Endogenous Growth and Property Rights Over Renewable Resources

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Endogenous Growth and Property Rights Over Renewable Resources

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Abstract

We analyze the general-equilibrium effects of alternative regimes of access rights over renewable natural resources – namely, open access versus full property rights – on the pace of development when economic growth is endogenously driven by both horizontal and vertical innovations. Resource exhaustion may occur under both regimes but is more likely to arise under open access. Under full property rights, positive resource rents increase expenditures and temporarily accelerate productivity growth, but also yield a higher resource price at least in the short-to-medium run. We characterize analytically the welfare effect of a regime switch induced by a failure in property rights enforcement: switching to open access is welfare reducing if the utility gain generated by the initial drop in the resource price is more than offset by the static and dynamic losses induced by reduced expenditure.

JEL Codes O11, O31, Q21

Keywords Endogenous growth, Innovation, Renewable Resources, Sustainable Development, Property Rights.

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1 Introduction

The gradual abandonment of primary inputs in fixed supply – especially, fossil fuels – in favor of alternative resources capable of natural regeneration is a primary task for most industries in both advanced and developing countries. Such "shift to renewables" raises a number of microeconomic issues concerning resource management, appropriability, externalities and potential market failure (see, e.g., Brown, 2000). Moreover, understanding how the use of renewables affects sustainability requires a complete macroeconomic analysis of the interplay between economic growth and resource exploitation. This paper studies how different regimes of access rights to renewable natural resources affect sustainability and welfare in the context of modern endogenous growth theory.

In resource economics there is a long tradition of studying access rights in partial equilibrium. The benchmark bioeconomic model – pioneered by Gordon (1954) and Schaeffer (1957), and fully characterized by Clark (1973) – typically considers the two polar cases of *open access,* in which the resource is accessible to atomistic harvesters that do not control the evolution of the aggregate resource stock, and *full property rights*, in which the sole owner, or a coordinated group of harvesters, controls the resource stock and therefore adjusts the time profile of harvesting to the dynamics of the resource base. The most popular result, known as the *Tragedy of the Commons* (Hardin, 1968), is that open access may induce resource exhaustion because atomistic harvesters maximize current rents neglecting the effects of current harvesting on future resource scarcity. Related contributions emphasize that, when both regimes yield positive resource stocks in the long run, the levels attained under different regimes depend on the specification of harvesting costs and discount rates (Zellner, 1962; Plourde, 1970). Importantly, because this literature focuses on partial-equilibrium models, its results are highly sensitive to the assumption that prices and the interest rate are exogenous (Clark, 2005). The Gordon-Schaeffer-Clark bioeconomic model has been seldom studied in the general equilibrium framework of modern growth theory. Consequently, we still lack of a satisfactory treatment of the dependence of growth on access rights. This gap in the existing literature motivates our analysis.

Notable attempts at integrating resource and growth economics that preceed ours are Tahvonen and Kuuluvainen (1991; 1993) and Ayong Le Kama (2001). Both introduce renewable resources and pollution in the neoclassical Solow-Ramsey model and study the interactions between harvesting and negative externalities. Bovenberg and Smulders (1995) analyse the same issues in the context of endogenous growth. Our analysis departs from these contributions in two fundamental respects. First, we abstract from pollution externalities and provide, instead, a detailed comparison of open access and full property rights over a renewable resource that exclusively plays the role of essential production input. Second, we employ a Schumpeterian model of endogenous growth in which different types of innovations coexist.

The distinctive feature of our framework is that productivity growth stems from innovations pursued by incumbent firms as well as by new firms entering the market.[1] Incumbent firms invest in projects aimed at increasing their own total factor productivity (vertical innovation). At the same time, entrants invest in projects that develop new products and set up production and marketing operations to serve the market (horizontal innovation). The rationale for using this approach is three-fold. First, this class of models is receiving strong empirical support in explaining historical patterns of innovation activity and economic growth (Madsen, 2010; Madsen and Timol, 2011). Second, the interaction between the mass of firms and technological change within the firm eliminates scale effects – that is, long-run growth rates do not depend on the size of endowments – a property that is empirically plausible (Laincz and Peretto, 2006; Ha and Howitt, 2007) and is furthermore realistic in the present context where input endowments include a stock of natural resources. Third, the model is analytically tractable, a highly desirable feature in the present context. A major reason for the lack of general-equilibrium analyses of access rights and economic growth is, as Brown (2000) put it, that "Introducing one more differential equation to account for renewable resource dynamics makes it difficult to get general analytical solutions and much of the profession continues to find it tasteless to rely on computer-aided answers". Our model yields a detailed characterization of the dynamics of the resource stock, income levels and productivity growth, both in the transition and in the long run. This allows us to compare equilibrium paths under both regimes and to obtain three sets of results.

The first set of results concerns the effect of property rights regimes on resource scarcity and sustainable resource use. Both regimes may yield resource exhaustion or sustained growth in the long run, but the condition for long-run sustainability is always more restrictive under open access: if the intrinsic regeneration rate of the natural resource falls within a specific interval of values, the economy experiences the Tragedy of the Commons under open access but sustained resource extraction – and, hence, sustainable economic growth – under full property rights. When natural regeneration is sufficiently intense to induce sus-

[1] The framework, pioneered by Peretto (1998) and Peretto and Connolly (2007), has been recently applied to study the role of resources in fixed supply – like, e.g., land – in a closed economy (Peretto 2012) and in a two-country, world general equilbrium model with asymmetric trade (Peretto and Valente 2012). In this paper we extend it to the case of renewable resources.

tained growth under both regimes, the resource stock is always higher under full property rights. Importantly, this last result does not imply that the resource price is necessarily lower under full property rights. The reason is that, in our model, the equilibrium value of resource rents is affected by both resource scarcity and income dynamics and – contrary to standard partial equilibrium models – income dynamics are driven by endogenous productivity growth. Specifically, full property rights induce a downward pressure on prices via scarcity effects (i.e., the resource stock tends to be preserved relatively to open access) as well as an upward pressure on prices via rent effects (i.e., resource harvesters with full ownership charge a higher price for given quantity).

The second set of results concerns the impact of resource property rights on market size, innovations and productivity growth. We show that, under full property rights, the market for manufacturing goods is always larger because strictly positive resource rents yield additional income that boost household spending. A larger market size, in turn, attracts entrants so that the economy converges to a steady state with a larger mass of firms. Productivity growth, however, is not faster because the process of entry in the manufacturing sector sterilizes the scale effect: in the long run, firm size and growth rates are the same in the two regimes.

The third set of results concerns the overall effect on consumption and welfare of a regime switch. A shift from full property rights to open access generates negative transitional effects – namely, a productivity slowdown and a gradual increase of natural resource scarcity – but also intantaneous level effects having a potentially ambiguous impact on consumption levels: the permanent reduction in expenditure is mitigated by a reduction in the resource price, because open access implies zero net rents from harvesting and thereby lower unit cost for resource inputs. Therefore, switching to open access is welfare reducing only if the utility gain generated by the initial drop in the resource price is more than offset by the static and dynamic losses induced by lower expenditures and transitional growth.

The plan of the paper is as follows. Section 2 describes the model setup. Section 3 derives the general equilibrium relationships that characterize the economy under each regime. Section 4 compares the two regimes in terms of equilibrium outcomes and studies the welfare impact of a regime switch. Section 5 concludes.

2 A Model of Renewable Resources and Endogenous Growth

The supply side of the economy comprises a final sector producing the consumption good, a manufacturing sector producing differentiated intermediate inputs, and a resource sector that supplies harvest goods to final producers. In the manufacturing sector, incumbents invest in R&D that raise own productivity (i.e., vertical innovations) while outside entrepreneurs develop new varieties of intermediate inputs and start new firms to serve the market (i.e., horizontal innovations). The resource sector may operate under two different regimes – open access or full property rights – that determine different time paths of resource rents and income levels as a result of households choices.

2.1 Final Producers

A representative competitive firm produces final output, Y, by means of H units of a "harvest good" drawn from a stock of a renewable natural resource, L_Y units of labor and n differentiated manufacturing goods. The technology is

$$Y(t) = H(t)^{\alpha} L_Y(t)^{\beta} \int_0^{n(t)} X_i(t)^{\gamma} \, di, \qquad \alpha + \beta + \gamma = 1, \tag{1}$$

where X_i is the quantity of manufacturing good i and $t \in [0, \infty)$ is the time index. The final producer demands inputs according to the usual conditions equating value marginal productivities to remuneration rates. The demand schedules for labor and resource read

$$L_Y(t) = \beta \frac{P_Y(t) Y(t)}{W(t)}, \tag{2}$$

$$H(t) = \alpha \frac{P_Y(t) Y(t)}{P_H(t)}, \tag{3}$$

where P_Y is the price of final output, W is the wage rate, and P_H is the resource price. The condition for X_i yields

$$X_i(t) = \left[\frac{\gamma P_Y(t) H(t)^{\alpha} L_Y(t)^{\beta}}{P_{Xi}(t)} \right]^{\frac{1}{1-\gamma}}, \qquad i \in [0, n(t)], \tag{4}$$

where P_{Xi} is the price of good i.

2.2 Manufacturing Sector: Incumbents

The manufacturing sector consists of single-product firms that supply differentiated goods under monopolistic competition. The typical firm produces with the technology

$$X_i(t) = Z_i(t)^{\theta} \cdot (L_{Xi}(t) - \phi), \qquad 0 < \theta < 1, \qquad \phi > 0 \tag{5}$$

where Z_i is firm-specific knowledge, θ is the associated elasticity parameter, L_{Xi} is labor employed in manufacturing production and ϕ is a fixed labor cost. Technology (5) exhibits constant returns to scale to labor but firm's productivity may increase over time by virtue of in-house R&D. Specifically, the firm's knowledge grows according to

$$\dot{Z}_i(t) = \kappa \cdot K(t) L_{Zi}(t), \quad \kappa > 0 \tag{6}$$

where κ is an exogenous parameter, L_{Zi} is labor employed in vertical R&D, and K is the stock of public knowledge available to all manufacturing firms. Public knowledge is the average knowledge in the manufacturing industry,

$$K(t) = \frac{1}{n(t)} \int_0^{n(t)} Z_j(t) \, dj, \tag{7}$$

which is taken as given at the firm level.[2] The firm maximizes

$$V_i(t) = \int_t^{\infty} \Pi_{Xi}(s) \, e^{-\int_t^s (r(v)+\delta) dv} ds, \quad \delta > 0 \tag{8}$$

subject to (6)-(7) and the demand schedule (4), where $\Pi_{Xi} = P_{Xi}X_i - WL_{Xi} - WL_{Zi}$ is the instantaneous profit, r is the interest rate and δ is the exogenous death rate. The solution to this problem, derived in the Appendix, yields a symmetric equilibrium where each firm produces the same output level and captures the same fraction $1/n$ of the market:

$$P_{Xi}(t) X_i(t) = \frac{1}{n(t)} \cdot \gamma P_Y(t) Y(t), \tag{9}$$

where $\gamma P_Y Y$ is the final producer's expenditure on manufacturing goods.

2.3 Manufacturing Sector: Entrants

Entrepreneurs develop new products and set up new firms to serve the market. These operations require labor in proportion to the value of the good to be produced.[3] Without loss of generality, we denote the entrant as i and write the sunk entry cost as

$$W(t) L_{Ni}(t) = \psi P_{Xi}(t) X_i(t), \quad \psi > 0 \tag{10}$$

where L_{Ni} is labor employed in entry operations and ψ is a parameter representing technological opportunity. A free-entry equilibrium requires that the value of the new firm equals the entry cost, that is,

$$V_i(t) = \psi P_{Xi}(t) X_i(t). \tag{11}$$

[2] Peretto and Smulders (2002) provide microeconomic foundations for the knowledge aggregator (7).

[3] Peretto and Connolly (2007) discuss the microfoundations of this assumption and of several alternatives that yield the same results.

In symmetric equilibrium the mass of firms grows according to

$$\frac{\dot{n}(t)}{n(t)} = \frac{1}{\psi\gamma} \cdot \frac{W(t)\, L_N(t)}{P_Y(t)\, Y(t)} - \delta, \tag{12}$$

where L_N is total employment in entry (see the Appendix).

2.4 Resource Dynamics

The resource stock, S, obeys the regeneration equation

$$\dot{S}(t) = G(S(t)) - H(t), \tag{13}$$

where $G(\cdot)$ is natural regeneration and H is harvesting. Following the benchmark model of renewable resources pioneered by Schaefer (1957), we assume that the regeneration function takes the logistic form

$$G(S(t)) = \eta S(t) \cdot \left(1 - \frac{S(t)}{\bar{S}}\right), \quad \eta > 0, \quad \bar{S} > 0 \tag{14}$$

where η is the intrinsic regeneration (or growth) rate and $\bar{S}$ is the carrying capacity of the habitat, i.e., the maximum level of the resource stock that the natural environment sustains when there is no harvesting.

The harvesting technology is

$$H(t) = BL_H(t) \cdot S(t), \quad B > 0 \tag{15}$$

where B is a productivity parameter also known as the "catchability coefficient" and L_H is the amount of labor employed in harvesting. Employment in harvesting is determined by the choices of the households, who behave like atomistic extractive firms and earn a flow of resource rents given by

$$\Pi_S(t) = P_H(t)\, H(t) - W(t)\, L_H(t), \tag{16}$$

where P_H is the price of the harvest good.

2.5 Household Behavior and Access Rights

We consider a representative household endowed with L units of labor that it can either sell in the market for the wage W or use to produce the harvest good that it can then sell in the market for the price P_H. The household has preferences

$$U(t) = \int_0^\infty \log C(t) \cdot e^{-\rho t} dt, \quad \rho > 0 \tag{17}$$

where $C(t)$ is consumption and ρ is the discount rate. Financial wealth, A, consists of ownership claims on firms that yield a rate of return r. The budget constraint reads

$$\dot{A}(t) = r(t)A(t) + W(t)L + \underbrace{[P_H(t)BS(t) - W(t)] \cdot L_H(t)}_{\Pi_S(t)=\text{resource rents}} - P_Y(t)C(t), \quad (18)$$

where the term in square brackets follows from (15)-(16). The household chooses the time paths of consumption, C, and employment in harvesting, L_H, to maximize (17) subject to (18). The choice over L_H depends on the regime of access rights.

Under *open access*, the household has no control over the total resource stock, the constraint (13) does not appear in the household problem, and the Hamiltonian reads

$$\mathcal{L}^{oa} \equiv \log C(t) + \lambda_a(t)\dot{A}(t), \quad (19)$$

where λ_a is the marginal shadow value of financial wealth. The first order condition with respect to L_H then yields maximization of current resource rents.

Under *full property rights*, instead, the household has full control over the resource stock and maximizes the present value of the stream of benefits in a forward-looking manner: equation (13) is an explicit constraint in the optimization problem and S is an additional state variable. Consequently, the current-value Hamiltonian reads

$$\mathcal{L}^{pr} \equiv \log C(t) + \lambda_a(t)\dot{A}(t) + \lambda_s(t)\dot{S}(t), \quad (20)$$

where λ_s is the marginal shadow value of the resource stock.

3 General Equilibrium

Open access and full property rights yield different harvesting plans, which, in equilibrium, induce different dynamics of consumption and innovation-led growth. Before analyzing in detail the two regimes, we describe the general equilibrium relations that hold in the economy independently of the type of access rights on natural resources. All the expressions discussed below are derived in the Appendix.

3.1 Main Features of the Equilibrium

The economy allocates labor across five activities: final production, manufacturing production, firm-specific knowledge accumulation (vertical R&D), entry (horizontal R&D), and resource harvesting. Because we have assigned the decision concerning employment in harvesting to the household, we have in fact modeled the household as determining labor supply as $L - L_H$. Consequently, the labor market clearing condition is

$$L - L_H(t) = L_Y(t) + L_X(t) + L_Z(t) + L_N(t), \tag{21}$$

where L_X and L_Z denote, respectively, total employment in production and vertical R&D. Labor mobility yields wage equalization across all activities. We take labor as the numeraire and set $W(t) \equiv 1$. We also denote expenditure on manufacturing goods by $y \equiv P_Y Y$.

The market for the final good clears when output equals consumption, $Y(t) = C(t)$. The household problem yields the Euler equation for consumption growth,

$$\frac{\dot{P}_Y(t)}{P_Y(t)} + \frac{\dot{C}(t)}{C(t)} = \frac{\dot{y}(t)}{y(t)} = r(t) - \rho. \tag{22}$$

From (8), the return to financial assets is

$$r(t) = \frac{\Pi_X(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)} - \delta. \tag{23}$$

The free-entry condition yields that financial wealth – the aggregate value of firms – is a constant fraction of the value of final output:

$$A(t) = n(t)V(t) = \psi\gamma \cdot y(t). \tag{24}$$

Equilibrium of the financial market requires that all rates of return be equal.

As discussed in detail in Peretto (1998) and Peretto and Connolly (2007), models of this class have well-defined dynamics also when one of the two R&D activities shuts down because it is return-dominated by the other, or even when they both shut down because they fail to generate the household's reservation rate of return on saving. For simplicity, we focus our analysis on the case in which both types of innovation are active and discuss the role of corner solutions in the Appendix. The growth rate of firm-specific knowledge is

$$\frac{\dot{Z}(t)}{Z(t)} = \kappa\theta\gamma \cdot \left(\gamma \frac{y(t)}{n(t)}\right) - (r(t) + \delta). \tag{25}$$

Equation (25) shows that larger firm size $\gamma y/n$ increases the typical firm's vertical R&D effort, boosting productivity growth. The gross growth rate of the mass of firms is

$$\frac{\dot{n}(t)}{n(t)} + \delta = \frac{1 - \gamma - \psi\rho}{\psi} - \frac{n(t)}{\gamma y(t)} \cdot \frac{1}{\psi} \cdot \left(\phi + \frac{1}{\kappa} \cdot \frac{\dot{Z}(t)}{Z(t)}\right). \tag{26}$$

The last term in (26) highlights that firm-specific knowledge accumulation reduces the expansion rate of product variety because it raises the anticipated post-entry incumbency cost.

Access rights influence the equilibrium of the economy via their effect on resource use and the income it generates. Equation (24) implies that expenditure is a constant fraction of labor and resource income:

$$y\left(t\right)=\frac{1}{1-\rho\psi\gamma}\cdot\left(L+\Pi_{S}\left(t\right)\right). \tag{27}$$

Different regimes of access rights over resources yield different dynamics of resource rents and, hence, of consumption expenditure. This mechanism has crucial implications for growth and welfare, as we show in the next section.

3.2 Equilibrium under Open Access

Under open access, the household chooses employment in harvesting in order to maximize *current* rents, while competition forces the price of the harvest good down to the marginal harvesting cost and thus resource rents to zero. From the Hamiltonian (19), we have

$$P_{H}^{oa}\left(t\right)=\frac{1}{BS^{oa}\left(t\right)}\quad\Longrightarrow\quad\Pi_{S}^{oa}\left(t\right)=0. \tag{28}$$

From (27), this constant time profile of resource income yields an equilibrium with constant expenditure on the final good which, via the saving rule (22), yields that the interest rate equals the discount rate:

$$y^{oa}\left(t\right)=y^{oa}\equiv\frac{L}{1-\psi\gamma\rho}\quad\text{and}\quad r^{oa}\left(t\right)=\rho. \tag{29}$$

This result has important implications for harvesting and innovation.

Proposition 1 *(Natural resource dynamics under Open Access) Harvesting is proportional to the existing resource stock:*

$$\frac{H^{oa}\left(t\right)}{S^{oa}\left(t\right)}=\alpha By^{oa}=\frac{\alpha BL}{1-\psi\gamma\rho}. \tag{30}$$

The regeneration equation (13) becomes

$$\dot{S}^{oa}\left(t\right)=\left(\eta-\frac{\alpha BL}{1-\psi\gamma\rho}\right)\cdot S^{oa}-\frac{\eta}{\bar{S}}\cdot\left(S^{oa}\right)^{2}, \tag{31}$$

and yields

$$\lim_{t\to\infty}S^{oa}\left(t\right)=S_{ss}^{oa}\equiv\begin{cases}\frac{\bar{S}}{\eta}\cdot\left(\eta-\frac{\alpha BL}{1-\psi\gamma\rho}\right) & \text{if}\quad\eta>\bar{\eta}^{oa}\equiv\frac{\alpha BL}{1-\psi\gamma\rho}\\ 0 & \text{if}\quad\eta\leqslant\bar{\eta}^{oa}\end{cases}. \tag{32}$$

There exists a condition on the parameters determining whether the economy experiences natural resource exhaustion or it reaches a steady state with a positive stock. The condition for long-run preservation, $\eta > \bar{\eta}^{oa}$, says that the intrinsic regeneration rate η must be sufficiently high to compensate for the adverse effects of consumers' impatience (a high ρ boosts current consumption and thereby harvesting), resource dependency in production (a high α yields a large demand for the harvesting good), and efficiency in harvesting (a high B raises the incentive to hire workers in harvesting). For $\eta \leqslant \bar{\eta}^{oa}$ the open-access economy experiences the *Tragedy of the Commons*: the rent-maximizing harvesting rule is unsustainable and the resource stock eventually vanishes.

The equilibrium paths of the innovation rates are as follows.

Proposition 2 *(Innovation dynamics under Open Access) The rate of accumulation of firm-specific knowledge is*

$$\frac{\dot{Z}^{oa}(t)}{Z^{oa}(t)} = \kappa\theta\gamma \cdot \frac{\gamma y^{oa}}{n^{oa}(t)} - (\rho + \delta). \tag{33}$$

The mass of firms follows a logistic process with constant coefficients,

$$\frac{\dot{n}^{oa}(t)}{n^{oa}(t)} = \nu \cdot \left[1 - \frac{n^{oa}(t)}{\tilde{n}^{oa}}\right], \qquad \nu \equiv \frac{1 - \gamma - \theta\gamma - \psi(\rho + \delta)}{\psi} \tag{34}$$

where ν is the intrinsic growth rate, and

$$\tilde{n}^{oa} \equiv \gamma y^{oa} \cdot \frac{1 - \gamma - \theta\gamma - \psi(\rho + \delta)}{\phi - (\rho + \delta) \cdot \kappa^{-1}}. \tag{35}$$

is the carrying capacity. In the long run, the mass of firms converges to the carrying-capacity level

$$n_{ss}^{oa} \equiv \lim_{t \to \infty} n^{oa}(t) = \tilde{n}^{oa}.$$

One implication of Proposition 2 is that the engine of growth in the long run is firm-specific knowledge accumulation. Entry generates transitional dynamics in the mass of firms but, due to the fixed operating cost ϕ, it is not self-sustaining. Consequently, in steady state the rate of entry exactly compensates for the death rate of firms δ. The long-run mass of firms n_{ss}^{oa}, on the other hand, determines long-run firm size, $\gamma y^{oa}/n_{ss}^{oa}$, and thus long-run growth. The interpretation is that, at any point in time, the equilibrium of factors market and the consumption/saving decision of the household determine the size of the market for manufacturing goods, γy^{oa}. This, in turn, determines the carrying capacity in the logistic equation characterizing the equilibrium proliferation of firms. Peretto and Connolly (2007) discuss in detail the intuition for why these logistic dynamics arise in a broad class of models and how they relate to the literature. The reason why these models exhibit logistic, instead of exponential, growth of the mass of frms is that they (re)introduce the fixed operating costs of the static theory of product variety that first-generation endogenous growth models set to zero.[4] In this paper's specific application of the Schumpeterian framework, the *finite* amounts of labor and of the natural resource are the force that limits the proliferation of a "specie" – firms/products – through the crowding effect implied by the fixed operating cost.

3.3 Equilibrium under Full Property Rights

Under full property rights, harvesting satisfies the Hotelling rule: the marginal net rent must grow over time at the rate of interest net of the marginal benefits from resource regeneration.[5] We show this result in the Appendix. Here, we focus on the components of the household harvesting plan that identify the key channels through which such plan affects macroeconomic outcomes.

First, resource rents are strictly positive because the household chooses the extraction path so to equalize the profits from harvesting to the marginal shadow value of the resource stock:

$$\Pi_S^{pr}(t) = y^{pr}(t) \cdot \lambda_s^{pr}(t) H^{pr}(t). \tag{36}$$

Second, the resource price is

$$P_H^{pr}(t) = \frac{1}{BS^{pr}(t)} + \underbrace{y^{pr}(t) \cdot \lambda_s^{pr}(t)}_{\text{scarcity rent}}. \tag{37}$$

Third, from (27) and (36) we obtain

$$y^{pr}(t) = \frac{L}{1 - \rho\psi\gamma - \lambda_s^{pr}(t) H^{pr}(t)}. \tag{38}$$

According to hese expressions, resource rents are not a constant fraction of consumption expenditure because the incentives to harvest depend on the marginal shadow value of the

[4] Setting $\phi = 0$ in (34) yields that the mass of firms does not follow a logistic process anymore but grows forever, exactly like in expanding-varieties models *à la* Grossman and Helpman (1991).

[5] The *Hotelling rule* – named after Hotelling (1931) – asserts that an efficient harvesting plan requires that the growth rate of the marginal net rents from harvesting equal the interest rate minus the shadow value of all the positive feedback effects that a marginal increase in the resource stock induces on current rents and on future consumption benefits from resource use. If the resource is non-renewable and harvesting costs are independent of the resource stock, the feedback effects are zero and the Hotelling rule asserts that the growth rate of the marginal net rents from harvesting equal the interest rate.

resource stock. Differently from the open access regime, full property rights induce an equilibrium path where expenditure and the interest rate are time-varying. The reason is that the economy's rate of return continuously adjusts to the dynamics of resource rents generated by the harvesting choices of forward-looking resource owners.

We can study the equilibrium path of the economy by constructing a two-by-two system that governs the joint dynamics of the *shadow value of the resource stock*, $m(t) \equiv \lambda_s^{pr}(t) S^{pr}(t)$, and the physical resource stock, $S^{pr}(t)$.

Proposition 3 *(Natural resource dynamics under Full Property Rights) Harvesting is a monotonously decreasing function of the shadow value of the resource stock:*

$$\frac{H^{pr}}{S^{pr}} = \Lambda(m) \equiv \frac{2\alpha BL}{1-\rho\psi\gamma+BLm+\sqrt{(1-\rho\psi\gamma+BLm)^2-4\alpha BLm}}. \tag{39}$$

The associated dynamical system consists of the costate equation generated by the Hamiltonian (20) and the regeneration equation (13) evaluated at the harvesting rule (39):

$$\frac{\dot{m}(t)}{m(t)} = \rho+\frac{\eta}{\bar{S}}S^{pr}(t)-\frac{\alpha}{m(t)}; \tag{40}$$

$$\frac{\dot{S}^{pr}(t)}{S^{pr}(t)} = \eta-\frac{\eta}{\bar{S}}S^{pr}(t)-\Lambda(m(t)). \tag{41}$$

The system is saddle-path stable and converges to:

$$\lim_{t\to\infty} m(t) = m_{ss} \equiv \begin{cases} \frac{\alpha}{\rho+(\eta/\bar{S})S_{ss}^{pr}} & \textit{if} \quad \eta > \bar{\eta}^{pr} \equiv \Lambda(m_{ss}) \\ \frac{\alpha}{\rho} & \textit{if} \quad \eta \leqslant \bar{\eta}^{pr} \end{cases},$$

$$\lim_{t\to\infty} S^{pr}(t) = S_{ss}^{pr} \equiv \begin{cases} \frac{\bar{S}}{\eta}\cdot(\eta-\Lambda(m_{ss})) & \textit{if} \quad \eta > \bar{\eta}^{pr} \equiv \Lambda(m_{ss}) \\ 0 & \textit{if} \quad \eta \leqslant \bar{\eta}^{pr} \end{cases}. \tag{42}$$

Figure 1 illustrates the dynamics in the two cases: $\eta > \bar{\eta}^{pr}$ yields positive resource stock in the steady state (left diagram) whereas $\eta < \bar{\eta}^{pr}$ leads to resource exhaustion (right diagram). The condition for long-run resource preservation is conceptually analogous to that obtained under open access: if the intrinsic regeneration rate is too low, resource exhaustion occurs. However, the intrinsic regeneration rate triggering exhaustion under full property rights, $\bar{\eta}^{pr} \equiv \Lambda(m_{ss})$, differs from that obtained under open access, $\bar{\eta}^{oa}$. We discuss this point in detail in section 4.

The convergence results in (42) imply that consumption expenditure and the interest rate are constant in the long run: from (38) and (22), we obtain

$$\lim_{t\to\infty} y^{pr}(t) = y_{ss}^{pr} \equiv \frac{L}{1-\psi\gamma\rho-m_{ss}\Lambda(m_{ss})} \quad \text{and} \quad \lim_{t\to\infty} r^{pr}(t) = \rho. \tag{43}$$

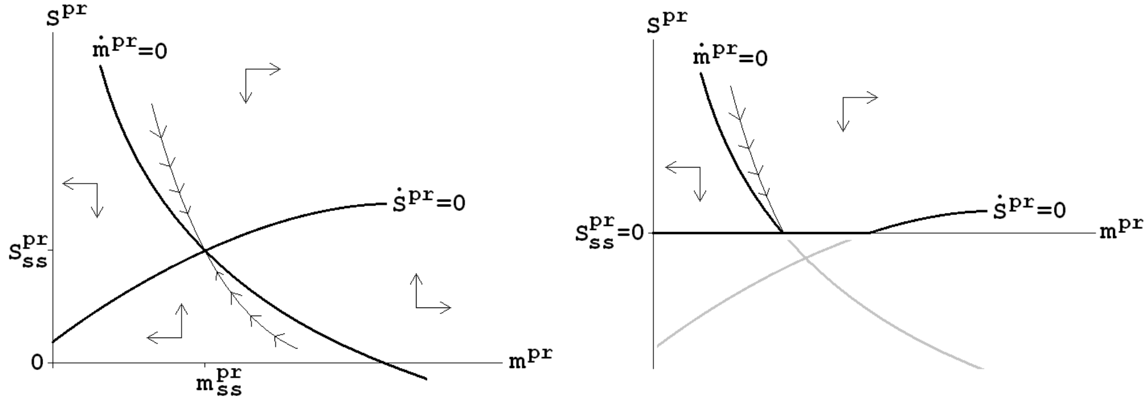

Figure 1: Dynamics under full property rights according to Proposition 3. Left graph: the case $\eta > \bar{\eta}^{pr}$ implies positive resource stock in the long run. Right graph: the case $\eta < \bar{\eta}^{pr}$ leads to long-run resource exhaustion.

In light of these results, we can characterize the dynamics of innovation as follows.

Proposition 4 *(Innovation dynamics under Full Property Rights) The rate of accumulation of firm-specific knowledge is*

$$\frac{\dot{Z}^{pr}(t)}{Z^{pr}(t)} = \kappa\theta\gamma \cdot \frac{\gamma y^{pr}(t)}{n^{pr}(t)} - \left(r^{pr}(t) + \delta\right).$$

The mass of firms follows the logistic process with time-varying carrying capacity

$$\frac{\dot{n}^{pr}(t)}{n^{pr}(t)} = \nu \cdot \left[1 - \frac{n^{pr}(t)}{\tilde{n}^{pr}(t)}\right], \qquad \nu \equiv \frac{1 - \gamma - \theta\gamma - \psi(\rho + \delta)}{\psi}$$

where ν is the intrinsic growth rate, and

$$\tilde{n}^{pr}(t) \equiv \gamma y^{pr}(t) \cdot \frac{1 - \gamma - \theta\gamma - \psi(\rho + \delta)}{\phi - \left(r^{pr}(t) + \delta\right) \cdot \kappa^{-1}}$$

is the carrying capacity. In the long run, since $y^{pr}(t) \to y^{pr}_{ss}$ and $r^{pr}(t) \to \rho$, we have

$$n^{pr}_{ss} \equiv \lim_{t\to\infty} n^{pr}(t) = \lim_{t\to\infty} \tilde{n}^{pr}(t) = \gamma y^{pr}_{ss} \cdot \frac{1 - \gamma - \theta\gamma - \psi(\rho + \delta)}{\phi - (\rho + \delta) \cdot \kappa^{-1}}. \tag{44}$$

Proposition 4 can be interpreted along similar lines as Proposition 2: the mass of firms follows a logistic process converging towards a stable carrying-capacity level n^{pr}_{ss}. As we noted above, the finite amounts of labor and of the natural resource limit the proliferation of a "specie" – firms/products – that would otherwise grow exponentially. Unlike the open-access regime, the carrying capacity of firms changes over time due to agents' internalization of the dynamics of the natural resource stock. More generally, human activity – i.e., harvesting – affects the evolution of the resource stock by modifying the habitat in which it grows. The evolution of the resource stock, in turn, affects the "economic habitat" in which firms grow in number and size. Productivity growth in the long run is driven by vertical innovation, whose incentives depend on firm size.

The crucial difference between the open access and the full property rights regimes is that in the former the economy lacks a price signal of scarcity capable of inducing an adaptive response of resource extractors to the changing habitat. This is why the Tragedy of the Commons occurs for a larger set of values of the natural regeneration rate than under full property rights, as we show in section 4 below.

3.4 Equilibrium Growth Rates

The model yields a clear characterization of equilibrium consumption and growth rates. In equilibrium, the logarithm of consumption in each instant equals

$$\log C(t) = \log \bar{a} + \underbrace{\log y(t)}_{\text{market size}} - \underbrace{\log P_H(t)^{\alpha}}_{\text{input cost}} + \underbrace{\log \left[(n(t))^{1-\gamma} (Z(t))^{\theta\gamma} \right]}_{\text{TFP}}, \tag{45}$$

where we have defined the constant $\bar{a} \equiv \alpha^{\alpha}\beta^{\beta}\gamma^{2\gamma}$. This expression shows that consumption is higher the higher is the value of final output (market-size effect), the lower is the resource price (input-cost effect), and the higher is total factor productivity (TFP) determined by the mass of firms and by the firm-specific knowledge stock. Accordingly, the growth rate of consumption is

$$g(t) \equiv \frac{\dot{C}(t)}{C(t)} = r(t) - \rho - \alpha \frac{\dot{P}_H(t)}{P_H(t)} + (1-\gamma)\frac{\dot{n}(t)}{n(t)} + \gamma\theta \frac{\dot{Z}(t)}{Z(t)}. \tag{46}$$

In the long run, the resource stock, the harvesting rate, the interest rate and the mass of manufacturing firms are all constant in both regimes. Consequently, the only source of consumption growth in the long run is firm-specific knowledge growth. An important implication of Propositions 2 and 4, then, is that the economy's steady-state growth rate is the same under open access and under full property rights.

Proposition 5 *(Steady-state growth) In the long run, firm size is the same in the two regimes, i.e.,*

$$\lim_{t\to\infty} \frac{\gamma y^{oa}}{n^{oa}(t)} = \lim_{t\to\infty} \frac{\gamma y^{pr}(t)}{n^{pr}(t)} = \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\phi-(\rho+\delta)\cdot\kappa^{-1}} \equiv \frac{\gamma y_{ss}}{n_{ss}}, \tag{47}$$

implying the same long-run growth rate in the two regimes:

$$\lim_{t\to\infty} g(t) = \lim_{t\to\infty} \gamma\theta \frac{\dot{Z}(t)}{Z(t)} = \gamma\theta \cdot \left[\kappa\gamma\theta \cdot \frac{\gamma y_{ss}}{n_{ss}} - (\rho + \delta)\right] > 0. \tag{48}$$

The reason why growth rates coincide is that, in our framework, the interaction between horizontal and vertical innovations fragments the intermediates' market into submarkets whose size does not depend on endowments: although the long-run levels of expenditures and of the mass of firms differ between the two regimes, the market share of each firm converges to the same equilibrium level – determined by expression (47). Armed with these results, we can investigate in detail the role of the regime of access rights.

4 Regime Comparison

In this section we compare the two regimes in four respects. First, we combine our previous results concerning long-run equilibria to compare the steady-state values of resource stocks, expenditures, and mass of firms, under the two regimes (subsection 4.1). Second, we show that different regimes of property rights induce contrasting effects on the equilibrium value of the resource price (subsection 4.2). Third, we distinguish between instantaneous and transitional effects of property rights regimes on consumption (subsection 4.3). Fourth, we characterize analytically the welfare impact of a regimes switch from full property rights to open access (subsection 4.4).

4.1 Long-Run Equilibria

We first focus on the dynamics of the resource stock and investigate the conditions for sustainable growth in the two regimes.

Proposition 6 *(Sustainability in the two regimes) The condition for long-run resource preservation is more restrictive under open access: $\bar{\eta}^{pr} < \bar{\eta}^{oa}$. Consequently, there are three cases:*

(i) For $\bar{\eta}^{pr} < \bar{\eta}^{oa} < \eta$, both regimes yield resource preservation and positive production in the long run, with

$$S_{ss}^{pr} > S_{ss}^{oa}, \quad y_{ss}^{pr} > y^{oa}, \quad n_{ss}^{pr} > n_{ss}^{oa}.$$

(ii) For $\bar{\eta}^{pr} < \eta < \bar{\eta}^{oa}$, the economy is sustainable under full property rights ($S_{ss}^{pr} > 0$) but experiences the Tragedy of the Commons under open access ($S_{ss}^{oa} = 0$).

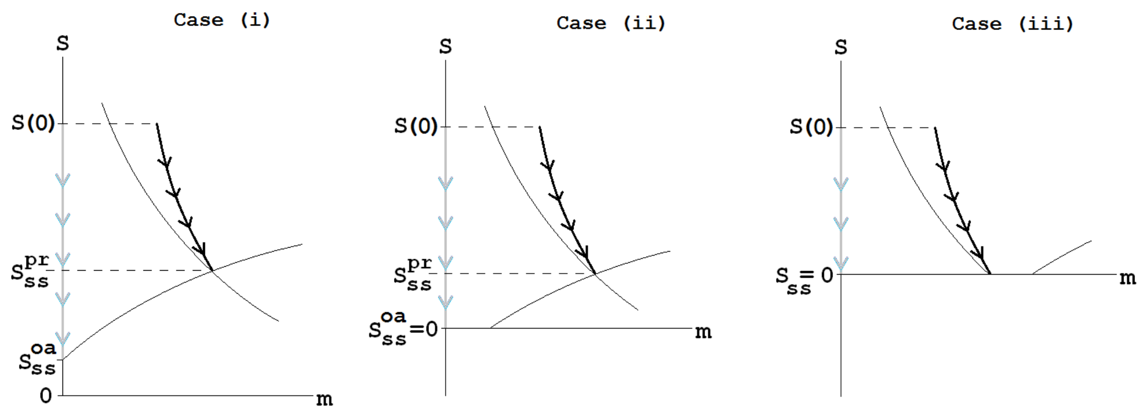

Figure 2: Regime Comparison according to Proposition 6. The black bold trajectory represents full property rights. The grey bold trajectory along the vertical axis represents open access.

(iii) For $\eta < \bar{\eta}^{pr} < \bar{\eta}^{oa}$*, both regimes yield resource exhaustion and zero production in the long run.*

The intuition behind the first statement in Proposition 6 follows immediately from the regeneration equation (13). To preserve the resource stock in the long run, the intrinsic regeneration rate η must be able to compensate for the depletion due to harvesting. Under open access, harvesting is more intense because agents do not consider the effects of current exploitation on future scarcity. Also, full property rights generate a higher level of expenditure that induces more intense entry during the transition and, consequently, more firms in the intermediate sector in the long run.

Figure 2 illustrates the dynamics of the resource stock $S(t)$ and of its shadow value $m(t)$ in the three scenarios listed in Proposition 6. The grey trajectories along the vertical axis – i.e., a zero shadow value in each instant – represent open access whereas the black trajectories are associated with full property rights. When (i) both regimes exhibit preservation, more intense harvesting under open access yields a lower resource stock in the long run. Alternatively, we may observe (ii) the Tragedy of the Commons under open access, or (iii) asymptotic exhaustion in both regimes. Figure 2 clarifies that *open access is a special case of full property rights* that obtains for $m(t) = 0$ because agents do not internalize the regeneration equation (13) in their intertemporal choices.

4.2 Resource Price: Scarcity versus Rent Effects

The equilibrium value of the resource price is affected by property rights regimes in two ways. First, the resource price at a given instant reflects current *scarcity* – i.e., the current level of the resource stock – and different regimes entail different degrees of resource preservation. Second, under full property rights, the resource price is also affected by income dynamics through the *rent effect* – that is, forward-looking extractors with full ownership make positive profits by charging a higher price than under open access given the same resource stock (cf. subsection 3.3). The interplay between scarcity effects and rent effects yields the following result.

Proposition 7 *(Resource price in the two regimes). For a given level of the resource stock $S^{oa}(t) = S^{pr}(t) = S(t)$, positive resource rents under under full property rights imply a higher resource price than under open access:*

$$P_H^{oa}(t) = \frac{1}{BS(t)} < P_H^{pr}(t) = \frac{1}{BS(t)} + \underbrace{y^{pr}(t)\,\lambda_s^{pr}(t)}_{\textit{Rent effect}}.$$

In long-run equilibria with positive preservation, the resource stock is higher under full property rights, $S_{ss}^{pr} > S_{ss}^{oa} > 0$, but the rent effect implies an ambiguous price gap:

$$\lim_{t\to\infty} P_H^{oa}(t) = \frac{1}{BS_{ss}^{oa}} \gtreqless \lim_{t\to\infty} P_H^{pr}(t) = \underbrace{\frac{1}{BS_{ss}^{pr}}}_{\textit{Scarcity effect}} + \underbrace{\lim_{t\to\infty} y^{pr}(t)\,\lambda_s^{pr}(t)}_{\textit{Rent effect}}.$$

In general, full property rights induce an upward pressure on prices via the rent effect as well as a downward pressure via scarcity effects. If we compare the two regimes at time zero, when the resource stock is given, the resource price is necessarily higher under full property rights because the rent effect is fully operative and is not mitigated by scarcity effects. As the two economies converge to their respective steady states, however, full property rights imply more intense resource preservation (cf. Figure 2), and the resulting scarcity effect may, but does not necessarily, determine a lower price than under open access. The implications of this tension between scarcity and rent effects for consumption and welfare may be substantial, as we show below.

4.3 Consumption: Expenditure-Price Tradeoff and Transitional Effects

Different harvesting regimes determine different paths of resource price, income and consumption. This mechanism has two main components. The first is *the expenditure-price* *tradeoff* captured by the first two terms in (45). High expenditure levels do not necessarily imply high consumption: if the resource price is also high, the positive impact of market size may be more than offset by the negative impact of input costs. This observation is immediately relevant to our regime comparison. On the one hand, full property rights yield higher expenditures relative to open access: from (27), positive resource rents imply $y^{pr}(t) > y^{oa}$ in each t. On the other hand, full property rights determine a higher resource price at time zero and, possibly, in the long run (cf. Proposition 7). The expenditure-price tradeoff thus suggests that full property rights do not necessarily enhance consumption at each point in time. In particular, open access can yield higher consumption in the short run.

The second source of consumption gaps between the two regimes is given by *differences in transitional growth rates*. Expression (46) captures the relevant components. On the one hand, equilibrium interest rates differ during the transition because full property rights yield positive and time-varying profits from harvesting (cf. subsection 3.3). On the other hand, productivity growth rates differ between regimes during the transition: entry in manufacturing proceeds at different speeds because, starting from a given initial condition $n(0)$, the mass of firms must reach different long-run levels, n_{ss}^{pr} or n_{ss}^{oa}, in the two regimes.

All these mechanisms jointly determine the overall impact of property rights regimes on consumption and thereby on present-value welfare. In particular, the expenditure-price tradeoff suggests that open access is not necessary welfare-reducing. Although open access is by definition a regime that fails to maximize present-value resource rents, it is not possible to conclude that full property rights are always Pareto-superior because access rights interact with other market failures – namely, monopolistic competition in manufacturing and non-decreasing returns to R&D – and the favorable impact of open access on resource prices can be substantial. The next subsection sheds further light on this issue by analyzing the welfare effects of a regime shift.

4.4 Regime Switch: From Property Rights to Open Access

Suppose that the economy is initially in the steady-state equilibrium of the full property rights regime with positive resource stock (i.e., $\bar{\eta}^{pr} < \eta$). At time $t = 0$, the economy suddenly shifts to open access – as a result of, e.g., failure in enforcing property rights. The overall impact of the regime switch on welfare depends on the combination of the instantaneous and transitional effects discussed below.

Instantaneous level effects. At time zero, the regime switch induces two instantaneous adjustments: expenditure jumps down, from y_{ss}^{pr} to y_{ss}^{oa}, and the resource price jumps down,

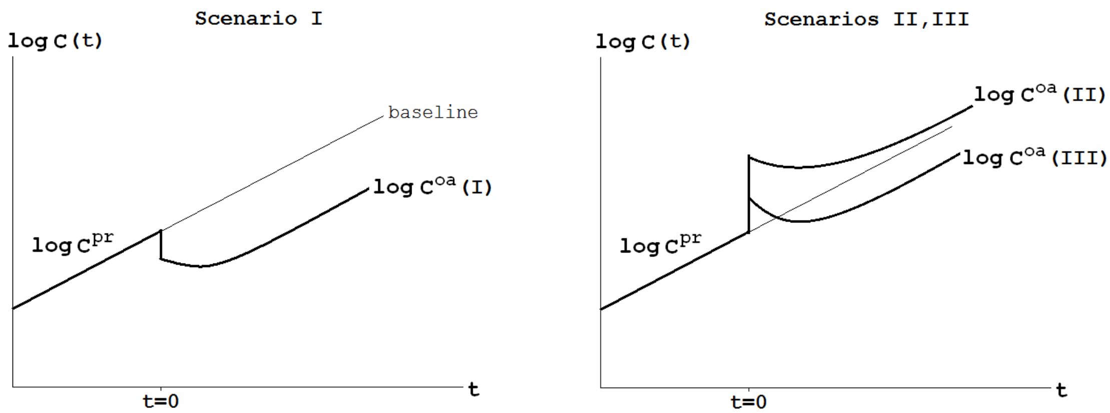

Figure 3: Effects of a regime switch from full property rights to open access at time $t = 0$. Scenario I: welfare loss (consumption is always below the baseline level). Scenario II: welfare gain (consumption is always above the baseline level). Scenario III: ambiguous welfare effect.

from $\left(P_H^{pr}\right)_{ss} = \left(1 + B y_{ss}^{pr} m_{ss}^{pr}\right)/BS_{ss}^{pr}$ to $P_H^{oa} = 1/BS_{ss}^{pr}$. From expression (45), the ratio between consumption levels (immediately) before and (immediately) after the switch is

$$\frac{C^{pr}\left(0^{-}\right)}{C^{oa}\left(0^{+}\right)} = \frac{y_{ss}^{pr}}{y_{ss}^{oa}} \cdot \left(\frac{P_H^{oa}\left(0^{+}\right)}{P_H^{pr}\left(0^{-}\right)}\right)^{\alpha} = \frac{y_{ss}^{pr}}{y_{ss}^{oa}} \cdot \left(\frac{1}{1 + B y_{ss}^{pr} m_{ss}^{pr}}\right)^{\alpha}$$

This ratio may be above or below unity in view of the expenditure-price tradeoff. Hence, the overall level effect is generally ambiguous: at the time of the regime switch, we may observe either an instantaneous drop or an instantaneous increase in consumption.

Transitional growth effects. After time 0, there are two types of transitional effects respectively induced by productivity growth and resource scarcity. First, there is a transitional slowdown in productivity growth via both horizontal and vertical innovations: the switch to open access reduces the mass of firms over time (n must move from the initial state n_{ss}^{pr} to the new steady state $n_{ss}^{oa} < n_{ss}^{pr}$) and also reduces the growth rate of firm-specific knowledge as a result of reduced expenditure. As a consequence, the transitional growth rate of TFP after the switch is smaller than the rate enjoyed before – in fact, it may even be negative because the mass if firms is shrinking. The second transitional effect results from increased scarcity: the resource stock moves from the initial state S_{ss}^{pr} to $S_{ss}^{oa} < S_{ss}^{pr}$, and this decline increases the resource price after the initial instantaneous drop.

Overall effect on welfare. After the switch, the consumption path generated by open access may be above or below the baseline path – i.e., the path, characterized by permanent full property rights, that the economy would have followed without the regime switch. The reason is the ambiguous impact of the instantaneous level effects: while the transitional growth effects (i.e., productivity slowdown and increased scarcity) tend to reduce consumption after the regime switch, the initial drop in the resource price may be strong enough to raise consumption above the baseline level at time zero. Figure 3 describes the possible outcomes according to three scenarios. If the initial jump in consumption is downward, the entire time profile of consumption for $t > 0$ is strictly below the baseline path – in which case, the switch to open access yields a welfare loss. If the initial consumption jump is upward, the impact on welfare is positive if consumption remains forever above the baseline path, and is generally ambiguous if consumption falls short of the baseline path at some finite time.

Since the model yields a closed-form solution for the equilibrium path after the regimes switch, we can assess the scope of possible ambiguities in welfare effects analytically:

Proposition 8 *The welfare change experienced by an economy that switches to the open access regime is*

$$\rho\left(U^{oa} - U^{pr}_{ss}\right) = \underbrace{\alpha B m^{pr}_{ss} y^{pr}_{ss}}_{\textit{Initial price drop}} - \underbrace{\left(1 - \frac{y^{oa}}{y^{pr}_{ss}}\right)\left(1 + \frac{\varphi}{\rho + \nu}\right)}_{\substack{\textit{Expenditure fall amplified} \\ \textit{by productivity slowdown}}} - \underbrace{\frac{\alpha}{\rho + \omega}\left(1 - \frac{S^{oa}_{ss}}{S^{pr}_{ss}}\right)}_{\textit{Increased scarcity}}. \tag{49}$$

Proposition 8 formally establishes that the switch to open access yields a welfare loss unless the positive effect of the initial drop in the resource price is large enough to compensate for the negative effects induced by (i) the instantaneous fall in expenditure due to the destruction of the flow of resource rents; (ii) the transitional slowdown of TFP growth induced by reduced expenditure; (iii) the gradual increase in resource scarcity. This result suggests a more general conclusion that abstracts from the experiment of regime switching: full property rights improve welfare relative to open access if the utility cost induced by positive resource rents is more than offset by the static gains generated by higher expenditure and the dynamic gains induced by faster (transitional) productivity growth.

It is self-evident that our results concerning the welfare impact of property-rights regimes firmly hinge on the endogeneous nature of both the resource price and the productivity growth rate. This property differentiates our analysis from the traditional resource economics literature, which typically employs partial equilibrium models.

5 Conclusion

This paper analyzed the impact of different regimes of access rights to renewable natural resources on sustainability conditions, innovation rates and welfare levels in a Schumpeterian model of endogenous growth. The crucial difference between open access and full property rights is that, in the former, the economy lacks a price signal of scarcity capable of inducing an adaptive response of resource extractors to the changing habitat. Consequently, the critical condition for long-run sustainability is always more restrictive under open access: the economy might experience the Tragedy of the Commons under open access and sustained economic growth under full property rights.

Full property rights yield positive rents from harvesting and therefore higher expenditure relative to open access: the bigger market size induces faster productivity growth during the transition via both horizontal and vertical innovations. However, positive rents also imply that the resource price is lower under open access given the same resource stock. Consequently, a failure in property-rights enforcement that induces a regime switch to open access generates negative transitional effects via slower productivity growth but also ambiguous level effects on consumption because reduced resource prices mitigate the impact of lower expenditures. The closed-form solution delivered by the model shows that switching to open access is welfare reducing if the utility gain generated by the initial drop in the resource price is more than offset by the static and dynamic losses induced by reduced expenditure.

The crucial role played by endogenous prices and endogenous productivity growth in our conclusions confirms that a proper understanding of the relationship between long-term sustainability and property-rights regimes requires a full general equilibrium analysis. In particular, the vertical structure of production that characterizes our model implies that prospects for sustainability hinge on the link between price formation in upstream extraction/harvesting and the incentives to innovate faced by downstream industries: this topic deserves further research at both the theoretical and the empirical levels.

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A Appendix

Manufacturing sector (incumbents): maximization problem. Using the demand schedule (4) and the technology (5), the incumbent firm's profit equals

$$\Pi_{Xi} = \left[\frac{\gamma P_Y H^{\alpha} L_Y^{\beta}}{P_{Xi}}\right]^{\frac{1}{1-\gamma}} \left[P_{Xi} - W Z_i^{-\theta}\right] - W L_{Zi} - W\phi. \tag{A.1}$$

The firm maximizes (8) subject to (A.1) and (6)-(7), using P_{Xi} and L_{Zi} as control variables, firm-specific knowledge Z_i as the state variable, taking public knowledge K as given. The current-value Hamiltonian is

$$\mathcal{L}_i^x \equiv \Pi_{Xi} = \left[\frac{\gamma P_Y H^{\alpha} L_Y^{\beta}}{P_{Xi}}\right]^{\frac{1}{1-\gamma}} \left[P_{Xi} - W Z_i^{-\theta}\right] - W L_{Zi} - W\phi + \lambda_i^x \cdot \kappa K L_{Zi}, \tag{A.2}$$

where λ_i^x is the dynamic multiplier associated to (6). Since the Hamiltonian is linear in L_{Zi}, we have a bang-bang solution. The necessary conditions for maximization read

$$1 = \frac{1}{1-\gamma}\left[\frac{P_{Xi} - W Z_i^{-\theta}}{P_{Xi}}\right], \tag{A.3}$$

$$\lambda_i^x \cdot \kappa K - W \leqslant 0 \qquad (<0 \text{ if } L_{Zi}=0, =0 \text{ if } L_{Zi}>0), \tag{A.4}$$

$$(r+\delta)\cdot \lambda_i^x - \dot{\lambda}_i^x = \theta \cdot X_i W Z_i^{-\theta-1}. \tag{A.5}$$

Condition (A.3) follows from $\partial \mathcal{L}^x / \partial P_{Xi} = 0$ and yields the standard mark-up rule

$$P_{Xi} = \frac{1}{\gamma}\cdot W Z_i^{-\theta}. \tag{A.6}$$

Condition (A.4) is the Kuhn-Tucker condition for R&D investment: in an interior solution, the marginal cost of employing labor in vertical R&D activity (W) equals the marginal benefit of accumulating knowledge ($\lambda_i^x \kappa K$). Condition (A.5) is the co-state equation for knowledge: with strict equality in (A.4), substitution of both $\lambda_i^x = W/(\kappa K)$ and (A.6) in (A.5) yields

$$r + \delta = \gamma\theta \cdot \frac{X_i P_{Xi}}{W}\cdot \kappa \frac{K}{Z_i} + \frac{\dot{W}}{W} - \frac{\dot{K}}{K}. \tag{A.7}$$

Manufacturing sector (incumbents): symmetry. The symmetry of the equilibrium is established in detail in Peretto (1998: Proposition 1) and Peretto and Connolly (2007). Applying the same proof to the present model, the mark-up rule (A.6) is invariant across varieties and implies the same price P_{Xi}, the same quantity X_i, and the same employment in production L_{Xi} for each $i \in [0, n]$. Therefore, we can combine (A.3) and (A.6) to write each firm's market share as in expression (9) in the main text. Concerning the knowledge stock, from (7) and (6), the equilibrium growth rate under symmetry is

$$\dot{K}/K = \dot{Z}_i/Z_i = \dot{Z}/Z = \kappa \cdot L_{Zi}, \tag{A.8}$$

where we can substitute $L_Z = nL_{Zi}$ to obtain

$$\frac{\dot{Z}}{Z} = \kappa \cdot \frac{L_Z}{n}. \tag{A.9}$$

Manufacturing sector (entry): derivation of (12). Given a constant death rate of firms δ, the mass of entrants in each instant equals the gross variation in the mass of firms $\dot{n}+\delta n$. This implies that total labor employed in entry activities equals $L_N = L_{Ni}\left(\dot{n}+\delta n\right)$, and equation (10) may be written as

$$WL_N = \left(\dot{n}+\delta n\right)\psi P_{Xi}X_i. \tag{A.10}$$

Rearranging terms, we have

$$\frac{\dot{n}}{n} = \frac{WL_N}{\psi n P_{Xi}X_i} - \delta, \tag{A.11}$$

where we can substitute (9) to obtain (12).

General equilibrium: derivation of (22). In both regimes of access rights – see the Hamiltonians (20) and (20) – the household problem yields the necessary conditions

$$1/C = \lambda_a P_Y, \tag{A.12}$$

$$\dot{\lambda}_a = \lambda_a\left(r-\rho\right), \tag{A.13}$$

from which we obtain the standard Keynes-Ramsey rule (22).

General equilibrium: derivation of (23). Time-differentiating (8) yields (23).

General equilibrium: derivation of (24). Combining (9) with (11), we obtain (24).

General equilibrium: derivation of (25). Substituting (A.8) in (A.7) yields

$$\frac{\dot{Z}}{Z} = \frac{\dot{W}}{W} + \kappa\theta\gamma^2 \cdot \frac{P_Y Y}{Wn} - \left(r+\delta\right). \tag{A.14}$$

Setting $W=1$ in (A.14) yields equation (25) in the text.

General equilibrium: derivation of (26). Time-differentiating the free entry condition (24), we obtain

$$\frac{\dot{V}_i}{V_i} = \frac{\dot{P}_Y}{P_Y} + \frac{\dot{Y}}{Y} - \frac{\dot{n}}{n}. \tag{A.15}$$

Substituting (A.15) in (23) to eliminate $\dot{V}_i/V_i$ yields

$$r+\delta+\frac{\dot{n}}{n}=\frac{\dot{P}_Y}{P_Y}+\frac{\dot{Y}}{Y}+\frac{\Pi_{Xi}}{V_i} \tag{A.16}$$

where, because $C=Y$, we can use the Keynes-Ramsey rule (22) to obtain

$$\frac{\dot{n}}{n}=\frac{\Pi_{Xi}}{V_i}-\rho-\delta. \tag{A.17}$$

Substituting (A.6) and (A.9) in the definition of profits Π_{Xi}, we have

$$\Pi_{Xi}=\gamma\left(1-\gamma\right)\cdot\frac{P_YY}{n}-W\phi-W\frac{1}{\kappa}\frac{\dot{Z}}{Z}. \tag{A.18}$$

Substituting (A.18) in (A.17), and using (24) to eliminate V_i, we have

$$\frac{\dot{n}}{n}=\frac{1-\gamma}{\psi}-W\frac{n}{\psi\gamma P_YY}\cdot\left[\phi+\frac{1}{\kappa}\frac{\dot{Z}}{Z}\right]-\rho-\delta,$$

which reduces to (26) for $W=1$.

General equilibrium: derivation of (27). Substituting $A=\psi\gamma\cdot P_YY$ from (24), as well as $Y=C$, in the wealth constraint (18), we obtain

$$\frac{\dot{P}_Y}{P_Y}+\frac{\dot{Y}}{Y}=r+\frac{L-y}{\psi\gamma\cdot y}+\frac{\Pi_S}{\psi\gamma\cdot y}. \tag{A.19}$$

The Keynes-Ramsey rule (22) then yields

$$y\left(1-\psi\gamma\rho\right)=L+\Pi_S, \tag{A.20}$$

which yields (27) in the text.

Equilibrium under open access: derivation of (28) and (29). Under open access, normalizing $W\equiv 1$ and recalling expression (18), the Hamiltonian (19) reads

$$\mathcal{L}^{oa}\equiv\log C^{oa}+\lambda_a^{oa}\cdot\left[r^{oa}A^{oa}+L-P_Y^{oa}C^{oa}+\left(P_H^{oa}BS^{oa}-1\right)\cdot L_H^{oa}\right], \tag{A.21}$$

where C^{oa} and L_H^{oa} are control variables and A^{pr} is the only state variable. The necessary conditions for maximization are:

$$1/C^{oa} = \lambda_a^{oa}P_Y^{oa}; \tag{A.22}$$

$$P_H^{oa}BS^{oa} = 1; \tag{A.23}$$

$$\dot{\lambda}_a^{oa} = \lambda_a^{oa}\cdot\left(\rho-r^{oa}\right). \tag{A.24}$$

From (A.23), we have $P_H^{oa} B S^{oa} = 1 \Longrightarrow \Pi_S^{oa} = 0$, which is expression (28) in the text. Substituting (28) in (27), we have

$$P_Y^{oa} Y^{oa} = L\left(1 - \psi\gamma\rho\right)^{-1}, \tag{A.25}$$

which, substituted into the Keynes-Ramsey rule (22), yields $r^{oa} = \rho$.

Equilibrium under open access: proof of Proposition 1. Combining (28) with (3), we obtain (30). Substituting (30) into (13), we obtain the differential equation (31) which converges to the unique steady state

$$\lim_{t\to\infty} S^{oa}\left(t\right) = \left(\bar{S}/\eta\right) \cdot \left(\eta - \frac{\alpha B L}{1-\psi\gamma\rho}\right).$$

Imposing the non-negativity restriction on physical quantities $S^{oa} \geqslant 0$ determines the critical threshold reported in (31).

Equilibrium under open access: proof of Proposition 2. As noted in the main text, Proposition 2 assumes that innovation activities are operative in each instant: that is, the economy uses positive amounts of labor in vertical R&D and entry activites ($L_Z > 0$ and $L_X > 0$) implying positive rates of public knowledge growth ($\dot{Z}\left(t\right) > 0$) and of gross entry ($\frac{\dot{n}(t)}{n(t)} + \delta > 0$). A detailed analysis of the implied restrictions on parameters is reported at the end of this Appendix. Expression (33) is obtained by substituting $y^{oa} = y^{oa}$ and $r^{oa} = \rho$ from (29) into equation (25). Substituting (33) into (26) yields (34), which is dynamically stable around the unique steady state $n_{ss}^{oa} \equiv \lim_{t\to\infty} n^{oa}\left(t\right) = \tilde{n}^{oa}$.

Equilibrium under Full Property Rights: the Hotelling rule. Under full property rights, normalizing $W \equiv 1$ and recalling expression (18), the Hamiltonian (20) reads

$$\mathcal{L}^{pr} \equiv \log C^{pr} + \lambda_a^{pr} \cdot \left[r^{pr} A^{pr} + L - P_Y^{pr} C^{pr} + \Pi_S^{pr}\right] + \lambda_s^{pr} \cdot \dot{S}^{pr},$$

that is,

$$\begin{aligned} \mathcal{L}^{pr} \equiv\ & \log C^{pr} + \lambda_a^{pr} \cdot \left[r^{pr} A^{pr} + L - P_Y^{pr} C^{pr} + \Pi_S^{pr}\left(S^{pr}, L_H^{pr}\right)\right] + \\ & + \lambda_s^{pr} \cdot \left[G\left(S^{pr}\right) - H^{pr}\left(S^{pr}, L_H^{pr}\right)\right], \end{aligned} \tag{A.26}$$

where C^{pr} and L_H^{pr} are the control variables, A^{pr} and S^{pr} are the state variables, and the functions

$$\Pi_S\left(S^{pr}, L_H^{pr}\right) \equiv \left(P_H^{pr} B S^{pr} - 1\right) \cdot L_H^{pr}, \tag{A.27}$$

$$G\left(S^{pr}\right) \equiv \eta S^{pr} \cdot \left[1 - \left(S^{pr}/\bar{S}\right)\right], \tag{A.28}$$

$$H\left(S^{pr}, L_H^{pr}\right) \equiv B L_H^{pr} S^{pr}, \tag{A.29}$$

directly follow from definitions (16), (14) and (15). The necessary conditions for maximization are:

$$1/C^{pr} = \lambda_a^{pr} P_Y^{pr}; \tag{A.30}$$

$$\frac{\partial \Pi_S\left(S^{pr}, L_H^{pr}\right)}{\partial L_H^{pr}} = \frac{\lambda_s^{pr}}{\lambda_a^{pr}} \cdot \frac{\partial H\left(S^{pr}, L_H^{pr}\right)}{\partial L_H^{pr}}; \tag{A.31}$$

$$\frac{\dot{\lambda}_a^{pr}}{\lambda_a^{pr}} = \rho - r^{pr}; \tag{A.32}$$

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} = \rho - \frac{\partial G\left(S^{pr}\right)}{\partial S^{pr}} + \frac{\partial H\left(S^{pr}, L_H^{pr}\right)}{\partial S^{pr}} - \frac{\lambda_a^{pr}}{\lambda_s^{pr}} \frac{\partial \Pi_S\left(S^{pr}, L_H^{pr}\right)}{\partial S^{pr}}; \tag{A.33}$$

along with the transversality conditions

$$\lim_{t\to\infty} \lambda_a^{pr}(t)\, A^{pr}(t)\, e^{-\rho t} = 0, \tag{A.34}$$

$$\lim_{t\to\infty} \lambda_s^{pr}(t)\, S^{pr}(t)\, e^{-\rho t} = 0. \tag{A.35}$$

Henceforth, we denote the *marginal net rent* from employing an additional unit of labor in harvesting as

$$\Pi_S' \equiv \frac{\partial \Pi_S\left(S^{pr}, L_H^{pr}\right)}{\partial L_H^{pr}} = \left(P_H^{pr} B S^{pr} - 1\right) = \frac{\Pi_S\left(S^{pr}, L_H^{pr}\right)}{L_H^{pr}}. \tag{A.36}$$

Time-differentiating (A.31), we obtain

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} - \frac{\dot{\lambda}_a^{pr}}{\lambda_a^{pr}} = \frac{\dot{\Pi}_S'}{\Pi_S'} - \frac{\dot{S}^{pr}}{S^{pr}},$$

where we can substitute (A.32) and (A.33) to obtain

$$\frac{\dot{\Pi}_S'}{\Pi_S'} = r^{pr} - \left\{ \frac{\lambda_a^{pr}}{\lambda_s^{pr}} \cdot \frac{\partial \Pi_S\left(S^{pr}, L_H^{pr}\right)}{\partial S^{pr}} + \left[\frac{\partial G\left(S^{pr}\right)}{\partial S^{pr}} - \frac{\partial H\left(S^{pr}, L_H^{pr}\right)}{\partial S^{pr}} \right] - \frac{\dot{S}^{pr}}{S^{pr}} \right\}. \tag{A.37}$$

Equation (A.37) is a generalized *Hotelling rule*: an efficient harvesting plan requires that the growth rate of the marginal net rents from resource harvesting equal the interest rate minus the term in curly brackets – which represents the shadow value of all the positive feedback effects that a marginal increase in the resource stock induces on current rents and on future consumption benefits from resource use. If the resource were non-renewable and harvesting costs were independent of the resource stock, the term in curly brackets would be zero: in that case, equation (A.37) would collapse to the basic Hotelling's (1931) rule $\dot{\Pi}_S'/\Pi_S' = \dot{P}_H/P_H = r^{pr}$.

Equilibrium under Full Property Rights: derivation of (36)-(37). From (A.27) and (A.29), the first order condition (A.31) can be re-written as

$$\lambda_a^{pr} \cdot \left(P_H^{pr} B S^{pr} - 1\right) = \lambda_s^{pr} \cdot B S^{pr},$$

where we can subtitute $\lambda_a^{pr} = 1/\left(P_Y^{pr} Y^{pr}\right)$ from (A.30), and multiply both sides by L_H^{pr}, to obtain

$$\left(P_H^{pr} BS^{pr} - 1\right) \cdot L_H^{pr} = \lambda_s^{pr} \cdot BS^{pr} L_H^{pr} \cdot y^{pr}, \tag{A.38}$$

which yields expression (37) in the main text. The left-hand side of (A.38) equals current net rents from harvesting, Π_S^{pr}. Therefore, substituting $H^{pr} = BS^{pr} L_H^{pr}$ from (15) into (A.38), we obtain equation (36) in the text.

Equilibrium under Full Property Rights: derivation of (38). From (27), we can rewrite the relation between expenditure and resource rents as

$$\Pi_S^{pr}(t) = y^{pr}(t)(1 - \rho\psi\gamma) - L. \tag{A.39}$$

Substituting Π_S^{pr} in (A.39) by means of (36), we obtain

$$y^{pr}(t) = \frac{L}{1 - \rho\psi\gamma - \lambda_s^{pr}(t) H^{pr}(t)}, \tag{A.40}$$

that is equation (38) in the text. For future reference, notice that – using the resource demand schedule (3) – resource rents can also be written as

$$\Pi_S^j(t) = P_H^j(t) H^j(t) - L_H^j(t) = \alpha y^j(t) - L_H^j(t). \tag{A.41}$$

Combining (A.39) with (A.41) under full property rights, it follows that

$$L - L_H^{pr}(t) = (1 - \rho\psi\gamma - \alpha) \cdot y^{pr}(t). \tag{A.42}$$

In any equilibrium with positive final output, we must have $L > L_H^{pr}(t)$ and, consequently, the parameter restriction

$$1 - \rho\psi\gamma - \alpha > 0. \tag{A.43}$$

Equilibrium under Full Property Rights: proof of Proposition 3. The proof hinges on three steps: (i) the derivation of the dynamic system (40)-(41); (ii) the proof of saddle-point stability; (iii) the proof of results (42).

(i) Dynamic system First, we derive equation (40). From (A.28) and (A.29), we have

$$\frac{\partial G(S^{pr})}{\partial S^{pr}} - \frac{\partial H\left(S^{pr}, L_H^{pr}\right)}{\partial S^{pr}} = \eta - 2\left(\eta/\bar{S}\right) \cdot (S^{pr}) - BL_H^{pr}. \tag{A.44}$$

From (A.30) and (A.27), we respectively have

$$\lambda_a^{pr} = 1/y^{pr} \text{ and } \frac{\partial \Pi_S\left(S^{pr}, L_H^{pr}\right)}{\partial S^{pr}} = P_H^{pr} BL_H^{pr}. \tag{A.45}$$

Substituting (A.44) and (A.45) into (A.33), as well as $H^{pr}/S^{pr} = BL_H^{pr}$ from (15), we have

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} = \rho - \eta + 2\left(\eta/\bar{S}\right) \cdot \left(S^{pr}\right) + \frac{H^{pr}}{S^{pr}} - \frac{1}{\lambda_s^{pr}} \cdot \frac{P_H^{pr} H^{pr}}{y^{pr} S^{pr}}. \quad \text{(A.46)}$$

Substituting P_H^{pr} by means of (3), we obtain

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} = \rho - \eta + 2\left(\eta/\bar{S}\right) S^{pr} + \frac{H^{pr}}{S^{pr}} - \frac{1}{\lambda_s^{pr}} \cdot \frac{\alpha}{S^{pr}}. \quad \text{(A.47)}$$

From (13) and (15), the growth rate of the resource stock is

$$\frac{\dot{S}^{pr}}{S^{pr}} = \eta - \left(\eta/\bar{S}\right) S^{pr} - \frac{H^{pr}}{S^{pr}}. \quad \text{(A.48)}$$

Equations (A.47)-(A.48) imply

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} + \frac{\dot{S}^{pr}}{S^{pr}} = \rho + \left(\eta/\bar{S}\right) S^{pr} - \frac{\alpha}{\lambda_s^{pr} S^{pr}}. \quad \text{(A.49)}$$

Equation (A.49) can be transformed into a differential equation governing the *shadow value of the resource stock*, $m \equiv \lambda_s^{pr} S^{pr}$, which depends on the resource stock:

$$\frac{\dot{m}}{m} = \rho + \left(\eta/\bar{S}\right) S^{pr}(t) - \frac{\alpha}{m}, \quad \text{(A.50)}$$

which is equation (40) in the text. We now derive (41). From (A.38), we have

$$P_H^{pr} B S^{pr} = 1 + \lambda_s^{pr} S^{pr} \cdot B y^{pr}. \quad \text{(A.51)}$$

Substituting $P_H^{pr} = \alpha y^{pr}/H^{pr}$ from (3) into (A.51), and using $m \equiv \lambda_s^{pr} S^{pr}$, we have

$$\frac{S^{pr}}{H^{pr}} = \frac{1}{\alpha B y^{pr}} + \frac{m}{\alpha}. \quad \text{(A.52)}$$

Using (A.40) to substitute y^{pr} in (A.52), and using $H^{pr} = H^{pr} \cdot S^{pr}/S^{pr}$,we obtain

$$\frac{S^{pr}}{H^{pr}} = \frac{1 - \rho\psi\gamma - m \cdot \frac{H^{pr}}{S^{pr}}}{\alpha BL} + \frac{m}{\alpha},$$

which generates the second-order static equation

$$\alpha BL \left(\frac{S^{pr}}{H^{pr}}\right)^2 - \left(1 - \rho\psi\gamma + BLm\right) \cdot \frac{S^{pr}}{H^{pr}} + m = 0. \quad \text{(A.53)}$$

Equation (A.51) determines, at each point in time, the equilibrium stock-flow ratio S^{pr}/H^{pr} for given m. The roots of (A.51) are

$$\frac{S^{pr}}{H^{pr}} = \frac{1 - \rho\psi\gamma + BLm \pm \sqrt{\left(1 - \rho\psi\gamma + BLm\right)^2 - 4\alpha BLm}}{2\alpha BL}. \quad \text{(A.54)}$$

Notice that, in order to ensure a real value for S^{pr}/H^{pr}, the term under the square root is constrained to be strictly positive:

$$\begin{aligned}(1-\rho\psi\gamma+BL\cdot m)^2-4\alpha BL\cdot m &= \\ (1-\rho\psi\gamma)^2+2\left(1-\rho\psi\gamma-4\alpha\right)BLm+(BLm)^2 &> 0.\end{aligned} \tag{A.55}$$

In order to isolate the admissible root in (A.54), notice that $H^{pr}=BL_H^{pr}S^{pr}$ from (15) and $L_H^{pr}<L$ from the requirement of strictly positive labor (see (A.43) above) imply that $H^{pr}<BLS^{pr}$ must hold in each instant in an equilibrium with positive harvesting and positive final production. Imposing this inequality in (A.54), we have

$$\frac{BLS^{pr}}{H^{pr}}=\frac{1-\rho\psi\gamma+BLm\pm\sqrt{(1-\rho\psi\gamma+BLm)^2-4\alpha BLm}}{2\alpha}>1. \tag{A.56}$$

The above inequality can only be satisfied by the solution exhibiting the plus sign in front of the square root.[6] Inverting the stock-flow ratio in (A.56), we can thus write

$$\frac{H^{pr}}{S^{pr}}=\Lambda\left(m\right) \tag{A.57}$$

in each instant t in which there is an equilibrium with positive production, where

$$\Lambda\left(m\right)\equiv\frac{2\alpha BL}{1-\rho\psi\gamma+BL\cdot m+\sqrt{(1-\rho\psi\gamma+BL\cdot m)^2-4\alpha BL\cdot m}}. \tag{A.58}$$

Notice that, given the restriction (A.55), definition (A.58) implies

$$\Lambda'\left(m\right)\equiv\frac{\partial\Lambda\left(m\right)}{\partial m}=-\frac{2\alpha B^2L^2\left\{1+2\left(1-\rho\psi\gamma-4\alpha\right)+2BLm\right\}}{\left[1-\rho\psi\gamma+BL\cdot m+\sqrt{(1-\rho\psi\gamma+BL\cdot m)^2-4\alpha BL\cdot m}\right]^2}<0. \tag{A.59}$$

These results allow us to complete the autonomous two-by-two system: equation (40) is (A.50) above; substituting result (A.57) into (A.48), we obtain (41).

(ii) Saddle-point stability The steady-state loci of system (40)-(41) are given by:

$$\dot{m}\left(t\right) = 0\rightarrow S^{pr}\left(t\right)=\frac{\bar{S}}{\eta}\cdot\left(\frac{\alpha}{m\left(t\right)}-\rho\right); \tag{A.60}$$

$$\dot{S}^{pr}\left(t\right) = 0\rightarrow S^{pr}\left(t\right)=\frac{\bar{S}}{\eta}\cdot\left[\eta-\Lambda\left(m\left(t\right)\right)\right]. \tag{A.61}$$

[6] The proof of this statement is by contradiction: picking the solution with the minus sign, inequality (A.56) would imply $4\alpha\left\{\alpha-\left(1-\rho\psi\gamma\right)\right\}>0$, which is not possible because we would violate the parameter restriction (A.43).

The steady state (m_{ss}, S_{ss}^{pr}) is therefore chcarcterized by:

$$S_{ss}^{pr} = \frac{\bar{S}}{\eta} \cdot \left(\frac{\alpha}{m_{ss}} - \rho \right); \tag{A.62}$$

$$\Lambda(m_{ss}) = \eta - \left(\eta / \bar{S}\right) S_{ss}^{pr}. \tag{A.63}$$

Therefore, there exists a steady state with positive resource stock if and only if parameters are such that

$$m_{ss} < \frac{\alpha}{\rho} \text{ and } \Lambda(m_{ss}) < \eta. \tag{A.64}$$

Linearizing system (40)-(41) around the steady-state (m_{ss}, S_{ss}^{pr}), we have

$$\begin{pmatrix} \dot{m}/m \\ \\ \dot{S}^{pr}/S^{pr} \end{pmatrix} \simeq \begin{pmatrix} \varsigma_1 \equiv \left(\alpha/m_{ss}^2\right) & \varsigma_2 \equiv \left(\eta/\bar{S}\right) \\ \\ \varsigma_3 \equiv -\Lambda'(m_{ss}) & \varsigma_4 \equiv -\left(\eta/\bar{S}\right) \end{pmatrix} \begin{pmatrix} m - m_{ss} \\ \\ S^{pr} - S_{ss}^{pr} \end{pmatrix},$$

where (recalling result (A.59) above), the coefficients have definite signs: $\varsigma_1 > 0$, $\varsigma_2 > 0$, $\varsigma_3 > 0$, $\varsigma_4 < 0$. These signs imply $(\varsigma_4\varsigma_1 - \varsigma_2\varsigma_3) < 0$. As a consequence, the characteristic roots of the linearized system, given by the eigenvalues

$$\frac{(\varsigma_1 + \varsigma_4) \pm \sqrt{(\varsigma_1 + \varsigma_4)^2 - 4(\varsigma_4\varsigma_1 - \varsigma_2\varsigma_3)}}{2},$$

are necessarily real and of opposite sign. The steady state (m_{ss}, S_{ss}^{pr}) thus displays saddle-point stability: given the initial state $S^{pr}(0) = S_0$, there is a unique trajectory determined by the jump variable $m(0)$ driving the system towards (m_{ss}, S_{ss}^{pr}). Ruling out explosive paths by standard arguments,[7] the saddle-path determines a unique equilibrium path which converges to a positive stationary level of the resource stock $S_{ss}^{pr} > 0$ provided that the restrictions (A.64) are satisfied.

(iii) Steady states Results (42) follow from the condition for positive steady-state resource stock implied by (41). If the parameters are such that $\eta > \Lambda(m_{ss})$, restrictions (A.64) are satisfied and saddle-point stability implies that $(m(t), S^{pr}(t))$ converge to the steady state (m_{ss}, S_{ss}^{pr}) with $S_{ss}^{pr} > 0$ determined by (A.62)-(A.63): see Figure 1, left graph. If parameters imply $\eta \leqslant \Lambda(m_{ss})$, instead, the steady state (A.62)-(A.63) is not feasible in view of restrictions (A.64) and the dynamics generated by the loci (A.60)-(A.61) imply that $(m(t), S^{pr}(t))$ converge to a steady state with $\lim_{t\to\infty} S^{pr}(t) = 0$ and $\lim_{t\to\infty} m(t) = \alpha/\rho$, as shown in Figure 1, right graph.

[7] Explosive paths would violate either the transversality condition $\lim_{t\to\infty} m(t) e^{-\rho t} = 0$ appearing in (A.35) or the intertemporal resource constraint (13).

Equilibrium under Full Property Rights: proof of Proposition 4. As noted in the main text, Proposition 4 assumes that innovation activities are operative in each instant (i.e., $\dot{Z}(t) > 0$ and $\frac{\dot{n}(t)}{n(t)} + \delta > 0$: see the further details reported at the end of this Appendix). The equilibrium growth rate of Z^{pr} follows directly from (25), and can be substituted into (26) to obtain

$$\frac{\dot{n}^{pr}(t)}{n^{pr}(t)} = \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\psi} - \frac{1}{\psi} \cdot \frac{n^{pr}(t)}{\gamma y^{pr}(t)} \cdot \left[\phi - \frac{1}{\kappa} \cdot \left(r^{pr}(t)+\delta\right)\right]. \tag{A.65}$$

Defining $\nu \equiv \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\psi}$ and $\tilde{n}^{pr}(t) \equiv \gamma y^{pr}(t) \cdot \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\phi-\frac{1}{\kappa}\cdot(r^{pr}(t)+\delta)}$, expression (A.65) reduces to

$$\frac{\dot{n}^{pr}(t)}{n^{pr}(t)} = \nu \cdot \left[1 - \frac{n^{pr}(t)}{\tilde{n}^{pr}(t)}\right]. \tag{A.66}$$

Having established that $\lim_{t\to\infty} y^{pr}(t) = y_{ss}^{pr}$ and $\lim_{t\to\infty} r^{pr}(t) = \rho$ in (43), the carrying capacity $\tilde{n}^{pr}(t)$ is asymptotically constant,

$$\lim_{t\to\infty} \tilde{n}^{pr}(t) = \gamma y_{ss}^{pr} \cdot \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\phi-(\rho+\delta)\cdot\kappa^{-1}},$$

implying that equation (A.66) is dynamically stable around the steady state $\lim_{t\to\infty} n^{pr}(t) = \lim_{t\to\infty} \tilde{n}^{pr}(t)$.

Equilibrium growth rates: derivation of (45). Symmetry in the manufacuting sector implies that final output (1) equals $Y = H^{\alpha} L_Y{}^{\beta} n X^{\gamma}$. Substituting the profit-maximizing conditions of the final sector, we have

$$Y = \left(\frac{\alpha P_Y Y}{P_H}\right)^{\alpha} \left(\frac{\beta P_Y Y}{W}\right)^{\beta} n \left(\frac{\gamma P_Y Y}{P_X}\right)^{\gamma}.$$

Observing that Y drops out and rearranging terms, we obtain

$$P_Y = \alpha^{\alpha}\beta^{\beta}\gamma^{2\gamma} \cdot P_H^{\alpha} \cdot W^{\beta} \cdot n^{-1+\gamma} \cdot P_X^{\gamma}.$$

Observing that $C = Y = y/P_Y$, setting $\bar{a} \equiv \alpha^{\alpha}\beta^{\beta}\gamma^{2\gamma}$, $W = 1$, and using the pricing rule (A.6) yields expression (45) in the text.

Equilibrium growth rates: derivation of (46). Time-differentiating (45) and substituting $\dot{y}/y$ by means of the Keynes-Ramsey rule (22), we obtain (46).

Equilibrium growth rates: proof of Proposition 5. Propositions 2 and 4 imply the result of identical firm size in the long run (47). Substituting (47) in (25), we obtain identical asymptotic rates of vertical innovation, $\lim_{t\to\infty} \frac{\dot{Z}(t)}{Z(t)} = \kappa\gamma\theta\frac{\gamma y_{ss}}{n_{ss}} - (\rho+\delta)$. Letting $t \to \infty$ in expression (46), we obtain $\lim_{t\to\infty} g(t) = \lim_{t\to\infty} \gamma\theta\frac{\dot{Z}(t)}{Z(t)}$ and therefore (48).

Long-Run Equilibria: proof of Proposition 6. The proof hinges on two steps: (a) proving that $\bar{\eta}^{oa} > \bar{\eta}^{pr}$, and (b) comparing the three subcases (i)-(iii).

(a) **Proof that $\bar{\eta}^{oa} > \bar{\eta}^{pr}$**. From (32) and (42), the difference between the critical levels $\bar{\eta}^{oa} - \bar{\eta}^{pr} = \frac{\alpha BL}{1-\psi\gamma\rho} - \Lambda(m_{ss})$. Substituting the defintion of $\Lambda(m_{ss})$ from the third expression in (42), we have

$$\bar{\eta}^{oa} - \bar{\eta}^{pr} \equiv \frac{\alpha BL}{1-\psi\gamma\rho} \cdot \left[1 - \frac{2(1-\psi\gamma\rho)}{1-\psi\gamma\rho + BLm_{ss} + \sqrt{(1-\rho\psi\gamma + BLm_{ss})^2 - 4\alpha BLm_{ss}}} \right].$$

The term in square brackets is strictly positive if and only if $1-\rho\psi\gamma > \alpha$, a condition that surely holds given the restriction (A.43).

(b) **Proof of subcases (i)-(iii)** Considering subcase (i), suppose that $\bar{\eta}^{pr} < \bar{\eta}^{oa} < \eta$. Then, both regimes yield positive stock in the long run with the following property. From (32) and (42),

$$S_{ss}^{oa} - S_{ss}^{pr} = \frac{\bar{S}}{\eta} \cdot \left[\Lambda(m_{ss}) - \frac{\alpha BL}{1-\psi\gamma\rho} \right] = \frac{\bar{S}}{\eta} \cdot (\bar{\eta}^{pr} - \bar{\eta}^{oa}) < 0,$$

because the last term is strictly negative ($\bar{\eta}^{oa} > \bar{\eta}^{pr}$). Hence, $S_{ss}^{pr} > S_{ss}^{oa}$. Under full property rights, positive harvesting in the long run implies a positive asymptotic shadow value of the resource stock: $m_{ss} > 0$ and $\Lambda(m_{ss})$. Consequently, (29) and (43) yield $y_{ss}^{pr} > y^{oa}$. Concerning the mass of firms, from (35) and (44), we have $\frac{n_{ss}^{oa}}{n_{ss}^{pr}} = \frac{y^{oa}}{y_{ss}^{pr}}$, which implies $n_{ss}^{pr} > n_{ss}^{oa}$. Considering subcase (ii), suppose that $\bar{\eta}^{pr} < \eta < \bar{\eta}^{oa}$. Then, we have $S_{ss}^{pr} > 0$ from (42) and $S_{ss}^{oa} = 0$ from (32); since the production function (1) implies that the resource is essential, resource exhaustion under open access yields zero production/consumption under open access. Considering subcase (iii), suppose that $\eta < \bar{\eta}^{pr} < \bar{\eta}^{oa}$. Then, we have $S_{ss}^{pr} = 0$ from (42) and $S_{ss}^{oa} = 0$ from (32), that imply zero production/consumption under both regimes.

Resource price: proof of Proposition 7. From (28) and (37), the resource prices under the two regimes read

$$P_H^{oa} = \frac{1}{BS^{oa}} \text{ and } P_H^{pr} = \frac{1 + Bmy^{pr}}{BS^{pr}}.$$

For a given level of the resource stock $S(t) = \tilde{S}$, the above expressions imply $P_H^{oa}|_{S=\tilde{S}} < P_H^{pr}|_{S=\tilde{S}}$ because $Bm(t)\,y^{pr}(t) > 0$. In the long-run equilibria of the two regimes, resource prices equal

$$\lim_{t\to\infty} P_H^{oa}(t) = \frac{1}{BS_{ss}^{oa}} \text{ and } \lim_{t\to\infty} P_H^{pr}(t) = \frac{1 + Bm_{ss}y_{ss}^{pr}}{BS_{ss}^{pr}}.$$

Consequently, the sign of the gap $\lim_{t\to\infty} P_H^{oa}(t) - \lim_{t\to\infty} P_H^{pr}(t)$ is determined by the inequality $\frac{1}{BS_{ss}^{oa}} \gtrless \frac{1+Bm_{ss}y_{ss}^{pr}}{BS_{ss}^{pr}}$, that is,

$$\frac{S_{ss}^{pr}}{S_{ss}^{oa}} \gtrless 1 + Bm_{ss}y_{ss}^{pr}. \tag{A.67}$$

Substituting S_{ss}^{oa} and S_{ss}^{pr} by (32) and (42), and eliminating y_{ss}^{pr} by (43), the above expression reduces to

$$\frac{\eta - \Lambda(m_{ss})}{\eta - \frac{\alpha BL}{1-\psi\gamma\rho}} \gtrless 1 + \frac{Bm_{ss}L}{1-\psi\gamma\rho - m_{ss}\Lambda(m_{ss})}. \tag{A.68}$$

The sign is generally ambiguous because, defining $\Upsilon \equiv 1-\psi\gamma\rho$ and $\Delta \equiv \Lambda(m_{ss})$, we can rewrite (A.68) as

$$\underbrace{(\Upsilon - m_{ss}\Delta)}_{\text{positive}}\underbrace{(\alpha BL - \Delta\Upsilon)}_{\text{positive}} - \underbrace{(m_{ss}BL)}_{\text{positive}}\underbrace{(\eta\Upsilon - \alpha BL)}_{\text{positive}} \gtrless 0.$$

Regime switch: proof Proposition 8. Rewrite (45) as

$$\log C(t) = \log \bar{a} + \log y(t) - \alpha \log P_H(t) + \log T(t), \tag{A.69}$$

where we have defined total factor productivity as

$$\text{TFP} = T(t) \equiv (n(t))^{1-\gamma}(Z(t))^{\theta\gamma}. \tag{A.70}$$

Starting from (A.69)-(A.70), the derivation of expression (49) involves three intermediate steps: deriving explicit expressions for (i) TFP, (ii) the resource price, and (iii) present-value utility, under the regime of open access.

(i) Total factor productivity For future reference, we denote the rate of vertical innovation by $\hat{Z}(t) \equiv \dot{Z}(t)/Z(t)$, its asymptotic value by $\hat{Z}_{ss} \equiv \lim_{t\to\infty}\hat{Z}(t)$, and the long-run growth rate of the economy by $g_{ss} \equiv \lim_{t\to\infty} g(t) = \theta\gamma\hat{Z}(t)$. Under open access, the TFP term can be re-expressed as follows. By definition,

$$\log T^{oa}(t) = \theta\gamma \log Z_0 + \theta\gamma \int_0^t \hat{Z}^{oa}(s)\,ds + (1-\gamma)\log n_0 + (1-\gamma)\log\left(\frac{n^{oa}(s)}{n_0}\right),$$

where we can add and subtract $\hat{Z}_{ss}$ from $\hat{Z}(t)$, obtaining

$$\log T(t) = \log\left(Z_0^{\theta\gamma} n_0^{1-\gamma}\right) + g_{ss}\cdot t + \theta\gamma\int_0^t \left[\hat{Z}(s) - \hat{Z}_{ss}\right] ds + (1-\gamma)\log\left(\frac{n(s)}{n_0}\right). \tag{A.71}$$

Denoting $x^j(t) \equiv \gamma y^j(t)/n^j(t)$, and recalling that $y^{oa}(t) = y^{oa}$ is constant over time, we have $\dot{n}^{oa}/n^{oa} = -\dot{x}^{oa}/x^{oa}$. Therefore, the differential equation for n^{oa} in (34) yields $\dot{x}^{oa} = \nu\left(x_{ss}^{oa} - x^{oa}\right)$, the solution of which is

$$x^{oa}(t) = x_0^{oa} e^{-\nu t} + x_{ss}^{oa}\left(1 - e^{-\nu t}\right). \tag{A.72}$$

Result (A.72) implies that

$$\begin{aligned} \theta\gamma \int_0^t \left(\hat{Z}(s) - \hat{Z}^*\right) ds &= \kappa(\theta\gamma)^2 \int_0^t \left(x(t) - x_{ss}\right) ds \\ &= \frac{\kappa(\theta\gamma)^2 x_{ss}}{\nu}\left(\frac{x_0}{x_{ss}} - 1\right)\left(1 - e^{-\nu t}\right). \end{aligned} \tag{A.73}$$

Also, from the solution (34), we have

$$\frac{n(t)}{n_0} = \frac{1 + \left(\frac{n_{ss}}{n_0} - 1\right)}{1 + \left(\frac{n_{ss}}{n_0} - 1\right) e^{-\nu t}},$$

where we can take logarithms and approximate the resulting terms to obtain

$$\log\left(\frac{n(s)}{n_0}\right) = \left(\frac{n_{ss}}{n_0} - 1\right)\left(1 - e^{-\nu t}\right). \tag{A.74}$$

Observing that $\frac{n_{ss}}{n_0} - 1 = \frac{x_0}{x_{ss}} - 1$, results (A.73) and (A.74) yield

$$\log T(t) = \log\left(Z_0^{\theta\gamma} n_0^{1-\gamma}\right) + g_{ss} \cdot t + \varphi\left(\frac{n_{ss}}{n_0} - 1\right)\left(1 - e^{-\nu t}\right), \tag{A.75}$$

where we have defined $\varphi \equiv \frac{\kappa(\theta\gamma)^2 x_{ss}}{\nu} + (1-\gamma)$.

(ii) Resource price Since open access implies a constant harvesting rate, the resource stock follows the logistic process

$$\frac{\dot{S}^{oa}(t)}{S^{oa}(t)} = \omega \cdot \left(1 - \frac{S^{oa}(t)}{S_{ss}^{oa}}\right), \tag{A.76}$$

where we have defined the constants $\omega \equiv \eta - BL_H$ and $S_{ss}^{oa} \equiv \bar{S} \cdot \frac{\eta - BL_H}{\eta}$. The solution of (A.76) is

$$\frac{S(t)}{S_0} = \frac{1 + \left(\frac{S_{ss}}{S_0} - 1\right)}{1 + \left(\frac{S_{ss}}{S_0} - 1\right) e^{-\omega t}},$$

where we can take logarithms and approximate the resulting terms to obtain $\log \frac{S(t)}{S_0} = \left(\frac{S_{ss}}{S_0} - 1\right)\left(1 - e^{-\omega t}\right)$. Since (28) implies $-\log \frac{P_H^{oa}(t)}{P_H^{oa}(0)} = \log \frac{S(t)}{S_0}$, we have

$$-\log \frac{P_H^{oa}(t)}{P_H^{oa}(0)} = \left(\frac{S_{ss}}{S_0} - 1\right)\left(1 - e^{-\omega t}\right). \tag{A.77}$$

(iii) present-value utility Under open access, expression (A.69) reads

$$\log C^{oa}(t) = \log \bar{a} + \log \left(\frac{y^{oa}}{\left(P_H^{oa}(0)\right)^{\alpha}} \right) - \alpha \log \left(\frac{P_H^{oa}(t)}{P_H^{oa}(0)} \right) + \log T^{oa}(t) . \qquad \text{(A.78)}$$

Without loss of generality, let us normalize $\log \bar{a} + \log \left(Z_0^{\theta\gamma} n_0^{1-\gamma} \right) \equiv 0$.[8] Substituting (A.75) and (A.77) in (A.78), we obtain

$$\begin{aligned} \log C^{oa}(t) &= \log y^{oa} - \alpha \log P_H^{oa}(0) + g_{ss} \cdot t \\ &\quad + \alpha \left(\frac{S_{ss}^{oa}}{S_0} - 1 \right) \left(1 - e^{-\omega t} \right) + \varphi \left(\frac{n_{ss}^{oa}}{n_0} - 1 \right) \left(1 - e^{-\nu t} \right) . \end{aligned} \qquad \text{(A.79)}$$

Substituting (A.79) in the welfare functional (17), and integrating, we obtain

$$U^{oa} = \frac{1}{\rho} \left[\log y^{oa} - \alpha \log P_H^{oa}(0) + \frac{g_{ss}}{\rho} + \frac{\alpha}{\rho + \omega} \left(\frac{S_{ss}^{oa}}{S_0} - 1 \right) + \frac{\varphi}{\rho + \nu} \left(\frac{n_{ss}^{oa}}{n_0} - 1 \right) \right] , \qquad \text{(A.80)}$$

which is the level of welfare associated to the transition dynamics under open access given generic initial conditions S_0, Z_0, n_0.

On the basis of these results, we consider the initial conditions at time zero as determined by the steady state of the full property rights regime. In particular, given the general expression (A.69), the *baseline consumption path* that the economy would follow if full property rights were maintained after time zero is given by

$$\log C^{pr}(t) = \log \bar{a} + \log y_{ss}^{pr} - \alpha \log P_{H,ss}^{pr} + \log T^{pr}(t) , \qquad \text{(A.81)}$$

where TFP grows at the asymptotic rate g_{ss} in each instant after time zero. Substituting (A.81) in the welfare functional (17), and integrating, we obtainWith these results in hand, we refine our exercise. The baseline path is the PR steady state and yields the *baseline present-value welfare*

$$U_{ss}^{pr} = \frac{1}{\rho} \left[\log y_{ss}^{pr} - \alpha \log \left(P_H^{pr} \right)_{ss} + \frac{g_{ss}}{\rho} \right] . \qquad \text{(A.82)}$$

Taking the difference between (A.80) and (A.82), we obtain expression (49).

Further details: operativeness of innovation activities. Both Proposition 2 and Proposition 4 assume that innovation activities are operative in each instant – that is, $\frac{\dot{Z}(t)}{Z(t)} > 0$ and $\frac{\dot{n}(t)}{n(t)} + \delta > 0$. The parameter restrictions that guarantee this outcome can be derived as follows. For simplicity, consider the open access regime. From (29), substitute

[8] This normalization only simplifies the notation and does not affect the results.

$y^{oa} = y^{oa}$ and $r^{oa} = \rho$ into equation (25): the non-negativity constraint on firm-specific R&D implies

$$\frac{\dot{Z}^{oa}(t)}{Z^{oa}(t)} = \begin{cases} \kappa\theta\gamma \cdot \frac{\gamma y^{oa}}{n^{oa}(t)} - (\rho+\delta) & \text{if} \quad n^{oa}(t) < \bar{n}^{oa} \equiv \frac{\kappa\theta\gamma^2}{\rho+\delta} y^{oa} \\ 0 & \text{if} \quad n^{oa}(t) \geqslant \bar{n}^{oa} \end{cases} . \tag{A.83}$$

Substituting this result in (26), we obtain

$$\frac{\dot{n}^{oa}(t)}{n^{oa}(t)} + \delta = \begin{cases} \frac{1-\gamma-\psi\rho-\theta\gamma}{\psi} - \frac{\phi-(\rho+\delta)\cdot\kappa^{-1}}{\psi\gamma L(1-\psi\gamma\rho)^{-1}} \cdot n^{oa}(t) & \text{if} \quad n^{oa}(t) < \bar{n}^{oa} \\ \frac{1-\gamma-\psi\rho}{\psi} - \frac{\phi}{\psi\gamma L(1-\psi\gamma\rho)^{-1}} \cdot n^{oa}(t) & \text{if} \quad n^{oa}(t) \geqslant \bar{n}^{oa} \end{cases} . \tag{A.84}$$

Imposing the non-negativity constraint on employment in entry, L_N, expression (A.84) implies two threshold levels on the mass of firms. First, when $\dot{Z}^{oa}(t) > 0$, there exists $n^{oa}_{T1} \equiv \frac{\gamma L(1-\gamma-\psi\rho-\theta\gamma)}{(1-\psi\gamma\rho)[\phi-(\rho+\delta)\kappa^{-1}]}$, such that

$$n^{oa}(t) < \bar{n}^{oa} \text{ and } \frac{\dot{n}^{oa}(t)}{n^{oa}(t)} + \delta = \begin{cases} \frac{1-\gamma-\psi\rho-\theta\gamma}{\psi} - \frac{\phi-(\rho+\delta)\cdot\kappa^{-1}}{\psi\gamma L(1-\psi\gamma\rho)^{-1}} \cdot n^{oa}(t) & \text{if} \quad n^{oa}(t) < n^{oa}_{T1} \\ 0 & \text{if} \quad n^{oa}(t) \geqslant n^{oa}_{T1} \end{cases} . \tag{A.85}$$

Second, when $\dot{Z}^{oa}(t) = 0$, there exists $n^{oa}_{T2} \equiv \frac{\gamma(1-\gamma-\psi\rho)L}{\phi(1-\psi\gamma\rho)}$, such that

$$n^{oa}(t) \geqslant \bar{n}^{oa} \text{ and } \frac{\dot{n}^{oa}(t)}{n^{oa}(t)} + \delta = \begin{cases} \frac{1-\gamma-\psi\rho}{\psi} - \frac{\phi}{\psi\gamma L(1-\psi\gamma\rho)^{-1}} \cdot n^{oa}(t) & \text{if} \quad n^{oa}(t) < n^{oa}_{T2} \\ 0 & \text{if} \quad n^{oa}(t) \geqslant n^{oa}_{T2} \end{cases} . \tag{A.86}$$

It follows from (A.85) and (A.86) that a sufficient condition for positive gross entry is

$$n^{oa}(t) < \min\{n^{oa}_{T1}, n^{oa}_{T2}\} . \tag{A.87}$$

Provided that (A.87) holds, the mass of firms obeys the logistic process described in (A.84). Proposition 2 thus assumes implicitly that condition (A.87) is satisfied. This assumption is without loss of generality: because the logistic process (34) always converges to a finite steady state, satisfying (A.87) is equivalent to imposing restrictions on the exogenous parameters appearing in (A.84) and on the initial condition $n^{oa}(0)$. The alternative cases in which parameters yield no horizontal R&D are a less interesting special case because the model collapses to an economy with vertical R&D only.