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Effluent taxes, market structure, and the rate and direction of endogenous technological change

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Abstract This paper studies the effects of effluent taxes on firms' allocation of resources to cost-reducing and emission-reducing R&D, and on entrepreneurs' decisions to develop new goods and enter the market. A tax set at an exogenous rate that does not depend on the state of technology reduces growth, the level of consumption of each good, and raises the number of firms. The induced increase in the variety of goods is a benefit not considered in previous analyses. In terms of environmental benefits, the tax induces a positive rate of pollution abatement that offsets the "dirty" side of economic growth. A tax set at an endogenous rate that holds constant the tax burden per unit of output, in contrast, has ambiguous effects on growth, the scale of activity of each firm and the number of firms. Besides being novel, the potential positive growth effect of this type of effluent tax is precisely what makes this instrument effective for welfare-maximizing purposes. The socially optimal policy, in fact, requires the tax burden per unit of output to equal the marginal rate of substitution between the growth rate of consumption and abatement. Moreover, a tax/subsidy on entry is needed, depending on whether the contribution of product variety to pollution dominates consumers' love of variety.

Keywords Growth · Market structure · R&D · Effluent taxes · Environment

JEL Classification E10 · L16 · O31 · O40

1 Introduction

Experience with the tradable permit program for SO_2 in Title IV of the 1990 Clean Air Act Amendments is responsible for a dramatic change in the approach to environmental policy: Economic incentives have come of age (Joskow et al. 1998; Stavins 1998; Schmalensee et al. 1998). One important aspect of the new emphasis on incentives is the realization

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that environmental policy potentially induces technical change that reduces abatement and compliance costs. As a result, one can now hear claims that the costs of policy interventions are a fraction of estimates that ignore the effects of incentives on technical change. Nowhere is this more apparent than in the debate on climate policy where, despite remarkably consistent estimates of the high incremental costs of control, the regulatory community persists in its claims.[1]

General equilibrium models for the mechanisms that give rise to these cost savings have lagged behind the rhetoric. An exception is Goulder and Schneider (1999) which offers a stylized analytical model, along with an extension to Goulder's Computable General Equilibrium model, that allows for technological change. The paper shows that a constant carbon tax induces a reallocation of R&D among industries, falling in some—with corresponding reductions in the pace of technological change—and increasing in others. These inter-sectoral effects are important and complex. Overall, the aggregate GDP costs of carbon taxes depend on the specification of the costs of attaining the knowledge-generating resources.

This paper aims at making further progress in understanding these issues by studying the effects of effluent taxes on the growth path of a model economy where market structure is endogenous and firms allocate resources to both cost-reducing and emission-reducing R&D. Three important observations motivate this focus. First, R&D resources can be allocated to a wide array of activities—new products, cost saving innovations, reducing emissions of the effluents responsible for pollution. Thus, an important dimension of the problem is the allocation of R&D resources across alternative types of technological change. In Goulder and Schneider, in contrast, the only way to reduce emissions is to improve the efficiency of producing energy, an assumption that removes the opportunity cost of abatement.

Equally important, since economic growth is the endogenous result of R&D decisions, modern theory puts the effects on such decisions among the most important consequences of policy. On this point as well the existing analysis comes up short. This is despite the fact that there is now a large theoretical and empirical literature on the interaction between growth and the environment.[2]

Finally, the modern literature on endogenous innovation has identified product differentiation as one of the crucial dimensions driving the growth process in that it produces satisfaction through love of variety for consumers and/or productivity gains through specialization for producers (see, e.g., Romer 1990). Moreover, and perhaps more importantly, it generates and sustains the profits that drive firms' incentives to undertake R&D. This insight provides the conceptual foundation for two extensions of the endogenous growth framework pursued in this paper in order to shed new light on the role of policy interventions.

The first is the integration—mentioned above and further discussed below—between growth theory and the literature on market structure and innovation. The second, is the

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[1] Among those who argue that regulations foster technical change and thus do not imply high costs are Porter (1991) and Porter and van der Linde (1995). For a criticism, see Palmer, Oates and Portney (1995). For a survey of the evidence, see Jaffe et al. (1995). A recent empirical test that rejects the so called Porter Hypothesis is in Smith and Walsh (2000).

[2] Over the last few years, this literature has grown so rapidly that any attempt at summarizing it here would do injustice to the many contributors. It is probably more productive to refer the reader to the recent reviews by Aghion and Howitt (1998, Ch. 5), Smulders (2000) and, in particular, Brock and Taylor (2005). A paper that deserves mention because some of the ground it covers is related to what is attempted here is Grimaud (1999). There is an even larger theoretical and empirical literature that exploits the notion of induced innovation to analyze the effects of environmental policies on the rate and direction of technological change; see the recent survey by Jaffe et al. (2003). This literature precedes and is not related to the literature on endogenous technological change initiated by Romer (1990).

idea that product differentiation is an independent source of environmental damage. This is because different products entail different production processes, which in turn entail different pollutants. Basic chemistry principles suggest that the variety of effluents plays a role in determining pollution because it affects the diversity and type of reactions that take place.[3] Building on this intuition, the paper proposes a simple extension of the traditional framework that allows one to make new progress in this direction.

The paper's analysis considers the effects of effluent taxes on incumbent firms' allocation of resources to cost-reducing and emission-reducing R&D, and on entrepreneurs' decisions to develop new goods that expand product variety and enter the market (i.e., set up new firms). The paper thus extends the framework of modern endogenous growth theory to the analysis of the rate *and* direction of technological change. Because of the distinction between R&D undertaken by incumbent firms and R&D undertaken by entrants, moreover, the paper also builds on the extensive literature on market structure and innovation (see, e.g., the reviews by Baldwin and Scott 1987; Cohen and Levin 1989) and on market structure and sunk costs (see, e.g., Sutton 1991). That literature, developed mostly in a partial equilibrium framework, has produced important insights that recently have been incorporated in the Schumpeterian version of endogenous growth theory (see, e.g., Aghion and Howitt 1998, 2005; Peretto 1996, 1998, 1999; Smulders and van de Klundert 1995; Thompson 2001).[4]

The analysis considers two types of effluent taxes. The first is a tax set at a flat, exogenous rate. The second is a tax set at an endogenous rate that reflects the state of technology so to hold constant the tax burden per unit of output faced by firms. The difference is that in the first case the market (i.e., the state of technology) determines the tax burden via its determination of the tax base, while in the second the tax rate adjusts to offset changes in the tax base.

The reasons for focusing on the first tax are very pragmatic: it is simple and thus helps to illustrate in a straightforward way how an emission charge affects the model economy considered here. In addition, one could argue that precisely because of it simplicity it is the kind of tax that is most likely to be implemented in reality. In line with intuition, this tax leads to lower productivity growth, but because it induces firms to undertake emission-reducing R&D, it generates positive effects due to improvements in abatement technology and therefore to a cleaner environment. This is the traditional trade-off on which the literature on growth and the environment referenced above has focused. Due to the endogenous market structure, however, in this paper there is also a trade-off between product and pollutant variety and the level of activity per firm. This shows up as a trade-off between growth and the number of firms, and as a related trade-off between emissions per firm and the number of polluting firms. This trade-off reflects the argument proposed by Carlton and Loury (1980, 1986) that a Pigouvian (per-unit) tax on emissions may not achieve social efficiency when the number of firms is endogenous, in which case an additional lump-sum tax or subsidy for participating firms is required. The reason is that the damage that an industry inflicts on the environment depends both on the scale of activity of the individual firm and on the number

[3] Similarly, the number of products is related to the number of production plants, which in turn determines the spatial distribution of production. One might argue, therefore, that this dimension too plays a role in determining overall pollution because it affects the geographical concentration of effluents.

[4] There is a related, very large, literature on partial equilbrium models that study the role of environmental policy instruments, in particular emission taxes, under a variety of assumptions (e.g., homogenous vs differentiated products, quantity vs price competition, atomistic vs oligopolistic behavior, free entry vs blocked entry). In this case as well it is nearly impossible to do justice here to all the contributions. The interested reader can consult the reviews already mentioned by Smulders (2000), Jaffee et al. (2003), and Brock and Taylor (2005). In addition, one might want to look at the collection of articles in Petrakis et al. (1999), in particular the one by Lange and Requate (1999).

of firms. There are thus two variables that the government must control and using a single instrument is not sufficient to correct all distortions.[5]

The reason for considering the second type of tax is that it is precisely the form that a welfare-maximizing policy must take. To summarize, the analysis highlights three main points.

- The exogenous-rate tax reduces growth and the scale of activity of each firm, and has a positive effect on the number of firms.[6] In line with economic intuition, therefore, the tax has costs: a lower growth rate of consumption and a lower level of consumption of each good. The induced increase in the variety of consumption goods is a benefit not considered in previous analyses. The endogenous-rate tax, in contrast, has ambiguous effects on growth, the scale of activity of each firm and the number of firms. Besides being novel, the potential positive growth effect of this type of effluent tax is precisely what makes it effective for welfare-maximizing purposes.
- In terms of environmental benefits, both types of tax generate a positive rate of pollution abatement that offsets the "dirty" side of economic growth. The exogenous-rate tax also raises product variety, while the endogenous-rate tax does not necessarily do so. What is important here is that product variety contributes to pollution so that one cannot determine the effect of these taxes on welfare without first taking a stand on the relative strength of this effect versus consumers' love of variety.
- The optimal tax rate is a function of the ratio between the stock of cost-reducing and emission-reducing knowledge. In particular, it sets the tax burden per unit of output equal to the marginal rate of substitution between the growth rate of consumption and abatement—a constant parameter coming from preferences. Moreover, because there are two distortions, in addition to the effluent tax welfare maximization requires a tax/subsidy on entry that affects the number of firms. Specifically, if product variety contributes to love-of-variety in consumption more than to pollution, a subsidy is needed; if it contributes more to pollution than to love-of-variety in consumption, a tax is needed.

As one can see, the paper contributes to the current debate on two counts. First, it shows that the opportunity costs of policy interventions do not disappear in a fully specified endogenous growth model that allows firms to respond by investing in environment-friendly technological change. The reason is that pollution abatement diverts resources from traditional cost reduction and product creation. Thus, although it is interesting to learn that there is a class of effluent taxes that can in fact raise growth their downside is that they do so by reducing product variety. Second, policy makers need to worry about two interdependent margins: the intensive margin where pollution per firm is determined, and the extensive margin, where the number of polluting firms is determined. The current debate ignores the second margin, implicitly assuming that product differentiation is not a determinant of environmental damages from emissions because all firms produce identical pollutants in identical relation to their output.

The paper is organized as follows. Section 2 discusses the setup of the model. Section 3 constructs the equilibrium of the market economy. Section 4 characterizes the effects of effluent taxes. Section 5 discusses the welfare implications of the tax and characterizes the

[5] As is well known, this argument has been criticized by Spulber (1985) who showed that the Carlton-Loury result breaks down if the damage function does not allow for an independent role of the number of firms. In the context of this paper the Spulber criticism is shown to correspond to a special case of a more general model.

[6] As is well known, this type of result is sensitive to the specific assumptions concerning the mode of competition, the degree of product differentiation and so on. See the literature referenced above on market structure and environmental policy.

optimal policy. Section 6 concludes. A Technical Appendix available on request provides details on the formal analysis that are not included in the text in order to streamline the presentation.

2 The model

This paper builds on previous work (Peretto 1999) that analyzed the interaction of growth and market structure. This section begins with a general overview of the framework. It then discusses in some detail the main components of the model studied here, and highlights the main innovations that allow one to apply the framework to the analysis of environmental problems.

2.1 Overview

The economy is closed and populated by a representative household that supplies inelastically labor services and consumption loans in competitive labor and assets markets. Manufacturing firms hire labor to produce differentiated consumption goods, undertake R&D, or, in the case of entrants, set up operations. The economy starts out with a given range of goods, each supplied by one firm. The household likes variety and is willing to buy as many differentiated goods as possible.

Entrants compare the present value of profits from introducing a new good to the entry cost. They target new product lines because entering an existing product line in Bertrand competition with the existing supplier would lead to zero flow profits, which would make them unable to recover the start-up cost. Once in the market, firms live forever and establish in-house R&D facilities to produce a stable flow of cost- and emission-reducing innovations. The steady state is reached when the economy settles into a stable industrial structure with a constant number of firms/products.

Firms pollute. The government imposes Pigouvian, per-unit taxes on emissions to force them to include pollution in their production costs. The paper is about the effects of these taxes on the market equilibrium and about the optimal form that they should take. It shows that the optimal policy mix requires a flat-rate effluent tax that depends on the state of technology *and* a tax/subsidy on entry. To keep the model simple, it is assumed that the revenues from the effluent and the entry tax are rebated in a lump-sum fashion to households. In case an entry subsidy is required, and exceeds the effluent tax revenues, it is financed with a lump-sum tax.

2.2 Consumption

At time 0, the representative household maximizes lifetime utility

$$U(0) = \int_0^\infty e^{-\rho t} \log u(t)\, dt,$$

subject to the budget constraint

$$\dot{B} = rB + WL + G - Y,$$

where $\rho > 0$ is the discount rate, B is assets holding, r is the interest rate, W is the wage rate, G is net government transfers to the household, and Y is consumption expenditure. The wage rate is the numeraire. The household is endowed with L units of labor and has no preference for leisure. Hence, the economy's labor supply is L.

The household has symmetric preferences over a continuum of differentiated goods and environmental quality,

$$\log u = \log \left[\int_0^N X_i^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} - \psi \log D\left(E_1, \ldots, E_N\right), \tag{1}$$

where $\epsilon > 1$ is the elasticity of product substitution, X_i is the household's purchase of each differentiated good, and N is the number of goods (the number of firms) existing at time t. Pollution externalities depend on emissions by firms: $\psi > 0$ is the elasticity of utility with respect to pollution externalities and E_i is firm i's output of its specific pollutant.

The solution for the optimal expenditure plan is well known. The household sets

$$\frac{\dot{Y}}{Y} = r - \rho \tag{2}$$

and taking as given this time-path of expenditure maximizes (1) subject to $Y = \int_0^N P_i X_i di$. This yields the demand schedule

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_j^{1-\epsilon} dj}. \tag{3}$$

With a continuum of goods, firms are atomistic and take the denominator of (3) as given. Hence, monopolistic competition prevails and firms face isoelastic demand curves.

2.3 Production, innovation, abatement and entry

The typical firm produces one differentiated consumption good with the technology

$$L_{X_i} = Z_i^{-\theta} X_i, \tag{4}$$

where L_{X_i} is total labor cost (the wage is the numeraire), which is linear in the firm's output, X_i, and decreasing in the firm's stock of cost-reducing knowledge, Z_i. The parameter $0 < \theta < 1$ is the elasticity of cost reduction. A by-product of the firm's manufacturing activity is pollution. Let

$$E_i = A_i^{-\theta} X_i, \tag{5}$$

stating that emissions are linear in the firm's output, X_i, and decreasing in the firm's stock of emission-reducing knowledge, A_i. For simplicity, the elasticity of emission reduction is assumed to be equal to the elasticity of cost reduction. Assume now that emissions are taxed on a per-unit basis at rate $\tau_i > 0$. Observe that because pollutants are firm specific, the tax rates are firm-specific. (4) and (5) then give rise to the cost function

$$C_i = L_{Xi} + \tau_i E_i = \left(Z_i^{-\theta} + \tau_i A_i^{-\theta} \right) X_i. \tag{6}$$

This equation emphasizes the role of the effluent tax: firms include emissions of pollutants in their cost structure and therefore view abatement as another form of cost-reducing R&D.

The firm accumulates cost-reducing knowledge according to the R&D technology

$$\dot{Z}_i = \xi L_{Z_i}^{\gamma} Z_i, \tag{7}$$

where $\dot{Z}_i$ measures the flow of knowledge generated by an R&D project employing L_{Z_i} units of labor for an interval of time dt, $\xi > 0$ is a parameter measuring the productivity of cost-reducing R&D, and Z_i is the stock of accumulated knowledge. Similarly, the firm accumulates emission-reducing knowledge according to

$$\dot{A}_i = \alpha L_{A_i}^{\gamma} A_i, \tag{8}$$

where $\dot{A}_i$ measures the flow of knowledge generated by an R&D project employing L_{A_i} units of labor for an interval of time dt, $\alpha > 0$ is a parameter measuring the productivity of emission-reducing R&D, and A_i is the stock of accumulated knowledge. These technologies are linear in the stocks of accumulated knowledge, a property that supports constant growth in steady state, and exhibit diminishing returns to scale to the flow of labor allocated to R&D, $0 < \gamma < 1$. For simplicity, the factor intensity of the two R&D technologies is assumed to be the same.

The parameters ξ and α reflect factors that affect the productivity of R&D (e.g., state of science). The parameter γ captures diminishing returns to R&D effort. Assuming different exponents across R&D types leaves the qualitative results unchanged. The same observation applies to the parameter θ which measures the elasticity of cost and emission reduction.

The simplest way to introduce entry in this environment is to assume that upon payment of a fixed, sunk cost $\beta > 0$ in units of labor an entrepreneur can create a new firm that starts out its activity with productivity and abatement capability equal to the industry average. Once in the market, the new firm operates technologies identical to the ones discussed above for incumbents. An interpretation of this assumption in line with previous work in endogenous growth theory is that entrepreneurs create new firms by running R&D projects that develop new differentiated goods and their manufacturing processes. Assuming that the targeted level of productivity and abatement capability of entrants is the industry's average, the corresponding entry technology is

$$\dot{N} = \frac{1}{\beta} L_N, \tag{9}$$

where $1/\beta > 0$ is the productivity of labor in entry and L_N is the amount of labor devoted to starting up $\dot{N}$ new firms in the interval of time dt. (Alternatively, as done above, one can refer to β as the cost of entry in units of labor.) Implicit in this formulation is a spillover from incumbents to entrants due to the fact that new firms can achieve the industry's average practice without compensating incumbents for their past efforts in developing such practice. For a detailed discussion of this setup, see Peretto (1998, 1999).

2.4 Pollution and other externalities

The utility index (1) posits pollution externalities. The function $D(\cdot)$ aggregates the emissions of N firms into a measure of environmental damage. The paper follows modern endogenous growth theory and focuses on product differentiation as a driver of innovation incentives. A natural extension of the notion that differentiation matters for growth because preferences (or technology) exhibit love-of-variety, is the notion that differentiation matters for pollution as well. Specifically, imagine that the damage function takes the following Dixit-Stiglitz-like form:

$$D(E_1, \ldots, E_N) = \left(\int_0^N E_i^{\delta} di \right)^{\mu},$$

where $\delta, \mu > 0$. This formulation exhibits love-of-variety due to pollutant differentiation whenever $\delta \neq 1$, and reduces to the familiar Dixit-Stiglitz expression for $\delta = 1/\mu$. (As is well known, the advantage of the two-parameter specification is that one can disentangle returns to variety from the elasticity of substitution between the arguments of the function.) In symmetric equilibrium the utility index (1) becomes

$$\log u = \log N^{\frac{\epsilon}{\epsilon-1}} X - \psi \log E^{\eta} N^{\mu},$$

where $\eta \equiv \delta\mu$ and μ are, respectively, the elasticity of pollution with respect to emissions per firm E and the number of firms N. If the damage function does not reduce to $D = (NE)^{\eta}$, that is if $\eta \neq \mu$, a Pigouvian tax on output (or emissions) is not sufficient and an additional instrument targeting the number of firms is needed.[7]

This setup accommodates quite naturally the Carlton and Loury (1980, 1986) argument mentioned in the introduction, and shows that the Spulber (1985) criticism corresponds to the special case $\eta = \mu$ (i.e., $\delta = 1$). One could argue that such a special case is more plausible, especially if the analysis focuses on remedies for the damages caused by some homogeneous pollutant (e.g., the carbon tax as a remedy to CO_2 emissions). On the other hand, one could argue that several chemicals are more toxic in combination rather than individually, or that emissions of some particulates neutralize part of the SO_2 discharged in the atmosphere. Ultimately, however, whether or not the damage function displays love-of-variety with respect to pollutant variety is an empirical question.

A good place to start assessing the proposed specification is the natural-science foundations of the mechanisms by which human activity affects the environment. Bolin (2003) reviews the state of the art. He opens the section on air quality and air pollution with the claim that "Pollution usually is a mixture of different pollutants causing different damages;... (p. 15)" He then discusses the role of a long list of pollutants. For each one, he emphasizes that complex interactions with the other pollutants ultimately determine what damage they cause. In the case of VOC, for example, he says: "Volatile organic carbons, which include non-methane hydrocarbons (NMHC), have short residence times in the atmosphere, but still play an important role in atmospheric chemistry, and they serve as precursors for aerosol formation. A number of hydrocarbons are naturally present in small amounts, but concentrations have increased markedly because of human emissions. Many extraneous compounds are being added because of leakage when extracting, refining, and using oil and natural gas, and also through biomass burning to provide energy in developing countries. They are common components of air pollution and play an important role in the complex web of chemical reactions in polluted air, for example, in the formation of smog (the interplay of smoke and fog). (p. 24)"

Later in the section he talks about "An integrated approach to the air pollution problem" and writes: "It is seldom sufficient to assess the impacts of one pollutant at a time, but rather the combined effect of several pollutants may be of more concern and there may be positive or negative feedback mechanisms in the system that need careful consideration. For example, increasing amounts of halocarbons in the atmosphere mean more target molecules for destruction by OH-radicles, which will reduce their availability for oxidation of other pollutants, e.g., methane. (p. 26)" In the rest of the chapter he provides several other examples and similar statements concerning depletion of the ozone layer, water pollution, acidification of water and soil, and climate change and global warming.

There is thus a solid foundation for thinking that pollutant variety and the interactions among different pollutants do matter in determining overall environmental damage.[8] It is

[7] One might notice that if $\mu, \delta < 0$, the damage function exhibits hate-of-variety. I ignore this case for simplicity, but one might want to keep in mind that the results that I discuss below are consistent with this property.

[8] In addition, one can find this emphasis on pollutant differentiation and interactions also in Helfand et al. (2003) who review the theoretical foundations of the damage function in the literature on pollution policy.

then appropriate to investigate the implications of the case $\eta \neq \mu$ instead of ruling it out arbitrarily. As the analysis below shows, the cost in terms of additional machinery is minimal.

Another important feature of this model's setup that is worth stressing here is that, if one abstracts from pollution externalities, in this economy market performance involves two dimensions: the rate of growth of consumption of each good and the variety of consumption goods. As discussed in Peretto (1999), in steady state both the growth rate *and* the number of firms are optimal. There is thus no reason for the government to intervene to eliminate distortions affecting R&D decisions. What this means is that the model is set up in a way that isolates environmental externalities from other considerations and thus allows this paper to study of the role of the effluent tax in a framework where there are no other distortions (at least in steady state).[9]

3 Equilibrium of the market economy

This section constructs the symmetric equilibrium of the manufacturing sector of the economy. It then imposes general equilibrium conditions to determine the aggregate dynamics of the economy.

3.1 Industry equilibrium

At time 0, the typical firm maximizes the present discounted value of profits

$$V_i(0) = \int_0^\infty e^{-\int_0^t r(s)ds} \Pi_i(t)dt.$$

Using the cost function (6), profits are

$$\Pi_i = \left(P_i - Z_i^{-\theta} - \tau_i A_i^{-\theta}\right) X_i - L_{Z_i} - L_{A_i}.$$

V_i is the value of the firm, the price of the ownership share of an equity holder. The firm maximizes V_i subject to the R&D technologies (7) and (8), the demand schedule (3), $Z_i(0) > 0$ and $A_i(0) > 0$ (the initial knowledge stocks are given), $Z_j(t)$ and $A_j(t)$ for $t \geq 0$ and $j \neq i$ (the firm takes as given the rivals' innovation paths), $\dot{Z}_i(t) \geq 0$ and $\dot{A}_i(t) \geq 0$ for $t \geq 0$ (innovation is irreversible). The solution of this problem yields the value of the firm given the time path of the number of firms.

Recall that upon payment of a sunk cost $\beta > 0$ entrepreneurs create new firms that start out with productivity and abatement levels equal to the industry's average levels, Z and A. Once in the market, entrants implement price and R&D strategies that solve a problem identical to the one outlined above. An equilibrium with free entry obtains when the maximized value of the firm equals the entry cost (see below).

To characterize firm i's strategy, form the Current Value Hamiltonian

$$CVH_i = \left(P_i - Z_i^{-\theta} - \tau_i A_i^{-\theta}\right) X_i - L_{Z_i} - L_{A_i} + q_{Z_i} \xi L_{Z_i}^{\gamma} Z_i + q_{A_i} \alpha L_{A_i}^{\gamma} A_i,$$

where q_{Z_i} and q_{A_i} are, respectively, the values of the marginal unit of cost-reducing and emission-reducing knowledge. The knowledge stocks, Z_i and A_i, are the state variables;

[9] It is not difficult to extend the analysis to the interaction between environmental distortions and other forms of market failure. I prefer to focus exclusively on environmental externalities, however, because it brings out most clearly the novel issues that models of this class allow one to study.

R&D investments, L_{Z_i} and L_{A_i}, and the product's price, P_i, are the control variables. The first order conditions are:

$$P_i = \frac{\epsilon}{\epsilon - 1}\left(Z_i^{-\theta} + \tau_i A_i^{-\theta}\right); \tag{10}$$

$$r = \frac{\dot{q}_{Z_i}}{q_{Z_i}} + \frac{\theta Z_i^{-\theta-1} X_i}{q_{Z_i}} + \xi L_{Z_i}^{\gamma}; \tag{11}$$

$$r = \frac{\dot{q}_{A_i}}{q_{A_i}} + \frac{\tau\theta A_i^{-\theta-1} X_i}{q_{A_i}} + \alpha L_{A_i}^{\gamma}; \tag{12}$$

$$L_{Z_i} = \left(\xi\gamma q_{Z_i} Z_i\right)^{\frac{1}{1-\gamma}}; \tag{13}$$

$$L_{A_i} = \left(\alpha\gamma q_{A_i} A_i\right)^{\frac{1}{1-\gamma}}. \tag{14}$$

In addition, one has the constraints on the state variables, (7) and (8), and the standard transversality conditions.

Equation (10) is the Bertrand-Nash price strategy, where the assumption that firms are atomistic allows approximation of the price elasticity of demand with the elasticity of product substitution, ϵ. Equations (11) and (12) define the rates of return to cost-reducing and emission-reducing R&D, respectively, as the ratio between revenues from the marginal unit of knowledge and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge. The revenue from the marginal unit of knowledge is given by the cost or emission reduction it yields times the scale of production to which it applies. Equations (13) and (14) define the R&D strategies for the firm according to the standard condition that the marginal benefit from one unit of R&D must equal its marginal cost.

If one ignores pollution externalities and emission taxes, under fairly general conditions the equilibrium of models of this class is symmetric (for a detailed discussion, see Peretto 1999). Hence, one can drop the subscript i and let all variables denote industry averages. In the present analysis, however, in order to focus on a symmetric equilibrium one needs the additional assumption that the tax rates τ_i be uniform across firms. Upon reflection, this assumption does not imply much loss of generality. The reason is that products and their associated pollutants enter the fundamental preferences and technology in a symmetric fashion so that all possible heterogeneity implied by product and pollutant differentiation shows up as love-of-variety effects fully captured by the number of products. This implies that the equilibrium without government intervention that provides the benchmark for welfare analysis is symmetric, while the optimal pollutant-specific tax rates are indeed symmetric. It is thus reasonable—not just convenient—to focus on uniform taxes and symmetric equilibria.

Now, imposing symmetry, taking logs and time derivatives of (13) and (14), using the production function (4), one obtains:

$$r = r_Z \equiv (1-\gamma)\frac{\dot{L}_Z}{L_Z} + \xi\theta\gamma L_X L_Z^{\gamma-1}; \tag{15}$$

$$r = r_A \equiv (1-\gamma)\frac{\dot{L}_A}{L_A} + \alpha\theta\gamma L_X L_A^{\gamma-1}\tau\left(\frac{Z}{A}\right)^{\theta}; \tag{16}$$

These equations define, respectively, the rates of return to cost- and emission-reducing R&D. Notice that no-arbitrage in the assets market requires that both rates of return equal the interest rate r.

Recall now that entry costs β and produces value V_i. The case $V_i > \beta$ yields infinite demand for labor in entry and is ruled out. The case $V_i < \beta$ yields zero entry. A free-entry equilibrium requires $V_i = \beta$, which implies indifference concerning the magnitude of the flow $L_N \geq 0$ (which is pinned down by the general equilibrium conditions). Differentiating the value of the firm yields

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V}_i}{V_i},$$

which is a no-arbitrage condition for equilibrium of the assets market. It requires that the rate of return to firm ownership equal the rate of return to a riskless loan of size V_i. The rate of return to firm ownership is the ratio between profits and the firm's stock market value plus the capital gain (loss) from the stock appreciation (depreciation). In an equilibrium with entry, $V_i = \beta$ implies $\dot{V}_i = 0$ and one obtains $r = \Pi_i/\beta$. Imposing symmetry and using the cost function (6) and the price strategy (10) yields

$$r = r_N \equiv \frac{1}{\beta}\left[\frac{L_X}{\epsilon - 1}\left(1 + \tau\left(\frac{Z}{A}\right)^{\theta}\right) - L_Z - L_A\right], \tag{17}$$

This equation defines the rate of return to entry. As for the returns to R&D, no-arbitrage requires that it be equal to the interest rate r.

It is useful to emphasize at this stage how the tax on emissions affects the behavior of firms. The expression for the rates of return to innovation, abatement and entry highlight that the transmission channel is the relative contribution of the firm's emission tax burden to its total cost. To see this, observe that using the production function (4) the cost function (6) can be written

$$C = \left[1 + \tau\left(\frac{Z}{A}\right)^{\theta}\right] Z^{-\theta} X = \left[1 + \tau\left(\frac{Z}{A}\right)^{\theta}\right] L_X.$$

This expression captures how the state of technology, measured by the knowledge ratio Z/A, and the flat-rate emission tax τ affect the firm's cost in relation to its scale of activity, measured either by output X or by employment in production operations L_X. As one can see, taxation of emissions raises both the rate of return to abatement (16) and the rate of return to entry (17). While the first property is obvious, the second might require a comment. By raising the firm's marginal cost, taxation of emissions raises prices and thus raises the revenue generated by each unit of labor assigned to production operations.

An important feature of this expression is that one cannot disentangle the effect of the tax from the effect of the knowledge ratio because they jointly determine the position of the cost function. In other words, what matter for the firm is the *tax burden per unit of output (labor)*, not the particular combination of tax rate and tax base that produces it. Accordingly, it is useful to define the factor

$$T \equiv \tau\left(\frac{Z}{A}\right)^{\theta}$$

that captures the contribution of the emission tax to the firm's cost of using a unit of labor in production operations. This notation suggests that two approaches are possible. The government can either set the tax rate τ at some exogenous level with the tax burden T determined by the state of technology; or it can set the tax burden T at some exogenous level with the tax rate τ determined by the state of technology. The analysis below considers both cases. The first because it is simple, the second because it is relevant for welfare.

3.2 General equilibrium

To characterize the general equilibrium of this economy, one needs to impose labor market clearing, which ensures that the resources constraint holds, and assets market clearing, which requires that the rate of return to saving be equal to the rate of return to investment. Labor market clearing requires

$$L = N\left(L_X + L_Z + L_A\right) + L_N, \tag{18}$$

where $L_N = \beta\dot{N}$. The rate of return to saving is given by Eq. (2), the rate of return to investment is given by any one of Eqs. (15)–(17).

Analysis of the system of differential equations describing this general equilibrium is beyond the scope of the paper. To illustrate the interesting results it is sufficient to discuss the steady state of the model. The next section focuses on this task. To ease the exposition the complete derivation of the general equilibrium system is relegated to the appendix.

4 The effects of effluent taxes

As discussed above, in the context of this model the government can either set the tax rate on emissions, τ, or the tax burden per unit of output, T. This section characterizes their effects on the market equilibrium. The next section discusses their welfare implications.

Let $g = g_Z$ denote the steady-state growth rate of the economy. Observe that the Euler equation (2) implies that in a steady state with constant expenditure, all rates of return must equal the discount rate ρ. In steady state the number of firms is constant so that $L_N = 0$. Also, $\dot{L}_Z = \dot{L}_A = 0$. One can then take the ratio of (15) and (16) to obtain

$$\frac{L_A}{L_Z} = \left(\frac{\alpha}{\xi}T\right)^{\frac{1}{1-\gamma}}.$$

This expression, the resources constraint and the fact that $g = \xi L_Z^{\gamma}$ allow one to rewrite (15) as

$$\rho = \xi\theta\gamma\left(\frac{g}{\xi}\right)^{-\frac{1-\gamma}{\gamma}}\left[\frac{L}{N} - \left(\frac{g}{\xi}\right)^{\frac{1}{\gamma}}\left[1 + \left(\frac{\alpha}{\xi}T\right)^{\frac{1}{1-\gamma}}\right]\right]. \tag{19}$$

This equation does *not* impose the free-entry condition while it imposes the resources constraint. Thus, it can be interpreted as the assets market clearing condition that the rate of return to saving be equal to the rate of return to investment characterizing a model with an exogenously fixed number of firms. The right hand side of the equation is downward sloping for two reasons. First, faster growth reduces the returns to R&D because there are diminishing returns to R&D effort. Second, faster growth implies that firms must reduce production, which depresses the returns to R&D since the value of innovation is increasing in the firm's scale of production. (Inside the bracket there is employment in production per firm, which is the difference between employment per firm, L/N, and total R&D employment per firm, $L_Z + L_A$.) Equation (19) also defines a negative relation between the number of firms and growth. The reason is that with a larger number of firms resources are spread more thinly and each firm can undertake less R&D effort.

Consider now the expressions for the returns to innovation, abatement and entry in (15), (16) and (17). The expression derived above for L_A/L_Z and the fact that $g = \xi L_Z^{\gamma}$ allow one to reduce these three equations to

$$\rho = \frac{\left(\frac{g}{\xi}\right)^{\frac{1}{\gamma}} \left[1 + \left(\frac{\alpha}{\xi}T\right)^{\frac{1}{1-\gamma}}\right]}{\frac{1+T}{\xi\theta\gamma(\epsilon-1)}\left(\frac{g}{\xi}\right)^{\frac{1-\gamma}{\gamma}} - \beta}. \tag{20}$$

This equation represents equilibrium of the assets market when free entry holds and the number of firms is endogenous. The rate of return to investment on the right hands side is U-shaped in growth because of the interaction of two forces. First, faster growth lowers the returns to R&D because of diminishing returns to R&D effort. Second, faster growth makes incumbency more costly which means that entrepreneurs demand a higher rate of return in order to enter.[10] Equilibrium exists if the right-hand side of (20) evaluated at its minimum is smaller than ρ. (The Technical Appendix discusses this condition formally.) There are two steady states. The one to the right can be ruled out because it is an unstable equilibrium of the assets market; see Peretto (1999) for details on this argument.

If the government sets the *tax burden per unit of output*, T, the determination of the steady state is straightforward. One uses Eq. (20) to obtain the growth rate; one then evaluates Eq. (19) at the equilibrium growth rate and solves for the number of firms. Inspection of Eqs. (19)–(20), or direct differentiation, reveals that a higher T has *ambiguous* effects on growth and the number of firms. (The detailed characterization of these effects is in the Technical Appendix.)

Besides being novel, this ambiguous growth effect is precisely what makes this policy effective for welfare. To see why, observe that—anticipating results derived below—the optimal policy might call for the government to *raise* the growth rate so that use of an emission tax is predicated on the property that a positive growth effect of the instrument is possible in the first place. As shown below, a policy that sets τ lacks this flexibility since its growth effect is unambiguously negative.

While it has an ambiguous effect on the *rate* of technological change, a policy that sets T has an unambiguous effect on its *direction*. This happens in two dimensions. First, a higher T lowers the rate of cost reduction relative to that of abatement. To see this, use the R&D technologies (7) and (8) to observe that

$$\frac{L_A}{L_Z} = \left(\frac{\alpha}{\xi}T\right)^{\frac{1}{1-\gamma}} \Leftrightarrow \frac{g_A}{g_Z} = \frac{\alpha}{\xi}\left(\frac{\alpha}{\xi}T\right)^{\frac{\gamma}{1-\gamma}}.$$

An interesting implication of this property is that under this policy the steady-state ratio Z/A is not constant. This too turns out to be important for welfare. Second, equation (19) says that there is a negative relation between growth, g, and product variety, N, and that this trade-off worsens with T in that holding constant g a higher T requires a reduction in N. All of this implies that along the transition to the new steady state any change in the rate of cost reduction is associated to a change of opposite sign in the rate of new product development.

10 This intuition explains why the locus exists only for sufficiently high growth rates. According to (15), firm growth is increasing in the firm's scale of production. Thus, the denominator is negative whenever growth is too low because the firm is too small and cannot generate cash flow sufficient to cover the entry cost. In this case the free entry condition cannot hold.

If the government sets the *tax rate on emissions*, τ, one further step is necessary to solve for the equilibrium. Observe that in contrast to the previous case the steady-state knowledge ratio Z/A must be constant. The R&D technologies (7) and (8) then yield that the allocation of R&D resources between cost reduction and abatement is such that

$$\frac{L_A}{L_Z} = \left(\frac{\xi}{\alpha}\right)^{\frac{1}{\gamma}}.$$

It follows that

$$\left(\frac{\alpha}{\xi}T\right)^{\frac{1}{1-\gamma}} = \left(\frac{\xi}{\alpha}\right)^{\frac{1}{\gamma}} \Rightarrow T = \tau\left(\frac{Z}{A}\right)^{\theta} = \left(\frac{\xi}{\alpha}\right)^{\frac{1}{\gamma}}.$$

Therefore, Eqs. (19)–(20) reduce to:

$$\rho = \xi\theta\gamma\left(\frac{g}{\xi}\right)^{-\frac{1-\gamma}{\gamma}}\left[\frac{L}{N} - \left(\frac{g}{\xi}\right)^{\frac{1}{\gamma}}\left[1 + \left(\frac{\xi}{\alpha}\right)^{\frac{1}{\gamma}}\right]\right]; \qquad (19')$$

$$\rho = \frac{\left(\frac{g}{\xi}\right)^{\frac{1}{\gamma}}}{\frac{1}{\xi\theta\gamma(\epsilon-1)}\left(\frac{g}{\xi}\right)^{\frac{1-\gamma}{\gamma}} - \frac{\beta}{1+\left(\frac{\xi}{\alpha}\right)^{\frac{1}{\gamma}}}}. \qquad (20')$$

Observe how in this case growth does not depend smoothly on the tax rate. The reason is that growth is determined by no-arbitrage between rates of return and, as one can see from the expressions above, they all depend on the tax only through the term T which, in steady state, is pinned down by the parameters of the innovation technologies. Thus, a flat-rate emission charge that is set exogenously changes the knowledge ratio but does not affect the allocation of resources across R&D types.[11]

To highlight the effects of these taxes, and provide a benchmark for the welfare analysis that follows, consider the equivalent of Equations (20) and (19) that one obtains when emissions are not taxed:

$$\rho = \xi\theta\gamma\left(\frac{g}{\xi}\right)^{-\frac{1-\gamma}{\gamma}}\left[\frac{L}{N} - \left(\frac{g}{\xi}\right)^{\frac{1}{\gamma}}\right]; \qquad (21)$$

$$\rho = \frac{\left(\frac{g}{\xi}\right)^{\frac{1}{\gamma}}}{\frac{1}{\xi\theta\gamma(\epsilon-1)}\left(\frac{g}{\xi}\right)^{\frac{1-\gamma}{\gamma}} - \beta}. \qquad (22)$$

Existence of this equilibrium requires

$$\frac{\gamma}{1-\gamma}\beta^{\frac{\gamma}{1-\gamma}}\left[\xi\theta\left(\epsilon-1\right)\right]^{\frac{1}{1-\gamma}} < \rho,$$

[11] This is a *steady-state* result. Along the transition, firms adjust their R&D portfolios and this drives the change in the ratio of the two knowledge stocks. The important property, therefore, is that changes in the tax affect the composition of R&D temporarily, while they have permanent effects on the composition of knowledge.

which ensures existence of the equilibria with effluent taxes (see the Technical Appendix for details). Comparing (22) to (20′) one sees that the locus with the emission tax is everywhere below the locus with no taxation. Consequently, the introduction of τ has a negative growth effect. The reason is that firms now split their R&D budgets between cost-reducing and emission-reducing R&D. This implies that abatement comes at the cost of lower productivity growth. Moreover, firms' overall R&D budgets are smaller.[12] Hence, the rate of cost reduction slows down because firms reduce overall R&D effort *and* disperse it over two activities. Moreover, the introduction of this tax has a positive effect on the number of firms, which further dilutes resources and thus R&D efforts across firms. (The Technical Appendix provides a formal proof.) Comparing (22) to (20), in contrast, one sees that the introduction of T does not unambiguously reduce growth since the locus with T can be either above or below the one with no taxation.

The difference between the two policies is due to the fact that the policy that sets T does not require the steady-state knowledge ratio Z/A to be constant and thus exhibits an additional degree of freedom. To see this, recall that the cost function of firms is

$$C = (1 + T)\, L_X.$$

The relation

$$T = \tau \left(\frac{Z}{A}\right)^{\theta} = \left(\frac{\xi}{\alpha}\right)^{\frac{1}{\gamma}}$$

then reveals the mechanism driving the two taxes. If the government sets T, the tax rate τ adjusts to satisfy the first equality while the second is irrelevant because the knowledge ratio Z/A does not need to be constant. More importantly, the cost of using a unit of labor is not just constant but *exogenously set by the government*. In contrast, if the government sets τ, the knowledge ratio adjusts to satisfy the second equality while T is redundant and can be eliminated altogether from the calculations. This is exactly what Eqs. (19′)–(20′) do. Crucially, the cost of using a unit of labor is *endogenous* and in steady state it must adjust in accordance with the ratio of R&D productivities. In other words, the government does not control it.

To gain intuition it is useful to review the main forces at play. The model captures the intricate interplay of market structure, productivity growth and emission reduction that is set in motion by the effluent tax. The tax forces firms to look at emissions as production costs and therefore induces them to spend on emission-reducing R&D. The tax, in other words, makes firms look at emission-reducing R&D as another kind of cost-reducing R&D. But precisely because it forces firms to pay for their emissions, the tax implies that firms face higher marginal costs of production. Holding constant R&D budgets, this means that firms reduce their scale of production. This is important because the fixed, sunk costs studied in this model—firms' R&D expenditures—are *endogenous* in the sense that they are chosen optimally by firms.[13] Since smaller firms can spread fixed costs over a smaller volume of output, they face weaker incentives to undertake R&D. As a result, their overall R&D budgets are smaller. The tension between these two forces—the effect of the tax on firms' marginal costs and on

[12] This can be shown formally by deriving the equivalent of equation (20) where the argument is the firm's total R&D budget $L_Z + L_A$.

[13] Sutton (1991), in the context of static, partial equilibrium models, and Peretto (1996, 1998, 1999), in the context of dynamic, general equilibrium models, have shown that this feature changes dramatically the relation between fundamentals and the endogenous number of firms with respect to models that allow only for exogenous fixed costs. This must be kept in mind when comparing these results with those from many of the models in, e.g., Petrakis et al. (1999).

their incentives to undertake endogenous fixed costs—suggests that the effect of the tax on the number of firms is potentially ambiguous. More specifically, the effect on the number of firms depends on how large is the reduction in the growth rate: if this is very large, it means that firms slash overall R&D budgets dramatically, and this produces an industry where there are more firms.[14] This is indeed what happens with the tax that does not depend on the state of technology. The reason is that the endogenous upward adjustment of the marginal cost of production provides an additional adverse effect on the return to innovation.

5 Welfare

As previously discussed, this paper extends to a general equilibrium model with product differentiation and endogenous growth the argument proposed by Carlton and Loury (1980, 1986) that two variables—emissions per firm and the number of firms—determine the damage that industry inflicts on the environment. This property has important implications for welfare. To see these, consider first the effects of the exogenous-rate tax discussed above on lifetime utility.

5.1 Welfare effects of the exogenous-rate tax

Suppose the economy is in steady state. In symmetric equilibrium, the utility index (1) can be written

$$\log u = \left(\frac{\epsilon}{\epsilon-1} - \psi\mu\right)\log N + (1-\psi\eta)\log L_X + \theta\,(1-\psi\eta)\log Z + \theta\psi\eta\log A.$$

To ensure that each one of the two dimensions of industrial activity, L_X and N, has a positive effect on welfare, let[15]

$$\frac{\epsilon}{\epsilon-1} - \psi\mu > 0 \quad \text{and} \quad 1-\psi\eta > 0.$$

Substituting into lifetime utility and integrating yields[16]

$$\rho U = \left(\frac{\epsilon}{\epsilon-1} - \psi\mu\right)\log N + (1-\psi\eta)\log L_X + \frac{\theta\,(1-\psi\eta)}{\rho}g_Z + \frac{\theta\psi\eta}{\rho}g_A. \quad (23)$$

[14] To see this, recall that this is a monopolistic competition model where, given their R&D budgets, firms face average cost curves that are decreasing everywhere and converging to marginal cost for a very large volume of output. Recall also that the tax does not affect (directly) the demand curve faced by each firm. The steady state of the model requires that each firm's average cost curve be tangent to the firm's demand curve—this is just the standard textbook representation of the long-run equilibrium of a monopolistic competition model. The rise in the firm's marginal cost pushes the average cost curve up, while the fall in the firm's fixed cost pulls it down. As a result, the new average cost curve can be tangent to a demand curve that is to the left or to the right of the initial one. When the tax induces a dramatic reduction in the firm's fixed cost, the overall effect on the average cost curve is to pull it down so that the new tangency point is with a demand curve to the left of the initial one. This demand curve is reached through entry that induces consumers to spread expenditure over a larger number of goods.

[15] These assumptions do not affect in a crucial way the qualitative results discussed below and the basic insight of the paper.

[16] For the purposes of this discussion it is useful to set initial values $Z(0) = A(0) = 1$ since they play no role in the analysis.

This is the level of steady-state lifetime utility that individuals achieve at the market equilibrium. It is the sum of three components: one related to the variety of goods, one related to the quantity of each good, and one related to the growth rate of the quantity of each good adjusted for the growth rate of emissions abatement.

To see what this means in practice, consider the effects of the tax that does not depend on the state of technology. To compare welfare in the regime where emissions are not taxed to welfare in the regime where they are, let a prime denote values for the first regime and a double prime denote values for the second regime. The utility differential between the two regimes is

$$\begin{aligned}\rho\left(U''-U'\right)=&\left(\frac{\epsilon}{\epsilon-1}-\psi\mu\right)\left(\log N''-\log N'\right)+\\&(1-\psi\eta)\left(\log L''_X-\log L'_X\right)+\\&\frac{\theta\,(1-\psi\eta)}{\rho}\left(g''-g'\right)+\\&\frac{\theta\psi\eta}{\rho}g'',\end{aligned}$$

where use has been made of the result, obtained in the previous section, that $g''_Z = g''_A = g''$ while $g'_Z = g'$ and $g'_A = 0$. Also, recall that $N'' > N'$.

The analysis in the previous section has shown that the tax reduces productivity growth because it redistributes the overall R&D budget of each firm across cost-reducing and emission-reducing R&D. Thus, while productivity growth is slower, the economy benefits from a positive rate of emissions abatement. In addition, the welfare effect of the tax depends on the coefficients $\frac{\epsilon}{\epsilon-1}-\psi\mu$ and $1-\psi\eta$ that capture the net effect on welfare of an increase in, respectively, the number of firms and the scale of production of each firm. Although both these coefficients are positive, so that an expansion of either dimension of industrial activity has a positive welfare effect, there is a trade-off between the effect of the tax on the intensive and the extensive margin. Thus, the change in utility brought about by the tax is the sum of four terms: the first is the gain associated with the rise in the number of firms; the second is the loss associated with the smaller firm size; the third is the loss associated with the lower growth rate; the fourth is the gain associated with the positive growth rate of abatement. As a result of these conflicting effects, evaluating the welfare effect of the tax requires taking a stand on the empirical magnitudes of these forces. This is still an underdeveloped area of research but one that, according to the theory discussed here, might deliver important insights.

The positive effects of the flat-rate emission tax that does not depend on the state of technology are interesting but do not answer the question: can such a tax achieve the social optimum? Comparing the two classes of taxes discussed in the previous section, one can conjecture that the answer to this question is: No. The reason is that this tax affects only one component of the factor, T, that drives firms' incentives. To address this issue in detail, the next subsection characterizes the optimal path of this economy.

5.2 The optimal path

The appendix shows that a planner wishing to maximize welfare chooses a combination of number of firms, N, production per firm, L_X, and R&D expenditures, L_Z and L_A, such that:

$$\frac{L}{N}=L_X+L_A+L_Z+\beta\frac{\dot{N}}{N}; \tag{24}$$

$$\rho = -\frac{\dot{L}_X}{L_X} - \frac{L}{\beta N} + \left(\frac{\epsilon}{\epsilon - 1} - \psi\mu\right)\frac{L_X}{\beta(1-\psi\eta)}; \tag{25}$$

$$L_Z = \left[\frac{\xi\gamma\theta}{\rho}L_X\right]^{\frac{1}{1-\gamma}}; \tag{26}$$

$$L_A = \left[\frac{\alpha\gamma\theta}{\rho}\frac{\psi\eta}{1-\psi\eta}L_X\right]^{\frac{1}{1-\gamma}}. \tag{27}$$

The ratio of (27) to (26) yields

$$\frac{L_A}{L_Z} = \left[\frac{\alpha}{\xi}\frac{\psi\eta}{1-\psi\eta}\right]^{\frac{1}{1-\gamma}}. \tag{28}$$

This is the socially optimal R&D ratio.

For the purposes of this discussion, it is useful to use the R&D technologies (7) and (8) to rewrite Eq. (28) as follows:

$$\left(\frac{\xi}{\alpha}\right)^{\frac{1}{\gamma}}\left(\frac{g_A}{g_Z}\right)^{\frac{1-\gamma}{\gamma}} = \frac{\psi\eta}{1-\psi\eta}. \tag{29}$$

Refer now to the steady-state welfare function (23). On the left hand side of (29) there is the slope of the economy's resources constraint in (g_A, g_Z) space; on the right hand side there is the slope of the welfare curve. The condition therefore states that the optimal ratio of growth rates, or equivalently the optimal R&D ratio, is obtained by equalizing the marginal rate of transformation of the growth rates of productivity and abatement to their marginal rate of substitution.

It should be clear by looking at Eq. (29) that the growth rates of cost-reducing and emission-reducing knowledge are *not* equal along the optimal path. An important feature of the optimal path, therefore, is that the knowledge ratio Z/A is not constant.

To fully characterize the steady-state optimal allocation of resources, one proceeds as in the previous section and obtains:

$$\rho = \xi\theta\gamma\left(\frac{g}{\xi}\right)^{-\frac{1-\gamma}{\gamma}}\left[\frac{L}{N} - \left(\frac{g}{\xi}\right)^{\frac{1}{\gamma}}\left[1 + \left(\frac{\alpha}{\xi}\frac{\psi\eta}{1-\psi\eta}\right)^{\frac{1}{1-\gamma}}\right]\right]; \tag{30}$$

$$\rho = \frac{\left(\frac{g}{\xi}\right)^{\frac{1}{\gamma}}\left[1 + \left(\frac{\alpha}{\xi}\frac{\psi\eta}{1-\psi\eta}\right)^{\frac{1}{1-\gamma}}\right]}{\frac{\frac{1}{\epsilon-1}-\psi(\mu-\eta)}{1-\psi\eta}\frac{1}{\xi\theta\gamma}\left(\frac{g}{\xi}\right)^{\frac{1-\gamma}{\gamma}} - \beta}. \tag{31}$$

The first equation determines the optimal relation between growth and the number of firms/products; the second determines the optimal growth rate. More importantly, the right hand sides of these equations define the social returns to investment in the case of, respectively, a fixed number of firms and an endogenous number of firms.

Figure 1 compares the market equilibrium with no government intervention (point M) to the social optimum (point S) in the case in which the number of firms is fixed. This exercise highlights two distortions. The first is obvious: with no taxation of emissions each firm chooses a price and R&D policy that does not take into account its own emissions. As

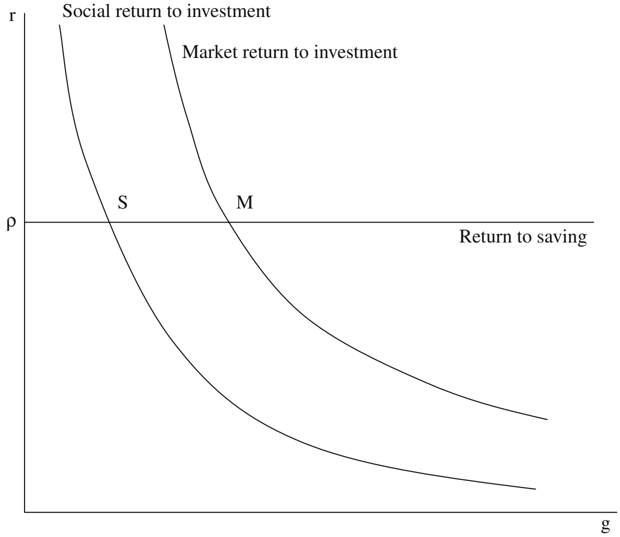

Fig. 1 Distortions with an exogenous number of firms

a consequence, the market equilibrium features $g_A = 0$ while the optimal growth rate of emission-reducing knowledge is positive. This distortion is captured by the term

$$1 + \left(\frac{\alpha}{\xi}\frac{\psi\eta}{1-\psi\eta}\right)^{\frac{1}{1-\gamma}} > 1$$

that appears in (30) and is absent from (21) so that, given the number of firms, the private return to R&D is higher than the social return. Accordingly, in (g, r) space the social optimum locus is everywhere below the market equilibrium locus capturing the fact that growth is too high because firms concentrate all R&D on cost reduction.[17] Interestingly, when one accounts for free entry the effect of this distortion is reversed. To see this, observe that the factor capturing the distortion appears at the numerator of (31) so that the social return to investment is higher than the private return.

Figure 2 illustrates the determination of the market equilibrium (point M) and the social optimum (point S) in the case in which there is free entry and the number of firms is endogenous. The top panel illustrates the determination of the growth rate. The bottom panel uses equations (21) and (30) in (g, N) space to illustrate the determination of the number of firms given the growth rate. The second distortion, therefore, is due to the fact that firms' entry decisions ignore the pollution externality associated with the number of firms. This is captured by the term

$$\frac{\frac{1}{\epsilon-1} - \psi(\mu-\eta)}{1-\psi\eta}$$

that appears in (31) and is replaced by $\frac{1}{\epsilon-1}$ in (22). The effect of this distortion depends on the role of product variety in people's preferences. In particular, the unregulated number of firms can be smaller or larger than the optimal one.

[17] Incidentally, one should notice that this locus is what one should focus on if one wishes to compare the predictions of this model to those of papers based on first-generation growth models that do not have an extensive margin.

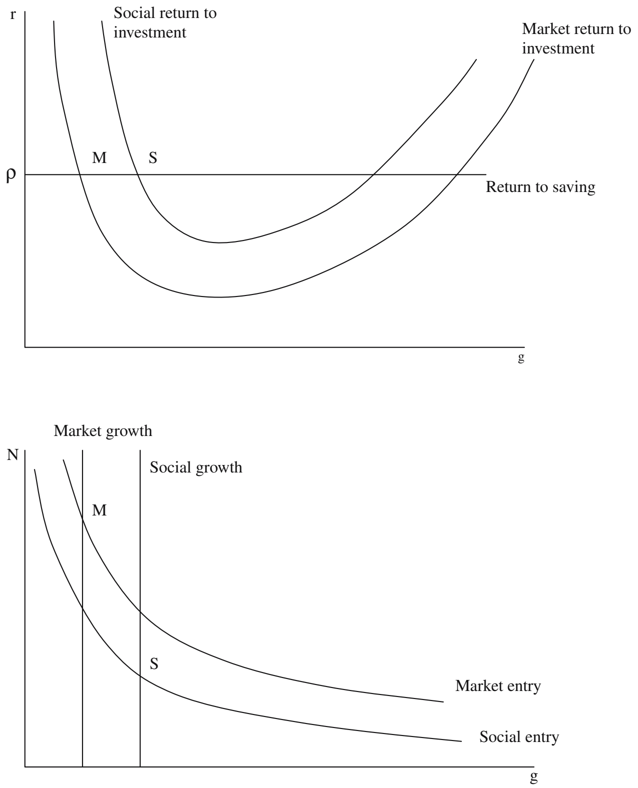

Fig. 2 Distortions with an endogenous number of firms

It is useful to isolate the role of the second distortion ignoring for a moment the first one. If the first distortion did not exist, the right hand side of Eq. (31) would be greater than the right hand side of Eq. (22) if

$$\frac{\frac{1}{\epsilon-1}-\psi\left(\mu-\eta\right)}{1-\psi\eta}<\frac{1}{\epsilon-1}\Longrightarrow\frac{\mu}{\eta}=\frac{1}{\delta}>\frac{\epsilon}{\epsilon-1}.$$

The left hand side of the second inequality is the ratio of the elasticities of pollution with respect to the number of firms and emissions per firm. The right hand side is the ratio of the elasticities of utility with respect to the variety of consumption goods and the quantity of each good (this elasticity is equal to one). The inequality then says that when product variety contributes to people's utility more through the pollution index than through the consumption index, the private return to investment is too low and the market grows too little. Since the market grows too little, there is excessive entry. This follows immediately from the fact that absent the first distortion equations (21) and (30) are identical.

If one takes into account both distortions, a sufficient condition for Eq. (31) to be above Eq. (22) is

$$\frac{\mu}{\eta}=\frac{1}{\delta}\geq\frac{\epsilon}{\epsilon-1}.$$

Figure 2 illustrates this case: In addition to not undertaking emission-reducing R&D, the market equilibrium features faster than optimal growth and larger than optimal product variety. Notice how in this case the first distortion amplifies the effects of the second since it works in the same direction by shifting Eq. (31) up in (g, r) space and Eq. (30) down in (g, N) space. It is instructive to observe that for $\eta = \mu$ (i.e., $\delta = 1$) the environmental damage function reduces to $D = (EN)^{\eta}$, which says that pollution depends on total emissions while the number of polluting firms plays no independent role. In this case, which is the typical one of homogeneous pollutants considered in the literature, the inequality above is *not* satisfied and the two distortions work in opposite directions. Hence, Eq. (31) *could be* below Eq. (22). In this case, growth at the market equilibrium is too high while the number of firms can be either below or above the socially optimal level.

The practical validity of these considerations rests on empirical evidence on the independent role of product variety in determining environmental damage. The analysis of this model, therefore, suggests yet again that this understudied area is likely to deliver important insights and guidelines for policy makers. To flesh out some of these insights, the next subsection examines what policies achieve the social optimum.

5.3 The optimal policy mix

As discussed above, a steady state for the market equilibrium with the flat-rate emission tax requires a constant knowledge ratio. The property that the optimal knowledge ratio is not constant then reveals that the flat-rate tax *cannot* deliver the social optimum precisely because it holds this ratio constant. So, what form should taxation of emissions take?

The social optimum requires the R&D ratio given by Eq. (28). No-arbitrage between the private returns to R&D in Eqs. (15) and (16) yields

$$\frac{\xi}{\alpha}\left(\frac{L_A}{L_Z}\right)^{1-\gamma} = \tau\left(\frac{Z}{A}\right)^{\theta} \equiv T. \tag{32}$$

It is straightforward to check that the left hand side of this equation is the marginal rate of transformation that enters condition (29) while the right hand side is the contribution of emissions to the firm's labor cost. The optimal policy, therefore, is to set the *tax burden per unit of output* equal to the marginal rate of substitution between the growth rates of productivity and abatement. Substituting (32) into (28) yields the optimal effluent tax rate

$$T \equiv \tau\left(\frac{Z}{A}\right)^{\theta} = \frac{\psi\eta}{1-\psi\eta} \Rightarrow \tau = \left(\frac{Z}{A}\right)^{-\theta}\frac{\psi\eta}{1-\psi\eta}, \tag{33}$$

which is *decreasing* in the knowledge ratio Z/A because when firms are relatively inefficient at abating emissions the tax base is large and a small tax rate is sufficient to induce them to spend relatively more on emission-reducing R&D. In other words, the optimal tax rate is decreasing in the knowledge ratio because the larger the knowledge ratio the larger the pain inflicted by the tax while the pain that the tax needs to inflict is constant. The other side of this coin is, of course, that the larger the knowledge ratio—and thus the larger the pain inflicted by the tax—the larger the firm's gain associated to emission-reducing R&D relative to cost-reducing R&D. This is how the optimal tax rate provides the appropriate incentives to firms. According to (15) and (16), the rate of return to cost-reducing R&D does not depend on the tax while the rate of return to emission-reducing R&D is proportional to the relative contribution of emissions to cost. The optimal tax rate reflects how people value the environment relative to consumption goods. As a result, firms implement the optimal path because the ratio of the rates of return to R&D now reflects their social value.

To complete the analysis of the optimal policy mix, let σ be an entry subsidy/tax so that the entry cost becomes $\beta(1-\sigma)$. Comparing the versions of (20) and (19) so modified to (31) and (30), one can see that the optimal effluent tax rate (33) makes (19) identical to (30). In order to have (20) identical to (31), one then needs

$$\sigma = \frac{\psi(\eta - \mu)}{1 - \psi\eta} \frac{1}{\beta\xi\theta\gamma} \left(\frac{g}{\xi}\right)^{\frac{1-\gamma}{\gamma}},$$

where g is the optimal growth rate determined by (31). If pollution is more sensitive to the number of firms than to emissions per firm, if $\eta < \mu$, the optimal policy mix requires a tax on entry. In contrast, if $\eta > \mu$ the optimal policy requires a subsidy to entry. Notice, moreover, that if $\eta = \mu$ the environmental damage function reduces to $D = (EN)^{\eta}$. This means that the utility loss due to pollution is $\psi\eta \log EN$. Thus, the environmental damage function in units of utility is of the type EN and the optimal policy requires $\sigma = 0$. As Carlton and Loury (1980, 1986) explain, only in this special case a Pigouvian tax on output is sufficient to achieve optimality because the damage from pollution depends on total emissions while the number of polluting firms plays no independent role.

6 Conclusion

This paper has studied the effects of flat-rate effluent taxes on the steady-state growth path a model economy where market structure is endogenous. Because this environment is different from the one examined in standard models of endogenous growth, the analysis yields new insights on how policy and regulations affect the economy. Specifically, the analysis considered the effects on incumbent firms' allocation of resources to cost-reducing and emission-reducing R&D, and on entrepreneurs' decisions to develop new goods that expand product variety and enter the market (i.e., set up new firms). Consequently, the paper provides an extension of the framework of modern endogenous growth theory to the analysis of the rate *and* direction of technological change.

The distinction between R&D undertaken by incumbent firms and R&D undertaken by entrants provides a conceptual link between the analysis undertaken in this paper and the extensive literature on market structure and innovation and on market structure and environmental regulation. One interesting aspect of this extension is that it allows investigation of the implications of the argument proposed by Carlton and Loury (1980, 1986) that a Pigouvian tax on emissions is not sufficient to achieve social efficiency when the number of firms is endogenous. In other words, policy makers need to worry about two interdependent margins: the intensive margin where pollution per firm is determined, and the extensive margin, where the number of polluting firms is determined. The current debate on the role of emission taxes ignores the second margin, implicitly assuming that product differentiation is not a determinant of environmental damages because all firms produce identical pollutants in identical relation to their output.

The paper's positive analysis has shown that the opportunity costs of policy interventions do not disappear in a fully specified endogenous growth model that allows firms to respond by investing in environment-friendly technological change. The reason is that pollution abatement diverts resources from traditional cost reduction and product creation. Thus, although it is interesting to learn that there is a class of effluent taxes that can *raise* growth their downside is that they do so by reducing product variety. The normative analysis has shown that the optimal tax rate sets the tax burden per unit of output equal to the marginal rate of substitution between the growth rate of consumption and abatement—a constant parameter coming from preferences. Moreover, a tax/subsidy on entry that affects the number of firms is also needed. If product variety contributes to love-of-variety in consumption more than to pollution, a subsidy is needed; if it contributes more to pollution than to love-of -variety in consumption, a tax is needed.

The practical validity of some of these normative considerations rests on empirical evidence on the independent role of pollutant variety in determining environmental damage. The analysis of this model, therefore, suggests that this understudied area is likely to deliver important insights and guidelines for policy makers. Regardless of this detail, however, the general message of this paper is that because the debate on environmental policy and regulations is now focusing on the role of technological change is important to understand better how the interaction between market structure and R&D incentives results in different patterns of economic growth and the related environmental impact.

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7 Appendix

7.1 The general equilibrium system for the market economy

The demand schedule (3), the price strategy (10), and symmetry allow one to rewrite (4) as

$$L_X = \frac{Y(\epsilon - 1)}{N\epsilon\left(1 + \tau\left(\frac{Z}{A}\right)^{\theta}\right)}. \tag{34}$$

This is the firm's demand of labor to employ in production. Substitute (34) into (15), (16), (17) and (18) to eliminate L_X. Next, substitute (17) into (2), (15) and (16) to eliminate r. This yields the following four differential equations:

$$\frac{1}{\beta}\left[\frac{Y}{N\epsilon} - L_Z - L_A\right] = (1-\gamma)\frac{\dot{L}_Z}{L_Z} + \xi\theta\gamma\frac{Y(\epsilon - 1)}{N\epsilon\left(1 + \tau\left(\frac{Z}{A}\right)^{\theta}\right)}L_Z^{\gamma-1}; \tag{35}$$

$$\frac{1}{\beta}\left[\frac{Y}{N\epsilon} - L_Z - L_A\right] = (1-\gamma)\frac{\dot{L}_A}{L_A} + \alpha\theta\gamma\frac{Y(\epsilon - 1)\tau\left(\frac{Z}{A}\right)^{\theta}}{N\epsilon\left(1 + \tau\left(\frac{Z}{A}\right)^{\theta}\right)}L_A^{\gamma-1}; \tag{36}$$

$$\rho + \frac{\dot{Y}}{Y} = \frac{1}{\beta}\left[\frac{Y}{N\epsilon} - L_Z - L_A\right]; \tag{37}$$

$$L = N\left[\frac{Y(\epsilon - 1)}{N\epsilon\left(1 + \tau\left(\frac{Z}{A}\right)^{\theta}\right)} + L_Z + L_A\right] + \beta\dot{N}. \tag{38}$$

In addition, there is the definition

$$T = \tau\left(\frac{Z}{A}\right)^{\theta}.$$

Under the policy that sets T this reduces to a system of four differential equations in four variables. In contrast, under the policy that sets τ there are four equations in five variables, and one needs an additional equation to close the system. To find it, notice that (7) and (8) yield

$$\frac{\dot{\left(\frac{Z}{A}\right)}}{\left(\frac{Z}{A}\right)} = \xi L_Z^{\gamma} - \alpha L_A^{\gamma}. \tag{39}$$

The steady state system is readily obtained upon use of (34) and the fact that rates of returns have to equal the discount rate ρ. Thus:

$$\rho = \xi\theta\gamma L_X L_Z^{\gamma-1}; \tag{35}$$

$$\rho = \alpha\theta\gamma L_X L_A^{\gamma-1}\tau\left(\frac{Z}{A}\right)^{\theta}; \tag{36}$$

$$\rho = \frac{1}{\beta}\left[\frac{L_X}{\epsilon-1}\left(1+\tau\left(\frac{Z}{A}\right)^{\theta}\right) - L_Z - L_A\right]; \tag{37}$$

$$\frac{L}{N} = L_X + L_Z + L_A; \tag{38}$$

$$\frac{L_A}{L_Z} = \left(\frac{\xi}{\alpha}\right)^{\frac{1}{\gamma}}. \tag{39}$$

The last equation does not apply to the policy that sets T.

7.2 The social planning problem

The Current Value Hamiltonian for the social planner is

$$\begin{aligned} CVH = &\left(\frac{\epsilon}{\epsilon-1} - \psi\mu\right)\log N + (1-\psi\eta)\log L_X + \\ &\theta(1-\psi\eta)\log Z + \theta\psi\eta\log A + \\ &\frac{q_N}{\beta}\left[L - N\left(L_X + L_A + L_Z\right)\right] + q_Z\xi Z L_Z^{\gamma} + q_A\alpha A L_A^{\gamma}, \end{aligned}$$

where q_Z and q_A are, respectively, the social shadow values of cost-reducing and emission-reducing knowledge. The knowledge stocks, Z and A, and the number of firms, N, are the state variables; R&D investments, L_Z and L_A, and the firm's scale of production, L_X, are the control variables. The first order conditions are:

$$\frac{1-\psi\eta}{L_X} = \frac{Nq_N}{\beta}; \tag{40}$$

$$L_Z = \left[q_Z\xi Z\gamma\frac{\beta}{Nq_N}\right]^{\frac{1}{1-\gamma}}; \tag{41}$$

$$L_A = \left[q_A\alpha A\gamma\frac{\beta}{Nq_N}\right]^{\frac{1}{1-\gamma}}; \tag{42}$$

$$\rho = \frac{\dot{q}_Z}{q_Z} + \xi L_Z^{\gamma} + \frac{\theta(1-\psi\eta)}{Zq_Z}; \tag{43}$$

$$\rho = \frac{\dot{q}_A}{q_A} + \alpha L_A^{\gamma} + \frac{\theta\psi\eta}{Aq_A}; \tag{44}$$

$$\rho = \frac{\dot{q}_N}{q_N} - \frac{L_X + L_A + L_Z}{\beta} + \left(\frac{\epsilon}{\epsilon - 1} - \psi\mu\right)\frac{1}{Nq_N}. \tag{45}$$

In addition, one has the constraints on the state variables, (7) and (8), the resources constraint (24), and the transversality conditions.

One can reduce this set of equations to a system in the state variable N and the control variable L_X. First, realize that (43) and (44) imply $Zq_Z = \frac{\theta(1-\psi\eta)}{\rho}$ and $Aq_A = \frac{\theta\psi\eta}{\rho}$ at all times. These expressions and (40) yield the policy functions:

$$L_Z = \left[\frac{\xi\gamma\theta}{\rho}L_X\right]^{\frac{1}{1-\gamma}};$$

$$L_A = \left[\frac{\alpha\gamma\theta}{\rho}\frac{\psi\eta}{1-\psi\eta}L_X\right]^{\frac{1}{1-\gamma}}.$$

It then follows that

$$\frac{L_A}{L_Z} = \left[\frac{\alpha}{\xi}\frac{\psi\eta}{1-\psi\eta}\right]^{\frac{1}{1-\gamma}}.$$

Now use (45) and (40) to obtain

$$\rho = -\frac{\dot{L}_X}{L_X} - \frac{L}{\beta N} + \left(\frac{\epsilon}{\epsilon-1} - \psi\mu\right)\frac{L_X}{\beta(1-\psi\eta)}.$$

Next, substitute the policy functions for L_Z and L_A into (24) to obtain

$$\frac{L}{N} = L_X + \left[\frac{\xi\gamma\theta}{\rho}L_X\right]^{\frac{1}{1-\gamma}}\left[1 + \left(\frac{\alpha}{\xi}\frac{\psi\eta}{1-\psi\eta}\right)^{\frac{1}{1-\gamma}}\right] + \beta\frac{\dot{N}}{N}.$$

This system describes the optimal path of the economy in (N, L_X) space. The analysis of the phase diagram is available on request.

References

Aghion P, Howitt P (1998) Endogenous growth theory. MIT University Press, Cambridge, MA

Aghion P, Howitt P (2005) Growth with quality-improving innovations: an integrated framework. In: Aghion P, Durlauf S (eds) Handbook of economic growth. North Holland, Amsterdam

Baldwin W, Scott J (1987) Market structure and technological change. Harwood Academy, New York

Bolin B (2003) Geophysical and geochemical aspects of environmental regulation. In: Maler K, Vincent J (eds) Handbook of environmental economics.. North Holland, Amsterdam

Brock W, Taylor M (2005) Economic growth and the environment: a review of theory and the empirics. In: Aghion P, Durlauf S (eds) Handbook of economic growth. North Holland, Amsterdam

Carlton D, Loury G (1980) The limitation of Pigouvian taxes as a long-run remedy for externalities. Q J Econ 95:559–566

Carlton D, Loury G (1986) The limitation of Pigouvian taxes as a long-run remedy for externalities: an extension of results. Q J Econ 101:631–634

Coehen W, Levin R (1989) Empirical studies of market structure and innovation. In: Schmalensee R, Willig R (eds) Handbook of industrial organization. North Holland, Amsterdam
Goulder L, Schneider S (1999) Induced technical change and the attractiveness of CO_2 emissions abatement policies. Resour Energy Econ 21:211–253
Grimaud A (1999) Pollution permits and sustainable development in a Schumpeterian growth model. J Environ Econ Manage 38:249–266
Helfand G, Berck P, Maull T (2003) The theory of pollution policy. In: Maler K, Vincent J (eds) Handbook of environmental economics. North Holland, Amsterdam
Jaffe A, Peterson S, Portney P, Stavins R (1995) Environmental regulation and the competitiveness of US manufacturing. J Econ Lit 33:132–163
Jaffe A, Newell R, Stavins R (2003) Technological change and the environment. In: Maler K, Vincent J (eds) Handbook of environmental economics. North-Holland, Amsterdam
Joskow P, Schmalensee R, Bailey E (1998) The market for sulfur dioxide emissions. Am Econ Rev 88:669–685
Lange A, Requate T (1999) Emission taxes for price setting firms: differentiated commodities and monopolistic competition. In: Petrakis E, Sartzetakis E, Xepapadeas A (eds) Environmental regulation and market power. Edward Elgar, Celthenham
Palmer K, Oates W, Portney P (1995) Tightening environmental standards: the benefit-cost or the No-cost paradigm?. J Econ Perspect 9:119–132
Peretto P (1996) Sunk costs, market structure and growth. Int Econ Rev 37:895–923
Peretto P (1998) Technological change and population growth. J Econ Growth 3:283–311
Peretto P (1999) Cost reduction, entry, and the interdependence of market structure and economic growth. J Monetary Econ 43:173–195
Petrakis E, Sartzetakis E, Xepapadeas A (eds) (1999) Environmental regulation and market power. Edward Elgar, Celthenham
Porter M (1991) America's green strategy. Scientific Am 264:168
Porter M, van der Linde C (1995) Toward a new conception of the environment-competitiveness relationship. J Econ Perspect 9:97–118
Romer P (1990) Endogenous technological change. J Polit Econ 98:S71–S102
Schmalensee R, Joskow P, Ellerman D, Montero J, Bailey E (1998) An interim evaluation of sulfur dioxide emission trading. J Econ Perspect 12:53–68
Smith VK, Walsh R (2000) Do painless environmental policies exist?. J Risk Uncertainty 21:73–94
Smulders S, van de Klundert T (1995) Imperfect competition, concentration and growth with firm-specific R&D. Eur Econ Rev 39:139–160
Smulders S (2000) Economic growth and environmental quality. In: Folmer H, Gabel LPrinciples of Environmental Economics. Edward Elgar, Celthenham
Spulber D (1985) Efficient regulation and long-run optimality. J Environ Econ Manage 12:103–116
Stavins R (1998) What can we learn from the grand policy experiments? Lessons from SO_2 allowances trading. J Econ Perspect 12:69–88
Sutton J (1991) Sunk costs and market structure. MIT University Press, Cambridge MA
Thompson P (2001) The microeconomics of an R&D-based model of endogenous growth. J Econ Growth 6:263–283