effects are sterilized by the endogenous adjustment of the mass of firms so that the short-run comovement between commodity prices and growth vanishes in the long run. Our results indicate that the overall substitutability between labor and the commodity is key to understanding how movements in commodity prices affect commodity-rich economies. The theory suggests that the commodity-labor substitutability properties of the multi-sector economy are subsumed in few sufficient statistics such as (i) the price elasticity of demand for materials in manufacturing, (ii) the price elasticity of the demand for the commodity in materials, and (iii) the commodity share in materials production costs. It would seem, then, important to obtain reliable estimates of those price elasticities and commodity cost-share to discipline quantitative variants of the theory developed here. We leave this task for future research.

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References

Alexeev, M. and Conrad, R. (2009). 'The elusive curse of oil', *Review of Economics and Statistics*, vol. 91(3), pp. 586–598.

Allcott, H. and Keniston, D. (2017). 'Dutch disease or agglomeration? The local economic effects of natural resource booms in modern America', *Review of Economic Studies*, in press.

Black, D., McKinnish, T. and Sanders, S. (2005). 'The economic impact of the coal boom and bust', *Economic Journal*, vol. 115(503), pp. 449–476.

Bolt, J. and van Zanden, J. (2014). 'The Maddison project: collaborative research on historical national accounts', *Economic History Review*, vol. 67(3), pp. 627–651.

Brunnschweiler, C. and Bulte, E. (2008). 'The resource curse revisited and revised: a tale of paradoxes and red herrings', *Journal of Environmental Economics and Management*, vol. 55(3), pp. 248–264.

Caselli, F. and Michaels, G. (2013). 'Do oil windfalls improve living standards? Evidence from Brazil', *American Economic Journal: Applied Economics*, vol. 5(1), pp. 208–238.

Charnavoki, V. and Dolado, J. (2014). 'The effects of global shocks on small commodity-exporting economies: lessons from Canada', *American Economic Journal: Macroeconomics*, vol. 6(2), pp. 207–237.

Christiano, L. and Fitzgerald, T. (2003). 'The band pass filter', *International Economic Review*, vol. 44(2), pp. 435–465.

Cuddington, J. and Jerrett, D. (2008). 'Super cycles in real metals prices?', *IMF Staff Papers*, vol. 55(4), pp. 541–565.

Deaton, A. and Miller, R. (1996). 'International commodity prices, macroeconomic performance, and politics in sub-saharan Africa', *Journal of African Economies*, vol. 5(3), pp. 99–191.

Erten, B. and Ocampo, J. (2013). 'Super cycles of commodity prices since the mid-nineteenth century', *World Development*, vol. 44, pp. 14–30.

Ethier, W. (1982). 'National and international returns to scale in the modern theory of international trade', *American Economic Review*, vol. 72(3), pp. 389–405.

Gelb, A. (1988). *Oil windfalls: blessing or curse?*, Oxford University Press.

Greasley, D. and Madsen, J. (2010). 'Curse and boon: natural resources and long-run growth in currently rich economies', *Economic Record*, vol. 86(274), pp. 311–328.

Hamilton, J. (1996). 'This is what happened to the oil price-macroeconomy relationship', *Journal of Monetary Economics*, vol. 38(2), pp. 215–220.

Hamilton, J. (2003). 'What is an oil shock?', *Journal of Econometrics*, vol. 113(2), pp. 363–398.

Hamilton, J. (2009). 'Causes and consequences of the oil shock of 2007-08', *Brookings Papers on Economic Activity*, vol. 2009(SPRING 2009), pp. 215–261.

Harding, T. and Venables, A. (2016). 'The implications of natural resource exports for non-resource trade', *IMF Economic Review*, in press.

Ismail, K. (2010). 'The structural manifestation of the 'Dutch disease': the case of oil exporting countries', *IMF Working Paper*, vol. 10(103), pp. 1–35.

Jacks, D. (2013). 'From boom to bust: a typology of real commodity prices in the long run', NBER Working Paper No. 18874, National Bureau of Economic Research.

Jerrett, D. and Cuddington, J. (2008). 'Broadening the statistical search for metal price super cycles to steel and related metals', *Resources Policy*, vol. 33(4), pp. 188–195.

Jin, H. and Jorgenson, D. (2010). 'Econometric modeling of technical change', *Journal of Econometrics*, vol. 157(2), pp. 205–219.

Jones, C. (1995). 'Time series tests of endogenous growth models', *Quarterly Journal of Economics*, vol. 110(2), pp. 495–525.

Kilian, L. (2008a). 'A comparison of the effects of exogenous oil supply shocks on output and inflation in the G7 countries', *Journal of the European Economic Association*, vol. 6(1), pp. 78–121.

Kilian, L. (2008b). 'Exogenous oil supply shocks: how big are they and how much do they matter for the U.S. economy?', *Review of Economics and Statistics*, vol. 90(2), pp. 216–240.

Kilian, L. (2009). 'Not all oil price shocks are alike: disentangling demand and supply shocks in the crude oil market', *American Economic Review*, vol. 99(3), pp. 1053–69.

Madsen, J. (2008). 'Semi-endogenous versus Schumpeterian growth models: testing the knowledge production function using international data', *Journal of Economic Growth*, vol. 13(1), pp. 1–26.

Peretto, P. (1998). 'Technological change and population growth', *Journal of Economic Growth*, vol. 3(4), pp. 283–311.

Peretto, P. (1999). 'Cost reduction, entry, and the interdependence of market structure and economic growth', *Journal of Monetary Economics*, vol. 43(1), pp. 173–195.

Peretto, P. and Connolly, M. (2007). 'The Manhattan metaphor', *Journal of Economic Growth*, vol. 12(4), pp. 329–350.

Peretto, P. and Smulders, S. (2002). 'Technological distance, growth and scale effects', *Economic Journal*, vol. 112(481), pp. 603–624.

Rajan, R. and Subramanian, A. (2011). 'Aid, Dutch disease, and manufacturing growth', *Journal of Development Economics*, vol. 94(1), pp. 106–118.

Sachs, J. and Warner, A. (1995). 'Natural resource abundance and economic growth', NBER Working Paper No. 5398, National Bureau of Economic Research.

Sachs, J. and Warner, A. (1999). 'The big push, natural resource booms and growth', *Journal of Development Economics*, vol. 59(1), pp. 43–76.

Sachs, J. and Warner, A. (2001). 'The curse of natural resources', *European Economic Review*, vol. 45(4), pp. 827–838.

Sala-i-Martin, X. and Subramanian, A. (2003). 'Addressing the natural resource curse: an illustration from Nigeria', NBER Working Paper No. 9804, National Bureau of Economic Research.

Smith, B. (2014). 'Dutch disease and the oil and boom and bust', Working Paper, Oxford Center for the Analysis of Resource Rich Economies.

Smith, B. (2015). 'The resource curse exorcised: evidence from a panel of countries', *Journal of Development Economics*, vol. 116, pp. 57–73.

Stokey, N. and Rebelo, S. (1995). 'Growth effects of flat-rate taxes', *Journal of Political Economy*, vol. 103(3), pp. 519–50.