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RESEARCH ARTICLE

Dynamic effects of patent policy on innovation and inequality in a Schumpeterian economy

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Abstract
This study explores the dynamic effects of patent policy on innovation and income inequality in a Schumpeterian growth model with endogenous market structure and heterogeneous households. We find that strengthening patent protection has a positive effect on economic growth and a positive or an inverted-U effect on income inequality when the number of differentiated products is fixed in the short run. However, when the number of products adjusts endogenously, the effects of patent protection on growth and inequality become negative in the long run. We also calibrate the model to US data to perform a quantitative analysis and find that the long-run negative effect of patent policy on inequality is much larger than its short-run positive effect. This result remains consistent with our empirical finding from a panel vector autoregression.

Keywords Patent policy · Income inequality · Innovation · Endogenous market structure

JEL Classification D30 · O30 · O40

1 Introduction

A recent study by Aghion et al. (2019) provides empirical evidence to show that innovation and income inequality have a positive relationship. However, innovation and income inequality are both endogenous variables; therefore, it would be interesting

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to see how they are both affected by an exogenous policy parameter, such as the level of patent protection. Therefore, this study analyzes the effects of patent policy on innovation and inequality. This analysis fills an important gap in the literature because previous studies, which explore the effects of patent policy on innovation in the macroeconomy, rarely consider its microeconomic implications on the income distribution. Furthermore, the Schumpeterian growth model that we develop allows us to analytically show how the effect of patent policy on the income distribution changes over time. The tractability of this dynamic analysis enables us to compare the transition path of income inequality derived from the growth model to the impulse response function estimated from a panel vector autoregression (VAR).

We introduce heterogeneous households into a Schumpeterian model with endogenous market structure to explore the effects of patent protection on economic growth and income inequality. The Schumpeterian model with endogenous market structure is based on Peretto (2007, 2011) and features both horizontal innovation (i.e., the development of new products) and vertical innovation (i.e., the quality improvement of products). Although endogenous market structure gives rise to transition dynamics in the aggregate economy, the wealth distribution of households is stationary (as an equilibrium outcome) along the entire transition path due to the stationary consumption-output and consumption-wealth ratios. This useful property makes our analysis tractable. Upon deriving the autonomous dynamics of the average firm size, we are able to also derive the dynamics of economic growth and the evolution of the income distribution (given a general wealth distribution).

In this growth-theoretic framework, we find that strengthening patent protection leads to a higher growth rate and causes a positive or an inverted-U effect on income inequality when the number of differentiated products is fixed in the short run. However, when the number of products adjusts endogenously, the effects of patent protection on economic growth and income inequality become negative in the long run. The intuition of these results can be explained as follows.

Stronger patent protection confers more market power to monopolistic firms, which then charge a higher markup and earn more profits. As a result, strengthening patent protection has a positive effect on innovation and economic growth when the number of firms is fixed in the short run. However, the increased profitability also attracts the entry of new firms, which in turn reduces the market share captured by each firm. Given that it is the firm size that determines the incentives for quality-improving innovation,[1] the entry of new firms caused by stronger patent protection stifles quality-improving innovation,[2] which determines long-run growth.[3] These contrasting effects of patent protection on economic growth at different time horizons have novel implications on the dynamics of income inequality.

In our model, households own different amounts of wealth. This wealth inequality gives rise to income inequality; see Piketty (2014) for evidence on the importance of wealth inequality on income inequality. Given that asset income is determined

[1] See Cohen and Klepper (1996a, b) and Laincz and Peretto (2006) for empirical evidence.

[2] See Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008) for evidence. Boldrin and Levine (2008) even suggest to abolish the patent system entirely.

[3] See Peretto and Connolly (2007) for a theoretical explanation on why vertical innovation, instead of horizontal innovation, drives growth in the long run and Garcia-Macia et al. (2019) for empirical evidence.

by the rate of return on assets and the value of assets, an increase in either the real interest rate or asset value would raise income inequality; see Madsen (2017) for evidence that asset returns are an important determinant of income inequality. As a result, strengthening patent protection has the following effects on income inequality in the short run. The positive effect of patent protection on the equilibrium growth rate leads to a higher interest rate through the Euler equation of the households; therefore, strengthening patent protection has a positive effect on income inequality by increasing the equilibrium growth rate and the real interest rate in the short run. This dynamic-general-equilibrium effect is also present in previous studies, such as Chu (2010b) and Chu and Cozzi (2018), who focus on quality improvement without variety expansion. In our model, endogenous entry gives rise to a novel effect. The larger markup as a result of stronger patent protection reduces the demand for intermediate goods, which in turn reduces the value of assets through the entry condition of new products. Therefore, strengthening patent protection also has a negative effect on income inequality.

The above positive and negative effects together generally give rise to an inverted-U relationship between patent protection and income inequality in the short run. However, it is also possible to have only a positive relationship between patent protection and income inequality over the permissible range of the policy instrument. In the long run, the effect of patent protection on economic growth becomes negative (due to endogenous market structure) as explained before. Therefore, the effect of patent protection on the real interest rate also becomes negative, and hence, strengthening patent protection has a negative effect on income inequality by decreasing the equilibrium growth rate and the real interest rate in the long run. Finally, we calibrate the model to US data to perform a quantitative analysis and find that the long-run negative effect of patent protection on income inequality is much larger than its short-run positive effect. This dynamic pattern of income inequality is consistent with the impulse response function estimated from a panel VAR.

This study relates to the patent-design literature.[4] In this literature, the seminal study is Nordhaus (1969), who analyzes the implications of patent length (i.e., the statutory term of patent). In the US, the statutory term of patent is 20 years; however, the vast majority of patents are not renewed until the end of the statutory term, rendering an extension of patent length ineffective in most industries.[5] Subsequent studies, such as Gilbert and Shapiro (1990) and Klemperer (1990), instead explore patent breadth (i.e., the broadness or scope of patent protection).[6] In this study, we also explore the effects of patent breadth but consider a dynamic general-equilibrium model of innovation and economic growth, which differs from the partial-equilibrium models in this literature.

Therefore, this study also relates to the literature on innovation and economic growth. Romer (1990) develops the seminal R&D-based growth model in which economic growth is driven by the invention of new products. Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom et al. (1990) consider an alternative growth engine that is the innovation of higher-quality products and develop the

[4] See Scotchmer (2004) for a comprehensive review of this literature.

[5] See for example Chu (2010a) for an analysis of patent length extension in an R&D-based growth model.

[6] When an inventor applies for a patent, he/she makes a number of claims about the invention in the patent application. The level of patent breadth is determined by how broadly these claims are to be interpreted by patent judges when it comes to enforcing the patent in courts.

Schumpeterian growth model. Subsequent studies, such as Smulders and van de Klundert (1995), Peretto (1998, 1999) and Howitt (1999), develop the second-generation Schumpeterian model with both vertical and horizontal innovation.[7] This study contributes to the literature by developing a second-generation Schumpeterian model with heterogeneous households to explore the effects of patent protection.

Other studies also explore the effects of patent protection on innovation in the R&D-based growth model; see for example, Cozzi (2001), Li (2001), Goh and Olivier (2002), Furukawa (2007), Futagami and Iwaisako (2007), Horii and Iwaisako (2007), Chu (2009, 2011), Acemoglu and Akcigit (2012), Iwaisako (2013), Iwaisako and Futagami (2013), Kiedaisch (2015), Chu et al. (2016) and Yang (2018, 2020). These studies focus on models with a representative household; therefore, they do not consider the effects of patent protection on income inequality. This study contributes to the literature by applying an R&D-based growth model with heterogeneous households to explore the effects of patent protection on income inequality in addition to innovation and economic growth.

Some studies in the literature consider heterogeneous workers and explore the effects of innovation on the skill premium or more generally wage inequality; see for example, Acemoglu (1998, 2002), Spinesi (2011), Cozzi and Galli (2014) and Grossman and Helpman (2018). This study complements them by assuming wealth heterogeneity rather than worker heterogeneity and by analyzing income inequality rather than wage inequality. A recent study by Madsen and Strulik (2020) explores the evolution of inequality (measured by the ratio of land rent to wages) in a unified growth model.[8] Our study differs from their interesting work by considering other measures (i.e., the coefficient of variation, the Gini coefficient and the top income share) of income inequality in a Schumpeterian growth model, in which innovation is the engine of technological progress and economic growth.

Some studies in the literature also explore the relationship between income inequality and innovation in the R&D-based growth model; see for example, Chou and Talmain (1996), Zweimuller (2000), Foellmi and Zweimuller (2006), Jones and Kim (2018) and Aghion et al. (2019). Our study relates to these interesting studies by exploring how patent policy influences the relationship between innovation and inequality. Chu (2010b), Chu and Cozzi (2018) and Kiedaisch (2020) also explore the effects of patent policy on innovation and inequality.[9] However, all the above-mentioned studies feature either vertical or horizontal innovation; as a result, they do not feature endogenous market structure. Furthermore, instead of focusing on a stationary income distribution, the tractability of our model allows us to analytically derive the evolution of the income distribution without imposing any parametric assumption on the wealth distribution. We find that endogenizing the market structure has novel implications on the dynamic effects of patent protection on income inequality.

[7] See Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008, 2010) and Ang and Madsen (2011) for empirical evidence that supports the second-generation Schumpeterian model.

[8] See Galor (2011) for a comprehensive review of unified growth theory.

[9] Chu et al. (2019) explore the effects of monetary policy in a monetary Schumpeterian growth model with heterogenous households. Like this study, their model features a complete market, which is different from the interesting framework in Bilbiie and Ragot (2020) and Bilbiie et al. (2020), who consider heterogeneous households in the monetary New Keynesian model with idiosyncratic shocks and incomplete markets.

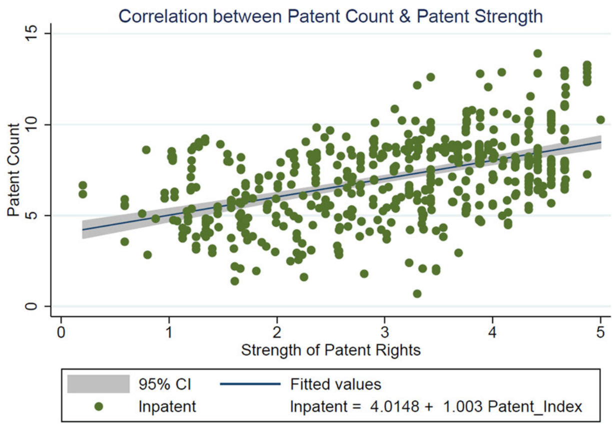


Fig. 1 Correlation between patent counts and patent strength

The rest of this study is organized as follows. Section 2 presents some stylized facts. Section 3 presents the model. Section 4 analyzes the dynamics of the model. Section 5 explores the effects of patent policy. Section 6 concludes. "Appendix A" contains the proofs.

2 Stylized facts

This study examines whether changes in the strength of patent protection affect income inequality. The Ginarte–Park index of patent rights is a standard measure of patent strength across countries; see Ginarte and Park (1997). Many studies use this index to estimate the effects of patent strength on innovation;[10] however, only a few studies explore the effects of patent strength on income inequality. A notable example is Adams (2008) who considers static panel regressions and finds that patent strength has a positive effect on income inequality, which is consistent with the positive short-run effect (but does not capture the negative long-run effect) from our panel VAR analysis.

Although the Ginarte–Park index is very influential in the literature, it is not available at an annual frequency (available at a quinquennial frequency only), which prevents us from using the index in our panel VAR analysis. Instead, we measure patent protection by using total patent counts, which is an annual time series being useful for a shock analysis. We have plotted the correlation between patent counts and the Ginarte–Park index in Fig. 1, which is clearly positive on average, indicating that countries with stronger patent rights tend to have higher patent counts. This empirical correlation may be driven by many forces, but it is consistent with our theoretical model in which stronger patent protection increases the number of patented products.[11]

[10] See for example Park (2005, 2008) for a discussion.

[11] See the discussion in Footnote 25.

Table 1 Descriptive statistics

	Mean	p50	SD	Min	Max	Obs
patent_strength	3.215	3.425	1.110	0.200	5.000	461
log_patents	7.133	7.196	2.230	0.693	13.913	2465
inequality	0.460	0.462	0.069	0.174	0.762	2465

Table 2 Panel unit-root tests

	Inverse χ^2(x)-P	Modified Inverse χ^2(x)-Pm
patent_strength	529.636***	13.666***
log_patents	299.930***	7.663***
inequality	220.351***	3.242***

H0: panel variable contains unit root; H1: panel variable is stationary. The Fisher-type unit-root test based on Phillips–Perron tests examines the null hypothesis of a unit root against the stationary alternative. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels respectively

We compile country-level data on income inequality and patent counts. The data series are in annual frequency, giving us an unbalanced panel of 89 countries from 1980 to 2017. The Gini index of household income inequality comes from the Standardized World Income Inequality Database, whereas the number of patents is taken from the World Development Indicators of the World Bank. Table 1 reports the descriptive statistics, whereas Table 2 performs a panel unit-root test to confirm the stationarity of the data.

We carry out a shock analysis in a panel VAR to examine the dynamic relationship between income inequality and patents.[12] We estimate a recursive panel VAR with a maximum of 3 lags to capture the dynamics in the data and identify a patent shock by applying the usual Choleski decomposition on the variance-covariance matrix of residuals. We estimate the panel VAR using the GMM estimator in Abrigo and Love (2016),[13] which can better deal with unobserved country heterogeneity, especially in fixed t and large n settings, providing a consistent estimate of the mean effects across countries. We specify the following ordering for the 2×1 vector of variables [patents, inequality] in order to identify the patent shock. The reason behind this specific recursive ordering stems from the theoretical ordering of the variables that should run from the more exogenous variable to the less exogenous one. The variable, patents, is ordered first and followed by inequality. By undertaking a panel VAR-Granger causality Wald test, we find patents to be exogenous among the variables.

Our aim here is to track the response of income inequality due to a shock in patents, using a panel VAR in a bivariate setting as a benchmark: the log of patents and income inequality. As efficiency can be improved by including a longer set of lags in GMM estimation, we estimate the VAR using 3 lags and plot the estimated response coeffi-

[12] See “Appendix C” for a formal description of the panel VAR.

[13] This estimator is essentially a difference GMM, but the differencing is based on forward orthogonal deviations, instead of the usual first-differencing.

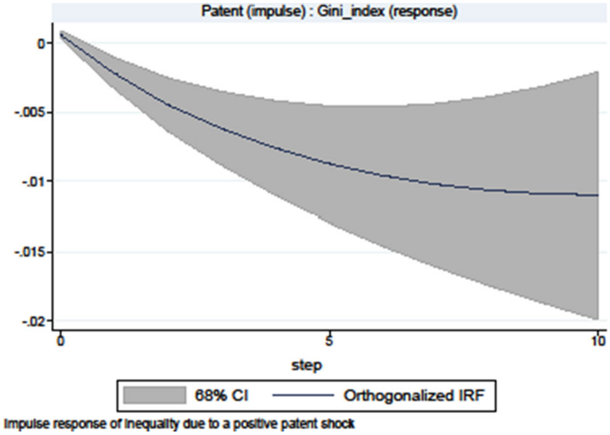
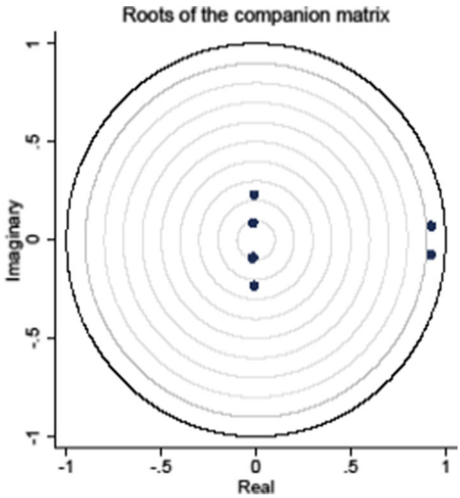


Fig. 2 Impulse response of income inequality to a positive patent shock

Fig. 3 Eigenvalue stability condition graph



cients up to a forecast horizon of 10 years. The panel VAR approach helps us assess the common response for the countries to a patent shock.

Figure 2 shows the bootstrapped impulse responses to a patent shock, together with plus/minus one standard-error confidence bands, obtained by bootstrapping (1000 draws). For a one standard deviation positive shock in patents, income inequality initially increases and then the median response converges to a negative level in the long run. The shaded curves represent the confidence interval around the estimated response functions, computed from a typical Monte Carlo integration exercise with 1000 draws, for statistical significance. Following Uhlig (2005) and Alessandri and Mumtaz (2019), we construct 68% confidence bands around the median estimate. The eigenvalue stability condition graph in Fig. 3 suggests that as all the eigenvalues lie inside the unit circle, the panel VAR satisfies the stability condition. Although the short-run positive response of income inequality to a patent shock is small, the novel finding here is the large negative response of inequality in the long run, which is consistent with our simulation results and remains robust even if we extend the panel VAR to a multivariate setting or consider top income inequality as an alternative measure of income inequality; see the robustness checks in "Appendix C".

In the rest of this section, we further examine the robustness of the negative relationship between patents and income inequality by considering an IV panel regression:

$$\sigma_{i,t} = \varphi_0 + \varphi_1 N_{i,t-1} + \Omega_i + \Omega_t + \epsilon_{i,t},$$

where $\sigma_{i,t}$ is income inequality in country i at time t and $N_{i,t-1}$ is the log patent counts at time $t-1$. Ω_t and Ω_i denote year and country fixed effects. In order to capture the variation in patent counts from the variation in patent strength, we use patent strength $\mu_{i,t-1}$ as an instrumental variable for patent counts $N_{i,t-1}$. Table 3 shows that φ_1 is negative and significant, providing further support for the negative effect of patent strength on income inequality via patent counts. The larger absolute value of φ_1 in the regression with IV than the one without IV shows that the variation in patent counts coming from patent strength has an even more significant negative effect on income inequality. Unfortunately, due to the quinquennial frequency of the Ginarte–Park index, we are not able to use patent strength in the panel VAR to capture its dynamics at a higher frequency.

Table 3 Regression results

	FE (1) Inequality	FE (2) Inequality	IV (3) Inequality
log_patents	−0.002**	−0.005***	−0.038**
	(0.001)	(0.001)	(0.017)
patent_strength			1st stage:
			log_patents
			0.249***
			(0.0798)
Country FE	YES	YES	YES
Year FE	No	YES	YES
Observations	2434	2434	443
No. of countries	115	115	74
F statistics	5.887**	11.966***	4.901***
First stage F test			9.81***
Under-id test			9.76***
Weak-id test			9.40***

Standard errors are in parentheses. The variable, log_patents, is instrumented by using patent strength. Other covariates include country and year dummies. Under-id and Weak-id tests report the Anderson canon. corr. LM statistic and Anderson-Rubin Wald test statistic with rejection implying identification. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels respectively

3 A Schumpeterian growth model with heterogeneous households and endogenous market structure

The Schumpeterian model with in-house R&D and endogenous market structure is based on Peretto (2007, 2011), which features creative accumulation instead of creative destruction.[14] Chu et al. (2016) introduce patent protection into the Peretto model to explore its effects on innovation and economic growth. We further introduce heterogeneous households into the model to analyze the effects of patent protection and endogenous market structure on economic growth and income inequality. Our analysis provides a complete closed-form solution for economic growth and the income distribution on the transition path and the balanced growth path.

3.1 Heterogeneous households

The economy features a unit continuum of households, which are indexed by $h \in [0, 1]$. The households have identical homothetic preferences over consumption but own different levels of wealth. The utility function of household h is given by[15]

$$U(h) = \int_{0}^{\infty} e^{-\rho t} \ln c_t(h) dt, \tag{1}$$

where the parameter $\rho > 0$ determines the rate of subjective discounting and $c_t(h)$ is household h's consumption of final good (numeraire). Household h maximizes (1) subject to

$$\dot{a}_t(h) = r_t a_t(h) + w_t L - c_t(h). \tag{2}$$

$a_t(h)$ is the real value of assets owned by household h, and r_t is the real interest rate. Household h supplies L units of labor to earn a real wage rate w_t.[16] From standard dynamic optimization, the familiar Euler equation is

$$\frac{\dot{c}_t(h)}{c_t(h)} = r_t - \rho, \tag{3}$$

which shows that the growth rate of consumption is the same across households such that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t = r_t - \rho$, where $c_t \equiv \int_0^1 c_t(h)dh$ is aggregate consumption.

[14] See Garcia-Macia et al. (2019) for evidence supporting the notion that creative accumulation is the main driving force of innovation.

[15] For simplicity, we consider inelastic labor supply. However, our results are robust to the extension of elastic labor supply; see "Appendix D" for derivations.

[16] Our results are robust to allowing for population growth. Derivations are available upon request.

3.2 Final good

Competitive firms produce final good Y_t using the following production function:

$$Y_t = \int_0^{N_t} X_t^{\theta}(i)[Z_t^{\alpha}(i)Z_t^{1-\alpha}L_t/N_t]^{1-\theta}di, \tag{4}$$

where $\{\theta, \alpha\} \in (0, 1)$. $X_t(i)$ denotes the quantity of non-durable intermediate good $i \in [0, N_t]$, and N_t is the mass of available intermediate goods at time t. The productivity of intermediate good $X_t(i)$ depends on its own quality $Z_t(i)$ and also on the average quality $Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_t(i)di$ of all intermediate goods capturing technology spillovers. The private return to quality is determined by α, and the degree of technology spillovers is determined by $1-\alpha$. The term L_t/N_t captures a congestion effect of variety and removes the scale effect in the model.[17]

Profit maximization yields the following conditional demand functions for L_t and $X_t(i)$:

$$L_t = (1-\theta)Y_t/w_t, \tag{5}$$

$$X_t(i) = \left(\frac{\theta}{p_t(i)}\right)^{1/(1-\theta)} Z_t^{\alpha}(i)Z_t^{1-\alpha}L_t/N_t, \tag{6}$$

where $p_t(i)$ is the price of $X_t(i)$. Competitive producers of final good pay $\theta Y_t = \int_0^{N_t} p_t(i)X_t(i)di$ for intermediate goods. The market-clearing condition for labor implies $L_t = L$ for all t.

3.3 Intermediate goods and in-house R&D

The monopolistic firm in industry i produces the differentiated intermediate good with a linear technology that requires $X_t(i)$ units of final good to produce $X_t(i)$ units of intermediate good $i \in [0, N_t]$. Furthermore, the firm in industry i incurs $\phi Z_t^{\alpha}(i)Z_t^{1-\alpha}$ units of final good as a fixed operating cost. To improve the quality of its product, the firm also devotes $R_t(i)$ units of final good to R&D. The innovation specification is given by[18]

$$\dot{Z}_t(i) = R_t(i). \tag{7}$$

In industry i, the monopolistic firm's (before-R&D) profit flow at time t is

$$\Pi_t(i) = [p_t(i) - 1]X_t(i) - \phi Z_t^{\alpha}(i)Z_t^{1-\alpha}. \tag{8}$$

[17] Our results are robust to parameterizing this congestion effect as $L_t/N_t^{1-\xi}$, where $\xi \in (0, 1)$, as in Peretto (2015). See the discussion in Footnote 29.

[18] Here we consider homogeneous research productivity normalized to unity; see for example Minniti et al. (2013) and Marsiglio and Tolotti (2018) for different aspects of research heterogeneity in the Schumpeterian growth model.

The value of the monopolistic firm in industry i is

$$V_t(i) = \int_t^\infty \exp\left(-\int_t^s r_u du\right)[\Pi_s(i) - R_s(i)]\,ds. \tag{9}$$

The monopolistic firm in industry i maximizes (9) subject to (6), (7) and (8). The current-value Hamiltonian for this optimization problem is

$$H_t(i) = \Pi_t(i) - R_t(i) + \eta_t(i)\dot{Z}_t(i), \tag{10}$$

where $\eta_t(i)$ is the co-state variable on (7).

We solve this optimization problem in "Appendix A" and derive the unconstrained profit-maximizing markup ratio given by $1/\theta$. To analyze the effects of patent breadth, we introduce a policy parameter $\mu > 1$, which determines the unit cost for imitative firms to produce $X_t(i)$ with the same quality $Z_t(i)$ as the monopolistic firm in industry i.[19] In general, the parameter μ captures the market power of monopolistic firms. Here we consider the case, in which a larger patent breadth μ increases the production cost of imitative firms and allows the monopolistic producer of $X_t(i)$, who owns the patent, to charge a higher markup without losing her market share to potential imitators.[20] Therefore, the equilibrium price becomes

$$p_t(i) = \min\{\mu, 1/\theta\}. \tag{11}$$

We assume $\mu < 1/\theta$. In this case, a larger patent breadth μ leads to a higher markup, and this implication is consistent with Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

We follow previous studies to consider a symmetric equilibrium in which $Z_t(i) = Z_t$ for $i \in [0, N_t]$. In this case, the size of intermediate-good firms is also identical across all industries, such that $X_t(i) = X_t$.[21] From (6) and $p_t(i) = \mu$, the quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L}{N_t}. \tag{12}$$

We define the following transformed variable:[22]

$$x_t \equiv \mu^{1/(1-\theta)} \frac{X_t}{Z_t} = \theta^{1/(1-\theta)} \frac{L}{N_t}. \tag{13}$$

[19] Here we assume a diffusion of knowledge from the monopolistic firm to imitators.

[20] Intuitively, the presence of monopolistic profits attracts potential imitators. However, stronger patent protection increases the production cost of imitative products and allows monopolistic firms to charge a higher markup without losing market share to these potential imitators; see also Li (2001), Goh and Olivier (2002), Chu (2011) and Iwaisako and Futagami (2013) for a similar formulation.

[21] Symmetry also implies $\Pi_t(i) = \Pi_t$, $R_t(i) = R_t$ and $V_t(i) = V_t$.

[22] This definition has the advantage that a change in μ does not directly affect x_t.

x_t is a state variable that is determined by the quality-adjusted firm size X_t/Z_t, which in turn depends on L/N_t.[23] Lemma 1 derives the rate of return on quality-improving R&D, which is increasing in x_t and μ.

Lemma 1 *The rate of return to in-house R&D is given by*

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right]. \tag{14}$$

Proof See "Appendix A". ∎

3.4 Entrants

Following previous studies, we assume that entrants have access to aggregate technology Z_t to ensure symmetric equilibrium at any time t. A new firm pays βX_t units of final good to set up its operation and enter the market with a new product (which will be protected by a patent). $\beta > 0$ is a cost parameter, and the cost function βX_t captures the case in which the setup cost is increasing in the initial output volume of the firm. The asset-pricing equation determines the rate of return on assets as

$$r_t = \frac{\Pi_t - R_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{15}$$

Intuitively, the asset-pricing equation equates the interest rate to the rate of return from V_t, which is given by the monopolistic profit Π_t net of the R&D cost R_t plus the capital gain $\dot{V}_t$. The free-entry condition is given by[24]

$$V_t = \beta X_t. \tag{16}$$

Substituting (7), (8), (13), (16) and $p_t(i) = \mu$ into (15) yields the return on entry as

$$r_t^e = \frac{\mu^{1/(1-\theta)}}{\beta} \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t} \right] + \frac{\dot{x}_t}{x_t} + z_t, \tag{17}$$

where $z_t \equiv \dot{Z}_t/Z_t$ is the growth rate of aggregate quality.

3.5 General equilibrium

The equilibrium is a time path of allocations $\{a_t, c_t, Y_t, X_t(i), R_t(i)\}$ and prices $\{r_t, w_t, p_t(i), V_t(i)\}$ such that the following conditions are satisfied:

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[23] Given a fixed L, the number of firms N_t converges to a steady state, at which point the firm size x_t also reaches a steady state.

[24] We treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also βX_t); therefore, $V_t(i) = \beta X_t$ always holds. If $V_t > \beta X_t$ ($V_t < \beta X_t$), then there would be an infinite number of entries (exits).

- households maximize utility taking $\{r_t, w_t\}$ as given;
- competitive firms produce Y_t and maximize profits taking $\{p_t(i), w_t\}$ as given;
- monopolistic firms produce $X_t(i)$ and choose $\{p_t(i), R_t(i)\}$ to maximize $V_t(i)$ taking r_t as given;
- entrants make entry decisions taking V_t as given;
- the value of all existing monopolistic firms adds up to the value of the households' assets such that $N_t V_t = \int_0^1 a_t(h)dh \equiv a_t$;
- the market-clearing condition of labor holds such that $L_t = L$; and
- the following market-clearing condition of final good holds:

$$Y_t = c_t + N_t(X_t + \phi Z_t + R_t) + \dot{N}_t \beta X_t. \tag{18}$$

3.6 Aggregation

Substituting (6) into (4) and imposing symmetry yield the following aggregate production function:

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} Z_t L, \tag{19}$$

which also uses markup pricing $p_t(i) = \mu$. Therefore, the growth rate of output is

$$\frac{\dot{Y}_t}{Y_t} = z_t, \tag{20}$$

which is determined by the quality growth rate z_t.

4 Dynamics

In this section, we analyze the dynamics of the model. Section 4.1 presents the dynamics of the aggregate economy. Section 4.2 summarizes the dynamics of the wealth distribution. Section 4.3 derives the dynamics of the income distribution. Section 4.4 considers the consumption distribution.

4.1 Dynamics of the aggregate economy

We now analyze the dynamics of the economy. In "Appendix A", we show that the consumption-output ratio c_t/Y_t jumps to a unique and stable steady-state value. This equilibrium property simplifies the analysis of transition dynamics and ensures the stationarity of the wealth distribution even on the transition path.

Lemma 2 *The consumption-output ratio jumps to a unique and stable steady-state value:*

$$\frac{c_t}{Y_t} = \frac{\beta\theta\rho}{\mu} + 1 - \theta. \tag{21}$$

Proof See "Appendix A". ∎

Equation (21) implies that for any given μ, consumption and output grow at the same rate given by

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho, \tag{22}$$

where the last equality uses the Euler equation in (3). Substituting (14) into (22) yields the growth rate of output given by

$$g_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right] - \rho, \tag{23}$$

which depends on the state variable x_t. Then, (20) implies that the quality growth rate is also given by

$$z_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right] - \rho, \tag{24}$$

which is positive if and only if

$$x_t > \overline{x} \equiv \frac{\mu^{1/(1-\theta)}}{\mu - 1} \left(\frac{\rho}{\alpha} + \phi \right). \tag{25}$$

Intuitively, innovation requires the firm size to be large enough so that it is profitable for firms to do in-house R&D. For the rest of the analysis, we assume that $x_t > \overline{x}$. In this case, the dynamics of x_t is derived in Lemma 3.

Lemma 3 *The dynamics of x_t is determined by an one-dimensional differential equation:*

$$\dot{x}_t = \mu^{1/(1-\theta)} \left[\frac{(1-\alpha)\,\phi - \rho}{\beta} \right] - \frac{(1-\alpha)\,(\mu - 1) - \beta\rho}{\beta} x_t. \tag{26}$$

Proof See "Appendix A". ∎

Proposition 1 *Under the parameter restriction $\rho < \min\{(1-\alpha)\phi, (1-\alpha)(\mu-1)/\beta\}$, the dynamics of x_t is globally stable and x_t gradually converges to a unique steady-state value. The steady-state values $\{x^*, g^*\}$ are given by*

$$x^*(\underline{\mu}) = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - \rho}{(1-\alpha)(\mu-1) - \beta\rho} > \overline{x}, \tag{27}$$

$$g^*(\underline{\mu}) = \alpha \left[(\mu - 1) \frac{(1-\alpha)\phi - \rho}{(1-\alpha)(\mu-1) - \beta\rho} - \phi \right] - \rho > 0. \tag{28}$$

Proof See "Appendix A". ∎

The differential equation in (26) shows that given an initial value x_0, the state variable x_t gradually converges to its steady-state value denoted as x^*, which also determines $N^* = \theta^{1/(1-\theta)} L/x^*$.[25] On the transition path, the firm size determines the rate of quality-improving innovation and the equilibrium growth rate g_t according to (23). When x_t evolves toward the steady state, g_t also gradually converges to its steady-state value g^*. The steady-state values of $\{x^*, g^*\}$ are derived in Proposition 1.

4.2 Dynamics of the wealth distribution

In this section, we show that for any given x_t at any time t, the wealth distribution is stationary and determined by its initial distribution that is exogenously given at time 0.[26] It is useful to recall that the aggregate economy features transition dynamics determined by the evolution of x_t. However, the wealth distribution is stationary despite the transition dynamics in the aggregate economy because the consumption-output ratio c_t/Y_t is stationary, which in turn implies that the consumption-wealth ratio c_t/a_t is also stationary as shown in the proof of Lemma 2.

Aggregating (2) across all households yields the following aggregate asset-accumulation equation:

$$\dot{a}_t = r_t a_t + w_t L - c_t. \tag{29}$$

Let $s_{a,t}(h) \equiv a_t(h)/a_t$ denote the share of wealth owned by household h. Then, the growth rate of $s_{a,t}(h)$ is given by

$$\frac{\dot{s}_{a,t}(h)}{s_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{c_t - w_t L}{a_t} - \frac{s_{c,t}(h)c_t - w_t L}{a_t(h)}, \tag{30}$$

where $w_t L = (1-\theta)Y_t$ and $s_{c,t}(h) \equiv c_t(h)/c_t$. Given that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t = r_t - \rho$, the consumption share $s_{c,t}(h)$ of any household $h \in [0, 1]$ is stationary such that $s_{c,t}(h) = s_{c,0}(h)$, which is endogenous. Proposition 2 derives the dynamics of $s_{a,t}(h)$ and shows that the wealth distribution of households is also stationary (i.e., $s_{a,t}(h) = s_{a,0}(h)$, which is exogenously given at time 0). This stationarity is due to the stationary consumption-output c_t/Y_t and consumption-wealth c_t/a_t ratios along the transition path of the aggregate economy.

Proposition 2 *The dynamics of $s_{a,t}(h)$ is given by an one-dimensional differential equation:*

$$\dot{s}_{a,t}(h) = \rho[s_{a,t}(h) - s_{a,0}(h)]. \tag{31}$$

Also, the wealth distribution is stationary and remains the same as the initial distribution.

[25] Therefore, the number of patented products N^* is increasing in patent protection μ, which is consistent with Fig. 1.

[26] Here we take wealth inequality as given; see for example Wan and Zhu (2019) for an analysis in which government policy affects wealth inequality.

Proof See "Appendix A". ∎

4.3 Dynamics of the income distribution

In this section, we show that the income distribution is endogenous and nonstationary but still analytically tractable. Although the wealth distribution is stationary, the transition dynamics in the aggregate economy (in particular, the transition dynamics of the real interest rate) gives rise to an endogenous evolution of the income distribution. Therefore, once we trace out the transition dynamics of the real interest rate, we can also trace out the transition dynamics of income inequality.

Income received by household h is given by

$$I_t(h) = r_t a_t(h) + w_t L. \tag{32}$$

Aggregating (32) yields the aggregate level of income as

$$I_t = r_t a_t + w_t L. \tag{33}$$

Let $s_{I,t}(h) \equiv I_t(h)/I_t$ denote the share of income received by household h. Then, we have

$$s_{I,t}(h) = \frac{r_t a_t(h) + w_t L}{r_t a_t + w_t L} = \frac{r_t a_t}{r_t a_t + w_t L} s_{a,0}(h) + \frac{w_t L}{r_t a_t + w_t L}, \tag{34}$$

which determines the evolution of the share of income received by household h and allows us to derive any moment of the income distribution. For example, the coefficient of variation of income is defined as[27]

$$\sigma_{I,t} \equiv \sqrt{\int_0^1 [s_{I,t}(h) - 1]^2 dh} = \frac{r_t a_t}{r_t a_t + w_t L} \sigma_a, \tag{35}$$

where $\sigma_a \equiv \sqrt{\int_0^1 [s_{a,0}(h) - 1]^2 dh}$ is the coefficient of variation of wealth that is exogenously given at time 0. Here we do not impose any parametric assumption on the distribution of $s_{a,0}(h)$ except that it is non-degenerate and has a well-defined standard deviation; for example, it may capture the case in which only the top 1% households own intangible capital from innovation as in Aghion et al. (2019).[28]

27 In "Appendix B", we show that the Gini coefficient of income is also given by $\sigma_{I,t} = \frac{r_t a_t}{r_t a_t + w_t L} \sigma_a$ when σ_a is defined as the Gini coefficient of wealth.

28 From (34), the top ε income share at time t is given by

$$\int_{1-\varepsilon}^1 s_{I,t}(h) dh = \frac{r_t a_t}{r_t a_t + w_t L} \int_{1-\varepsilon}^1 s_{a,0}(h) dh + \frac{w_t L}{r_t a_t + w_t L} \varepsilon = \frac{\sigma_{I,t}}{\sigma_a} \left[\int_{1-\varepsilon}^1 s_{a,0}(h) dh - \varepsilon \right] + \varepsilon,$$

which is increasing in $\sigma_{I,t}$ if and only if $\int_{1-\varepsilon}^1 s_{a,0}(h) dh > \varepsilon$. In the US, the top 1% wealth share is 40%.

Equation (35) shows that income inequality $\sigma_{I,t}$ is increasing in the asset-wage income ratio $r_t a_t/(w_t L)$ given that wealth inequality drives income inequality in our model. Proposition 3 derives the equilibrium expression for $\sigma_{I,t}$ at any time t. Let's define a composite parameter $\Theta \equiv (1-\theta)/(\theta\beta)$.

Proposition 3 *The degree of income inequality at any time t is given by*

$$\sigma_{I,t} = \frac{1}{1+\mu\Theta/r_t}\sigma_a = \frac{1}{1+\mu\Theta/(\rho+g_t)}\sigma_a. \tag{36}$$

Proof See "Appendix A". ∎

4.4 Dynamics of the consumption distribution

In this section, we show that the consumption distribution is also analytically tractable. From (2), we can show that the consumption of household h is given by

$$c_t(h) = \left[r_t - \frac{\dot{a}_t(h)}{a_t(h)}\right]a_t(h) + w_t L = \rho a_t(h) + w_t L, \tag{37}$$

where we have used

$$\frac{\dot{a}_t(h)}{a_t(h)} = \frac{\dot{a}_t}{a_t} = \frac{\dot{c}_t}{c_t}. \tag{38}$$

In (38), the first equality is based on the stationarity of the wealth distribution, whereas the second equality is based on the stationarity of the c_t/a_t ratio as shown in the proof of Lemma 2.

Aggregating (37) yields the aggregate level of consumption as

$$c_t = \rho a_t + w_t L. \tag{39}$$

Let $s_{c,t}(h) \equiv c_t(h)/c_t$ denote the share of consumption by household h. Then, we have

$$s_{c,t}(h) = \frac{\rho a_t(h) + w_t L}{\rho a_t + w_t L} = \frac{\rho a_t}{\rho a_t + w_t L}s_{a,0}(h) + \frac{w_t L}{\rho a_t + w_t L}, \tag{40}$$

which determines the evolution of the share of consumption by household h. The coefficient of variation of consumption is defined as

$$\sigma_{c,t} \equiv \sqrt{\int_0^1 [s_{c,t}(h)-1]^2 dh} = \frac{\rho a_t}{\rho a_t + w_t L}\sigma_a, \tag{41}$$

where σ_a is once again the coefficient of variation of wealth that is exogenously given at time 0. Finally, we derive the equilibrium expression for $\sigma_{c,t}$ at any time t as

$$\sigma_{c,t} = \frac{1}{1 + \mu\Theta/\rho}\sigma_a, \tag{42}$$

where the composite parameter $\Theta \equiv (1-\theta)/(\theta\beta)$ is defined as before and we have used $w_t L/a_t = \mu\Theta$ as in Proposition 3.

5 Effects of patent breadth on growth and inequality

This section analyzes the effects of patent breadth μ on economic growth and inequality. Equation (23) shows that the initial impact of a larger μ on the growth rate g_t is positive because x_t is fixed in the short run. This is the standard positive *profit-margin* effect, captured by $(\mu-1)/\mu^{1/(1-\theta)}$ in (23), of patent breadth on monopolistic profits and innovation as in previous studies, such as Li (2001) and Chu (2011), which feature an exogenous market structure. However, in our model, the market structure is endogenous and the number of firms gradually adjusts. The higher profit margin attracting entry of new firms reduces the size x_t of each firm and the rate of return r_t^q on quality-improving innovation as (14) shows.[29] In the long run, this negative *entry* effect dominates the positive profit-margin effect such that the new steady-state growth rate g^* in (28) is lower than the initial steady-state growth rate; see Fig. 4 for an illustration in which patent breadth increases at time t. In summary, endogenous market structure gives rise to opposite short-run and long-run effects of patent protection on growth.

The above contrasting effects of patent protection on economic growth at different time horizons have novel implications on income inequality, which is determined by the rate of return on assets and the value of assets as (35) shows. The initial impact of a larger patent breadth μ has both a positive effect and a negative effect on income inequality $\sigma_{I,t}$. The positive effect arises because a larger patent breadth initially increases the growth rate g_t and the interest rate r_t as in Chu (2010b) and Chu and Cozzi (2018), who focus on quality improvement without endogenous entry. We refer to this positive effect as the *interest-rate* effect as in Chu and Cozzi (2018). In our model, endogenous entry gives rise to a novel negative effect of patent breadth on income inequality because a larger patent breadth reduces the demand for intermediate goods X_t, which in turn reduces asset value via the entry condition in (16). We refer

[29] As shown in "Appendix E", this result is robust to parameterizing the congestion effect $\xi \in (0,1)$ as $L/N_t^{1-\xi}$ in (4), which yields $Y_t = (\theta/\mu)^{\theta/(1-\theta)} Z_t N_t^{\xi} L$ in (19). In this case, the output growth rate in (20) becomes $\dot{Y}_t/Y_t = z_t + \xi \dot{N}_t/N_t$ as in Peretto (2015). Although a larger patent breadth increases the variety growth rate $\dot{N}_t/N_t$, which contributes to the output growth rate as empirical studies tend to find (see e.g., Garcia-Macia et al. 2019), the overall effect of a larger patent breadth on $\dot{Y}_t/Y_t$ eventually becomes negative because the effect of μ on z_t eventually becomes negative and dominates its positive effect on $\dot{N}_t/N_t$; see the analysis of patent breadth on economic growth in Chu et al. (2020). This negative effect arises because more entries reduce average firm size x_t and z_t is increasing in firm size; see Cohen and Klepper (1996a, b) and Laincz and Peretto (2006) for evidence on the relationship between firm size and innovation.

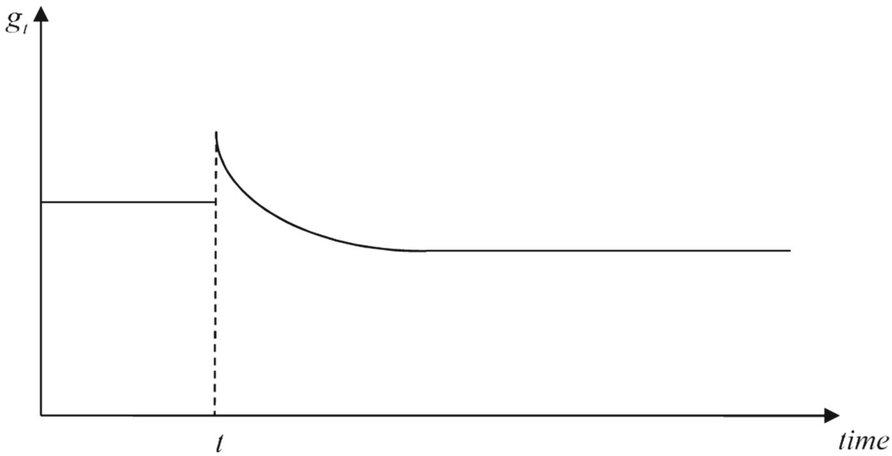

Fig. 4 Transitional effects of patent breadth on economic growth

to this negative effect as the *asset-value* effect, which is captured by the term $\mu\Theta$ in the denominator of (36).[30]

These positive interest-rate and negative asset-value effects together generally give rise to an inverted-U relationship between patent protection and income inequality in the short run. However, it is also possible to yield only a positive relationship between patent protection and income inequality over the permissible range of patent breadth μ. In the long run, the effect of a larger patent breadth on the growth rate g_t and the interest rate r_t becomes negative due to endogenous market structure. Therefore, the interest-rate effect of patent breadth becomes negative in the long run. Together with the negative asset-value effect, increasing patent breadth unambiguously reduces income inequality in the long run; see Fig. 5 for an illustration in which case 1 (case 2) refers to a small (large) increase in patent breadth at time t. Proposition 4 summarizes these results.

Proposition 4 *Strengthening patent protection has the following effects on economic growth and income inequality at different time horizons: (a) it causes a positive effect on economic growth and a positive or an inverted-U effect on income inequality in the short run; and (b) it causes a negative effect on both economic growth and income inequality in the long run.*

Proof See "Appendix A". ∎

Finally, we explore the effect of patent breadth on consumption inequality $\sigma_{c,t}$. Equation (42) shows that an increase in the level of patent breadth causes a one-time permanent decrease in consumption inequality as shown in Fig. 6. This decrease is due to the asset-value effect captured by the term $\mu\Theta$ in the denominator of (42).

30 It is useful to note that the Schumpeterian growth model without endogenous entry in Chu and Cozzi (2018) also features an asset-value effect, which however is positive. The difference in our model with endogenous entry is due to the entry condition in (16) under which the a_t/Y_t ratio is decreasing in μ as shown in the proof of Lemma 2.

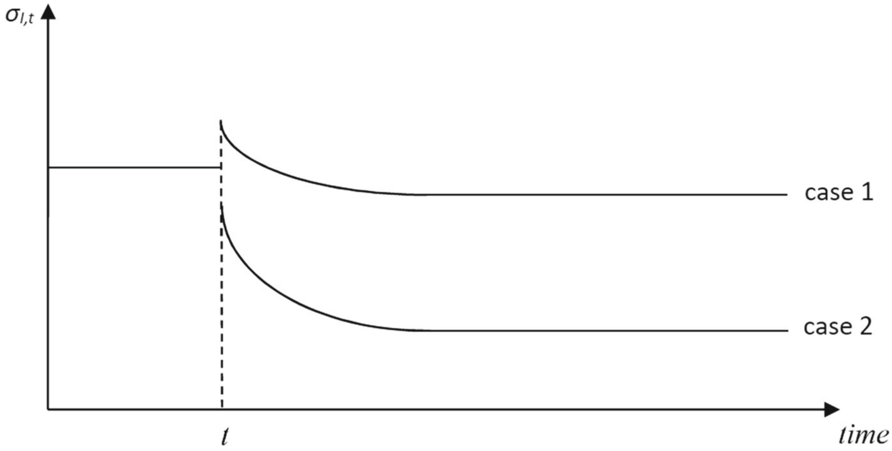

Fig. 5 Transitional effects of patent breadth on income inequality

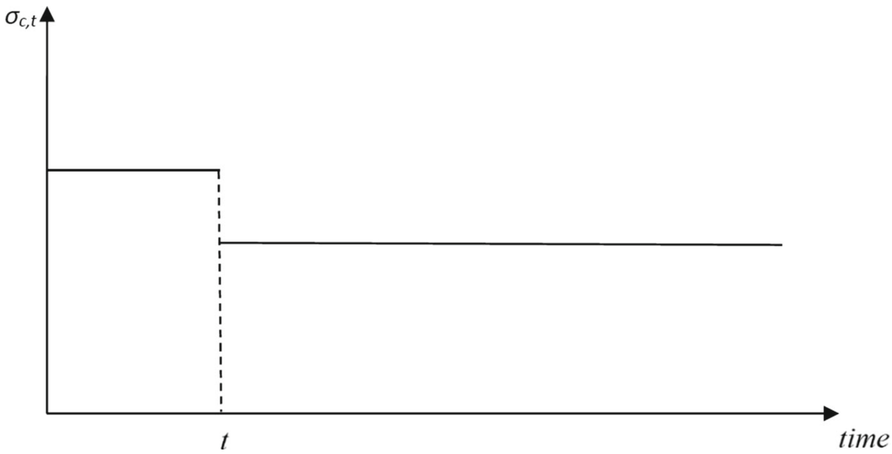

Fig. 6 Transitional effects of patent breadth on consumption inequality

Interestingly, in the case of consumption inequality, the interest-rate effect is absent because $r_t - \dot{c}_t/c_t = \rho$ in (37). However, this property is due to the log utility function. In the case of a more general iso-elastic utility function, the elasticity of intertemporal substitution would determine the sign of the interest-rate effect; see the analysis in Chu (2010b).

5.1 Quantitative analysis

In this section, we calibrate the model to aggregate US data in order to perform a quantitative analysis. The model features the following parameters: $\{\alpha, \rho, \theta, \beta, \phi, \mu\}$. We follow Iacopetta et al. (2019) to set the degree of technology spillovers $1 - \alpha$ to 0.833. We set the discount rate ρ to 0.03 and the markup μ to 1.40, which is at the

Table 4 Calibrated parameter values

α	ρ	θ	β	ϕ	μ
0.167	0.030	0.400	4.667	0.499	1.400

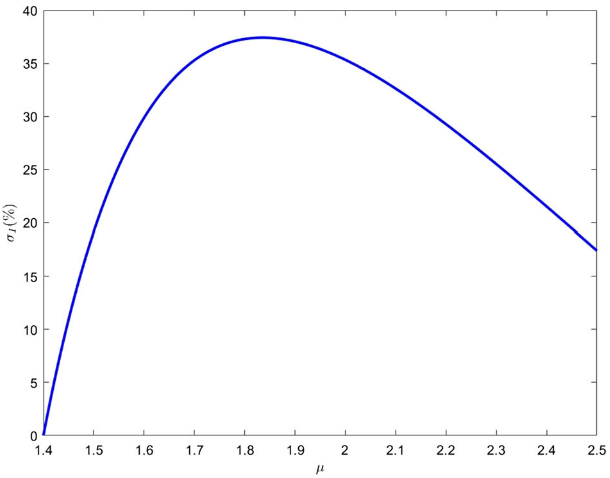

Fig. 7 The short-run effects on income inequality

Table 5 Effects of patent protection on economic growth

	Short-run effects				Long-run effects		
$\rho =$	0.03	0.04	0.05		0.03	0.04	0.05
$\mu = 1.20$	2.28%	2.34%	2.40%	1.20	1.15%	0.97%	0.79%
1.30	2.22%	2.26%	2.31%	1.30	1.64%	1.56%	1.48%
1.40	2.17%	2.21%	2.25%	1.40	1.77%	1.72%	1.67%

upper bound of the range of values reported in Jones and Williams (2000).[31] Then, we calibrate $\{\theta, \beta, \phi\}$ by matching the following moments in the US economy. First, labor income as a share of output is 60%. Second, the consumption share of output is 64%. Third, the growth rate of output per capita is 2%. Table 4 summarizes the calibrated parameter values.

Based on these parameter values, we first simulate the relationship between patent breadth and income inequality in the short run by using (23) and (36).[32] Specifically, we consider the moment when the level of patent breadth μ changes by holding x_t constant in (23). Figure 7 plots the percent changes in income inequality when the markup μ deviates from its initial value of 1.40 and shows that the value of μ that maximizes income inequality in the short run is about 1.84.

[31] We will examine a range of paramater values in a robustness check.

[32] In "Appendix F", we also simulate the relationship between patent breadth and cosumption inequality.

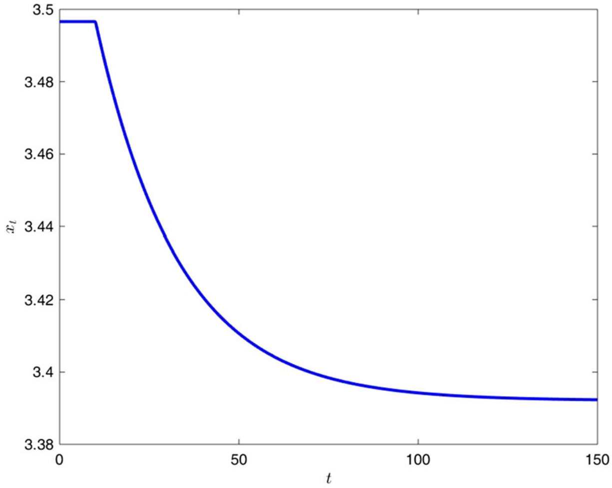

Fig. 8 Transitional path of the firm size

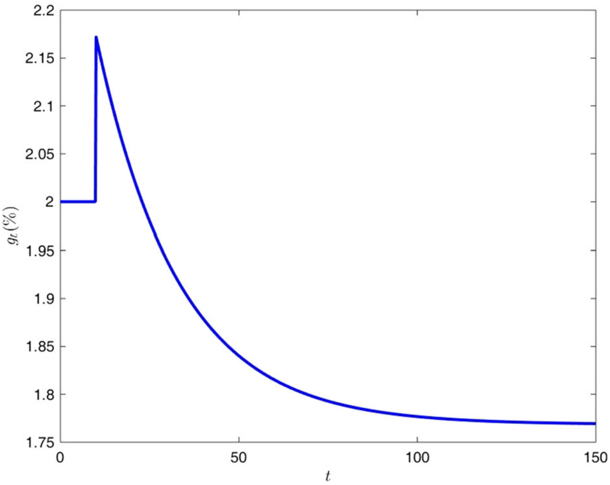

Fig. 9 Transitional path of the growth rate

Table 6 Effects of patent protection on income inequality

	Short-run effects				Long-run effects		
$\rho =$	0.03	0.04	0.05		0.03	0.04	0.05
$\mu = 1.20$	4.18%	4.28%	4.35%	1.20	−16.19%	−16.52%	−16.74%
1.30	3.19%	3.27%	3.32%	1.30	−7.24%	−7.39%	−7.49%
1.40	2.43%	2.49%	2.54%	1.40	−4.80%	−4.90%	−4.96%

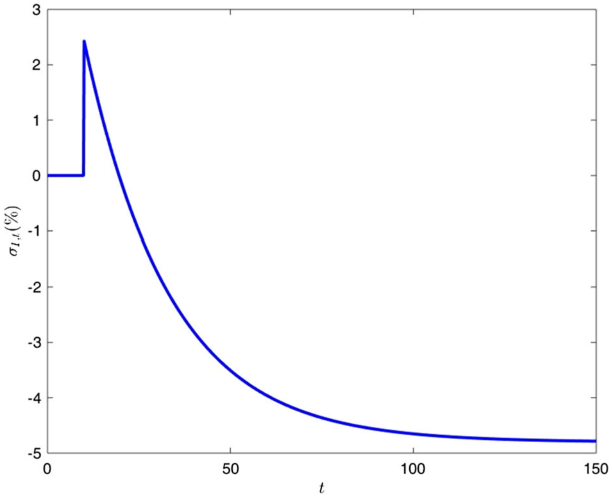

Fig. 10 Transitional path of income inequality

Next we simulate the dynamic effects of patent breadth μ on the quality-adjusted firm size x_t, the growth rate g_t and income inequality $\sigma_{I,t}$. The baseline value of markup μ is 1.40, and we raise μ by 0.01 to 1.41. Figure 8 presents the transitional path of the quality-adjusted firm size x_t. Figure 9 presents the transitional path of the growth rate g_t. Figure 10 presents the transitional path of income inequality $\sigma_{I,t}$ in terms of percent changes from its initial value. When patent protection strengthens, the growth rate increases from 2.00 to 2.17%, which in turn raises income inequality by 2.43% on impact. Gradually, more products enter the market, resulting into a gradual decrease in the quality-adjusted firm size x_t from 3.50 to 3.39. This smaller firm size leads to a decrease in the steady-state growth rate to 1.77%, which in turn decreases income inequality by 4.80% in the long run. Therefore, the negative effect of patent breadth on income inequality in the long run is much larger in magnitude than its positive effect in the short run. This result (especially the novel negative effect of patent protection on income inequality in the long run) is consistent with the stylized facts documented in Sect. 2. In the US, the level of patent protection has gradually increased since the end of 1970's.[33] This period of strengthening patent protection coincides with a period of rising income inequality during which the Gini index rises from 0.43 in 1979 to 0.51 in 2017. Our simulation results imply that when the strengthening of patent protection stops, its positive effect on income inequality will eventually become negative after a few decades.

In this numerical exercise, we consider a conservatively low discount rate ρ and a relatively large markup μ. Considering a larger ρ or a smaller μ would lead to an even more significant decrease in economic growth g and income inequality σ_I in the long run. In the following tables that report results for $\rho \in \{0.03, 0.04, 0.05\}$ and $\mu \in \{1.20, 1.30, 1.40\}$,[34] we present the equilibrium growth rates and the percent

[33] For example, the Ginarte–Park index of patent rights increases from 3.83 in 1975 to 4.88 in 2015.

[34] Here we recalibrate the other parameters $\{\theta, \beta, \phi\}$ to match the same moments as before.

changes in income inequality on impact when μ increases by 0.01 and also when the economy reaches the new balanced growth path. The tables show that strengthening patent protection can lead to a decrease in the steady-state growth rate to as low as 0.79% and a decrease in income inequality by as much as 16.74% in the long run. Therefore, we present the relatively conservative results under $\rho = 0.03$ and $\mu = 1.40$ as our benchmark.

6 Conclusion

This study introduces heterogeneous households into a Schumpeterian growth model with endogenous market structure. Although endogenous market structure causes the aggregate economy to feature transition dynamics, the wealth distribution of households is stationary, which in turn allows us to derive the dynamics of the income distribution. In summary, we find that strengthening patent protection increases economic growth and causes a positive or an inverted-U effect on income inequality in the short run when the number of differentiated products is fixed. However, when the number of products adjusts endogenously, the effects of patent protection on economic growth and income inequality eventually become negative. This finding highlights the importance of endogenous market structure, which gives rise to different effects of patent policy on innovation and inequality at different time horizons. Therefore, previous studies that neglect the endogenous adjustment of the market structure may have identified only the short-run effects of patent policy on innovation and inequality. Finally, to maintain the tractability of the dynamics of income inequality, we have focused on the effects of the aggregate economy on the evolution of the income distribution, without adding to the model a potential feedback effect from the income distribution to the aggregate economy. We leave this interesting extension to future research.

Appendix A: Proofs

Proof of Lemma 1 The current-value Hamiltonian for monopolistic firm i is given by (10). To introduce the upper bound μ on price $p_t(i)$, we modify (10) as follows:

$$H_t(i) = \Pi_t(i) - R_t(i) + \eta_t(i)\dot{Z}_t(i) + \omega_t(i)[\mu - p_t(i)], \qquad (10')$$

where $\omega_t(i)$ is the multiplier on $p_t(i) \leq \mu$. Substituting (6)–(8) into (10'), we can derive

$$\frac{\partial H_t(i)}{\partial p_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial p_t(i)} = \omega_t(i), \qquad (A1)$$

$$\frac{\partial H_t(i)}{\partial R_t(i)} = 0 \Rightarrow \eta_t(i) = 1, \qquad (A2)$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha\left\{[p_t(i)-1]\left[\frac{\theta}{p_t(i)}\right]^{1/(1-\theta)}\frac{L_t}{N_t}-\phi\right\}Z_t^{\alpha-1}(i)\,Z_t^{1-\alpha}$$
$$= r_t\eta_t(i) - \dot{\eta}_t(i). \tag{A3}$$

If $p_t(i) < \mu$, then $\omega_t(i) = 0$. In this case, $\partial\Pi_t(i)/\partial p_t(i) = 0$ yields $p_t(i) = 1/\theta$. If the constraint on $p_t(i)$ is binding, then $\omega_t(i) > 0$. In this case, we have $p_t(i) = \mu$, proving (11). Given that we assume $\mu < 1/\theta$, $p_t(i) = \mu$ always holds. Substituting (A2), (13) and $p_t(i) = \mu$ into (A3) and imposing symmetry yield (14). ∎

Proof of Lemma 2 Substituting (16) into the total asset value $a_t = N_t V_t$ yields

$$a_t = N_t\beta X_t = (\theta/\mu)\beta Y_t, \tag{A4}$$

where the second equality uses $\theta Y_t = N_t(\mu X_t)$.[35] Differentiating A4 with respect to t yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{a}_t}{a_t} = r_t + \frac{w_t L}{a_t} - \frac{c_t}{a_t}, \tag{A5}$$

where the second equality uses (2) with $a_t \equiv \int_0^1 a_t(h)dh$ and $c_t \equiv \int_0^1 c_t(h)dh$. Using (3) for r_t, (5) for w_t, and A4 for a_t, we can rearrange A5 to obtain

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{a}_t}{a_t} = \frac{c_t}{a_t} - \left[\rho + \frac{\mu(1-\theta)}{\beta\theta}\right], \tag{A6}$$

the right-hand side of which is increasing in c_t/a_t with a strictly negative y-intercept. Therefore, c_t/a_t must jump to the steady state. Then, we have (21), noting A4. ∎

Proof of Lemma 3 Substituting $z_t = r_t - \rho = r_t^e - \rho$ into (17) yields

$$\frac{\dot{x}_t}{x_t} = \rho - \frac{\mu^{1/(1-\theta)}}{\beta}\left[\frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t}\right], \tag{A7}$$

where we have also used the expression of z_t in (24) to obtain (26). ∎

Proof of Proposition 1 One can rewrite (26) simply as $\dot{x}_t = d_1 - d_2 x_t$. This linear system for x_t has a unique (non-zero) steady state that is globally (and locally) stable if

$$d_1 \equiv \mu^{1/(1-\theta)}\left[\frac{(1-\alpha)\phi - \rho}{\beta}\right] > 0, \tag{A8a}$$

$$d_2 \equiv \frac{(1-\alpha)(\mu-1) - \beta\rho}{\beta} > 0, \tag{A8b}$$

[35] We derive this by using $p_t(i) = \mu$ and $X_t(i) = X_t$ for $\theta Y_t = \int_0^{N_t} p_t(i)X_t(i)di$.

from which we obtain $\rho < \min\{(1-\alpha)\phi, (1-\alpha)(\mu-1)/\beta\}$. Then, $\dot{x}_t = 0$ yields the steady-state value $x^* = d_1/d_2$, which gives (27). Substituting (27) into (23) yields (28). ∎

Proof of Proposition 2 Manipulating (2) yields

$$\frac{\dot{a}_t(h)}{a_t(h)} = r_t + \frac{w_t L}{a_t(h)} - \frac{c_t(h)}{a_t(h)}. \tag{A9}$$

Then, the growth rate of $s_{a,t}(h) \equiv a_t(h)/a_t$ is

$$\frac{\dot{s}_{a,t}(h)}{s_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{w_t L - c_t(h)}{a_t(h)} - \frac{w_t L - c_t}{a_t}, \tag{A10}$$

which becomes

$$\dot{s}_{a,t}(h) = \frac{c_t - w_t L}{a_t} s_{a,t}(h) - \frac{s_{c,t}(h)c_t - w_t L}{a_t}. \tag{A11}$$

We use (5) for w_t, (21) for c_t/Y_t and A4 for a_t/Y_t in A11 to derive

$$\dot{s}_{a,t}(h) = \rho s_{a,t}(h) - s_{c,t}(h)\frac{\beta\theta\rho + \mu(1-\theta)}{\beta\theta} + \frac{\mu(1-\theta)}{\beta\theta}. \tag{A12}$$

To achieve stability of $s_{a,t}(h)$, $\dot{s}_{a,t}(h) = 0$ must hold for any $t \geq 0$ because $s_{a,t}(h)$ is a pre-determined variable and its coefficient is positive. We can achieve this if and only if $s_{c,t}(h)$ jumps into a stationary level at $t = 0$ that ensures $s_{a,t}(h)$ to be stationary. Then, we have

$$s_{c,0}(h) = \frac{\beta\theta\rho s_{a,0}(h) + \mu(1-\theta)}{\beta\theta\rho + \mu(1-\theta)}, \tag{A13}$$

and $s_{c,t}(h) = s_{c,0}(h)$ for any $t \geq 0$. Substituting A13 into A12 yields (31). ∎

Proof of Proposition 3 By (35), we have

$$\sigma_{I,t} = \frac{1}{1 + [w_t L/(r_t a_t)]}\sigma_a. \tag{A14}$$

Using (5) for w_t and A4 for a_t/Y_t, we obtain

$$\frac{w_t L}{r_t a_t} = \mu\left(\frac{1-\theta}{\beta\theta}\right)\frac{1}{r_t}, \tag{A15}$$

where $r_t = \rho + g_t$. Combining A14 and A15 yields (36). ∎

Proof of Proposition 4 With $r_t^q = r_t$, it is straightforward to show from (14) that for a given x_t, r_t is increasing in $\mu \in (1, 1/\theta)$. Thus, the short-run effect of μ on $r_t = g_t + \rho$ is positive. To see the short-run effect of μ on inequality, we use A14 and A15 to write

$$\sigma_{I,t} = \frac{(r_t/\mu)}{(r_t/\mu) + \Theta}\sigma_a, \tag{A16}$$

noting $r_t = g_t + \rho$. It shows that $\sigma_{I,t}$ is increasing in r_t/μ, in which[36]

$$\frac{r_t}{\mu} = \frac{\alpha}{\mu}\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}}x_t - \phi\right], \tag{A17}$$

which uses (14) and $r_t^q = r_t$. For a given x_t, we can show that

$$\frac{d}{d\mu}\left(\frac{r_t}{\mu}\right) > 0 \Leftrightarrow (\mu - 1) - \frac{\phi\mu^{1/(1-\theta)}}{x_t} - \frac{1-\mu\theta}{1-\theta} \equiv \varkappa(x_t, \mu) < 0. \tag{A18}$$

It is useful to note that for a given x_t, $\varkappa(x_t, \mu)$ is a monotonically increasing function in both x_t and μ.[37] At both ends of the original domain of $\mu \in (1, 1/\theta)$, the signs of $\varkappa(x_t, \mu)$ are opposite such that

$$\lim_{\mu \to 1} \varkappa(x_t, \mu) = -\left(\frac{\phi}{x_t} + 1\right) < 0 \tag{A19a}$$

and

$$\lim_{\mu \to 1/\theta} \varkappa(x_t, \mu) = \left(\frac{1-\theta}{\theta}\right)\left[1 - \frac{\alpha\phi}{\alpha\phi + \rho}\frac{\overline{x}}{x_t}\right] > 0, \tag{A19b}$$

noting $\overline{x}/x_t < 1$. As shown in Fig. 11, there uniquely exists a threshold value of μ, denoted as $\hat{\mu}(x_t) \in (1, 1/\theta)$, such that the effect of μ on $\sigma_{I,t}$ is positive for a sufficiently small $\mu \in (1, \hat{\mu}(x_t))$ and negative for a sufficiently large $\mu \in (\hat{\mu}(x_t), 1/\theta)$. This implies that the unconstrained short-run effect of μ on $\sigma_{I,t}$ follows an inverted-U shaped. However, to ensure $x^* > \overline{x}$, there is an upper bound of μ, that is,

$$\mu < 1 + \beta(\alpha\phi + \rho) \equiv \overline{\mu}. \tag{A20}$$

Thus, if $\overline{\mu} < \hat{\mu}(x_t)$, then only the positive part of an inverted-U effect appears in the feasible range of $\mu \in (1, \overline{\mu})$.

[36] The lower bound of the right-hand side of A17 at $x_t = \overline{x}$, defined in (25), is strictly positive, which implies $r_t/\mu > 0$.

[37] $\varkappa(x_t, \mu)$ being increasing in x_t is obvious. As for μ, note

$$\frac{d}{d\mu}\varkappa(x_t, \mu) = \frac{1}{1-\theta}\frac{1}{x_t}\left[x_t - \overline{x}\left(\frac{\alpha\phi}{\alpha\phi + \rho}\right)\left(1 - \frac{1}{\mu}\right)\right] > 0,$$

in which the inequality always holds due to $x_t > \overline{x}$ in (25).

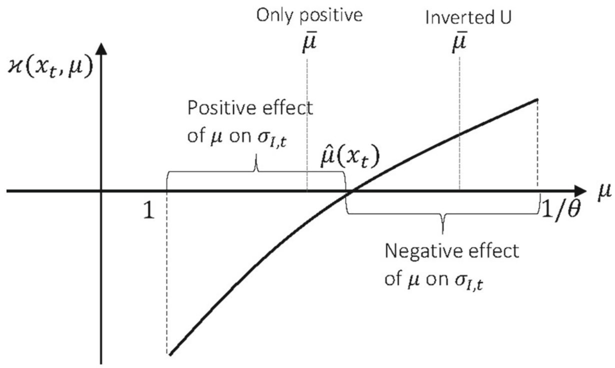

Fig. 11 Proof of Proposition 4

Finally, concerning the long-run effects of μ, we differentiate (28) with respect to μ to derive

$$\frac{d}{d\mu}g^* = -\frac{\alpha\beta\rho\,[(1-\alpha)\phi-\rho]}{[(1-\alpha)(\mu-1)-\beta\rho]^2} < 0, \tag{A21}$$

showing the negative effect of μ on the long-run growth rate g^*. Given that $r^* = g^* + \rho$, an increase in μ leads to a decrease in the long-run interest rate r^* and also a decrease in the steady-state ratio r^*/μ. Therefore, the long-run effect of μ on income inequality $\sigma_{I,t}$ is also negative. ∎

Appendix B: Gini coefficient

Income received by household h is given by

$$I(h) = ra(h) + wL = s_a(h)ra + wL, \tag{B1}$$

where the identity index h is uniformly distributed between 0 and 1. We now order the households in an ascending order of income. The Gini coefficient of income is given by $\sigma_I = 1 - 2b_I$, where

$$b_I \equiv \int_0^1 \mathcal{L}_I(h)dh. \tag{B2}$$

The Lorenz curve $\mathcal{L}_I(h)$ of income is given by

$$\mathcal{L}_I(h) \equiv \frac{\int_0^h I(\chi)d\chi}{\int_0^1 I(\chi)d\chi} = \frac{ra\int_0^h s_a(\chi)d\chi + wL\int_0^h 1d\chi}{ra+wL}, \tag{B3}$$

where $\int_0^h 1d\chi = h$ and $\int_0^h s_a(\chi)d\chi$ is the Lorenz curve $\mathcal{L}_a(h)$ of wealth. To see this,

$$\mathcal{L}_a(h) \equiv \frac{\int_0^h a(\chi)d\chi}{\int_0^1 a(\chi)d\chi} = \frac{\int_0^h a(\chi)d\chi}{a} = \int_0^h s_a(\chi)d\chi. \tag{B4}$$

Substituting B3 and B4 into B2 yields

$$b_I = \frac{ra}{ra + wL}\int_0^1 \mathcal{L}_a(h)dh + \frac{wL}{ra + wL}\int_0^1 hdh, \tag{B5}$$

where $\int_0^1 hdh = 0.5$ and $\int_0^1 \mathcal{L}_a(h)dh \equiv b_a$. Recall that the Gini coefficient of wealth is given by $\sigma_a = 1 - 2b_a$. Therefore, substituting B5 into $\sigma_I = 1 - 2b_I$ yields the Gini coefficient of income given by

$$\sigma_I = \frac{ra}{ra + wL}\sigma_a, \tag{B6}$$

which is the same as (35) except that σ_a is now the Gini coefficient of wealth.

Appendix C: Panel VAR and robustness checks

In this appendix, we provide a formal description of the panel VAR, which extends the traditional VAR to panel data and allows for unobserved individual heterogeneity denoted as Λ_n for country n. A first-order panel VAR model can be specified as follows:

$$Ay_{n,t} = \Lambda_n + \Lambda(L)y_{n,t-1} + \varepsilon_{n,t},$$

where $y_{n,t}$ is a $k \times 1$ vector of endogenous variables for country n at time t. As this equation cannot be estimated directly due to contemporaneous correlations between $y_{n,t}$ and $\varepsilon_{n,t}$, the standard reduced form can be derived by pre-multiplying the system by A^{-1} as follows:

$$y_{n,t} = \Gamma_n + \Gamma(L)y_{n,t-1} + e_{n,t},$$

where $\Gamma_n = A^{-1}\Lambda_n$, $\Gamma(L) = A^{-1}\Lambda(L)$ and $e_{n,t} = A^{-1}\varepsilon_{n,t}$. The impulse response functions can now be derived on the basis of the moving average representation of the system as follows:

$$y_{n,t} = \gamma_n + \sum_i \Gamma^i(L)e_{n,t-i} = \gamma_n + \sum_i \Phi_i(L)\varepsilon_{n,t-i},$$

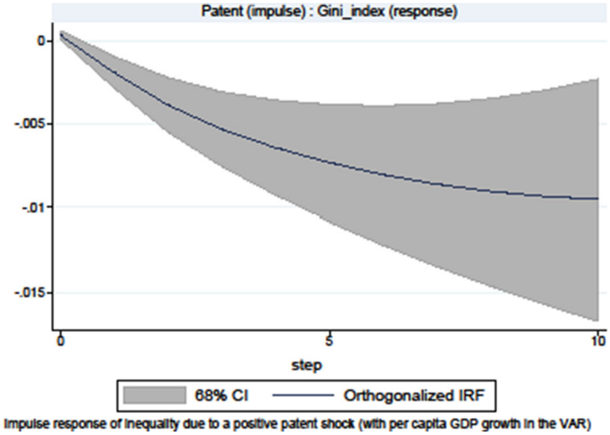

Fig. 12 Three-variable VAR

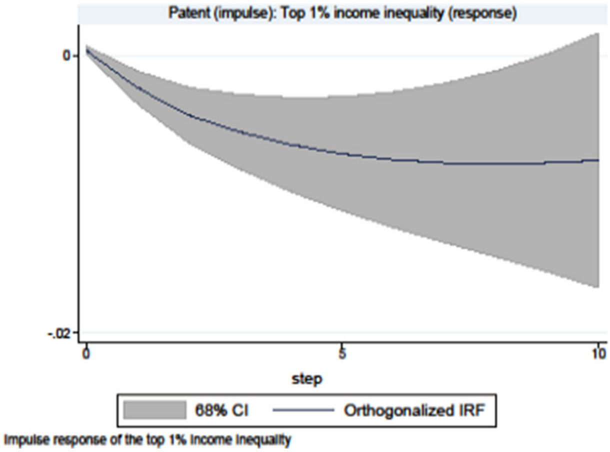

Fig. 13 Top 1% income inequality

where Φ_i are the impulse response functions.

Here we first extend the bivariate setting to a multivariate setting by including per capita GDP growth in the analysis. Figure 12 presents the impulse response function. The initial impact of income inequality in response to a patent shock continues to be positive. More importantly, we continue to see a significant negative response for a 10 year forecast horizon. The result also holds even if we exclude non-resident patents.

We further estimate the effects of patents by changing the inequality measure. We now consider income inequality at the top 1% or the 99th percentile. The impulse response function using this alternative measure is shown in Fig. 13, which shows a similar response as the benchmark in Fig. 2. Specifically, the initial positive response disappears at some point, giving rise to a negative response subsequently.

Appendix D: Elastic labor supply

Our baseline model features inelastic labor supply. In this appendix, we allow for elastic labor supply to confirm the robustness of our results. First, we generalize the utility function of household $h \in [0, 1]$ as follows:

$$U(h) = \int_0^\infty e^{-\rho t} \{\ln c_t(h) + \kappa \ln[L - l_t(h)]\} dt, \tag{D1}$$

where $l_t(h)$ is household h's labor supply. The parameter $\kappa > 0$ measures the importance of leisure $L - l_t(h)$. We replace L by $l_t(h)$ in household h's asset-accumulation equation in (2):

$$\dot{a}_t(h) = r_t a_t(h) + w_t l_t(h) - c_t(h). \tag{D2}$$

Utility maximization yields household h's labor-supply function as follows:

$$l_t(h) = L - \frac{\kappa c_t(h)}{w_t}, \tag{D3}$$

which shows that $l_t(h)$ converges to L as $\kappa \to 0$. Then, we replace L by $l_t \equiv \int_0^1 l_t(h)dh$ on the production side. The rest of the model is the same as our baseline model except that we redefine the state variable x_t as

$$x_t \equiv \mu^{1/(1-\theta)} \frac{X_t}{Z_t} \frac{L}{l_t} = \theta^{1/(1-\theta)} \frac{L}{N_t} \tag{D4}$$

to ensure that the state variable x_t does not jump when l_t changes.

We now explore the dynamics of the aggregate economy. Aggregating D3 across $h \in [0, 1]$ and combining it with $(1 - \theta)Y_t = w_t l_t$ yield aggregate employment:

$$\underset{+}{l(\mu)} = \left(1 + \frac{\kappa}{1-\theta} \frac{c_t}{Y_t}\right)^{-1} L = \left[1 + \frac{\kappa}{1-\theta} \left(\frac{\beta\theta\rho}{\mu} + 1 - \theta\right)\right]^{-1} L, \tag{D5}$$

in which the consumption-output ratio c_t/Y_t always jumps to (21). Therefore, D5 implies that aggregate employment l is stationary and increasing in μ. Then, the growth rate of output becomes

$$g_t = z_t = \alpha \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} \frac{l(\mu)}{L} x_t - \phi\right) - \rho, \tag{D6}$$

which is increasing in μ for a given x_t. Given an initial value x_0, the state variable x_t gradually converges to a steady-state value given by $x^* L/l(\mu)$, where x^* is given in (27). Therefore, the steady-state equilibrium growth rate g^* is the same as before (due to the scale-invariant property of the model) and continues to be decreasing in μ. The dynamics in x_t determines the dynamics in the growth rate g_t as before.

We now explore the dynamics of the income distribution. It can be shown that the dynamics of household h's wealth share $s_{a,t}(h)$ is still given by the one-dimensional differential equation in (31).[38] As for household h's income share $s_{I,t}(h)$, it is given by

$$s_{I,t}(h) = \frac{r_t a_t(h) + w_t l_t(h)}{r_t a_t + w_t l_t}, \tag{D7}$$

where $w_t l_t(h) = w_t L - \kappa c_t(h)$ from D3. Combining D3 and $c_t(h) = (r_t - g_t)a_t(h) + w_t l_t(h)$ from D2 yields

$$w_t l_t(h) = \frac{1}{1+\kappa}\left[w_t L - \kappa(r_t - g_t)a_t(h)\right], \tag{D8}$$

which shows that wealthier households supply less labor and receive less wage income. Despite the presence of this wage inequality, the equilibrium expression for overall income inequality is largely the same as before. To see that, we substitute D8 into D7 to derive

$$s_{I,t}(h) = \frac{(r_t + \kappa g_t)a_t}{(r_t + \kappa g_t)a_t + w_t L} s_{a,0}(h) + \frac{w_t L}{(r_t + \kappa g_t)a_t + w_t L}, \tag{D9}$$

which shows that the dynamics of household h's income share is now determined by the dynamics of the ratio $(r_t + \kappa g_t)a_t/(w_t L)$, rather than $r_t a_t/(w_t L)$. The additional term $\kappa g_t a_t$ captures the effect of elastic labor on income inequality.

The degree of income inequality is given by

$$\sigma_{I,t} = \frac{(r_t + \kappa g_t)a_t}{(r_t + \kappa g_t)a_t + w_t L}\sigma_a. \tag{D10}$$

One can show that the ratio $w_t L/a_t$ becomes

$$\frac{w_t L}{a_t} = \frac{\mu}{l(\mu)}\Theta L, \tag{D11}$$

where Θ is defined as in Sect. 4.3 and $l(\mu)$ is given in D5. Substituting D11 into D10 yields a similar expression for $\sigma_{I,t}$ as in (36), except for the extra terms κ and l/L.

$$\sigma_{I,t} = \left[1 + \frac{\Theta L}{\rho + (1+\kappa)g_t}\frac{\mu}{l(\mu)}\right]^{-1}\sigma_a. \tag{D12}$$

It is useful to note from D5 that $\mu/l(\mu)$ is increasing in μ. Together with the result that g^* is decreasing in μ, a larger patent breadth decreases income inequality in the

[38] Derivations are available upon request.

long run. As for the short run, the positive effects of μ on $\mu/l(\mu)$ and g_t (for a given x_t) give rise to an overall ambiguous effect of μ on $\sigma_{I,t}$ in the short run.[39] Therefore, the dynamic effects of patent protection on income inequality are the same as before. Finally, one can substitute D8 into $c_t(h) = (r_t - g_t)a_t(h) + w_t l_t(h)$ and follow the steps in Sect. 4.4 to derive the degree of consumption inequality as

$$\sigma_{c,t} = \left[1 + \frac{\Theta L}{\rho} \frac{\mu}{l(\mu)}\right]^{-1} \sigma_a, \tag{D13}$$

which is decreasing in μ as before.

Appendix E: Congestion effect

In this appendix, we extend the model by parameterizing the congestion effect $\xi \in (0, 1)$ as $L/N_t^{1-\xi}$ in (4), which modifies (19) as

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} Z_t N_t^{\xi} L. \tag{E1}$$

In this case, the output growth rate becomes

$$\dot{Y}_t/Y_t = z_t + \xi n_t, \tag{E2}$$

where $n_t \equiv \dot{N}_t/N_t$ is the variety growth rate and contributes to the output growth rate when $\xi > 0$. Following the same derivations as in the proof of Lemma 1, one can derive the rate of return to in-house R&D given by

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi\right], \tag{E3}$$

which is the same as (14) except that $x_t = \theta^{1/(1-\theta)} L/N_t^{1-\xi}$. Following the same derivations as in the proof of Lemma 2, one can also show that the consumption-output ratio c_t/Y_t jumps to unique and stable steady-state value, which in turn implies that

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho, \tag{E4}$$

as in (22). Substituting E3 into E4 yields

$$g_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi\right] - \rho = z_t + \xi n_t, \tag{E5}$$

[39] We have simulated $\sigma_{I,t}$ and find that it is either increasing or inverted-U in μ in the short run. We can also prove this result analytically under some sufficient conditions; derivations are available upon request.

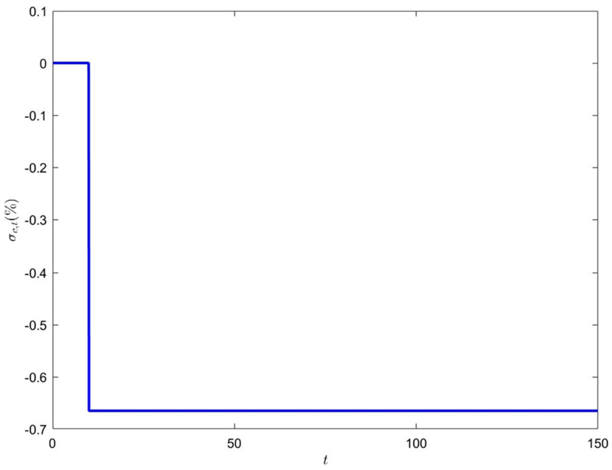

Fig. 14 Transitional path of consumption inequality

where the second equality follows from E2. In other words, although the output growth rate g_t depends on both the quality growth rate z_t and the variety growth rate n_t, it is still determined by the rate of return to in-house R&D in equilibrium.

Equation E5 shows that for a given x_t, a larger patent breadth μ gives rise to a higher growth rate g_t in the short run. Gradually, the entry of new firms reduces the average firm size x_t. It can be shown that the linearized dynamics of x_t is given by[40]

$$\dot{x}_t = (1-\xi)\left\{\mu^{1/(1-\theta)}\left[\frac{(1-\alpha)\,\phi-\rho}{\beta}\right]-\frac{(1-\alpha)\,(\mu-1)-\beta\rho}{\beta}x_t\right\}. \tag{E6}$$

Therefore, the steady-state value of x_t is given by (27) as before. Substituting (27) into E5 yields the same steady-state equilibrium growth rate g^* as in (28), which is decreasing in the level of patent breadth μ as before.

Appendix F: Consumption inequality

In this appendix, we present the quantitative effects of patent breadth on consumption inequality. First, we simulate the dynamic effects of patent breadth μ on consumption inequality $\sigma_{c,t}$. As before, the baseline value of markup μ is 1.40, and we raise μ by 0.01 to 1.41. Figure 14 shows that consumption inequality decreases by 0.67% permanently.

Then, we report results under other values of $\rho \in \{0.03, 0.04, 0.05\}$ and $\mu \in \{1.20, 1.30, 1.40\}$. Table 7 shows that strengthening patent protection can lead to a decrease in consumption inequality by up to 0.78% permanently.

40 Derivations are available upon request.

Table 7 Effects on consumption inequality

$\rho =$	0.03	0.04	0.05
$\mu = 1.20$	-0.78%	-0.78%	-0.78%
1.30	-0.72%	-0.72%	-0.72%
1.40	-0.67%	-0.67%	-0.67%

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